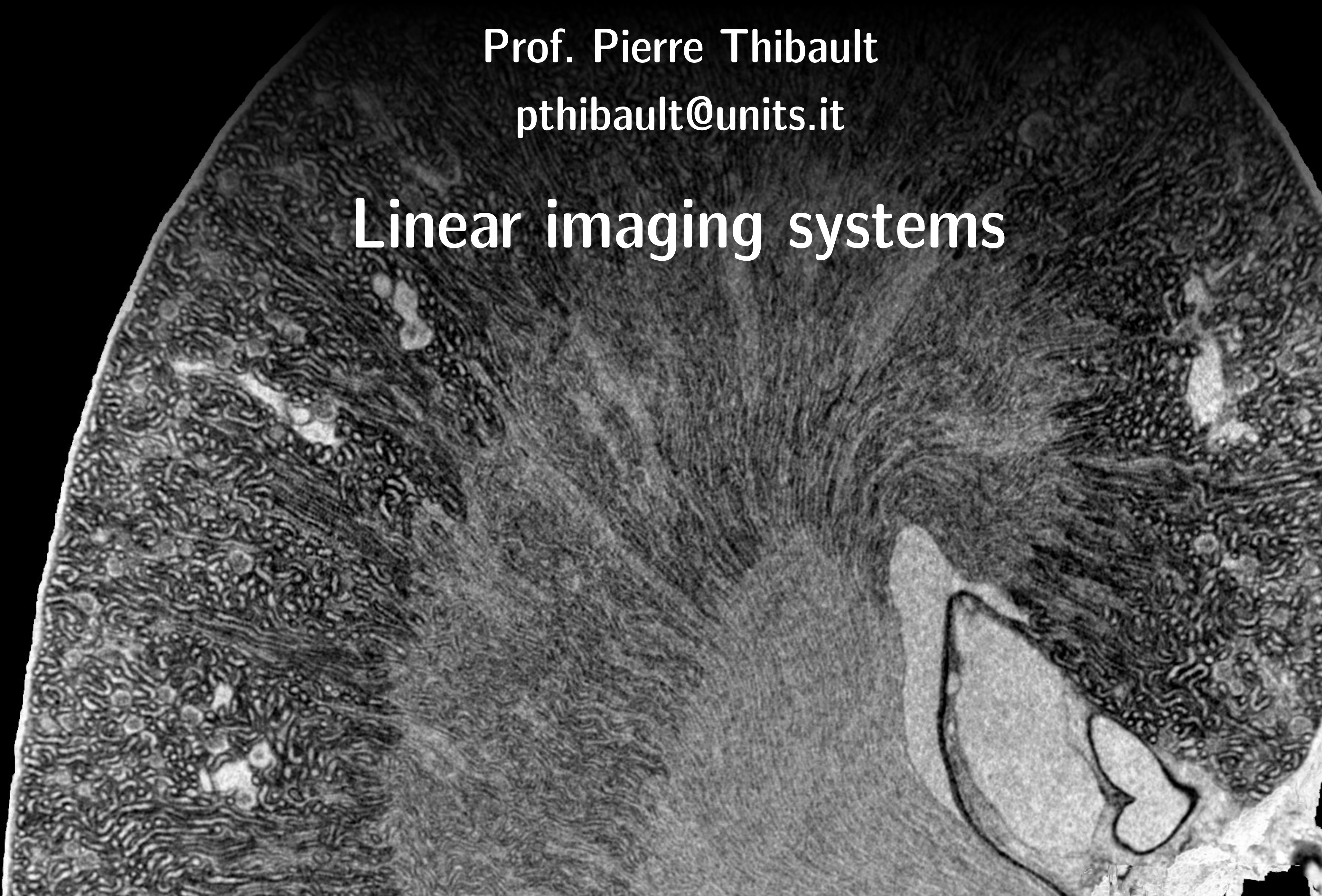
Image Processing for Physicists

Prof. Pierre Thibault

pthibault@units.it

Linear imaging systems



Overview

- Definition of resolution
- Imaging systems:
  - Linear transfer model
  - Noise

Resolution

"the smallest detail that can be distinguished"

- No unique definition
  - Numerical aperture   microscopy, photography, telescopes, ...
  - Pixel size
  - Other criteria (PSF, MTF)
- What is "detail"?
- What is "distinguish"?

Resolution

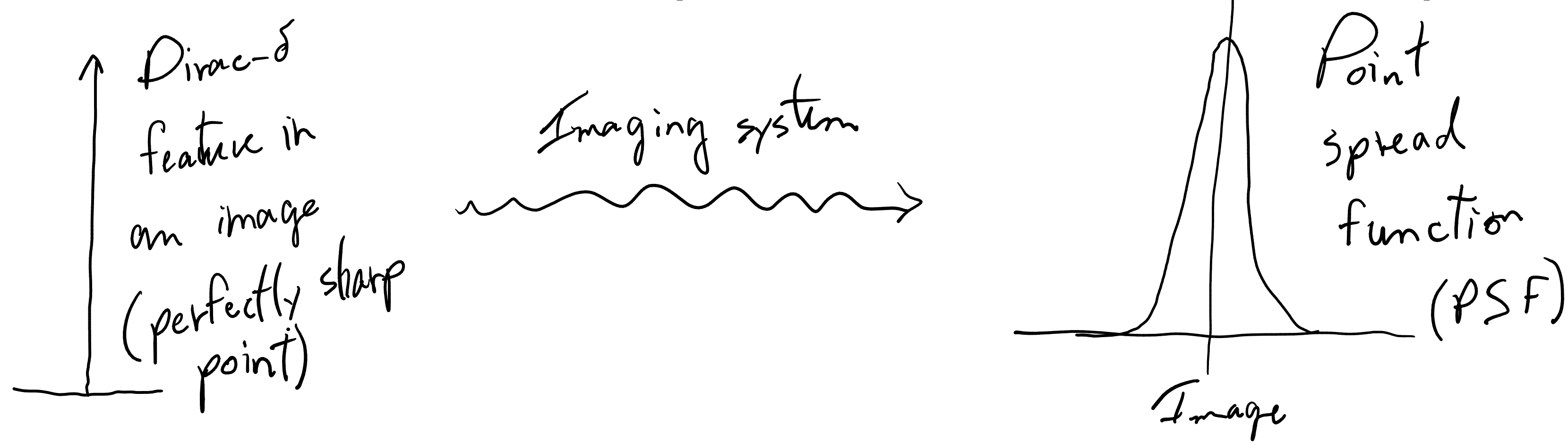
1280 x 1280

640 x 640

- **not** simply given by pixel size (i.e. sampling rate)
- light quality, optics quality, detector quality, algorithm quality, noise, ...

Linear translation-invariant systems

- Point spread function ("impulse response")



- LTI system: convolution with PSF

Input: $f(x,y) = \int dx'dy' \, f(x',y') \, \delta(x-x')\,\delta(y-y')$

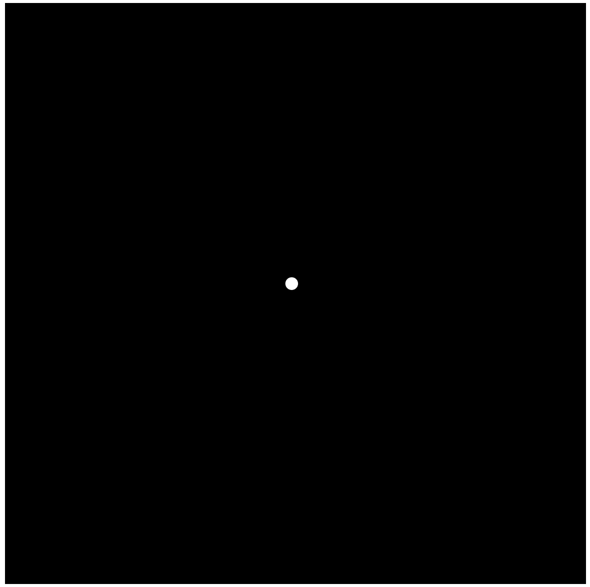
Imaging system

PSF

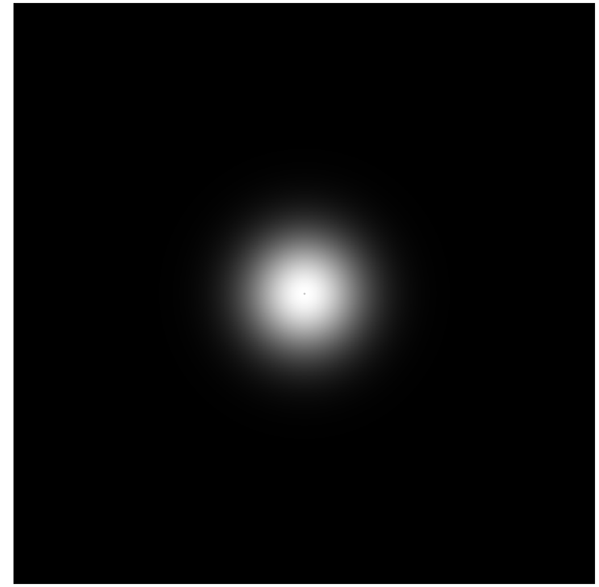
Output: $S\{f\} = \int dx'dy' \, f(x',y') \, h(x-x',y-y') = f * h$

linear translation invariant operator representing the imaging system

Point spread function



"point source"



PSF

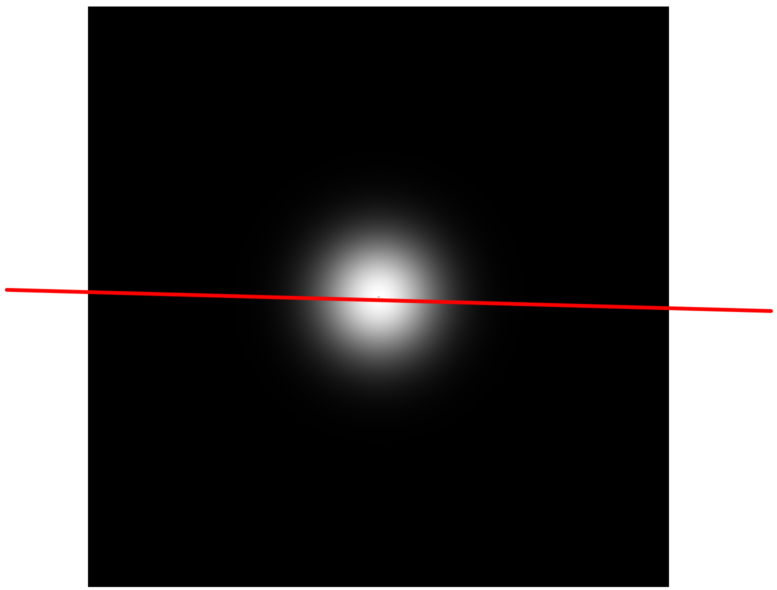
"true image"

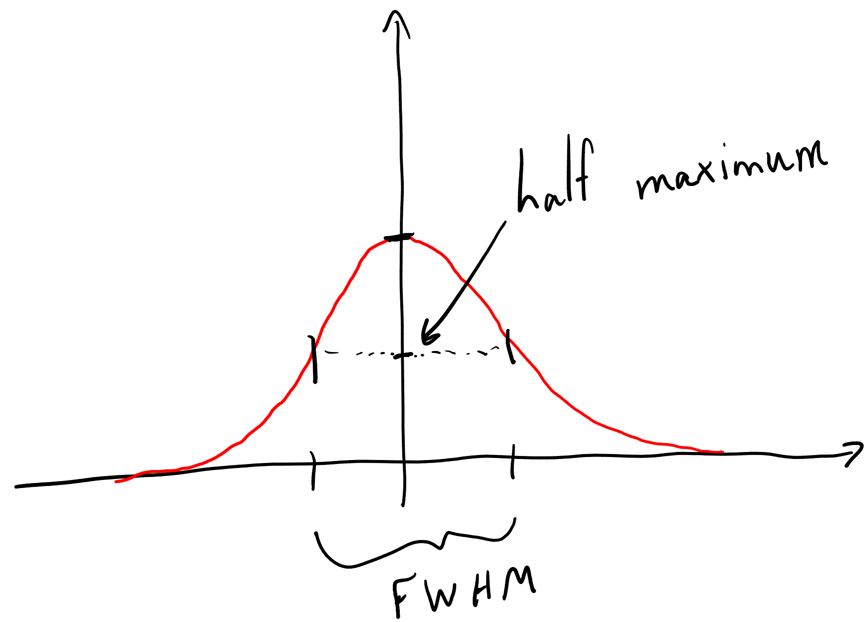
measured image

PSF and resolution

→ width of PSF

Commonly, resolution from PSF given by FWHM "full width at half maximum"

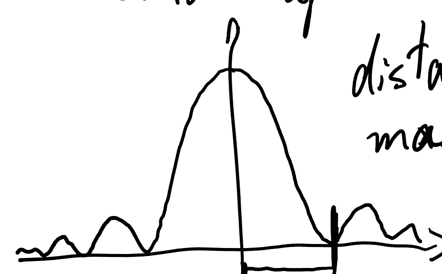




Rayleigh criterion:

applies to imaging systems with a circular aperture

PSF: Airy disc



distance from max to first 0

(more later)

Measurement of the PSF

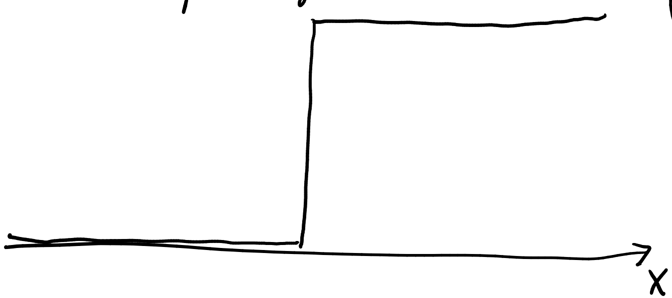
- Direct measurement from impulse

Generate a sharp point → output = PSF!

(Astronomy: easy! Just pick a bright star!)

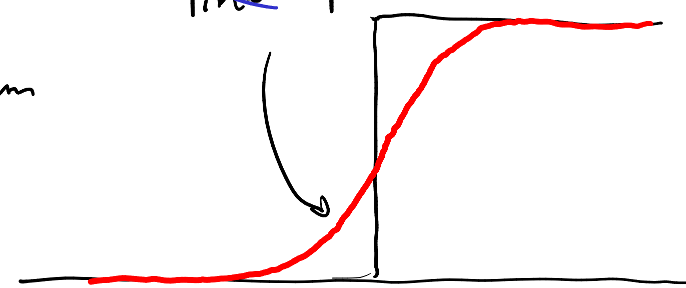
- Line-spread function

Sharp edge "knife edge"



Imaging system

~~line~~ edge-spread function



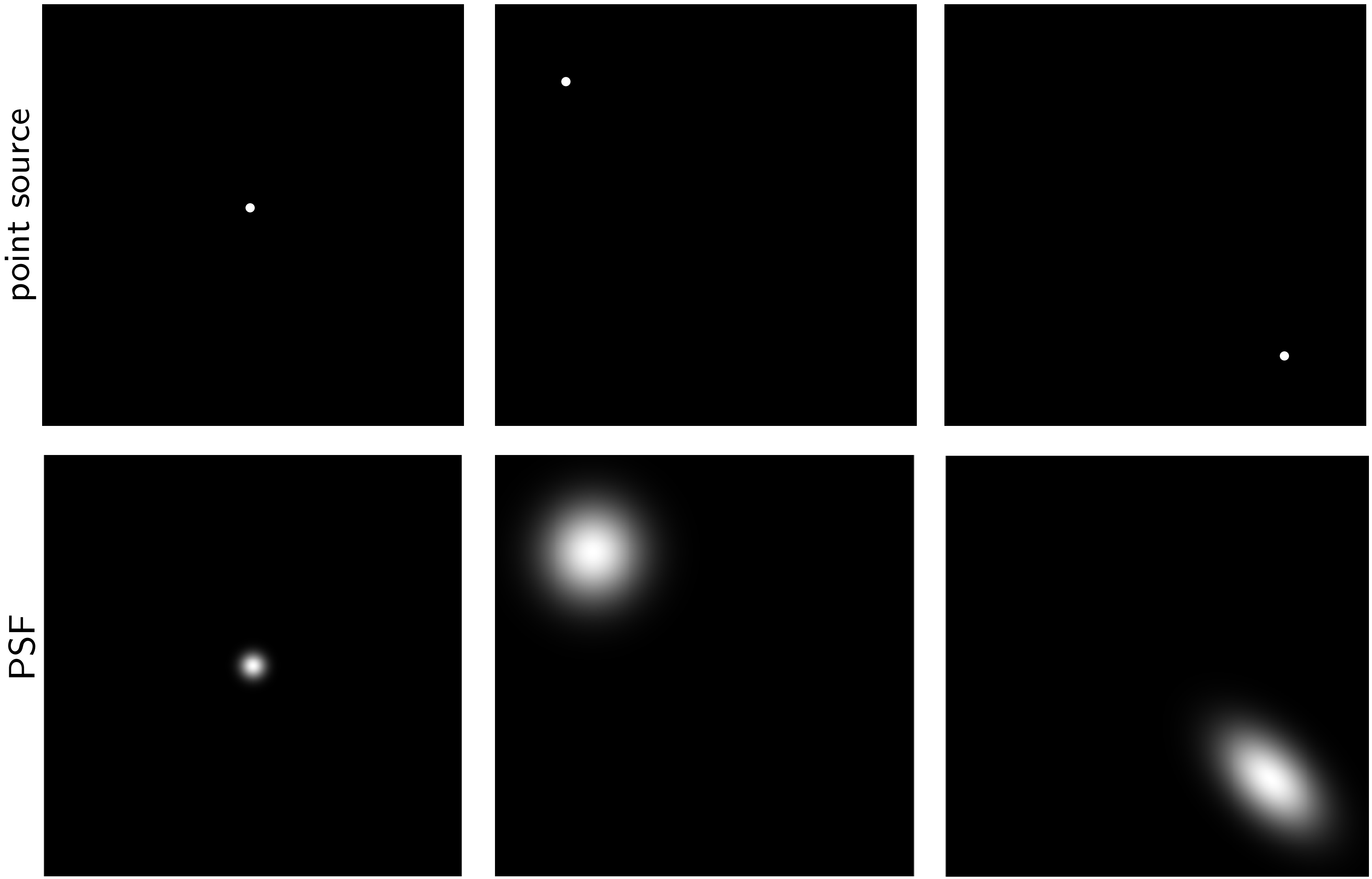
$$H(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

Heavyside step function

since $\frac{\partial H}{\partial x} = \delta(x)$

PSF = derivative of ~~line~~ edge-spread function

PSF and translation invariance



- Not translation invariant → PSF depends on position → not a convolution
- Useful to model system imperfections, lens aberrations, ...

