Simmetries of QCD Lagrangian

Let's consider the QCD Lag. with generated masses

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \sum_{I=u,d,c,s,t,b} \left(\bar{q}^I i \not{D} q^I - m^I \bar{q}^I q^I \right)$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_3 f^{abc} A^b_\mu A^c_\nu$$

$SU(3)_c$ generators

$q^I \in \underline{3}$ of $SU(3)_c$ $\qquad D_\mu q^I = \left(\partial_\mu + i g_3 A^a_\mu \frac{\lambda^a}{2} \right) q^I$

The masses for c, b, t are larger than Λ_{QCD}, so they decouple before g_s comes to STRONG COUPLING REGIME.
$\Rightarrow$ Low en. physics governed by u, d, s, whose masses are below Λ_{QCD}, but to (see lattice QCD).

$m_u, m_d, m_s \ll \Lambda_{QCD}$. In the approx that they are all massless, i.e. $m_u \sim m_d \sim m_s \simeq 0$, $\mathcal{L}_{QCD}$ is INVARIANT under $U(3)_L \times U(3)_R$ FLAVOUR sym. (i.e. global sym.)

$\rightarrow$ we can rotate L-handed and R-handed fermions independently, and we would expect classically a conservation of j^μ_L and j^μ_R, with currents in the adjoint of $U(N_f)$ $(N_f = 3)$

$$G_{Flav} = SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A$$

$j^\mu_5 = \bar{\psi} \gamma^\mu \gamma^5 \psi$

$j^\mu = \bar{\psi} \gamma^\mu \psi$

$$j^{\alpha\mu}_{L,R} = \bar{\psi} \gamma^\mu \frac{1 \mp \gamma_5}{2} t^\alpha \psi = \frac{1}{2}\left(j^{\alpha\mu} \mp j^{\alpha\mu}_5 \right)$$

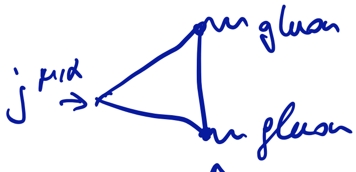
t^α: generators of $SU(N_f)$

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

Are these symmetries good sym. at the quantum level?
I.e. which ones are anomalous?

- $U(1)_V$ is coupled to the PHOTON (i.e. to a gauge field)
  $\Rightarrow$ it cannot be anomalous.

- $SU(N_f)_V$ or $SU(N_f)_A$; their anomaly may have a contrib. proportional to

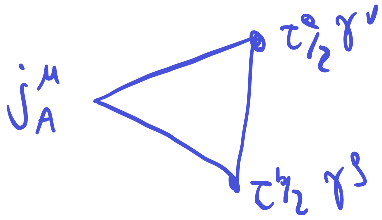


the same if we couple to other gauge fields of SM

$$\mathrm{Tr}\left[t^\alpha\left\{\frac{\tau^a}{2},\frac{\tau^b}{2}\right\}\right](\dots) = 0$$

$\mathrm{tr}(A\otimes B) = (\mathrm{tr}A)(\mathrm{tr}B)$

- $U(1)_A$ however IS ANOMALOUS!
  It is exactly the SINGLET ANOMALY discussed before.



$\leftarrow$ ANOMALY $\propto F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$

$\uparrow$ gluons

Spontaneous breakdown of an APPROXIMATE sym lead to low-mass spinless particles, called PSEUDO-GOLDSTONE BOSONS.

In QCD there is a non-zero quark condensate (lattice)

$$\langle \bar{q}_L^I q_R^J \rangle = -c\,\delta^{IJ} \qquad c \sim \Lambda_{QCD}^3$$

$\Rightarrow$ the VACUUM is NOT invariant and the sym $SU(N_f)_L \times SU(N_f)_R$ is spont. broken. to $SU(N_f)_V$

$\rightsquigarrow$ we expect $2\dim SU(N_f) - \dim SU(N_f) = N_f^2 - 1$ (pseudo-) Goldstone bosons.

When $N_f = 3$, one has 8 (quasi-) massless scalars.

If the vacuum is s.t. $\langle \bar{q}_L^I q_R^J \rangle = -c\,\delta^{IJ}$

then unbroken sym is (L, R) s.t. $L = R$

$\hookrightarrow$ massless deformations are obtained by applying sym group with parameters depending on x. These live in

$$\frac{SU(N_f)_L \times SU(N_f)_R}{SU(N)_V} \simeq SU(N_f)$$

(These are the massless fluctuations around the vac.)

$\Rightarrow$ massless particles are parametrized by

$$U(x) = e^{2\sqrt{2}\, i\, \pi(x)/f_\pi} \qquad (\pi(x) = \pi^a(x)\, T^a)$$

Identify with MESONS ?

observed hundred of MESONS in Nature.
Simple model of mesons views them as BOUND STATES of $q\,\bar{q}$. Since quarks have spin 1/2, MESONS have integer spin (BOSONS).

SPIN 0 : all pseudo-scalars (parity = −1)
masses $\lesssim m_p \approx 938$ MeV

MESON		quark content	m (MeV)	lifetime (sec)	
PION	π^+	$u\bar{d}$	140	10^{-8}	complex
PION	π^0	$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	135	10^{-16}	real
ETA	η	$\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$	548	10^{-19}	real
ETA'	η'	$\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$	958	10^{-21}	real
KAON	K^+	$u\bar{s}$	494	10^{-8}	complex
KAON	K^0	$d\bar{s}$	498	$10^{-8} - 10^{-11}$	complex

Why do these mesons arise? Why does this particular pattern of masses emerge?

