18 November

$$\left((1+x)^2\right)' = 2(1+x)\,(1+x)'$$
$$= 2(1+x)$$

$$f(x) = \frac{1+x+e^x}{1+3(1+x)^2}$$

$$f'(x) = \frac{(1+x+e^x)'\left(1+3(1+x)^2\right) - (1+x+e^x)\left(\cancel{1}+3(1+x)^2\right)'}{\left(1+3(1+x)^2\right)^2}$$

$$= \frac{(1+e^x)\left(1+3(1+x)^2\right) - (1+x+e^x)\, 3 \cdot 2\,(1+x)}{\left(1+3(1+x)^2\right)^2}$$

$$f(x) = \lg\left(\frac{1+e^x+3x^2}{\lg(1+x^2)}\right) = \qquad x \neq 0$$

$$= \lg(1+e^x+3x^2) - \lg(\lg(1+x^2))$$

$$f'(x) = \lg'(1+e^x+3x^2)\,(1+e^x+3x^2)' - \lg'(\lg(1+x^2))\,\lg'(1+x^2) \cdot (1+x^2)'$$

$$= \frac{1}{1+e^x+3x^2}(e^x+6x) - \frac{1}{\lg(1+x^2)}\,\frac{1}{1+x^2}\,2x$$

Def $f:[a,b)\to\mathbb{R}$ e sia $x_0\in[a,b)$

Se esiste ed è finito

$$\lim_{x\to x_0^+}\frac{f(x)-f(x_0)}{x-x_0}$$

allora ~~chiamo~~ denoto il limite con $f'_d(x_0)$ e chiamo derivata destra di f nel punto x_0.

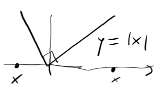
$f:(a,b]\to\mathbb{R}$ c'è una nozione $\lim_{x\to x_0^-}\frac{f(x)-f(x_0)}{x-x_0}=f'_s(x_0)$

Esempio Sia $f:I\to\mathbb{R}$ x_0 un punto interno sono equivalenti le seguenti due proposizioni

1) $f'(x_0)$ esiste
2) $f'_d(x_0)$ ed $f'_s(x_0)$ esistono e sono uguali

Quando 1) e 2) sono vere allora $f'(x_0)=f'_s(x_0)=f'_d(x_0)$.

Esempio $|x|:\mathbb{R}\to\mathbb{R}$ $\quad |x|=\begin{cases}x & \text{se } x\ge 0\\ -x & \text{se } x<0\end{cases}$



per $x>0$

$$(|x|)'=(|x|_{|\mathbb{R}_+})'=(x)'=1$$

per $x<0$ $\quad (|x|)'=(|x|_{|\mathbb{R}_-})'=(-x)'=-1$

$(|x|)'_d(0)=\lim_{x\to 0^+}\frac{|x|-\cancel{0}}{x}$ $\quad f'_d(0)$ $\quad f'(0^+)=\lim_{x\to 0^+}f'(x)$

$f(0^+)=\lim_{x>0,\, x\to 0}f(x)$

$=\lim_{x\to 0^+}\frac{x}{x}=\lim_{x\to 0^+}1=1$

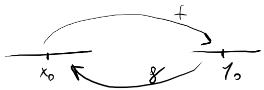
$(|x|)'_s(0)=\lim_{x\to 0^-}\frac{|x|-0}{x}=\lim_{x\to 0^-}\frac{|x|}{x}=\lim_{x\to 0^-}\frac{-x}{x}$

$=\lim_{x\to 0^-}(-1)=-1$

$(|x|)'(0)$ non esiste

Teor $f: I \to J$ strettamente monotona e suriettiva

Sia $g: J \to I$. Sia $x_0 \in I$ e sia $y_0 = f(x_0)$



Se esiste $f'(x_0)$ e se $f'(x_0) \neq 0$, allora esiste

$g'(y_0)$ e si ha $g'(y_0) = \dfrac{1}{f'(x_0)}$

Dim $f'(x_0) = \lim_{x \to x_0} \dfrac{f(x)-f(x_0)}{x-x_0}$

Se pongo $y = f(x)$

$\iff x = g(y)$

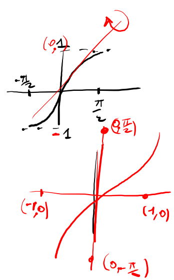
$$\frac{g(y)-g(y_0)}{y-y_0} = \frac{x-x_0}{f(x)-f(x_0)}$$

$$= \frac{1}{\dfrac{f(x)-f(x_0)}{x-x_0}}$$

$$g'(y_0) = \lim_{y \to y_0} \frac{g(y)-g(y_0)}{y-y_0} = \lim_{x \to x_0} \frac{1}{\dfrac{f(x)-f(x_0)}{x-x_0}} = \frac{1}{f'(x_0)}$$