The Fourier picture

$\mathcal{F}\{f * h\} = F(u) \cdot H(u)$ describes how an oscillating signal changes through the imaging system.

Consider a single spatial frequency u_0:

Imaging system

$S\{e^{2\pi i u_0 x}\} = H(u_0)\, e^{2\pi i u_0 x}$

eigenvalue

oscillating terms are eigenfunctions of the (LTI) imaging system

F.T. of PSF : OTF "Optical Transfer Function"

Optical transfer function

Response of a system to an oscillating signal with well-defined frequency

$$OTF(u) = \mathcal{F}\{PSF(x)\}$$

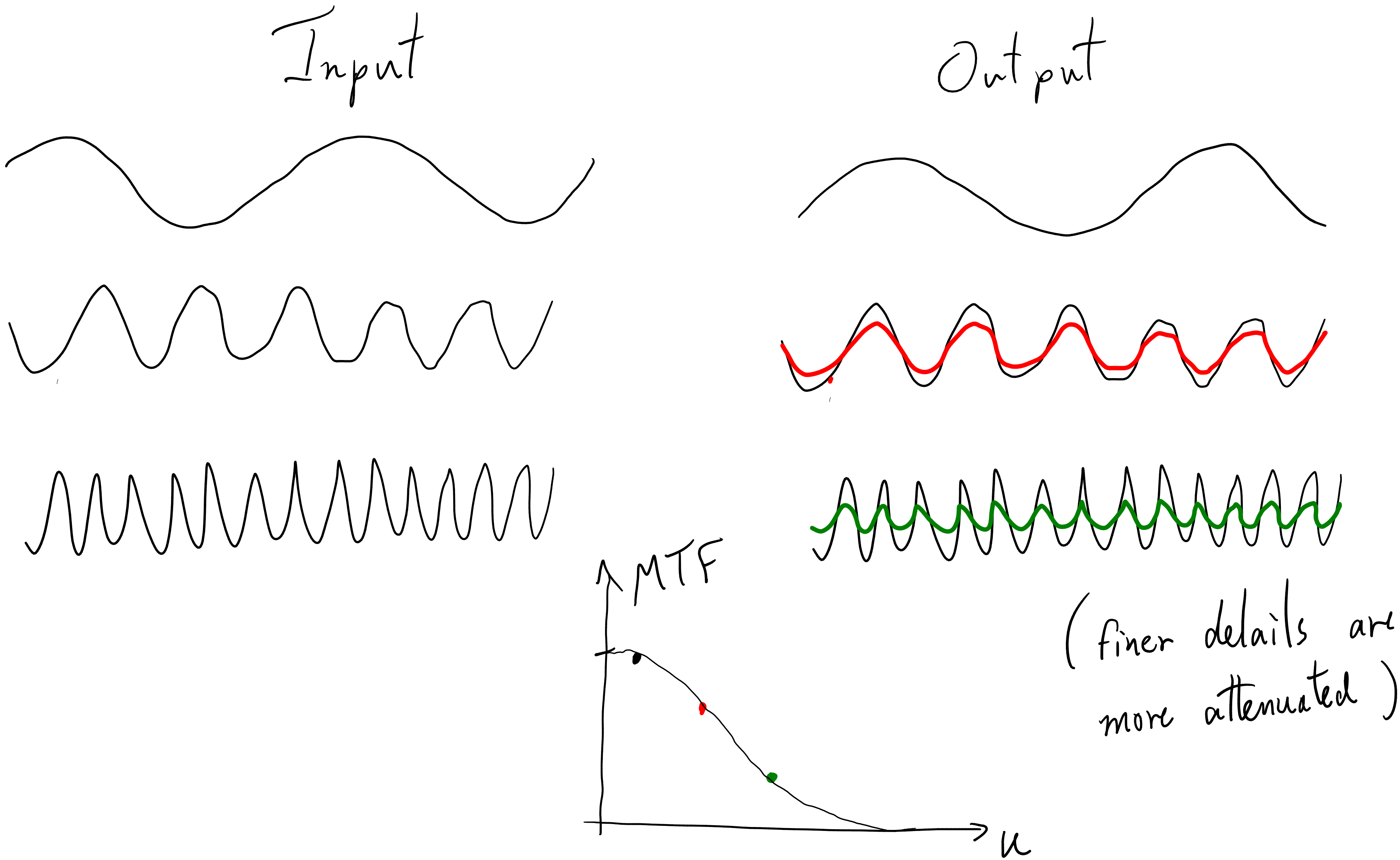
Amplitude: $|OTF| = MTF$ "modulation tranfer function"

Phase: $\arg\{OTF\} = PTF$ "phase transfer function"

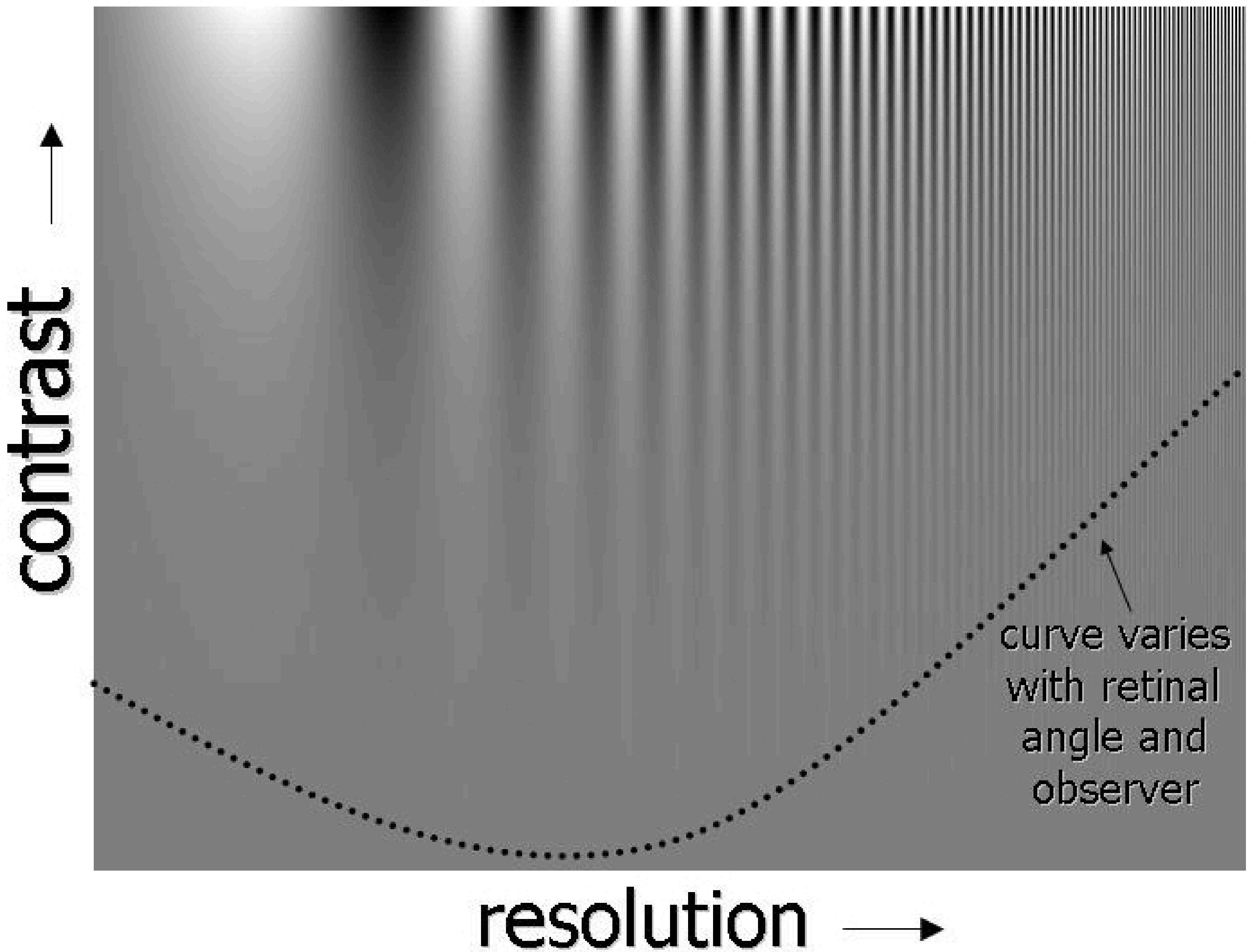
$$OTF = MTF\, e^{i\,PTF}$$

Modulation transfer function

Amplitude change of an oscillating signal for a given frequency



Eye MTF


contrast
curve varies
with retinal
angle and
observer
resolution

Campbell-Robson curve

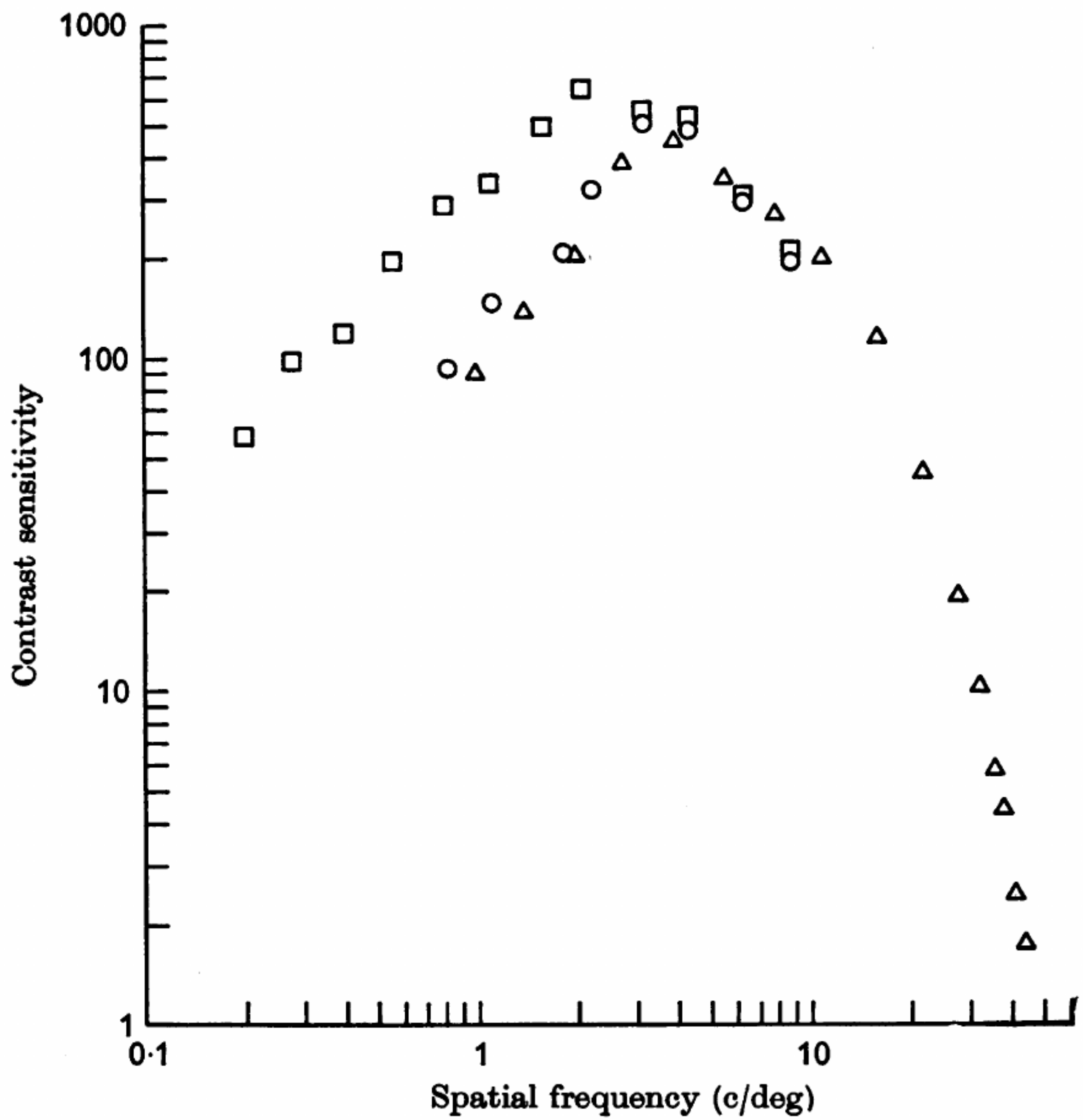
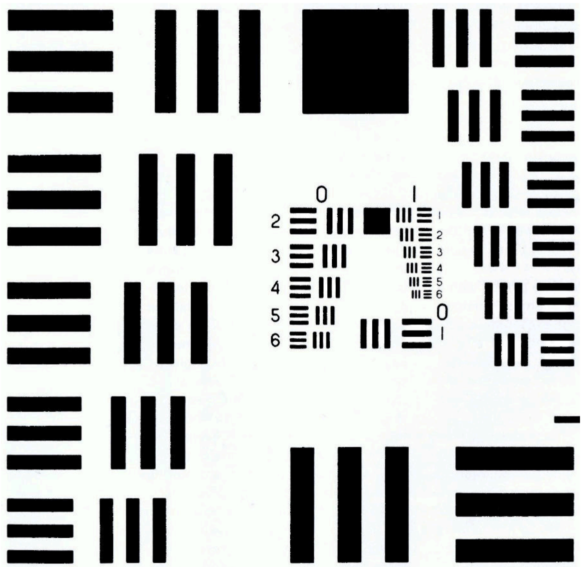
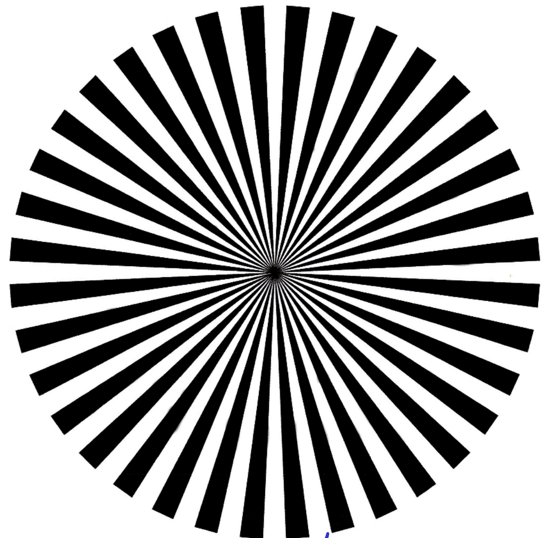


Fig. 2. Contrast sensitivity for sine-wave gratings. Subject F.W.C., luminance 500 cd/m². Viewing distance 285 cm and aperture $2° \times 2°$, △; viewing distance 57 cm, aperture $10° \times 10°$, □; viewing distance 57 cm, aperture $2° \times 2°$, ○.

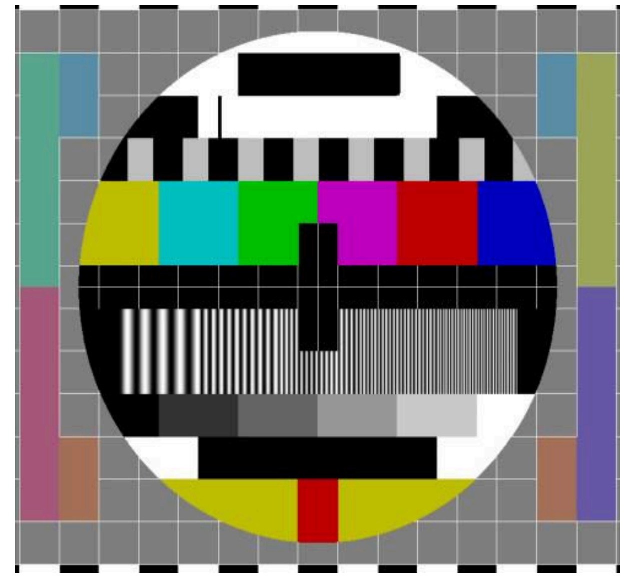
Measurement of MTF



USAF



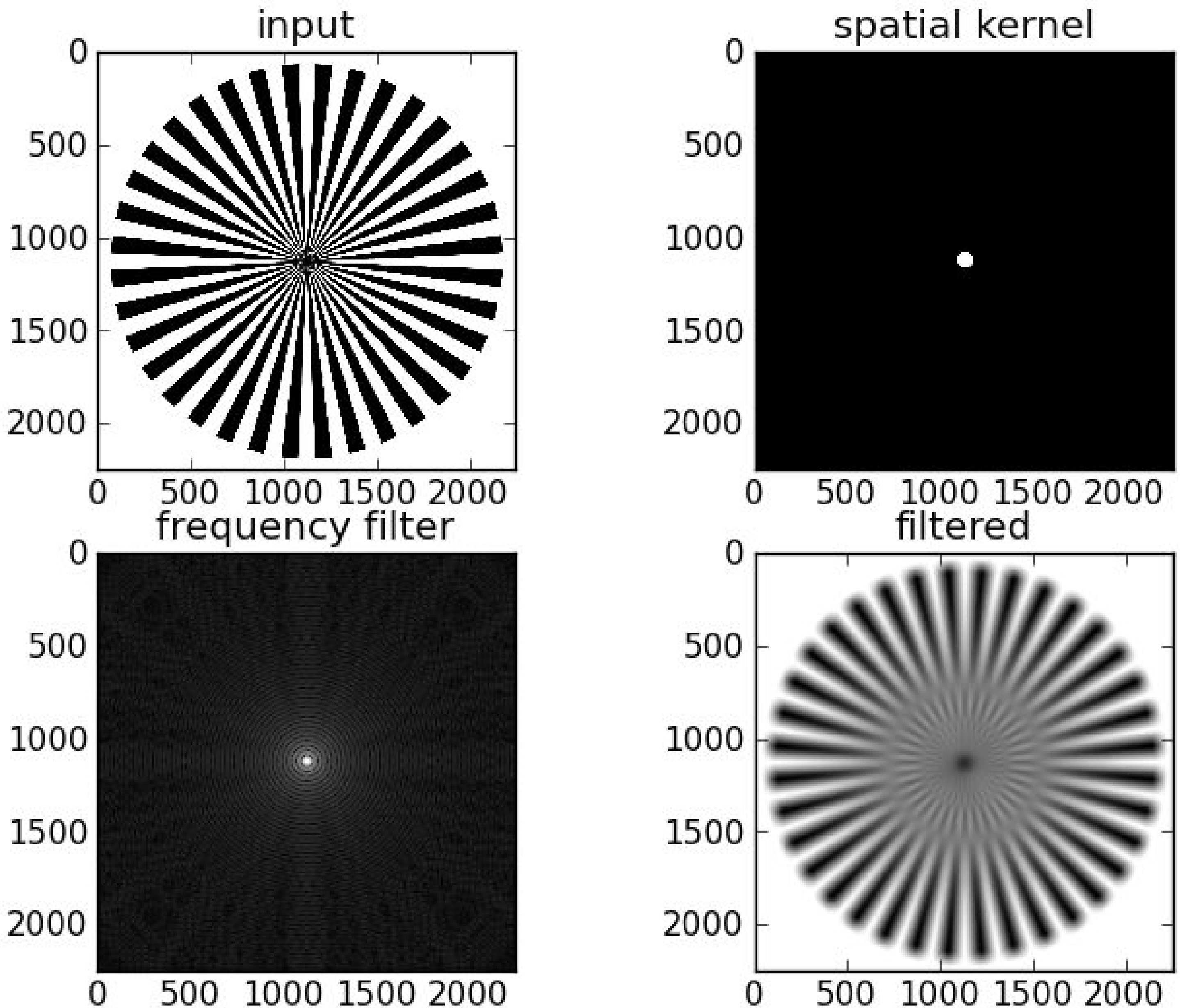
"Siemens star"



