Predittore ottimo ad 1 e 2 passi a partire da dati osservati

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$$\underbrace{E[y]}_{\bar{y}} = \frac{1}{4}\underbrace{E[y(t-2)]}_{\bar{y}} + \cancel{E[e(t)]} - \cancel{2E[e(t-1)]}$$

$$E[y] = ?\ 0 \qquad \sigma_y^2 = ?$$

$\boxed{\text{Es.}}$

$$y(t) = \frac{1}{4}y(t-2) + e(t) - 2e(t-1)$$

$$e(\cdot) \sim WN(0,1)$$

	1	2	3	4	5	6
y	$5/2$	$7/5$	-1	$-7/5$	$-3/10$	$-4/5$

$$\hat{y}(7|6) = ? \qquad \hat{y}(7|5) = ?$$

$$\hat{y}(t+1|t) \qquad \hat{y}(t+2|t)$$

$$W(z) = \frac{1-2z^{-1}}{1-\frac{1}{4}z^{-2}} \Longrightarrow \frac{z^2-2z}{z^2-\frac{1}{4}} = \frac{z\,(z-2)}{(z+\frac{1}{2})(z-\frac{1}{2})}$$

$$\hat{W}(z) = \frac{\cancel{1-2z^{-1}}}{(1+\frac{1}{2}z^{-1})\cancel{(1-\frac{1}{2}z^{-1})}} \cdot \frac{\cancel{(1-\frac{1}{2}z^{-1})}}{\cancel{(1-2z^{-1})}}$$

$$\hat{W}(z) = \frac{1}{1+\frac{1}{2}z^{-1}} \quad \leftarrow \ ?$$

$W(z) \longleftrightarrow \hat{W}(z)$

$e(\cdot) \sim WN(0,1)$ $\qquad$? $\eta(\cdot) \sim WN(0, \lambda^2)$

$$\phi_{yy}(z) = W(z) \cdot W(z^{-1}) \cdot \lambda_e^2 = \hat{W}(z)\, \hat{W}(z^{-1}) \cdot \lambda_\eta^2$$

$$\frac{1-2z^{-1}}{1-\frac{1}{4}z^{-2}} \cdot \frac{1-2z}{1-\frac{1}{4}z^{2}} \cdot 1 = \frac{1}{1+\frac{1}{2}z^{-1}} \cdot \frac{1}{1+\frac{1}{2}z} \cdot \lambda_\eta^2 \quad \Big/ \left(1-\frac{1}{4}z^{-2}\right) \cdot \left(1-\frac{1}{4}z^{2}\right)$$

$$(1-2z^{-1})(1-2z) = \left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{2}z\right)\lambda_\eta^2$$

$$\left[5-2\left(z+z^{-1}\right)\right] = \frac{1}{4}\lambda_\eta^2 \left[5-2\left(z+z^{-1}\right)\right]$$

$$\lambda_\eta^2 = 4$$

$\sigma_y^2 = ?$ $\quad E[y] = 0$

$$E[y^2] = E\left\{\left[\frac{1}{4}y(t-2) + e(t) - 2e(t-1)\right]^2\right\} =$$

$y^2(t)$

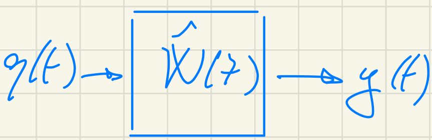
$$\sigma_y^2 = \frac{1}{16}\underbrace{E[y^2(t-2)]}_{\sigma_y^2} + \underbrace{E[e^2]}_{1} + 4\underbrace{E[e^2(t-1)]}_{1} +$$

$$+ 2\cdot\frac{1}{4}\cancel{E[y(t-2)\cdot e(t)]}_{0} - 2\cdot\frac{1}{4}\cdot 2\,\cancel{E[y(t-2)e(t-1)]}_{0}$$

$$-2\cdot 2\,\boxed{\cancel{E[e(t)\,e(t-1)]}}_{0}$$

$$\sigma_y^2 = \frac{1}{16}\sigma_y^2 + 5 \Rightarrow \sigma_y^2 = 16/3$$

$$\hat{W}(z) = \frac{1}{1 + \frac{1}{2}z^{-1}}$$



$\eta(\cdot) \sim WN(0, \lambda^2)$ $\quad \hat{\sigma}_y^2 = ?$

$$y(t) = -\frac{1}{2}y(t-1) + \eta(t) \qquad E[y] = 0$$

$$\hat{\sigma}_y^2 = \frac{1}{4}\hat{\sigma}_y^2 + \lambda^2 \Rightarrow \hat{\sigma}_y^2 = \frac{4}{3}\lambda^2$$

$$\sigma_y^2 = \frac{16}{3} \Rightarrow \frac{16}{3} = \frac{4}{3}\lambda^2 \Rightarrow \lambda^2 = 4$$

$$\boxed{\begin{array}{l}\eta(t) \rightarrow \hat{W}(z) \rightarrow y(t) \\ \sim WN(0,4)\end{array}}$$

forma canonica

$\boxed{AR(1)}$ $y(t) = -\frac{1}{2}y(t-1) + \eta(t) \qquad \eta \sim WN(0,4)$

$$\hat{y}(t+1|t) = ? \qquad \hat{y}(t+2|t) = ?$$

$$\hat{y}(t|t-1) = -\frac{1}{2}y(t-1) \qquad \hat{y}(7|6) = +2/5$$

$$\hat{y}(t+1|t) = -\frac{1}{2}y(t) \qquad \hat{\sigma}_{\hat{y}}^2 \rightarrow 4$$

$$y(t) = -\frac{1}{2}\, y(t-1) + \eta(t) \qquad \frac{1}{1+\frac{1}{2}z^{-1}} = \frac{C(z)}{A(z)}$$

$$\hat{y}(t+2|t) = ?$$

$$\begin{array}{l|l} 1 & 1+\frac{1}{2}z^{-1} \\ \hline -1-\frac{1}{2}z^{-1} & 1-\frac{1}{2}z^{-1} \\ \hline / \quad -\frac{1}{2}z^{-1} & \\ \quad +\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2} & \\ \hline / \quad \frac{1}{4}z^{-2} & \end{array}$$

$$\hat{W}(z) = \underbrace{\left[1-\frac{1}{2}z^{-1}\right]}_{E(z)} + z^{-2} \underbrace{\frac{\frac{1}{4}}{1+\frac{1}{2}z^{-1}}}_{\hat{W}_2(z)}$$

$$\hat{W}_2(z) = \frac{\frac{1}{4}}{1+\frac{1}{2}z^{-1}} \qquad W_2(z) = +\frac{1}{4}$$

$$\hat{y}(t+2|t) = \frac{1}{4}\, y(t)$$

$$\hat{y}(t+r|t) = a^r\, y(t) \qquad a = -\frac{1}{2} \quad r = 2$$

$$\hat{y}(7|5) = \frac{1}{4}\left(-\frac{3}{10}\right) = -\frac{3}{40}$$