Systems Dynamics

Course ID: 267MI – Fall 2022

Thomas Parisini
Gianfranco Fenu

University of Trieste
Department of Engineering and Architecture

267MI –Fall 2022

Lecture 4
Model Identification from Data

Lecture 4: Table of Contents

System Identification: an Introduction

Modelling
Identification
Prediction
  & filtering

→

Disciplines providing tools
to **estimate** variables and
unknown parameters
and to **design models** of
natural and artificial systems
using **experimental data**.

*Constructing models for a slice of reality and studying their properties is really what science is about. The **models** – "the hypotheses", "the laws of nature", "the paradigms" – can be of a more or less formal character, but they all have the **fundamental property** that they try **to link observations to some pattern**.*

L. Ljung, T. Glad, "Modeling of Dynamic Systems", Prentice Hall, 1994

System Identification: an Introduction

"Transparent Box" vs. "Black Box" Modeling Approach
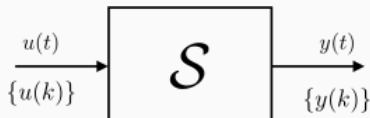
"Transparent Box" Modeling Approach

So far, approach undertaken to devise dynamical models:

Inputs ("causes")   Outputs ("effects")

$$u(t) \in \mathbb{R}^m$$ $$y(t) \in \mathbb{R}^p$$
$$\{u(k) \in \mathbb{R}^m\}$$ $$\{y(k) \in \mathbb{R}^p\}$$

$$\xrightarrow[\{u(k)\}]{u(t)} \quad \boxed{\mathcal{S}} \quad \xrightarrow[\{y(k)\}]{y(t)}$$

Definition of the "system" entity to be analysed $\implies$ Physical laws, a priori knowledge, heuristic considerations, statistical evidence, etc. $\implies$ *Mathematical models*: algebraic and/or differential and/or difference equations

A Different (Data-Based) Approach
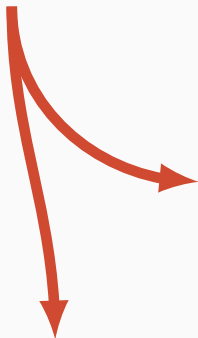

$\{u(k)\}$ → [black box] → $\{y(k)\}$

Experimental data, sensor measurements, historical data, etc.

"Black-box" modeling approach
Input, output and disturbance variables are characterized in terms of numerical sequences. These are the data to be used to determine the dynamical model.

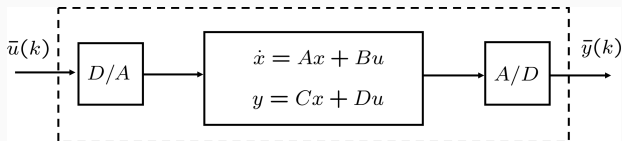# "Black-Box" Modeling Approach (Identification)

Given: $\{y(k)\}$, $\{u(k)\}$

Discrete-time model:

- continuous-time system and data obtained by sampling
- discrete-time system and data inherently discrete-time

Finite-difference equations

Continuous-Time System and Data Obtained by Sampling

Linear, time-invariant case:

$$u(t) = \bar{u}(k)$$
$$t_k \leq t < t_{k+1}$$

Recall the **_step-invariant transform_**

$$\bar{y}(k) = y(t_k)$$

$$\begin{cases} \bar{x}[(k+1)] = \bar{A}\bar{x}(k) + \bar{B}\bar{u}(k) \\ \\ \bar{y}(k) = \bar{C}\bar{x}(k) + \bar{D}\bar{u}(k) \end{cases}$$
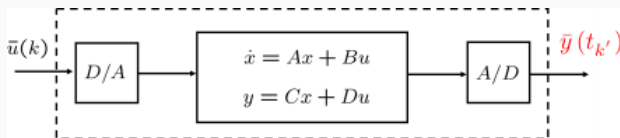
Letting $\quad t_{k+1} - t_k = \Delta$

$$\bar{A} = e^{A\Delta} \qquad \bar{B} = \int_0^{\Delta} e^{Ar} B \, dr \qquad \bar{C} = C \qquad \bar{D} = D$$

$u(t) = \bar{u}(k)$ What if the output is sampled at $\bar{y}(k) = y(t_{k'})$

$t_k \leq t < t_{k+1}$ $t_{k'} \neq t_k$ with $t_k \leq t_{k'} < t_{k+1}$?