source: http://fotomagazin.de

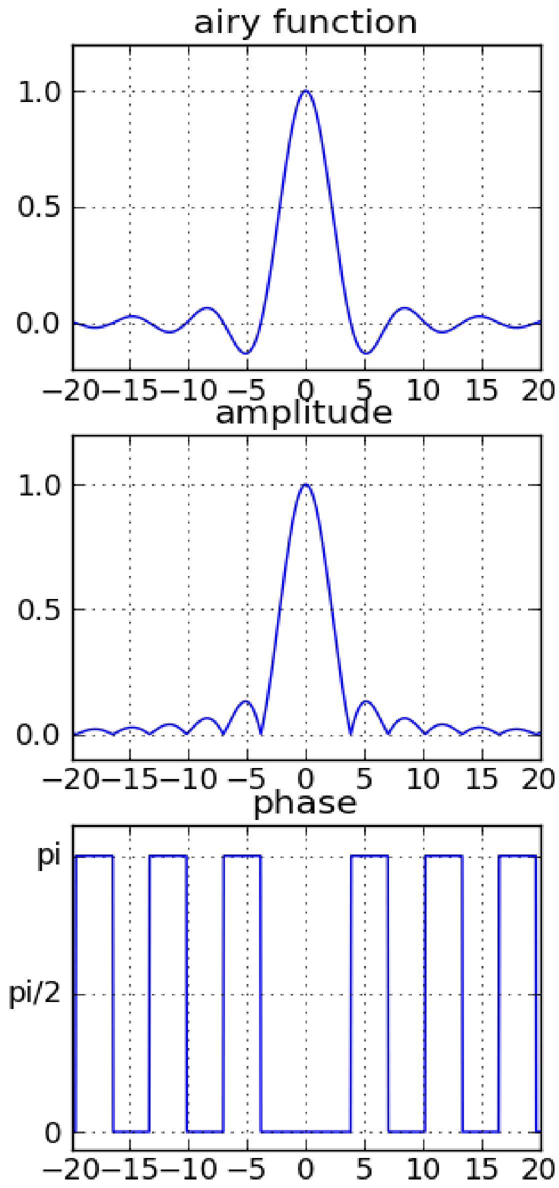
Phase transfer function

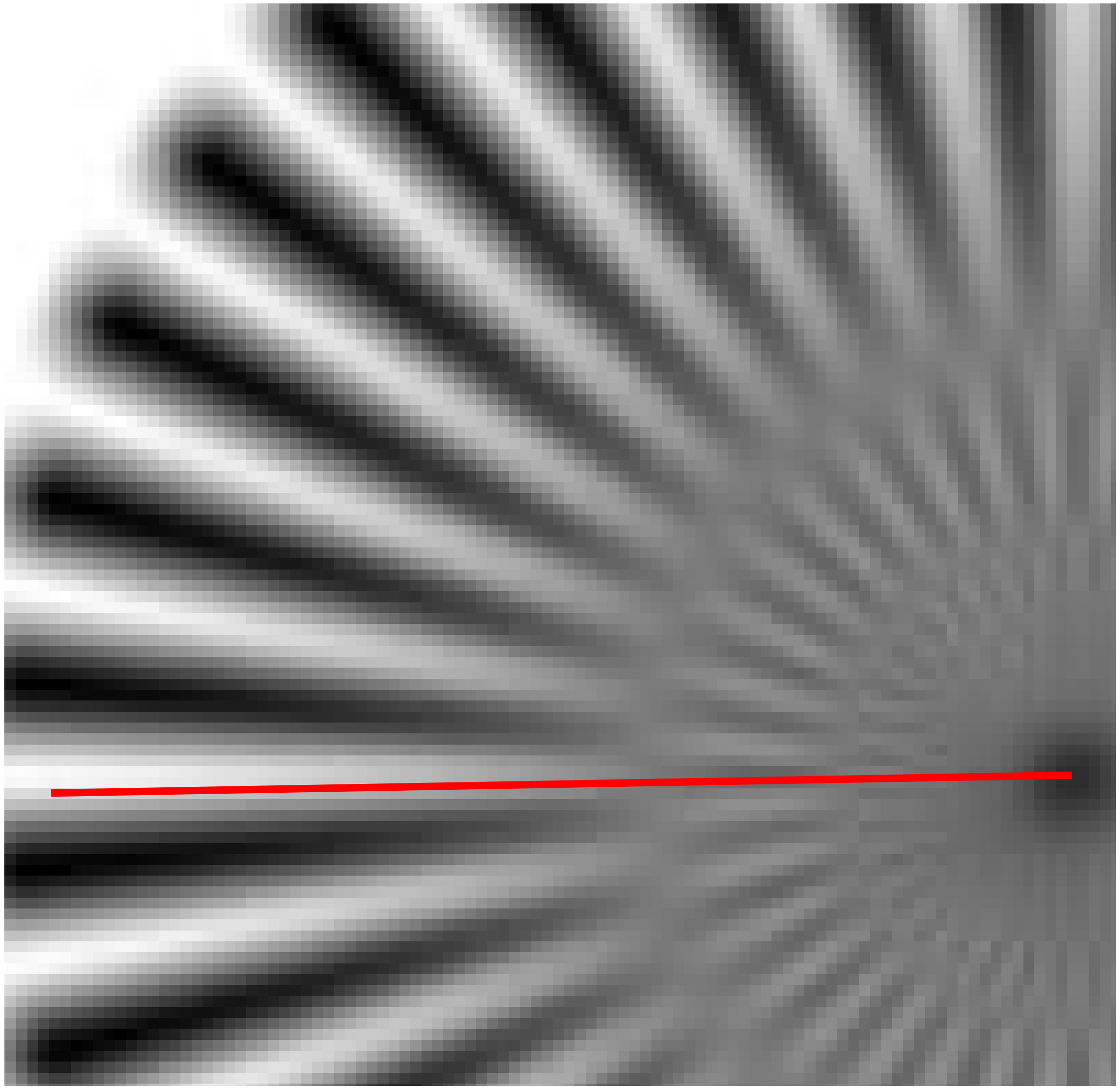
describes how an oscillating signal changes in phase due to system



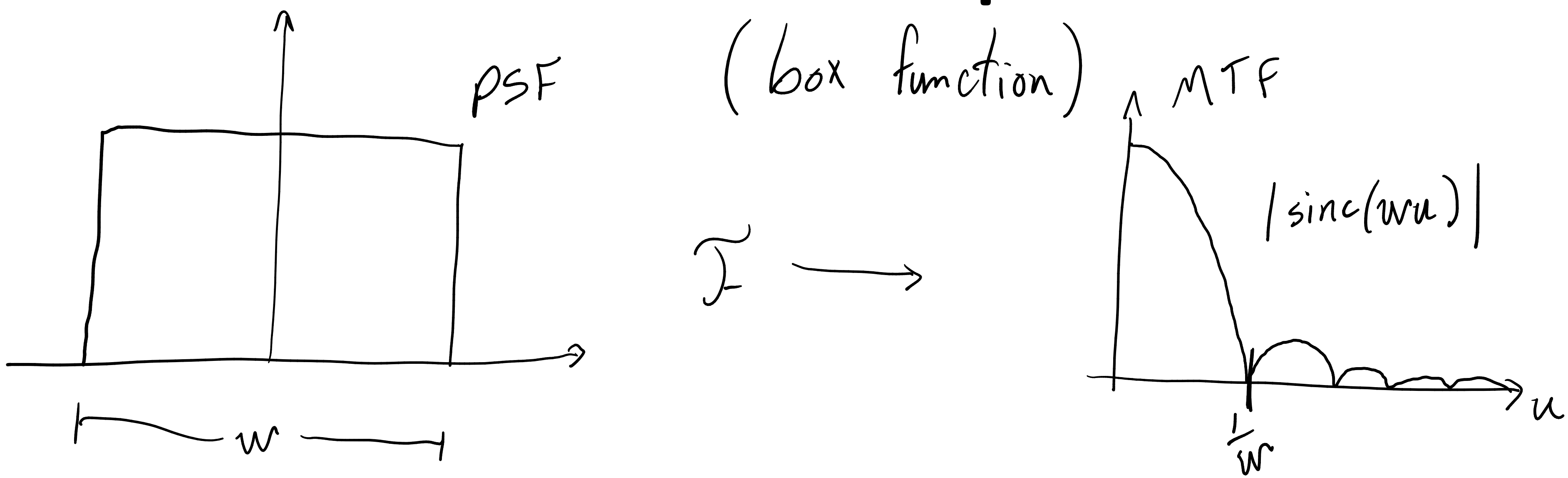
Phase transfer function

describes how an oscillating signal changes in phase due to system



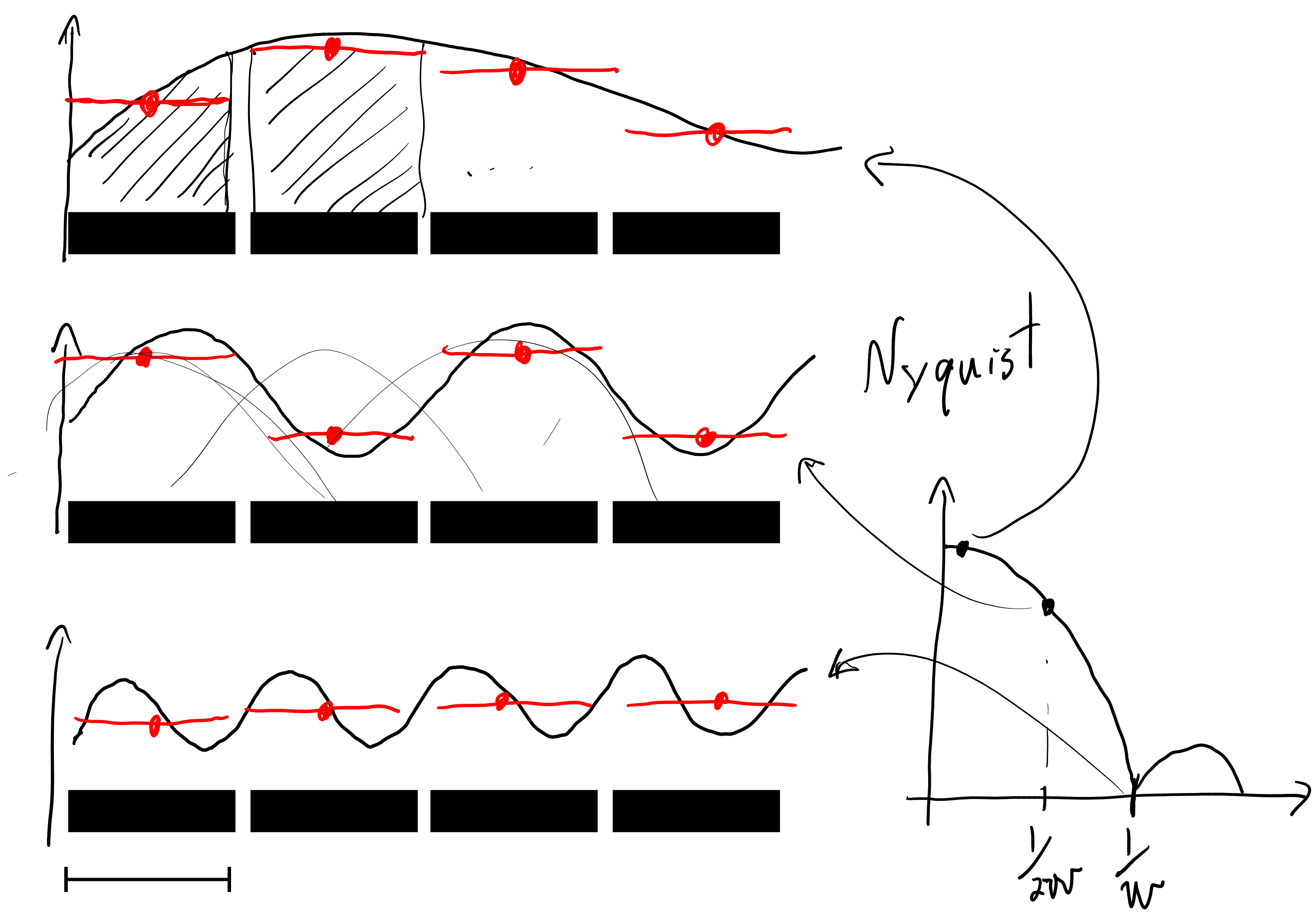


MTF of an "ideal pixel"



Pixel MTF

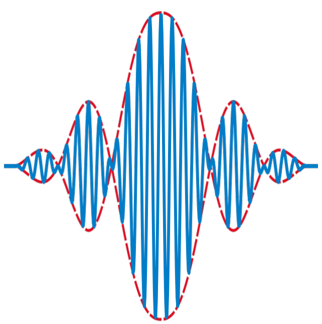
Modulation transfer function of a single detector pixel



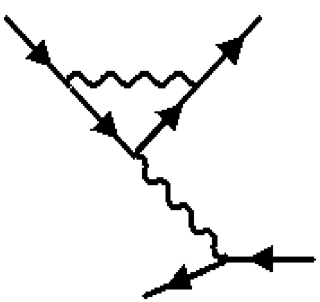
Imaging as a linear filter

$$\mathrm{Ouput}(u) = \mathrm{input}(u) \cdot \underbrace{MTF_{optics}(u) \cdot MTF_{detector}(u) \cdot MTF_{algorithm}(u) \cdot \ldots}_{\text{effective } MTF}$$