→ explanation from ssb of chiral sym.

In fact these mesons can be rearranged in a 3×3 matrix

$$\pi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 & K^0 \\ \bar{K}^- & \bar{K}^0 & \sqrt{\frac{2}{3}}\,\eta^0 \end{pmatrix}$$

By embedding $U(1)_{ISOSPIN} \times U(1)_{STRANGENESS}$ into $SU(3)_V$, these are identified with $\pi^a(x)$ above.

Chiral Lagrangian

The unique $SU(3)_L \times SU(3)_R$ - invariant term in the (pseudo) Goldstone boson Lagrangian (IR - effective theory) that is of second order in space-time derivatives is

$$\mathcal{L}_{2\partial} = \frac{1}{16} f_\pi^2 \, \mathrm{Tr} \left(\partial_\mu U \partial^\mu U^\dagger \right) \qquad (*)$$

This is actually invariant under $SU(3)_L \times SU(3)_R$ and the vacuum breaks it to $SU(3)_V$ (in fact, $\det U = 1$ and $UU^\dagger = \mathbb{1}$ $\rightsquigarrow$ up to $SU(3)_L \times SU(3)_R$ it is the $\mathbb{1}_3$ that breaks to $SU(3)_V$).

- From $(*)$ one obtains at quadratic order

$$-\frac{1}{2} \partial_\mu \pi^0 \partial^\mu \pi^0 - \partial_\mu \pi^+ \partial^\mu \pi^- - \partial_\mu K^+ \partial^\mu K^- - \partial_\mu K^0 \partial^\mu \bar{K}^0 - \frac{1}{2} \partial_\mu \eta^0 \partial^\mu \eta^0$$

- One obtains also interaction terms that goes like

$$-\frac{2}{f_\pi^2} \, \mathrm{tr} \left(\pi^2 (\partial \pi)^2 - (\pi \partial \pi)^2 \right) + \dots$$

$\rightarrow$ interaction for mesons ; comparing with experiments (looking at decays of mesons) one obtains

$$f_\pi = 184 \text{ MeV}$$

(suppressed by $\left(\frac{p}{\Lambda_{QCD}}\right)^N$)

- There are also higher derivative couplings of the form

$$l_1 \left[\mathrm{Tr} \left(\partial_\mu U \partial^\mu U \right) \right]^2 + l_2 \, \mathrm{Tr} \left(\partial_\mu U \partial_\nu U^\dagger \right) \mathrm{Tr} \left(\partial^\mu U \partial^\nu U^\dagger \right) + \dots$$

Also these contribute to mesons interactions.

- $\partial_\mu \to D_\mu = \partial_\mu + i A_\mu$ introduces interaction with photons (still there at low en.)

CURRENTS

We started with microscopic non-ab gauge theory and got very different effective theory at low energy.

It is useful to know how operators in UV get mapped to ops in IR. For CURRENTS, this map is easy.

$$J^a_{V\mu} = J^a_{L\mu} + J^a_{R\mu} \qquad J^a_{A\mu} = J^a_{L\mu} - J^a_{R\mu}$$

In UV: $\quad J^a_{V\mu} = \bar{\psi}_i T^a_{ij} \gamma_\mu \psi_j \qquad J^a_{A\mu} = \bar{\psi}_i T^a_{ij} \gamma_\mu \gamma^5 \psi_j$

In IR:

consider infinitesimal transf $L = 1 + i\alpha^a T^a$

$$\delta_L U = -i\alpha^a T^a U \qquad (U \mapsto L^\dagger U)$$

compute current in standard way: $\alpha \to \alpha(x)$ $\quad \delta\mathcal{L} = \partial_\mu \alpha^a J^a_{L\mu}$

$$\Rightarrow J^a_{L\mu} = \frac{i f_\pi^2}{4} \operatorname{tr}\left(U^\dagger T^a \partial_\mu U - (\partial_\mu U)^\dagger T^a U\right)$$
$$\approx -\frac{f_\pi}{2} \partial_\mu \pi^a$$

Similarly $\quad \delta_R U = i\alpha^a U T^a$

$$J^a_{R\mu} = \frac{i f_\pi^2}{4} \operatorname{tr}\left(-T^a U^\dagger \partial_\mu U + (\partial_\mu U^\dagger) U T^a\right)$$
$$\approx \frac{f_\pi}{2} \partial_\mu \pi^a$$

Both currents have $\neq 0$ matrix elem. between $|0\rangle$ and $|\pi^a(p)\rangle$

E.g. $\langle 0 | J^a_{L\mu} | \pi^b(p) \rangle = -i \frac{f_\pi}{2} \delta^{ab} p_\mu e^{-ix\cdot p}$ This tells us that $SU(N_f)_L$ is sp. broken (acting with Q^a_L on the vac is $\neq 0$)

However $\langle 0 | \underbrace{J^a_{L\mu} + J^a_{R\mu}}_{J^a_{V\mu}} | \pi^b(p) \rangle = 0$: $SU(N_f)_V$ is preserved