AG 3 - fourth assignment

Valentina Beorchia

November 13, 2022

1. Prove that the homogeneous coordinate ring $S(X) = \frac{\mathbb{K}[x_0,\ldots,x_n]}{I_H(X)}$ is not invariant under isomorphisms of projective varieties.

2. Let X, Y be irreducible topological spaces. Assume $X \times Y$ has a topology for which the inclusions

$$j_p : Y \to X \times Y, \quad b \to (p, b),$$

and

$$i_q : X \to X \times Y, \quad a \to (a, q),$$

are continuous for all $p \in X$ and for all $q \in Y$. Prove that $X \times Y$ is irreducible.

3. Let $\mathbb{K} = \overline{\mathbb{K}}$. Show that $\mathbb{A}^1_{\mathbb{K}} \setminus \{\text{point}\}$ is not isomorphic to a projective variety. Similarly, prove that $\mathbb{P}^1_{\mathbb{K}} \setminus \{\text{point}\}$ is not isomorphic to a projective variety.