input



interaction



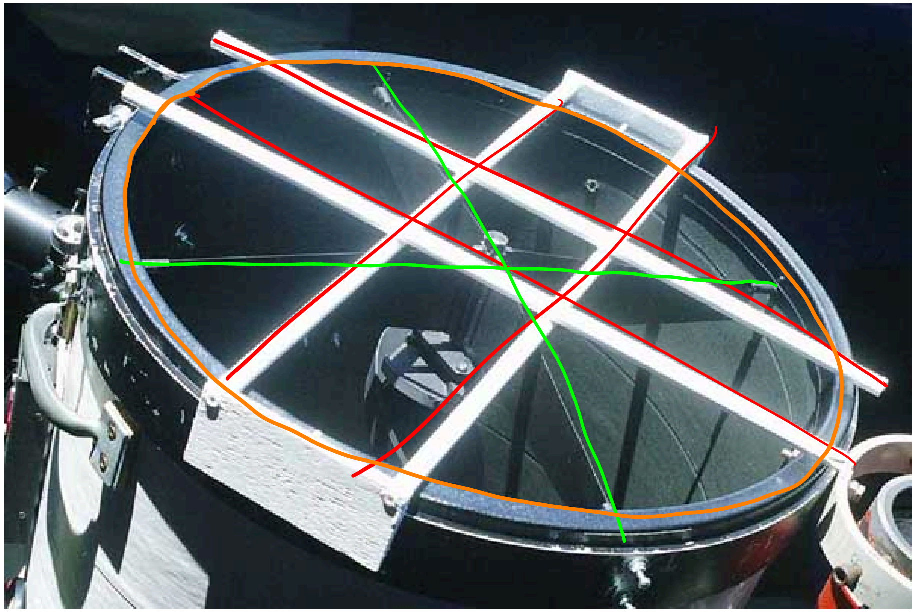
detector

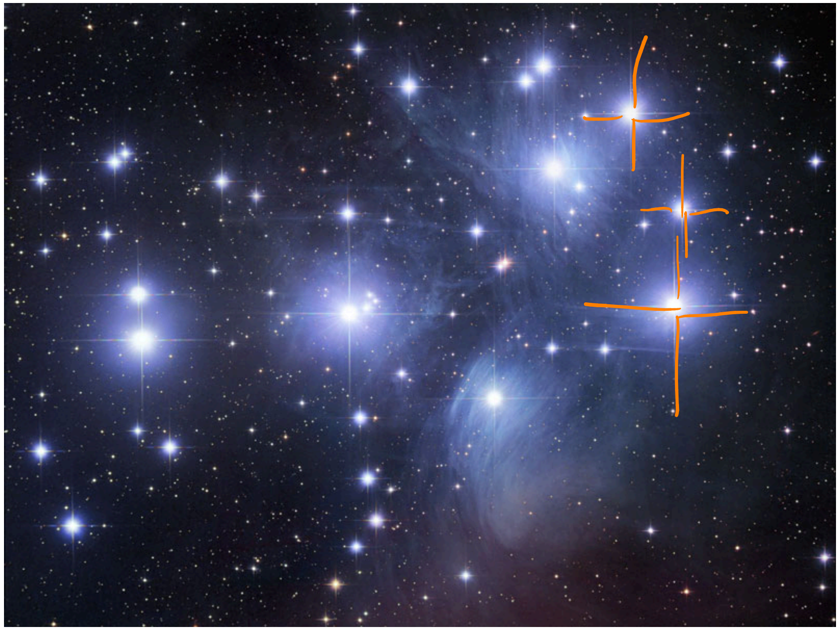
analysis
processing

display

PSF examples

- isolated stars are essentially PSFs

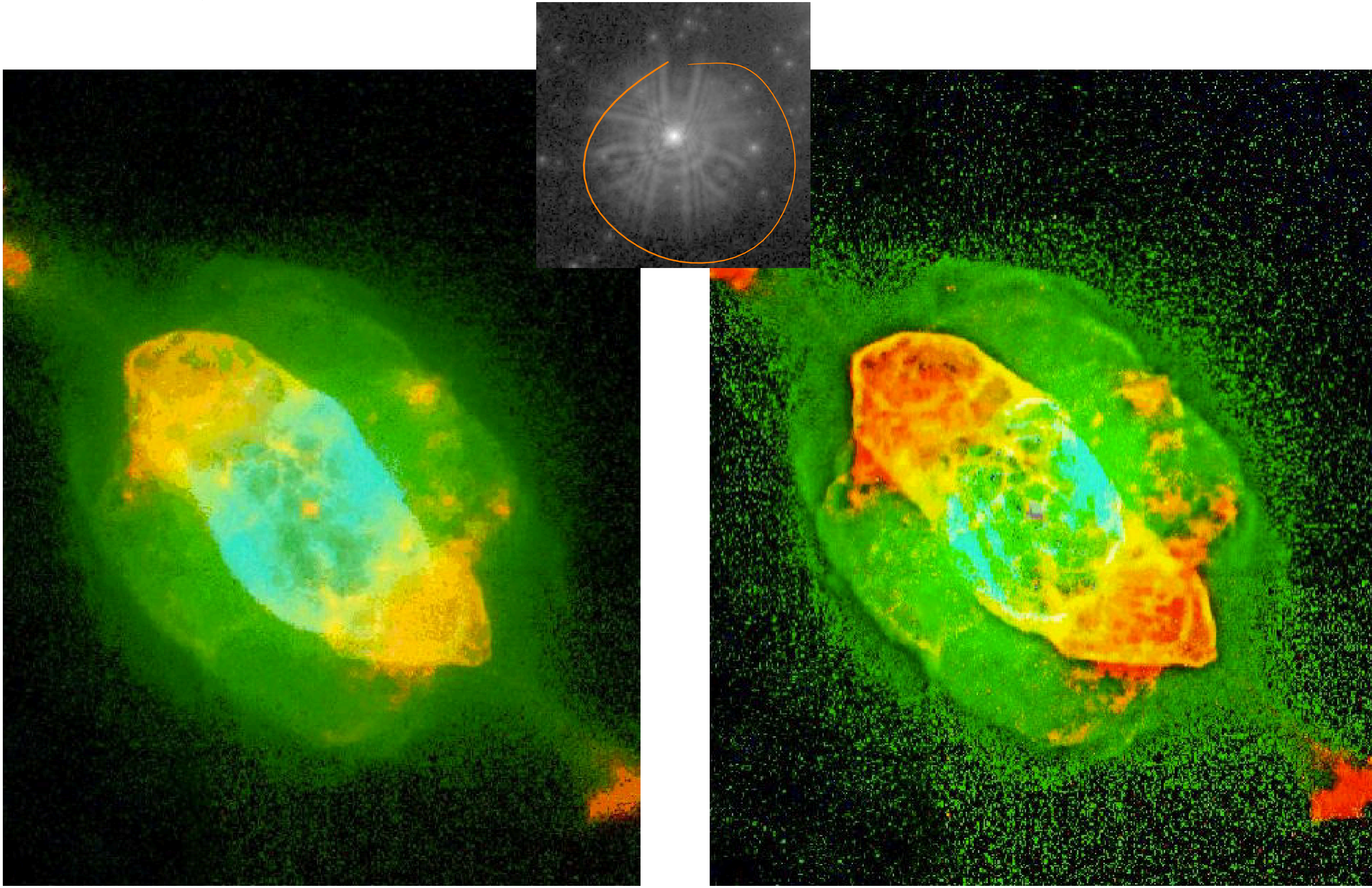




source: www.apod.nasa.gov

PSF examples

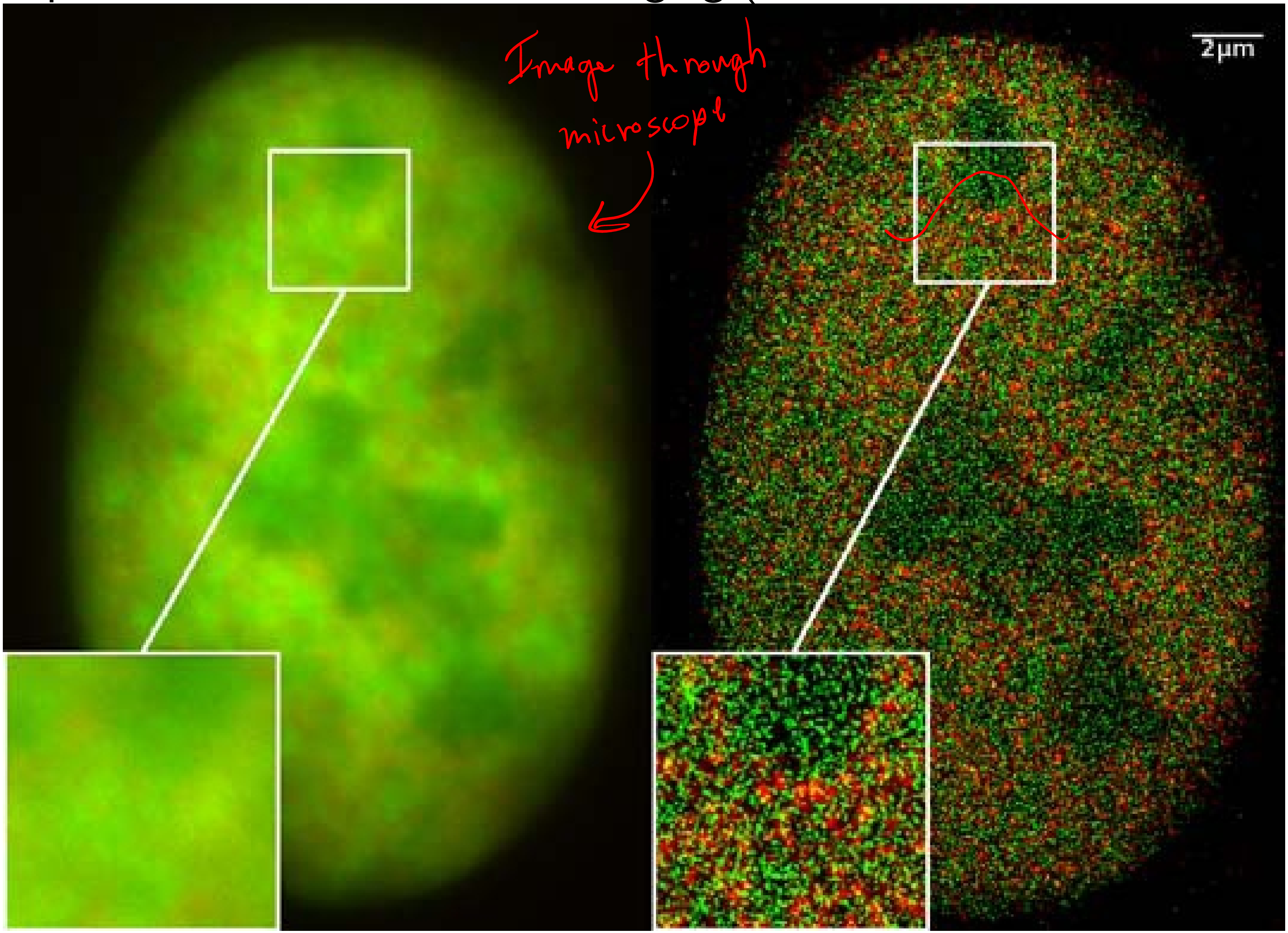
Hubble flawed mirror deconvolution (correction for spherical aberration)



source: www.wikipedia.org

PSF examples

Super-resolution fluorescence imaging (STORM, STED, PALM, ...)



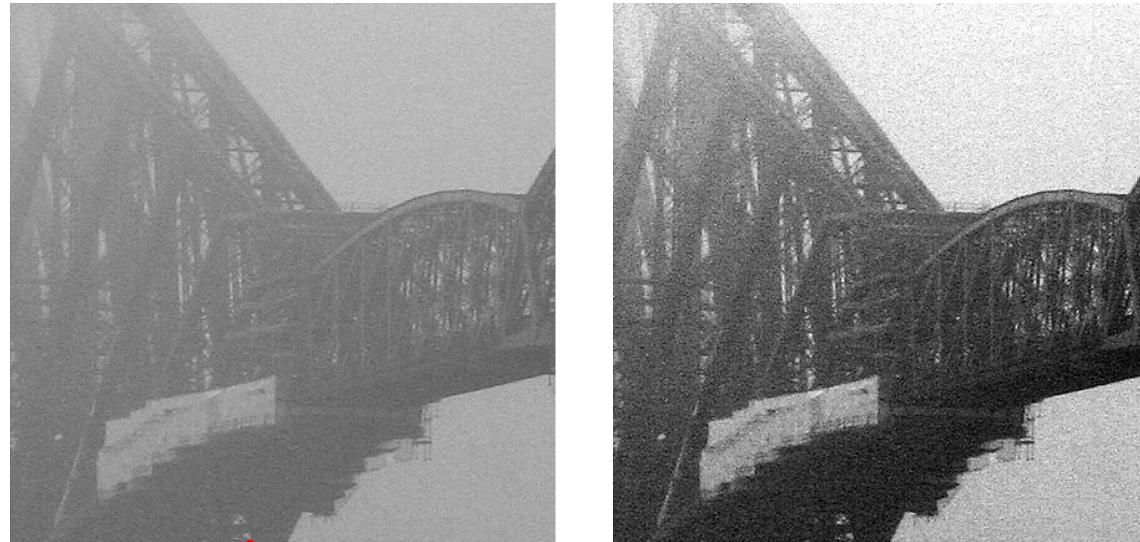
source: www.wikipedia.org

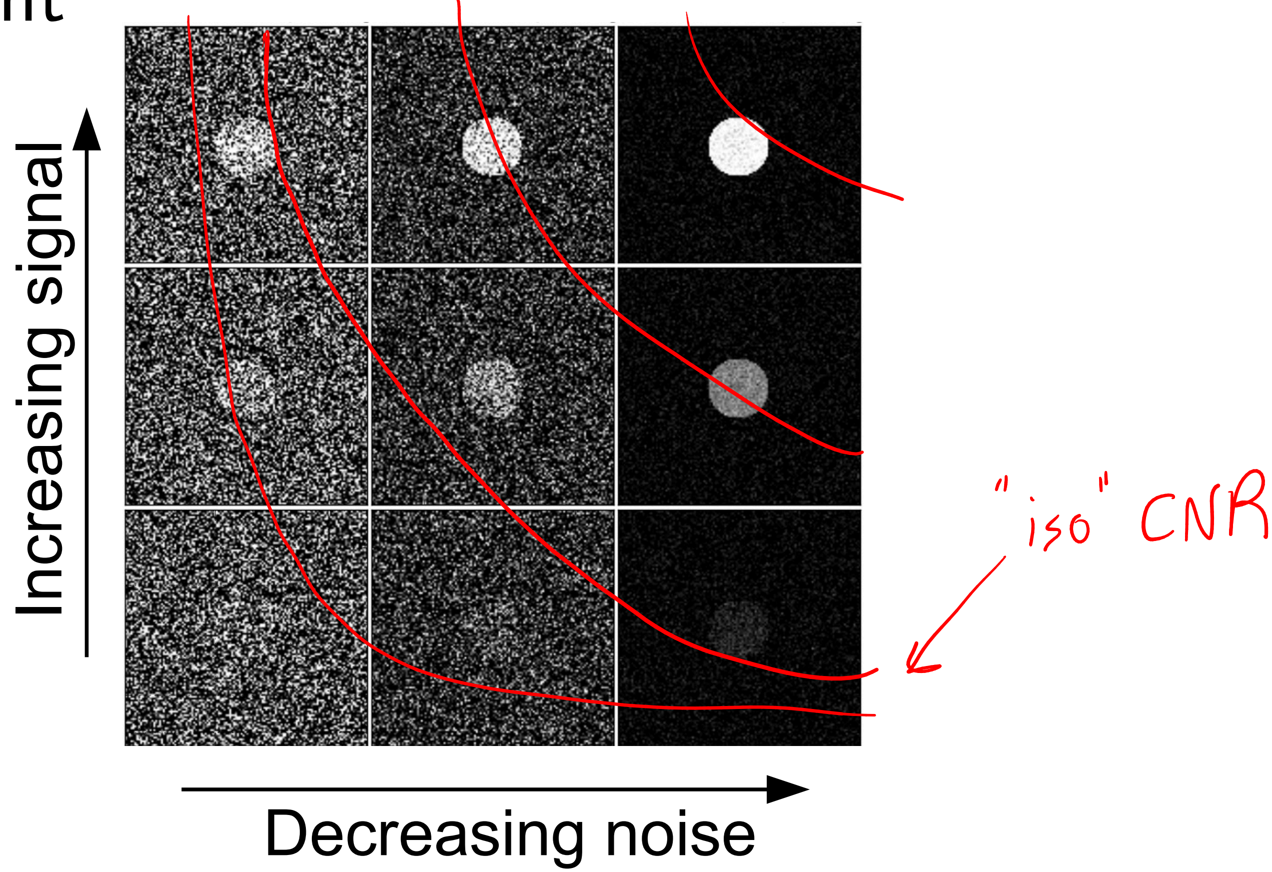
Contrast and noise

- Intensity operation: higher contrast, higher noise

CNR

- Contrast-to-noise remains constant





Random variables

- random variable, sample space

  X $\qquad\qquad$ Ω

  probability of measuring x : $p(x) \leq 1$

  $$p(\Omega) = 1$$

- probability density function "PDF"

  $$p(a < x < b) = \int_a^b \rho(x)\,dx$$

  probability density $\qquad \int_\Omega \rho(x)\,dx = 1$

- expectation value

  $$E[f(x)] = \langle f \rangle = \int_\Omega f(x)\rho(x)\,dx$$

  special case:

  $$E[x] = \langle x \rangle = \mu = \int x\,\rho(x)\,dx$$

  "mean"

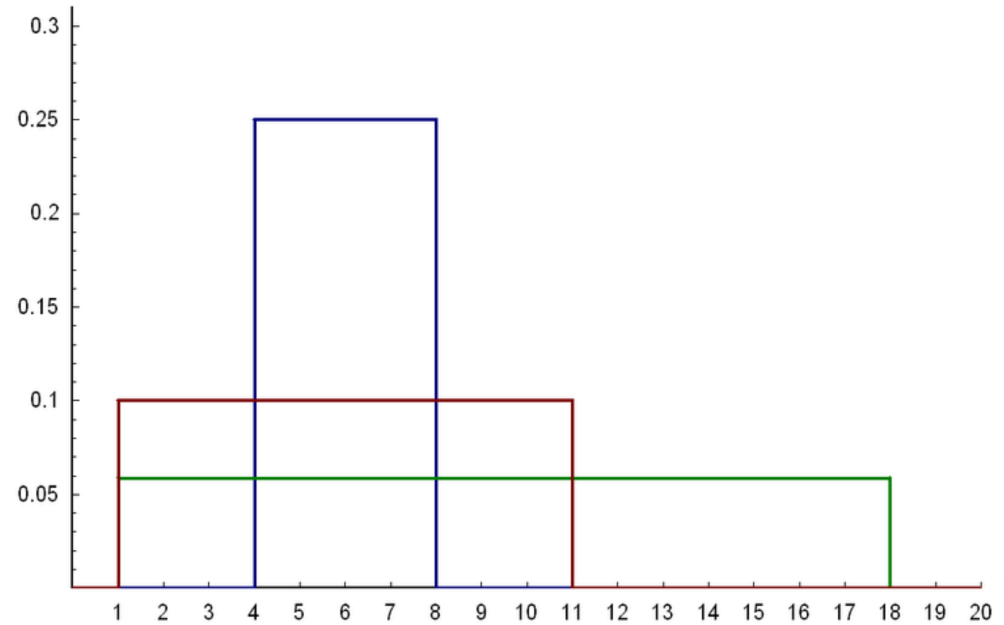
- variance

  $$\mathrm{var}(x) = V[x] = E\left[(x - E[x])^2\right] = \langle (x - \langle x \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