- Let's recall the expression

$$y(t) = C\, e^{A\,(t-t_0)}\, x\,(t_0) + C \int_{t_0}^{t} e^{A(t-\tau)} B u(\tau)\, d\tau + D u(t)$$

for the movement of the output of a continuous-time LTI system (from "*Fundamentals of Automatic Control*").

Continuous-Time System and Data Obtained by Sampling (cont.)

- Let's consider t_k as initial time instant (i.e $t_0 = t_k$), the instant $t_{k'}$ as final time instant and let's assume $t_{k'} - t_k = \alpha$. Recall also the stair-wise behavior of the input signal:
  $$u(t) = \bar{u}(k) \,, \ t_k \leq t < t_{k+1} \,.$$

  $$y\left(t_{k'}\right) = C \, e^{A\alpha} \, x\left(t_k\right) + C \left(\int_{t_k}^{t_{k'}} e^{A(t_{k'} - \tau)} B \, d\tau \right) u(t_k) + Du(t_k)$$

- Substitute $r = t_{k'} - \tau$ into the integral term and rewrite the expression

  $$y\left(t_{k'}\right) = C \, e^{A\alpha} \, x\left(t_k\right) + C \left(\int_0^\alpha e^{Ar} \, dr \right) Bu(t_k) + Du(t_k)$$

- Let's compare with the discrete-time output expression

  $$\bar{y}(k) = \bar{C}\bar{x}(k) + \bar{D}\bar{u}(k)$$

- If $t_{k'} \neq t_k$ then

$$\bar{C} = C\, e^{A\alpha} \qquad \bar{D} = C \left(\int_0^\alpha e^{Ar}\, dr \right) B + D \qquad t_{k'} - t_k = \alpha\, (< \Delta)$$

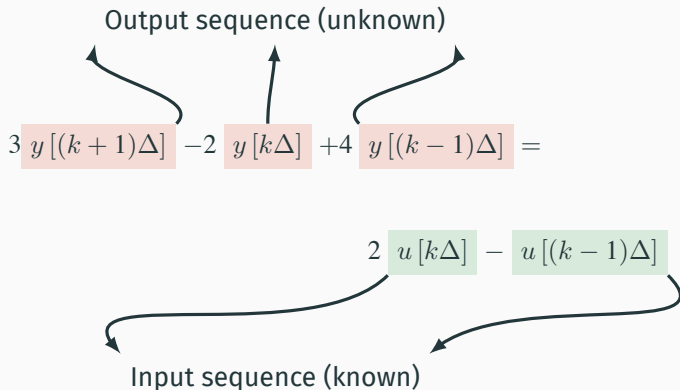- When $t_{k'} = t_k$ obviously

$$\bar{C} = C \qquad \bar{D} = D$$

- In both the cases the following expressions hold (remember the *step-invariant transform*)

$$\bar{A} = e^{A\Delta} \qquad \bar{B} = \int_0^\Delta e^{Ar} B\, dr \qquad \Delta = t_{k+1} - t_k\ \forall k$$

As usual, let's assume Δ as the sampling time.

Output sequence (unknown)

$$3 \; y\left[(k+1)\Delta\right] - 2 \; y\left[k\Delta\right] + 4 \; y\left[(k-1)\Delta\right] =$$

$$2 \; u\left[k\Delta\right] - u\left[(k-1)\Delta\right]$$

Input sequence (known)

- With sampling-time Δ enhanced:

$$3y\left[(k+1)\Delta\right] - 2y\left[k\Delta\right] + 4y\left[(k-1)\Delta\right] =$$
$$2u\left[k\Delta\right] - u\left[(k-1)\Delta\right]$$

- Compact:

$$3y_{k+1} - 2y_k + 4y_{k-1} = 2u_k - u_{k-1}$$

General Expression

- Typical framework: linear finite-difference equations with constant coefficients.

