AG 3 - fifth assignment

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1. Let X and way be a qp variety, and let $f : \dashrightarrow Y$ be a rational map. Prove that f is dominant if and only if for every representative pair (U, φ), we have that $\varphi(U)$ is dense in Y.

2. Let $X = V_P(x_1^2 + x_2^2 - x_0^2) \subseteq \mathbb{P}^2_{\mathbb{C}}$ be the projective closure of the unit circle. Let

$$U = X \setminus V_P(x_1, x_0 - x_2),$$

and consider the morphism $\varphi : U \to \mathbb{P}^1$ given by $\varphi(x_0 : x_1 : x_2) = (x_0 - x_2 : x_1)$. Show that:

   (a) the rational map $f = \langle U, \varphi \rangle$ is in fact a morphism on X;

   (b) the morphism f is an extension to the north pole of the stereographic projection;

   (c) find the image of the north pole;

   (d) the morphism f is an isomorphism, and find its inverse.

3. Consider the affine curve $C = V(y^4 + x^4 - x^2) \subseteq \mathbb{A}^2$ and determine its strict transform under the blow up of $\mathbb{A}^2$ at the origin.

4. **Optional** Show that the second projection

$$p_2 : \hat{\mathbb{A}}^2_{\mathbb{R}} \to \mathbb{P}^1_{\mathbb{R}}$$

gives the blow-up $\hat{\mathbb{A}}^2_{\mathbb{R}}$ of $\mathbb{A}^2_{\mathbb{R}}$ in the origin the structure of a vector bundle on $\mathbb{P}^1_{\mathbb{R}}$. What kind of vector bundle is it?