Uniform distribution

- probability density function

$$\rho(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$



- expectation value

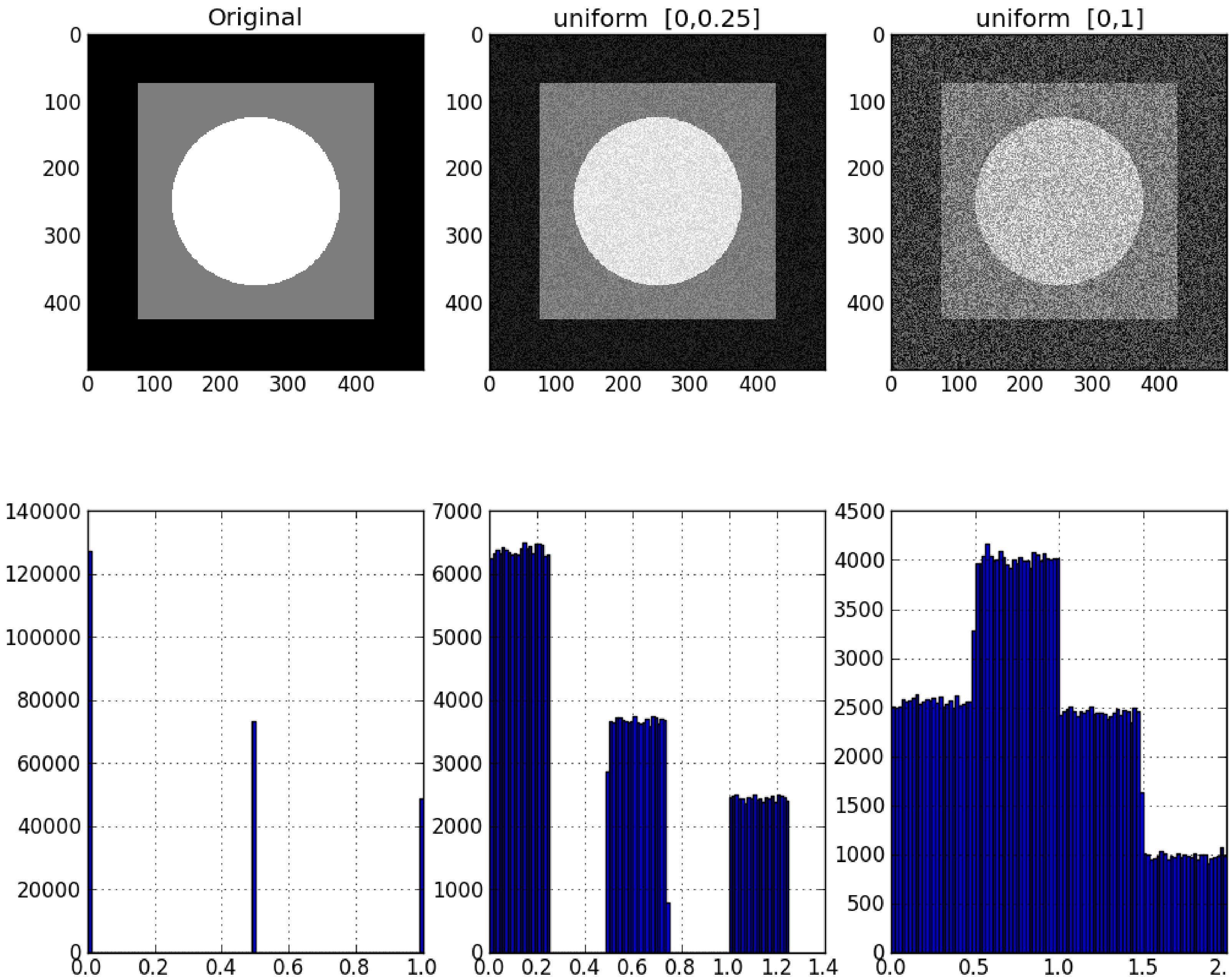
$$\langle x \rangle = \frac{1}{2}(a+b)$$

- variance

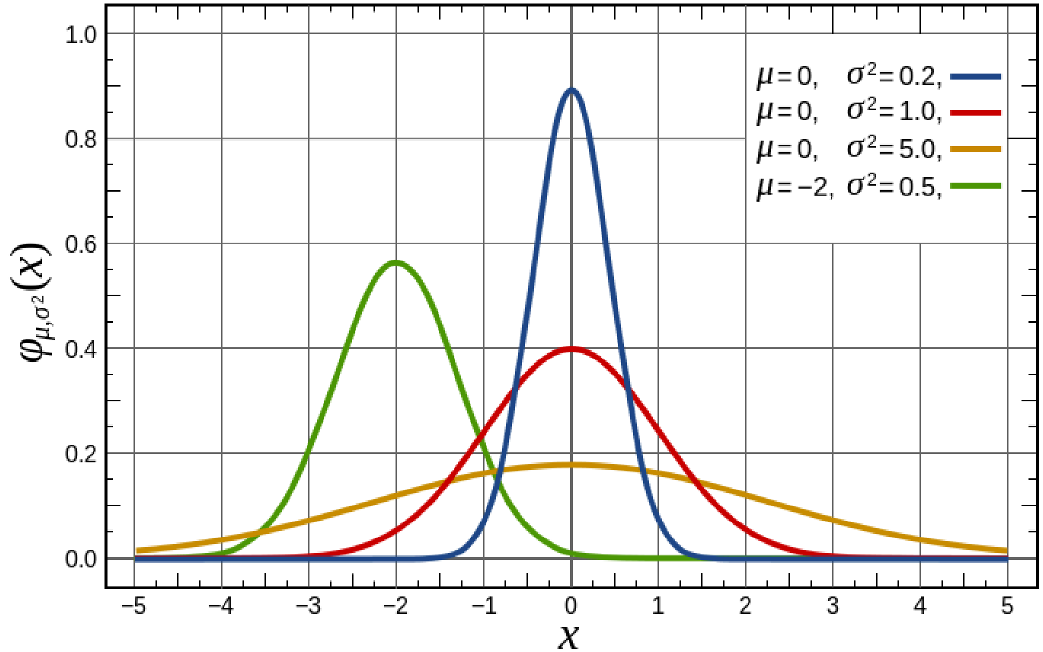
$$\operatorname{var} x = \frac{(b-a)^2}{12}$$

- occurrence not very common, but useful to construct other probability distributions

Uniform distribution



Gaussian distribution



- probability density function

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- expectation value

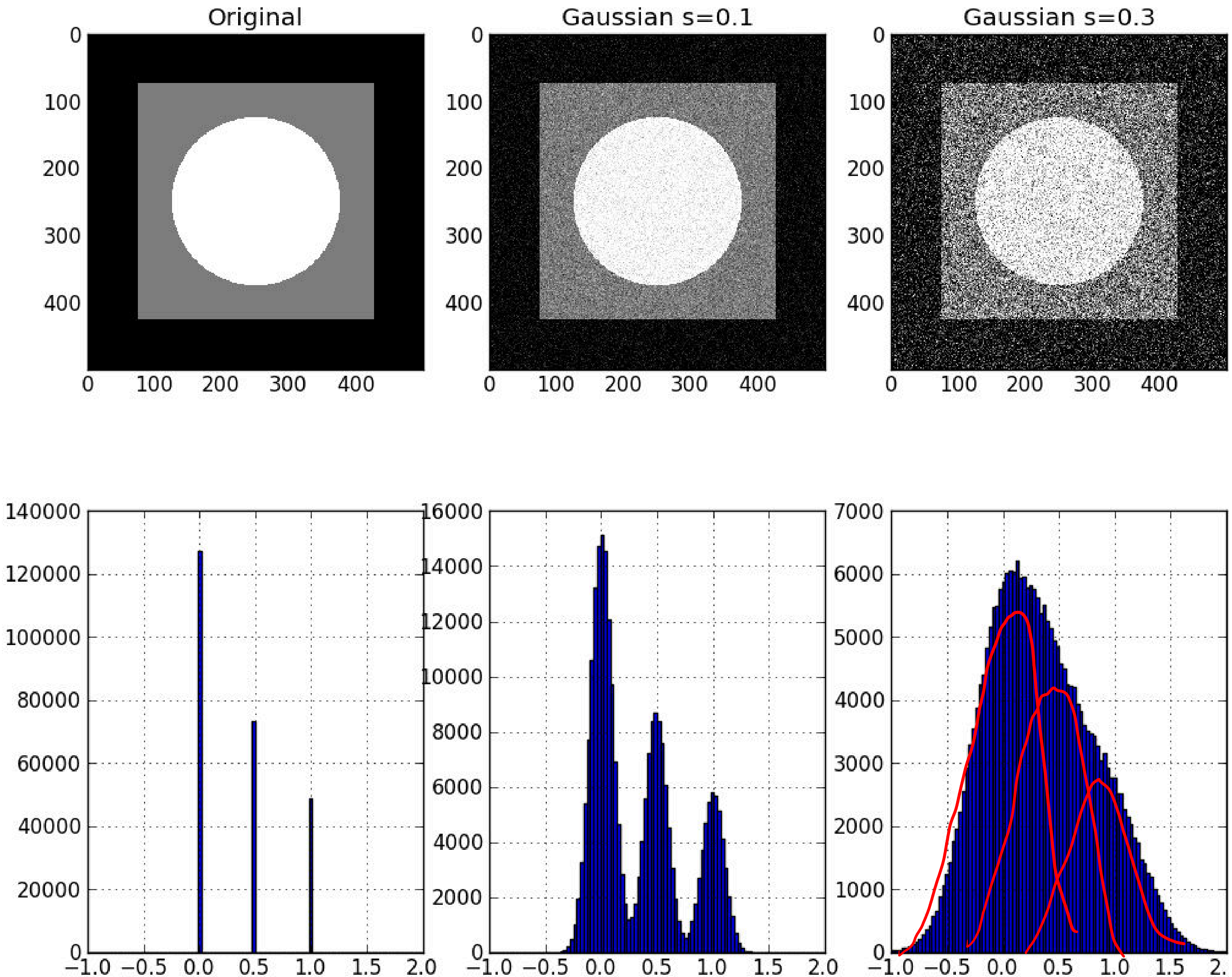
$$\langle x \rangle = \mu$$

- variance

$$\mathrm{var}\, x = \sigma^2$$

- occurrence very common (central limit theorem)

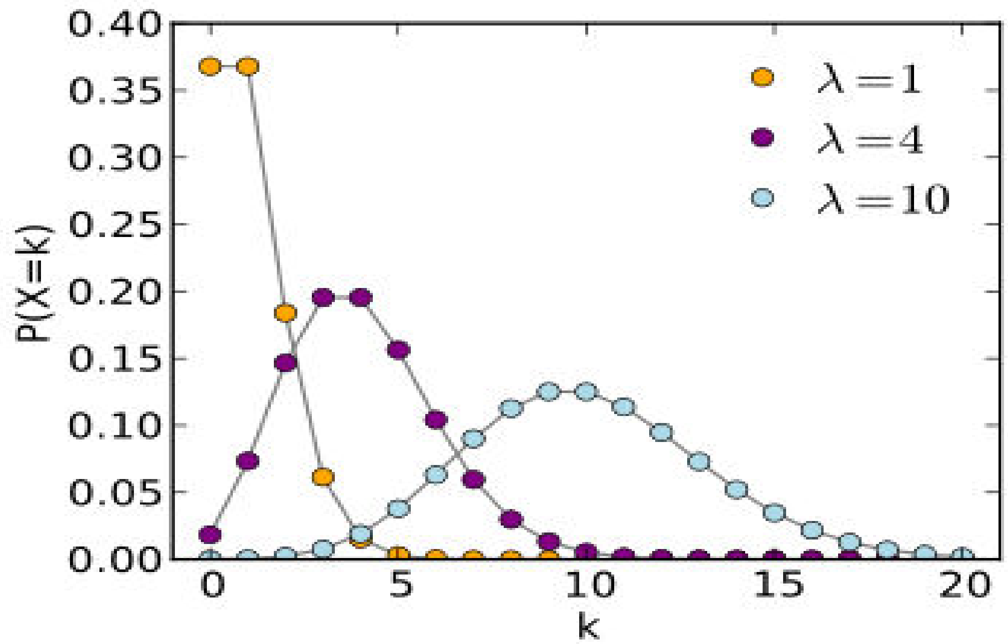
Gaussian distribution



Poisson distribution

- probability mass function

$$p(n) = \frac{1}{n!} \lambda^n e^{-\lambda}$$



S/N ratio $\frac{E[n]}{\sqrt{\mathrm{var}\, n}} = \frac{\lambda}{\sqrt{\lambda}} = \sqrt{\lambda}$

- expectation value

$$E[n] = \lambda$$

- variance

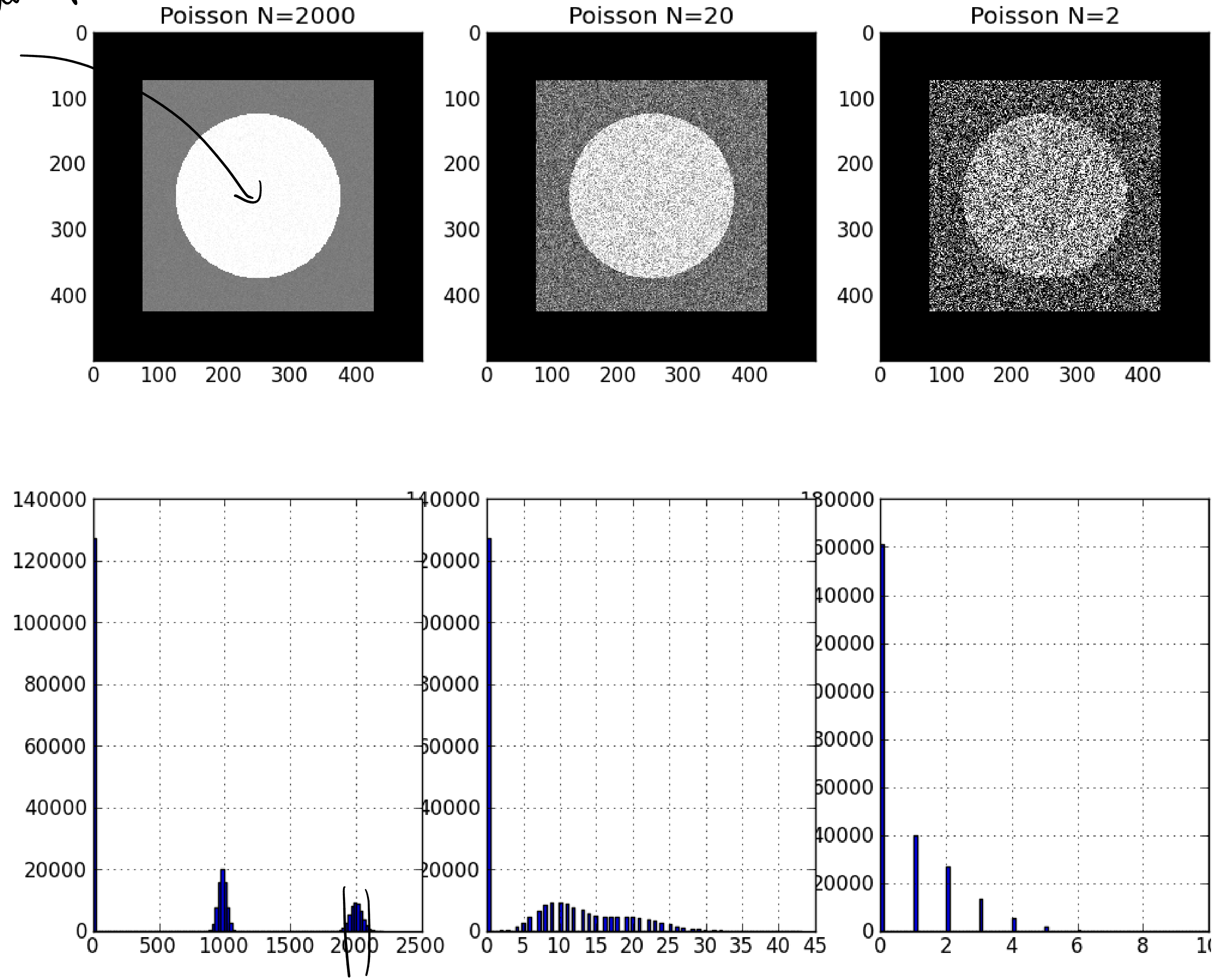
$$\mathrm{var}\, n = \lambda$$

- occurrence counting process (photons, electrons)

"shot noise"

Poisson distribution

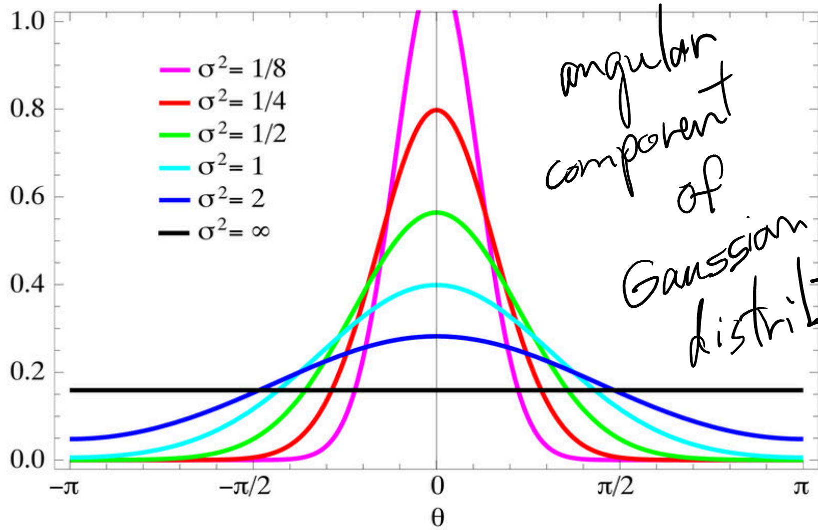
in average 2000 counts



Poisson distribution

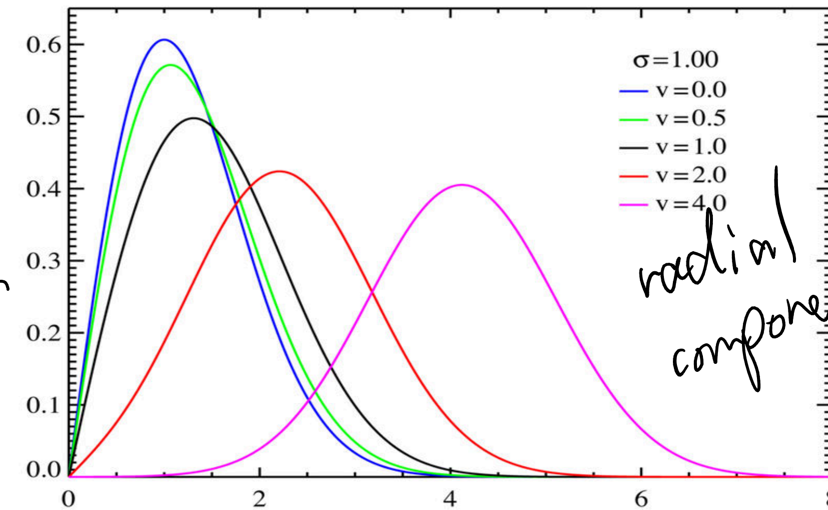
Many other distributions

Wrapped normal distribution



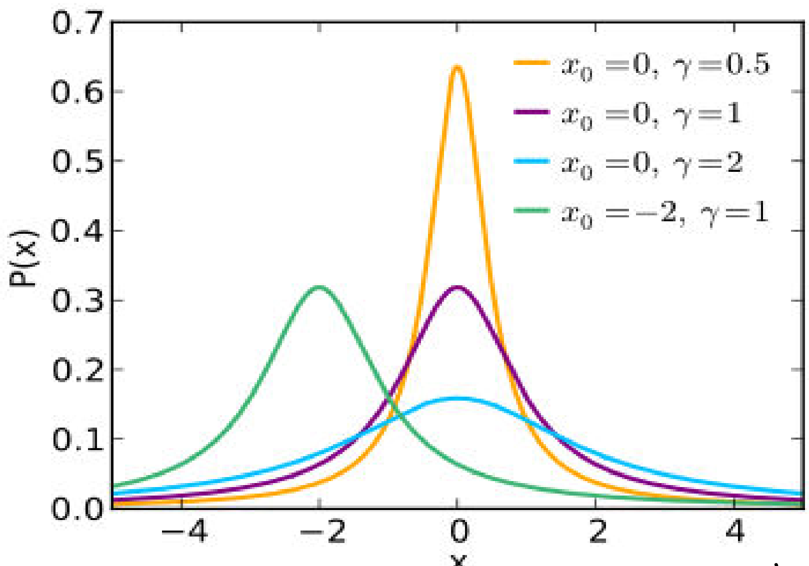
angular component of Gaussian distribution