$x = g(y)$

Esempi $\sin x : [-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1,1]$



$\arcsin(y) : [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$

per $-1 < y < 1$

$$(\arcsin y)' = \frac{1}{\sqrt{1-y^2}}$$

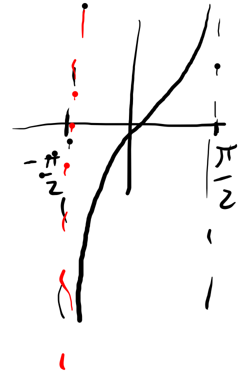
$x = \arcsin y$
$y = \sin x$

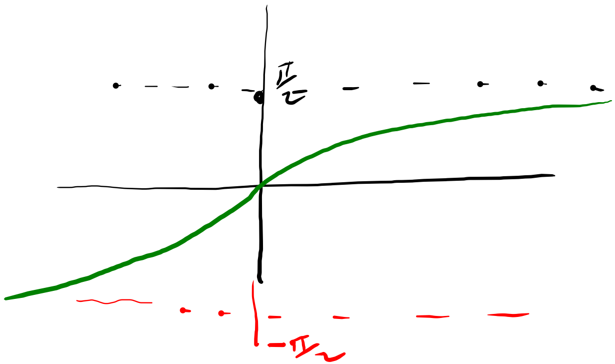
$(\sin x)' = \cos x > 0$ per $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$$(\arcsin y)' = \frac{1}{(\sin x)'} = \frac{1}{\cos x} = \frac{1}{\sqrt{1-\sin^2 x}} = \frac{1}{\sqrt{1-y^2}}$$

$$\tan x : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \longrightarrow \mathbb{R}$$

$$\arctan x : \mathbb{R} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$





$$(\tan x)' = 1 + \tan^2 x$$

$$y = \tan x$$

$$(\arctan y)' = \frac{1}{1+y^2}$$

$$(\arctan y)' = \frac{1}{(\tan x)'} = \frac{1}{1+\tan^2 x} = \frac{1}{1+y^2}$$

Funzioni iperboliche

$$\sinh x = sh\, x = \frac{e^{x} - e^{-x}}{2}$$

$$\cosh x = ch\, x = \frac{e^{x} + e^{-x}}{2}$$

$$\tanh x = th\, x =$$

$$= \frac{sh\, x}{ch\, x} = \frac{\frac{e^{x} - e^{-x}}{\cancel{2}}}{\frac{e^{x} + e^{-x}}{\cancel{2}}} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$sh(-x) = \frac{e^{-x} - e^{-(-x)}}{2} = \frac{e^{-x} - e^{x}}{2} = -\frac{e^{x} - e^{-x}}{2}$$
$$= -sh\, x$$

$$ch^2 x - sh^2 x = 1$$

$$\left(\frac{e^{x} + e^{-x}}{2}\right)^2 - \left(\frac{e^{x} - e^{-x}}{2}\right)^2 =$$

$$= \frac{\cancel{e^{2x}} + \cancel{e^{-2x}} + 2}{4} - \frac{\cancel{e^{2x}} + \cancel{e^{-2x}} - 2}{4}$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$\sin x = \frac{e^{ix} + e^{-ix}}{2i}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$sh(2x) = 2\, sh(x)\, ch(x)$$

$$sh(2x) = \frac{e^{2x} - e^{-2x}}{2}$$

$$2\, sh(x)\, ch(x) = 2\, \frac{e^{x} - e^{-x}}{2}\, \frac{e^{x} + e^{-x}}{2} =$$

$$= \frac{1}{2}\left(e^{x} - e^{-x}\right)\left(e^{x} + e^{-x}\right) = \frac{1}{2}\left(e^{2x} - e^{-2x}\right)$$