- General expression takes on the form:

$$a_n y_{k+n} + a_{n-1} y_{k+n-1} + \cdots + a_0 y_k =$$
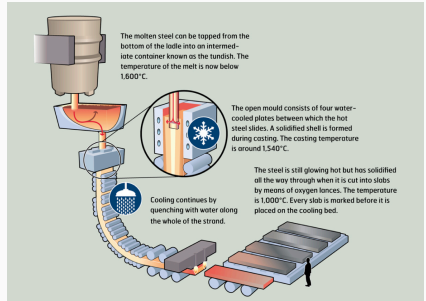$$b_m u_{k+m} + b_{m-1} u_{k+m-1} + \cdots + b_0 u_k$$

with given initial conditions
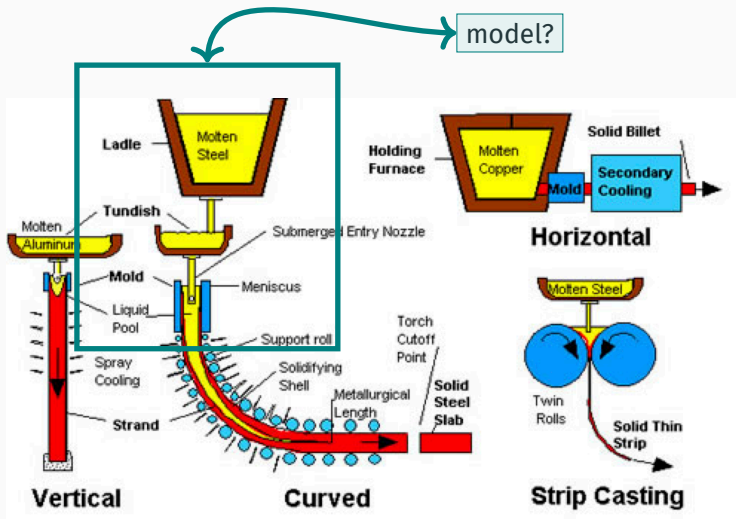
$$\{y_k : k = -n, \ -(n-1), \ \ldots, \ 0\}$$
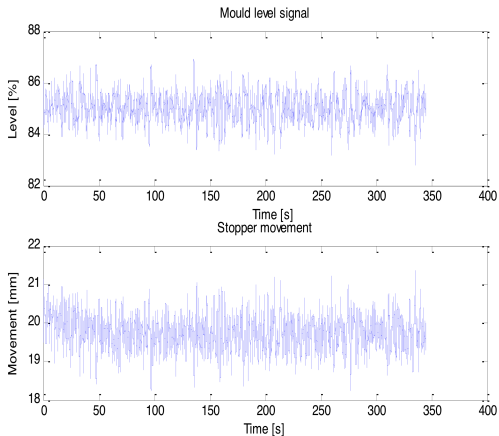
and known input sequence u_k.

An Example: a Real Application

The molten steel can be tapped from the bottom of the ladle into an intermediate container known as the tundish. The temperature of the melt is now below 1,600°C.

The open mould consists of four water-cooled plates between which the hot steel slides. A solidified shell is formed during casting. The casting temperature is around 1,540°C.

The steel is still glowing hot but has solidified all the way through when it is cut into slabs by means of oxygen lances. The temperature is 1,000°C. Every slab is marked before it is placed on the cooling bed.

Cooling continues by quenching with water along the whole of the strand.

model?

"Black-box" modeling approach

$\{u(k)\}$ → [ ] → $\{y(k)\}$

267MI –Fall 2022

Lecture 4
Model Identification from Data

END