Rice distribution

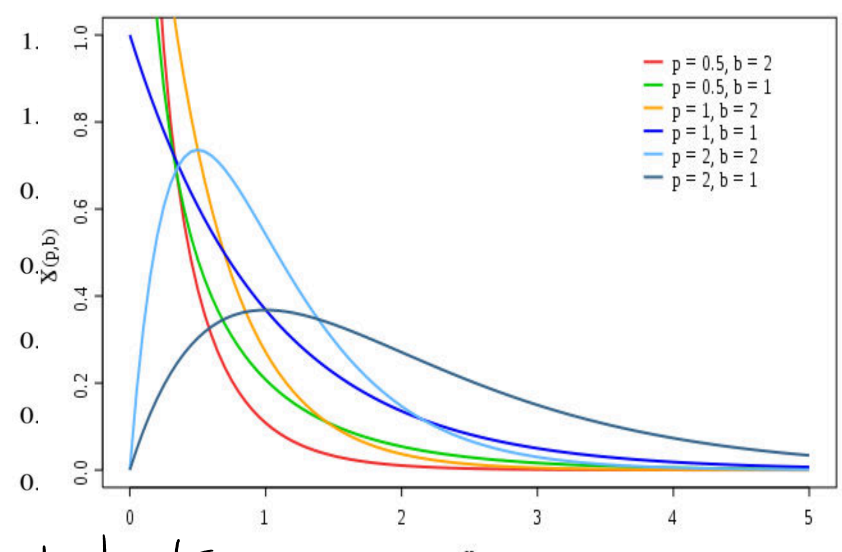


radial component

Lorentz distribution



Gamma distribution



long tail distributions

Detector noise (CCD)

- Various sources:
  - shot noise (photon statistics, Poisson)
  - dark current (thermal electronic fluctuations in semiconductor, Poisson)
  - readout noise (fluctuations during amplification and digitization, Gauss)
  - many other imperfections ... ← systematic errors

(same exposure time)

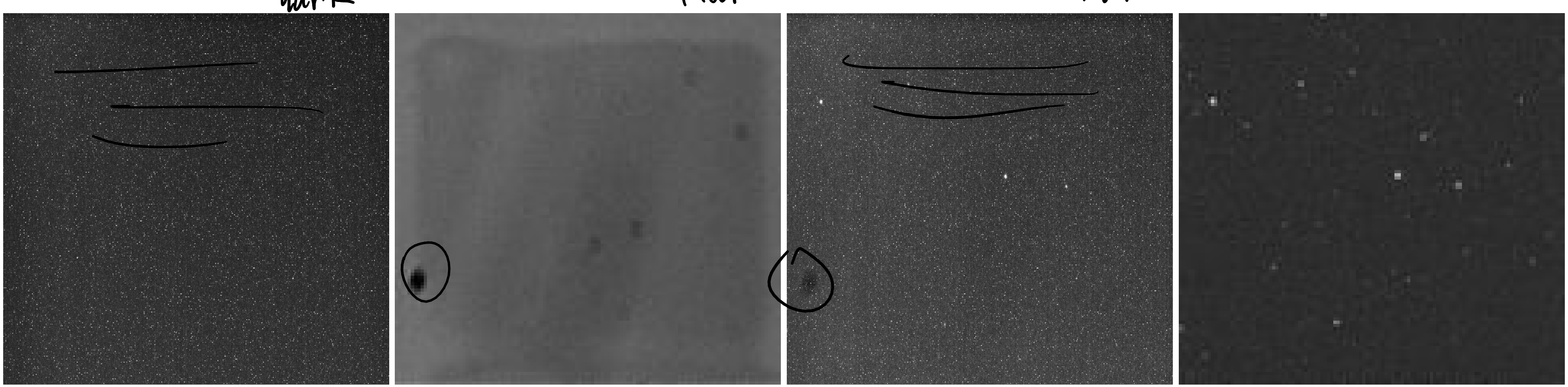
- dark frame measures detector noise, hot pixels, dead pixels

$$\frac{\text{raw} - \text{dark}}{\text{flat} - \text{dark}}$$

- bright frame measures gain differences and imperfections (dust, etc)

flat field uniform illumination

dark frame dark | *bright frame* flat-dark | *raw image* raw | *calibrated image*



source: H. Raab, Johannes-Kepler-Observatory, Linz

Correlation & Convolution

Convolution: $f * g = \int_{-\infty}^{\infty} f(x')\, g(x-x')\, dx'$

Convolution theorem: $\mathcal{F}\{f*g\} = F \cdot G$

if f is real,
$f^* = f$
$\Downarrow$
$\int F e^{2\pi i u x}$
$= \int F^* e^{-2\pi i u x}$

Correlation $f \circledast g = \int_{-\infty}^{\infty} f^*(x')\, g(x+x')\, dx'$

$$f \circledast g = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} F^*(u) e^{-2\pi i u x'} du \int_{-\infty}^{\infty} G(u') e^{2\pi i u'(x+x')} du'$$

$$= \iint_{-\infty}^{\infty} du\, du'\, F^*(u)\, G(u')\, e^{2\pi i u' x} \underbrace{\int_{-\infty}^{\infty} dx'\, e^{2\pi i x'(u'-u)}}_{\delta(u'-u)}$$

$$\boxed{\mathcal{F}\{f \circledast g\} = F^* G}$$

$$f(x) = \int_{-\infty}^{\infty} F(u)\, e^{2\pi i u x}\, du$$

$$f^*(x) = \int_{-\infty}^{\infty} F^*(u)\, e^{-2\pi i u x}\, du$$

if $f = f^* \Rightarrow F(u) = F^*(-u)$

Noise power spectrum

- power spectrum of pure noise image

$$\int_{-\infty}^{\infty} du\, F^*(u) \underbrace{\int_{-\infty}^{\infty} G(u')\, e^{2\pi i u' x}\, \delta(u'-u)}_{G(u)\, e^{2\pi i u x}}$$

$$= \int_{-\infty}^{\infty} F^*(u)\, G(u)\, e^{2\pi i u x} = \mathcal{F}^{-1}\{F^* G\}$$

- connection to auto-correlation

$n(x,y)$ noise "function" (actually a multivariate random variable)

$N(u,v)$: F.T. $\quad NPS = \langle |N(u,v)|^2 \rangle$ ← expectation value

noise power spectrum

$$|N(u,v)|^2 = N^*(u,v)\, N(u,v)$$

$$\Rightarrow \mathcal{F}\{|N(u,v)|^2\} = n \circledast n \quad = \text{auto-correlation}$$

Procedure for noise charterization:

1.) measure multiple realizations of the random variable $n(x,y)$
($\hookrightarrow$ take many dark frames) $n_i(x,y)$ $0 < i \leq M$

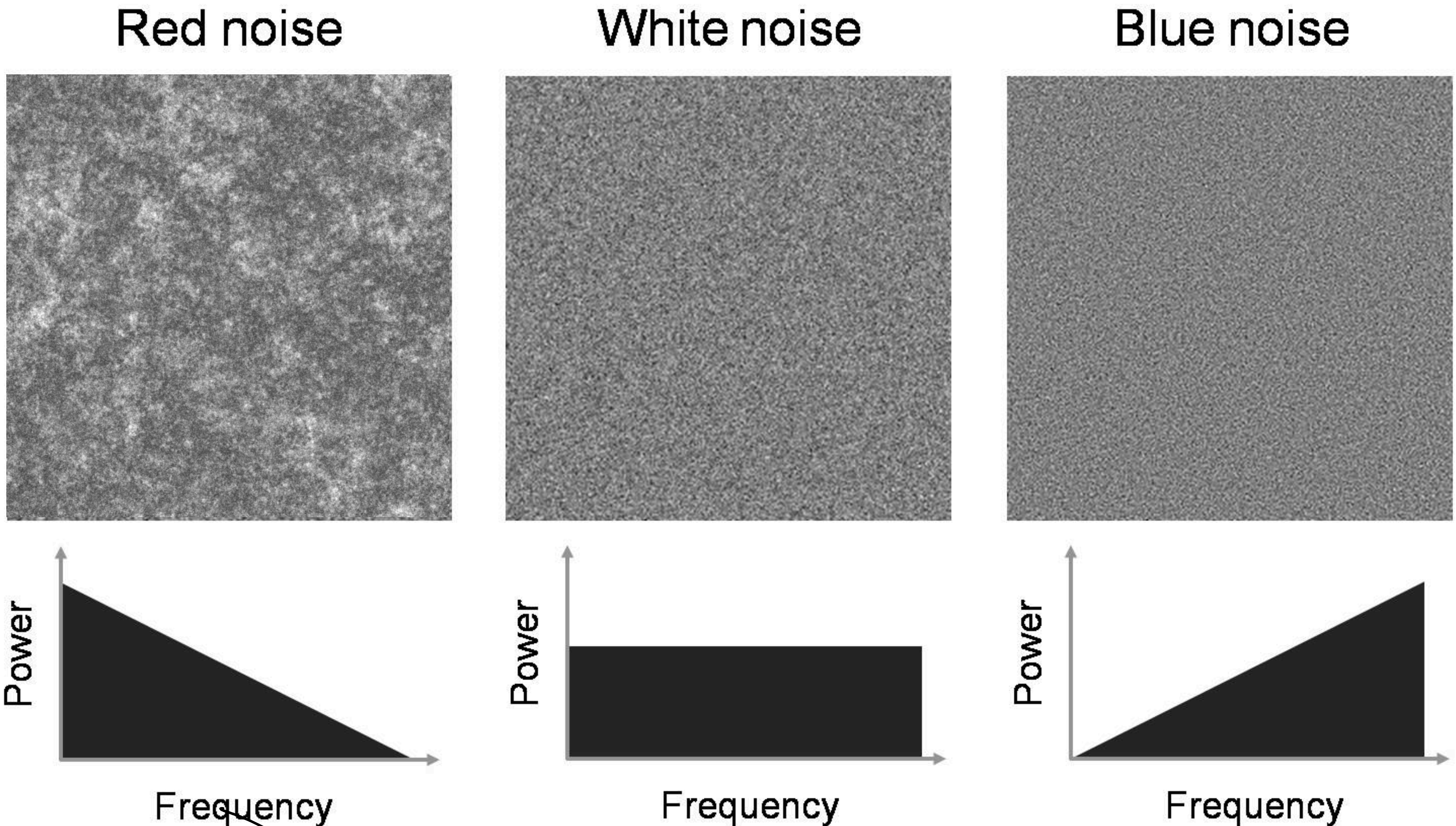
2) $N_i(u,v) = \mathcal{F}\{n_i(x,y)\}$

3) $\langle |N(u,v)|^2 \rangle \simeq \frac{1}{M} \sum_i |N_i(u,v)|^2$

$\hookrightarrow$ estimate of NPS

4) $\mathcal{F}^{-1}\{NPS\} \rightarrow$ estimate of noise autocorrelation

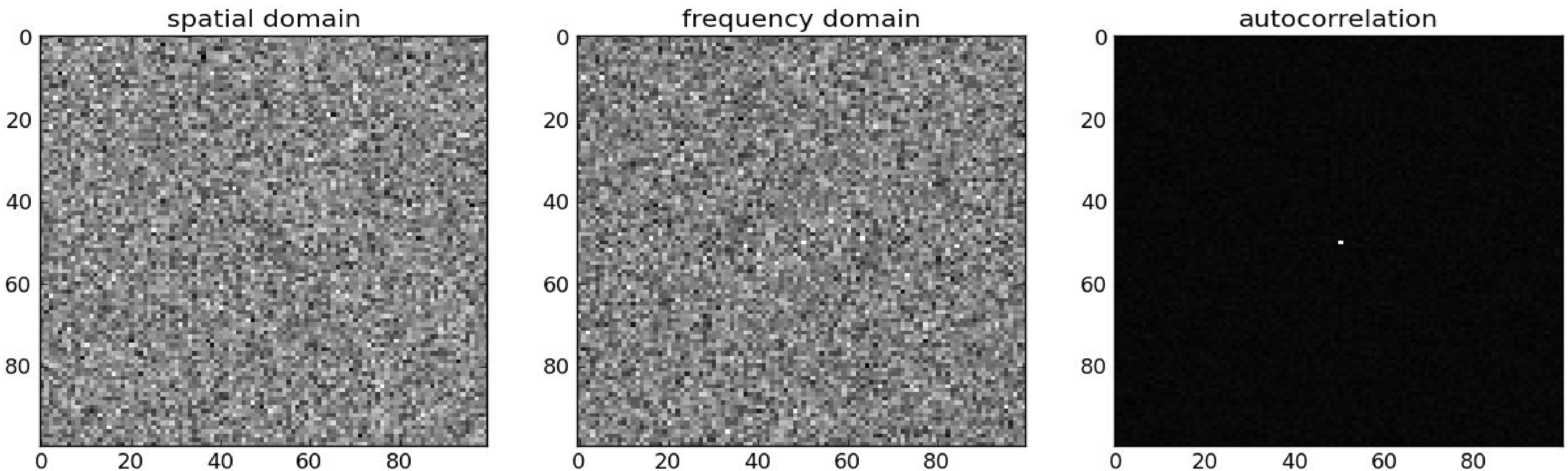
Noise power spectrum



NPS

source: http://scien.stanford.edu/pages/labsite/2008/psych221/projects/08/AdamWang/project_report.htm

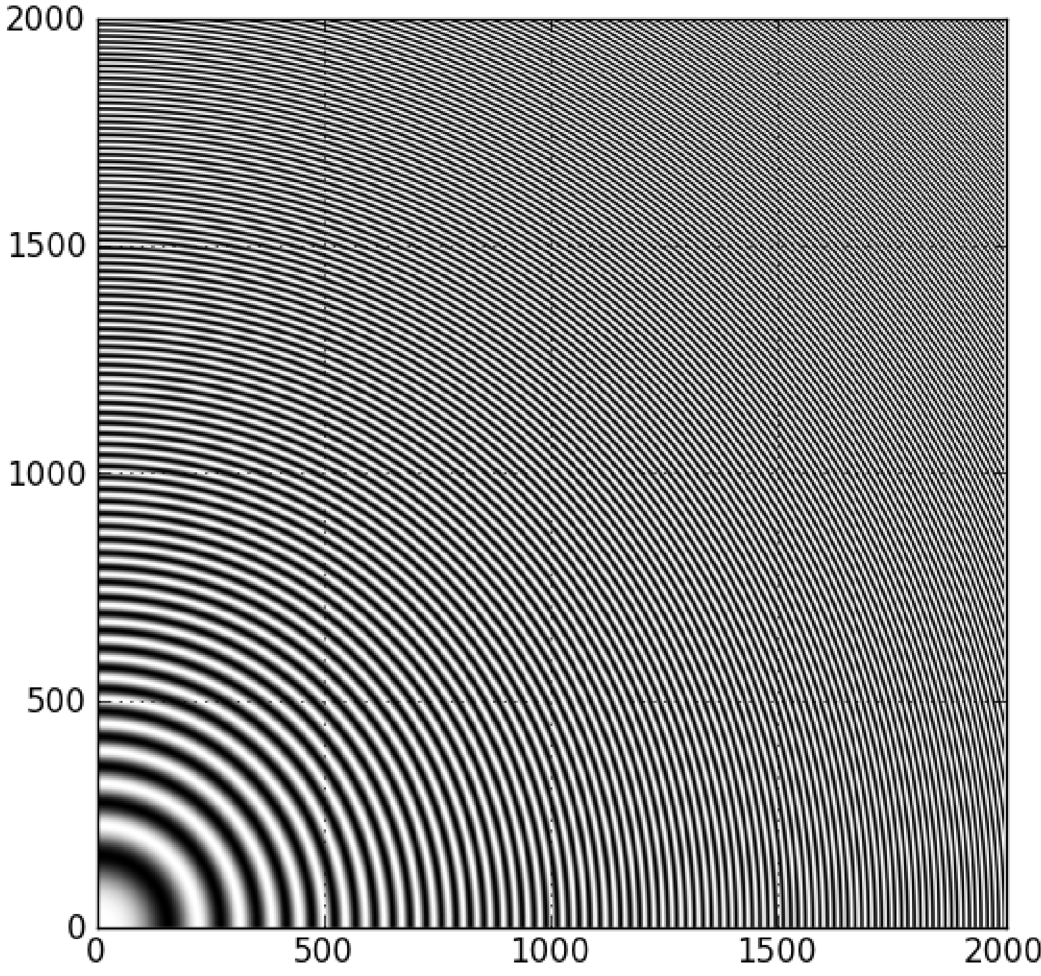
White noise

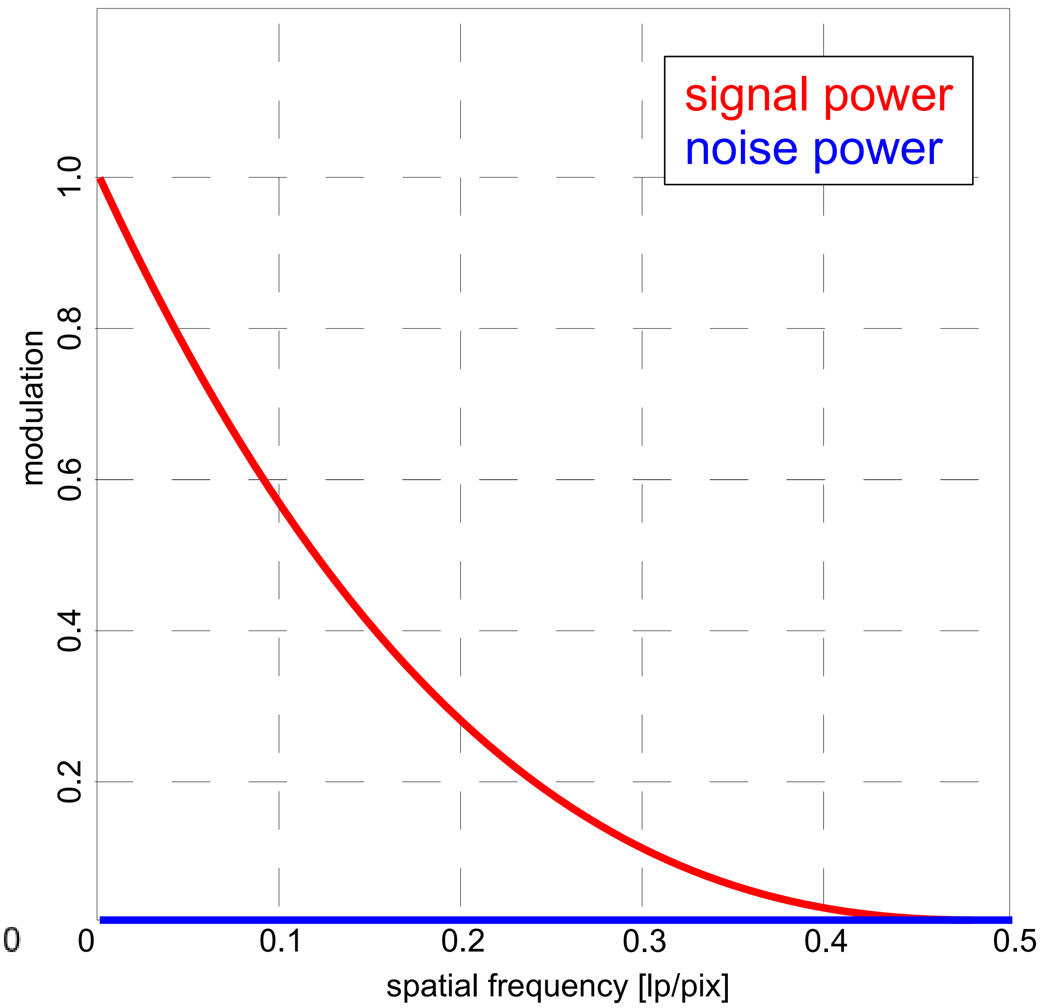


- white noise in spatial domain equals white noise in frequency domain
- white noise is perfectly uncorrelated
- all other types of noise are correlated to some degree
- white noise is an idealization

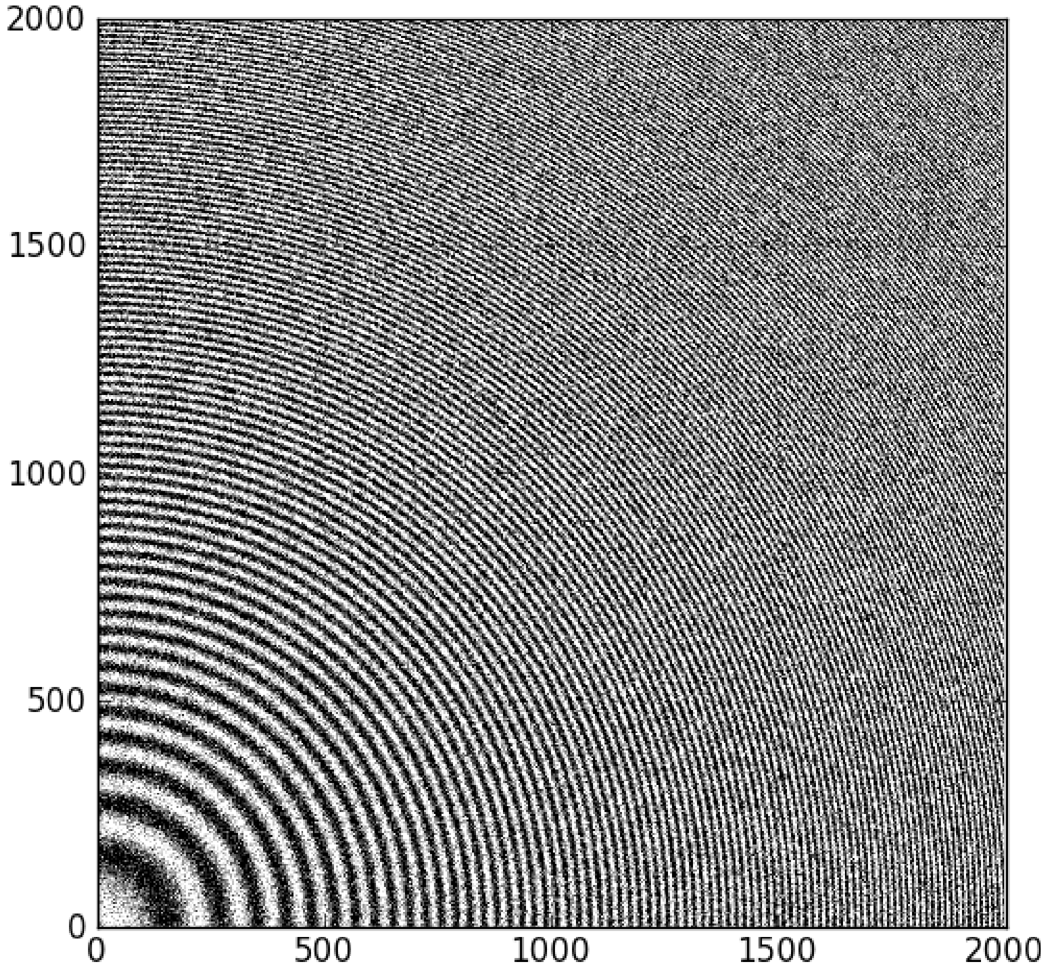
NPS is uniform over whole frequency range

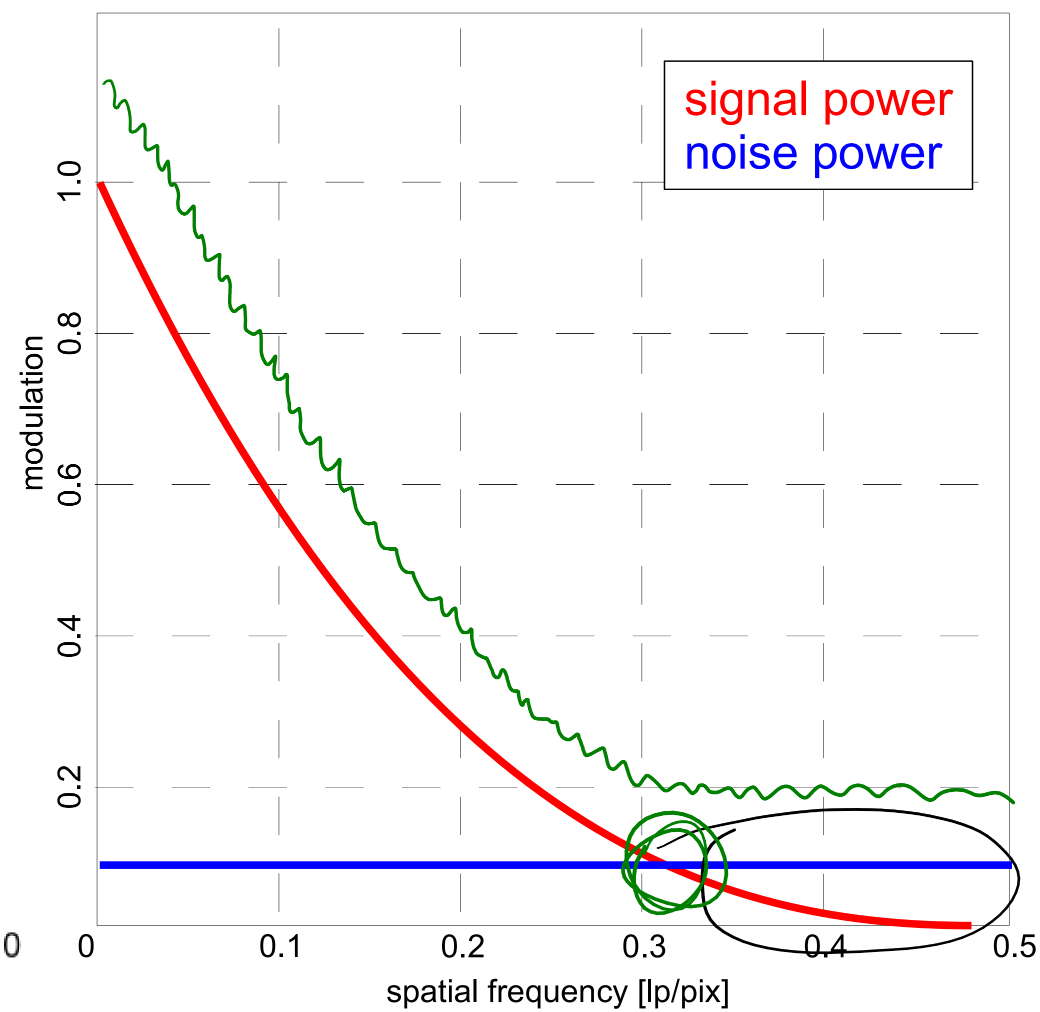
Signal power vs. noise power



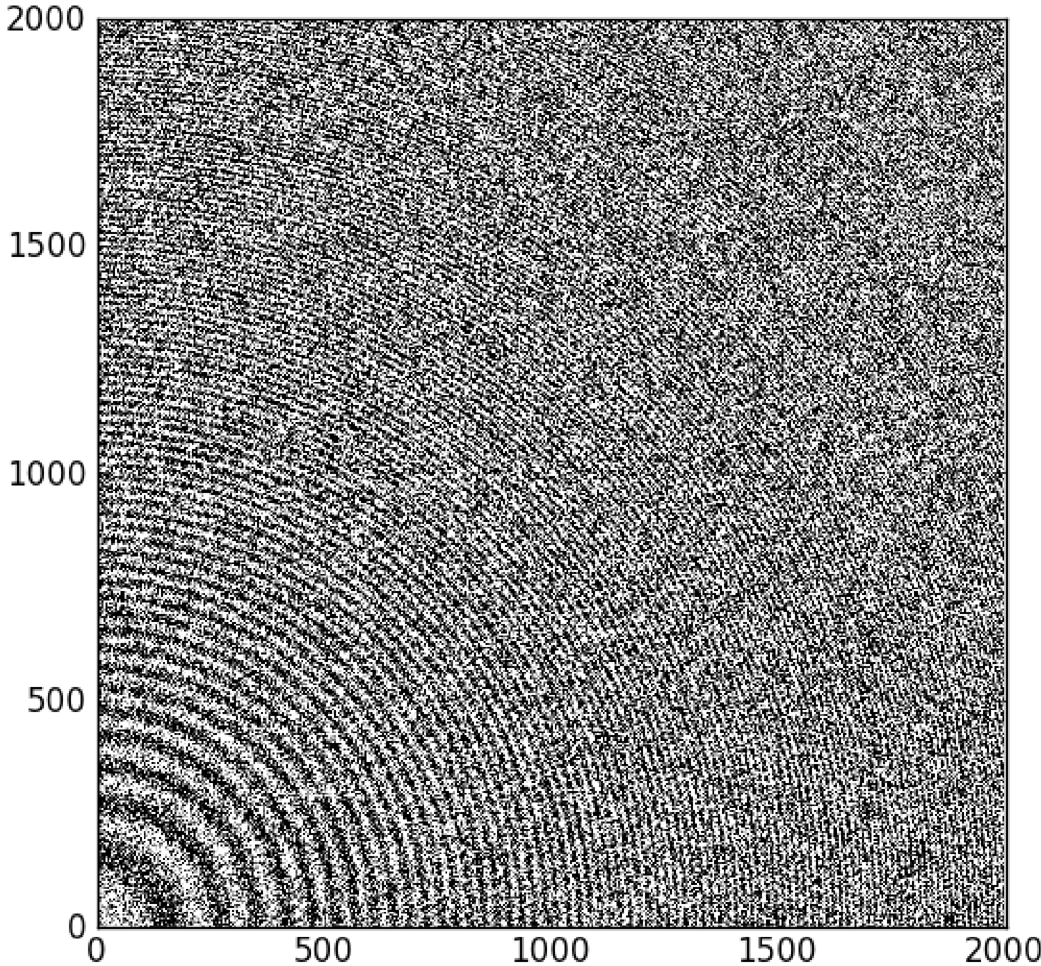


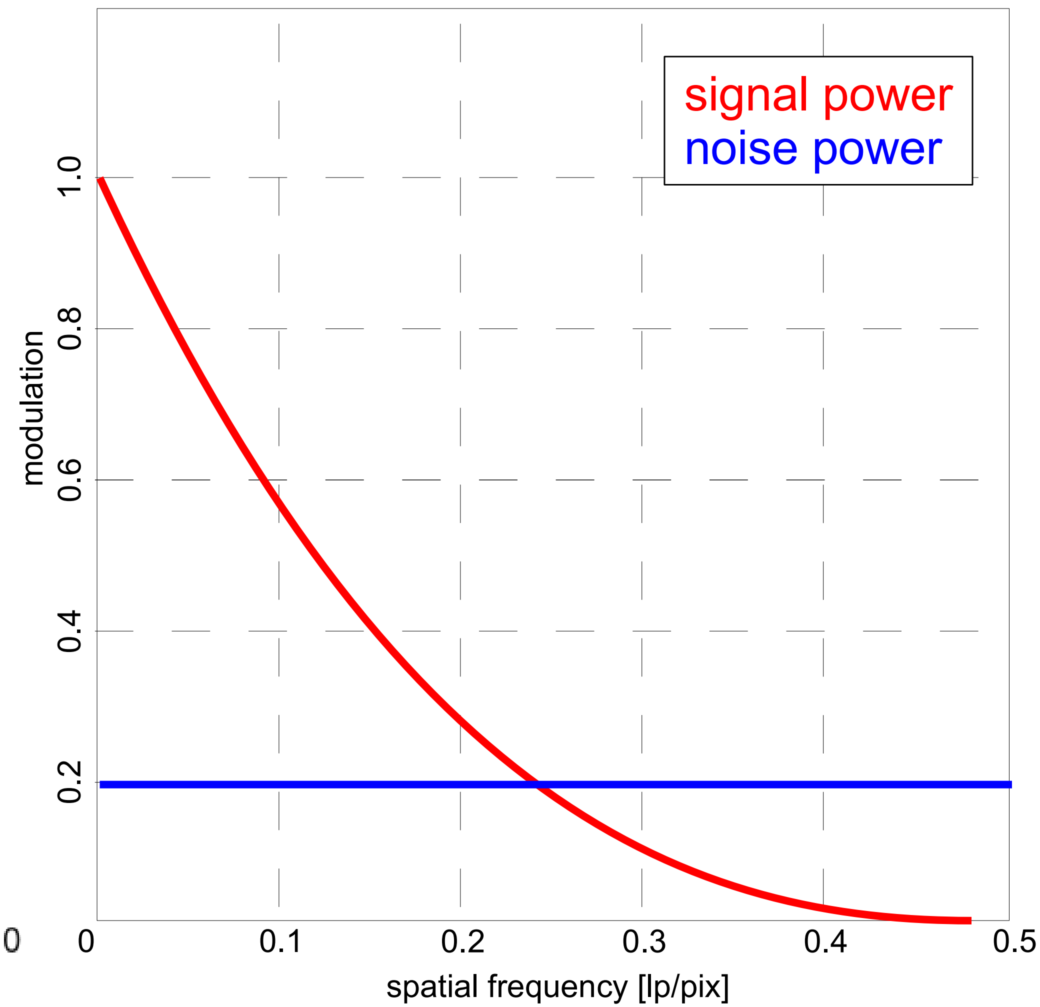
Signal power vs. noise power



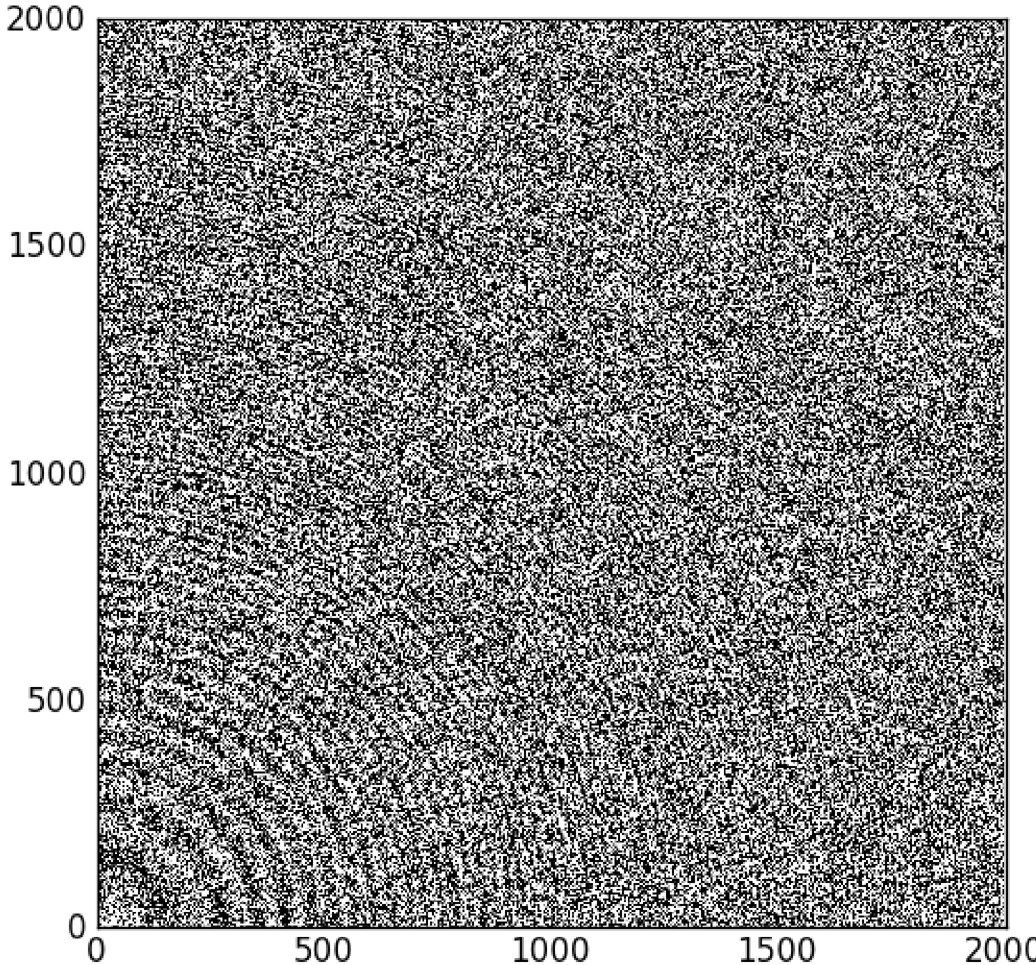


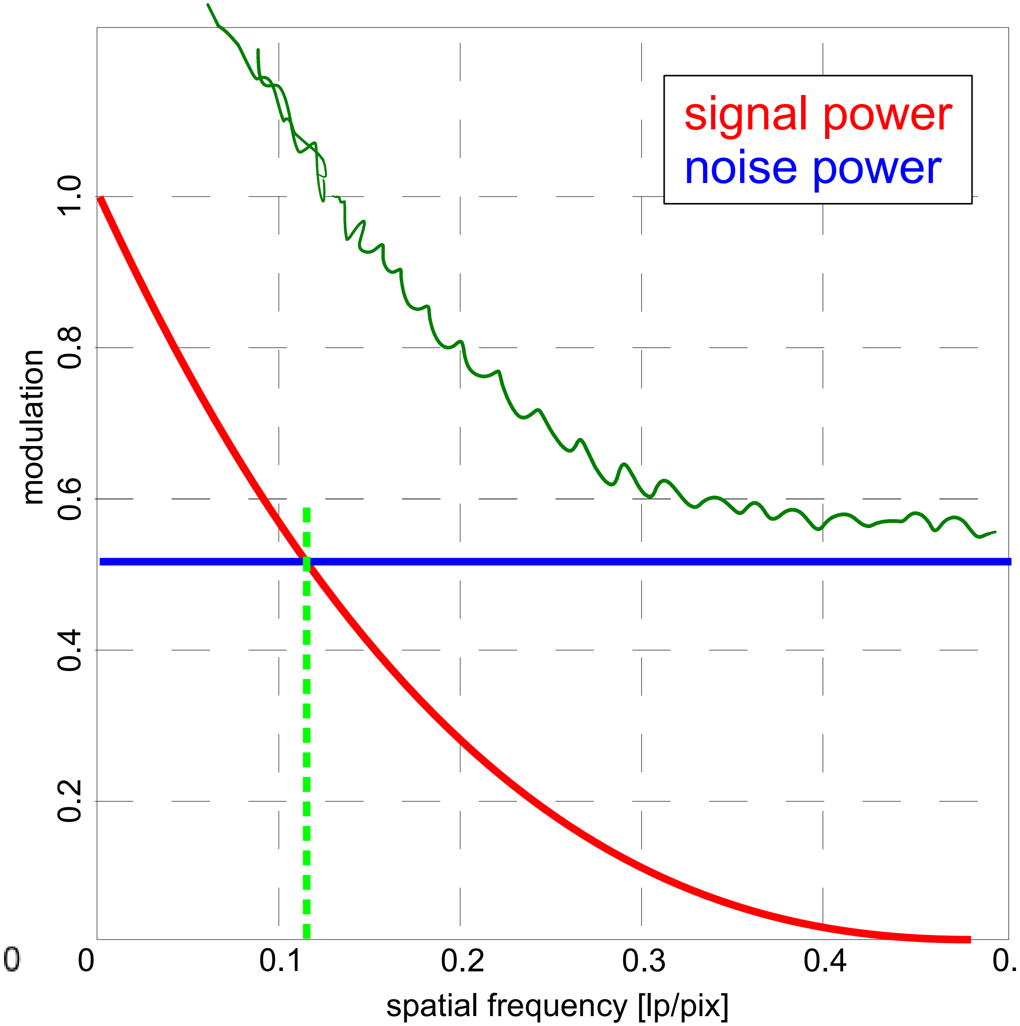
Signal power vs. noise power





Signal power vs. noise power

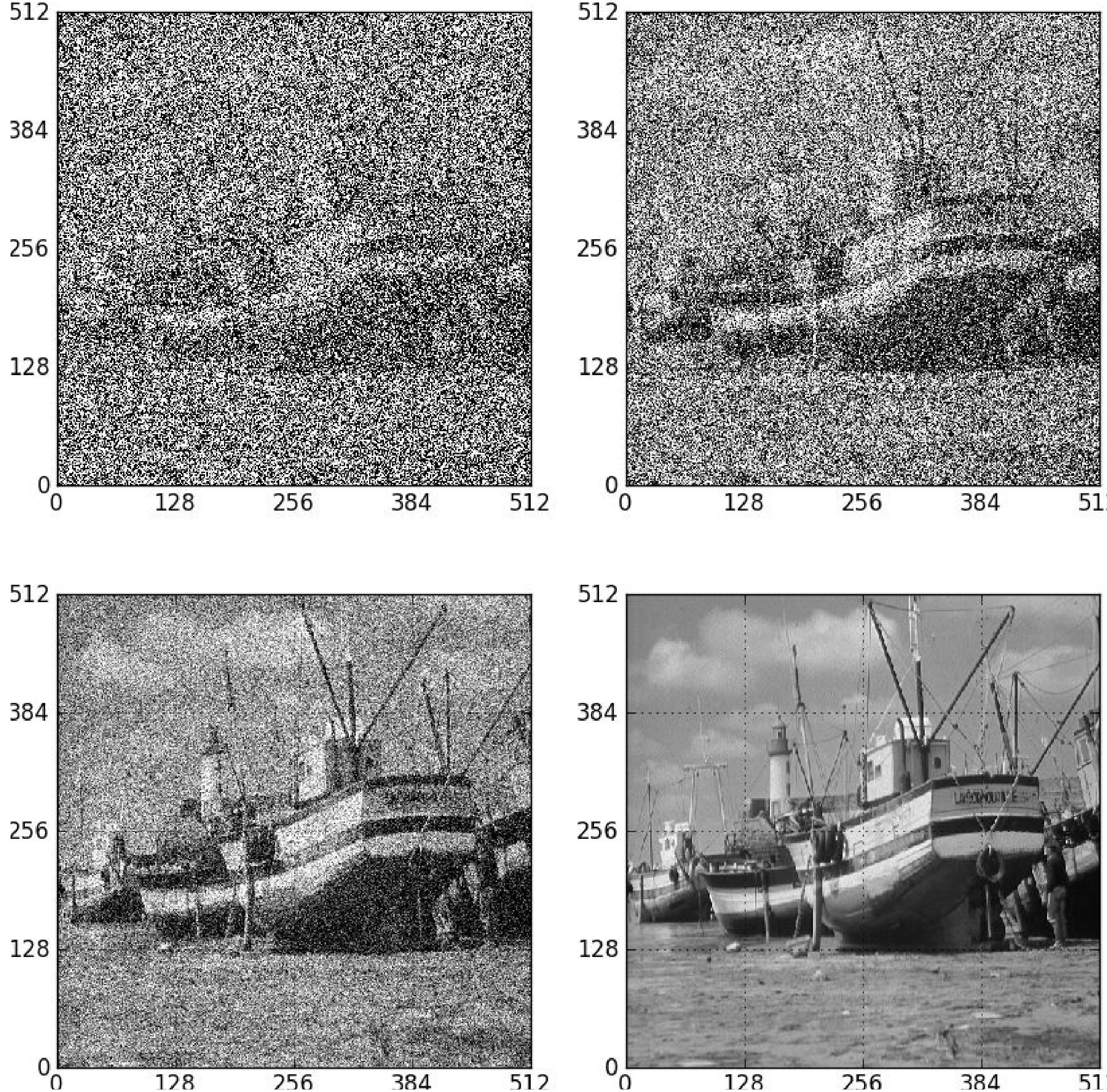




- Noise power exceeds signal power for high frequencies
- Small scale image details are lost in noise first

Gaussian filter removes more noise than signal → can be used for denoising

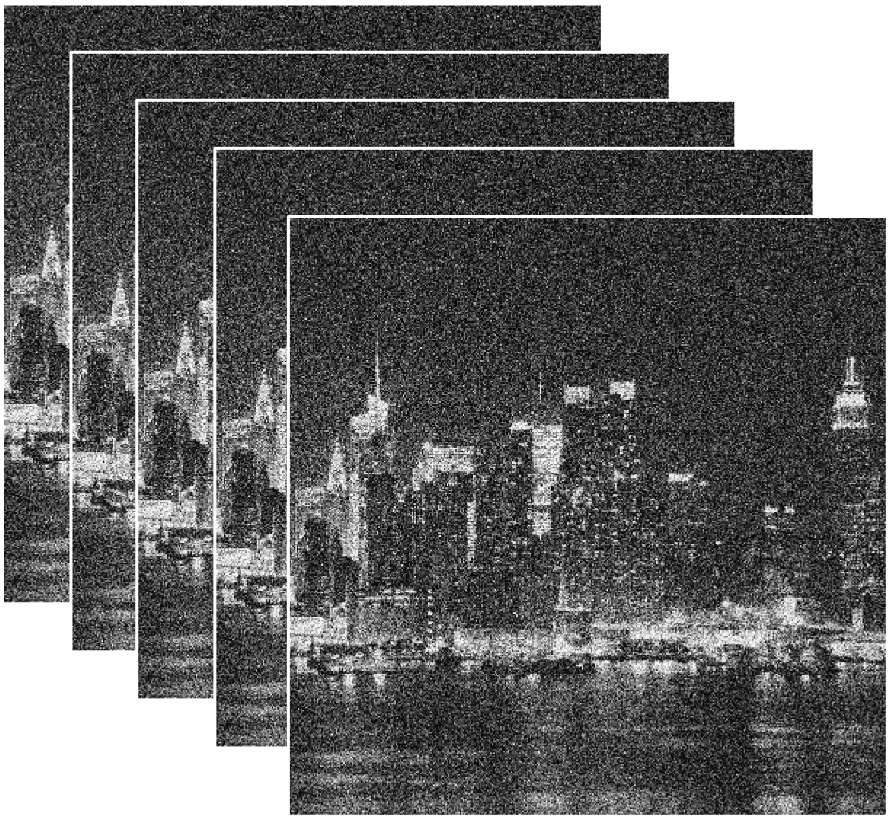
Signal power vs. noise power



Noise reduction by averaging

- Average multiple images

signal amplified, noise cancelling out



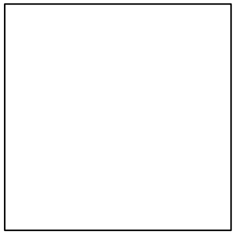
- requirement: additive noise, zero mean

Denoising by linear filtering

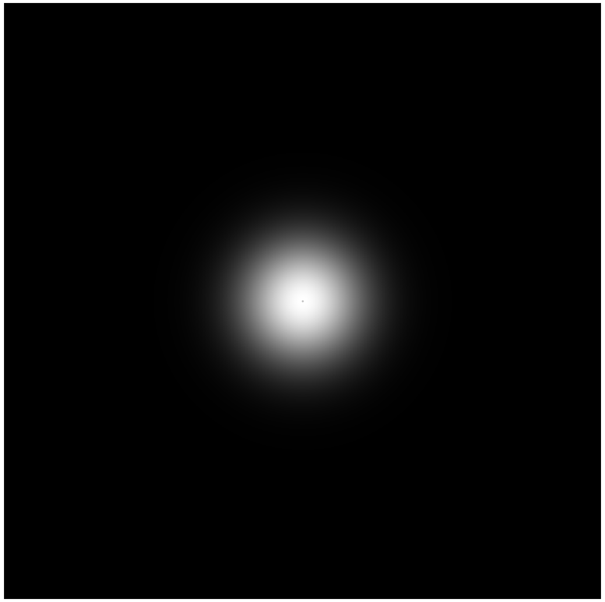
- use spatial convolution or frequency filtering to reduce noise
- noise reduction possible, but at cost of sharpness
- trade-off between noise reduction and resolution
- need fancier methods

original

convolution kernel

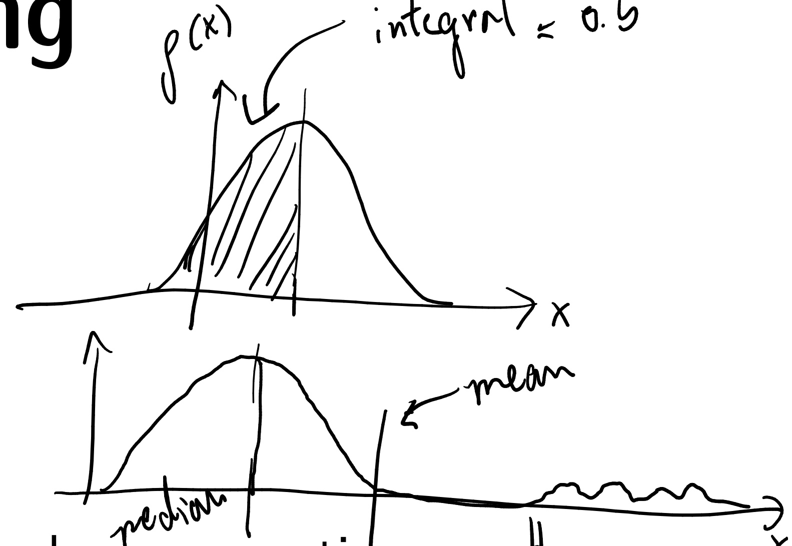


frequency filter



Resulting image

Median filtering



- Use median as estimator for fat tail distributions

median is less affected by outliers than mean. Especially useful for long-tail distribution

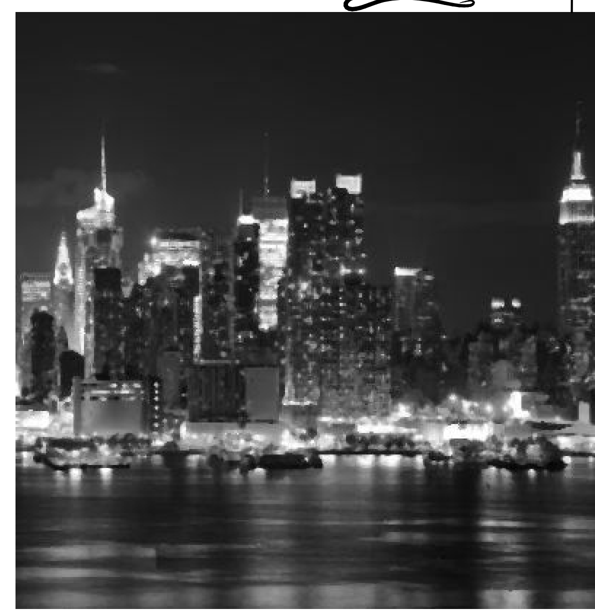
- less sensitive to outliers in pixel ensemble, better edge preservation

for each pixel, replace with median of →

Salt and pepper noise

Gauss sigma=1 pixel

Median 1 pixel



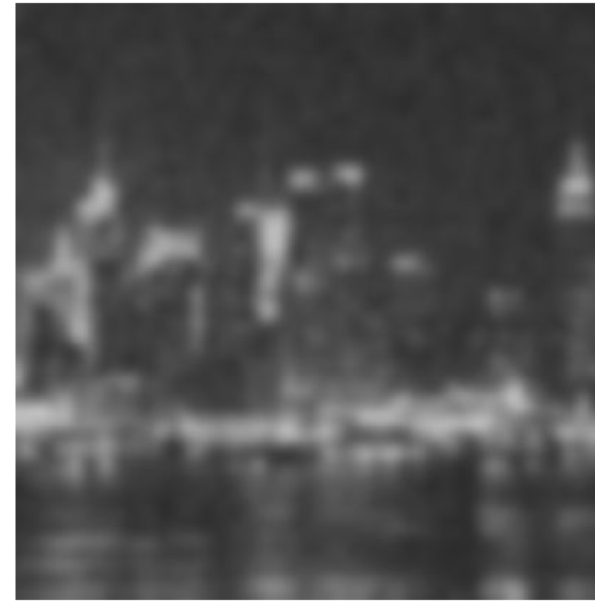
"zingers"

Median filtering

1x Gauss

2x Gauss

5x Gauss



1x Median

2x Median

5x Median

Common abbreviations

Abbreviation	Name	Definition
IRF	Impulse response function	Linear operator map of delta function
PSF	Point spread function	Image of point object (optical IRF)
OTF	Optical transfer function	Fourier transform of PSF
PTF	Phase transfer function	Phase part of OTF
MTF	Modulation transfer function	Amplitude of OTF
CTF	Contrast transfer function	MTF for non-sinusoidal objects
PDF	Probability density function	Probability distribution for a given random variable
SPS	Signal power spectrum	Amplitude squared of signal F.T.
NPS	Noise power spectrum	Amplitude squared of noise F.T.
SNR	Signal to noise ratio	Mean signal / mean noise
CNR	Contrast to noise ratio	Mean contrast / mean noise