25 Nov

$$\lim_{x\to+\infty} \frac{x^n}{e^x} = 0 \qquad \text{per qualsiasi } n \in \mathbb{N}$$

$$\lim_{x\to+\infty} \frac{x^R}{e^x} = 0 \qquad \forall\ R > 0$$

$$\frac{x^R}{e^x} = \frac{x^R}{x^{[R]+1}} \cdot \frac{x^{[R]+1}}{e^x} = x^{\overbrace{R-[R]-1}^{<0}} \cdot \frac{x^{[R]+1}}{e^x}$$

$$x^{R-[R]-1} \xrightarrow{x\to+\infty} 0 \qquad \frac{x^{[R]+1}}{e^x} \xrightarrow{x\to+\infty} 0$$

gerarchie all'infinito

Le funzioni della forma x^R, $R>0$, ~~sono~~ sono molto più piccole e^x per $x \gg 1$.

$$\lim_{x\to+\infty} \frac{x^\varepsilon}{\lg x} \qquad \varepsilon > 0 \qquad \frac{+\infty}{+\infty}$$

$$\lim_{x\to+\infty} \lg x = \sup\{\lg x : x>0\} = \sup \mathbb{R} = +\infty$$

perchè $\lg : \mathbb{R}_+ \to \mathbb{R}$ e' l'inversa di $e^x : \mathbb{R} \to \mathbb{R}_+$

$\varepsilon > 0$

Qui $\lim_{x\to+\infty} x^\varepsilon = +\infty$ perché

$$x^\varepsilon = e^{\varepsilon \lg x}$$

$$= \lim_{x\to+\infty} e^{\varepsilon \lg x} \underset{y=\varepsilon \lg x}{=} \lim_{y\to+\infty} e^y = +\infty$$

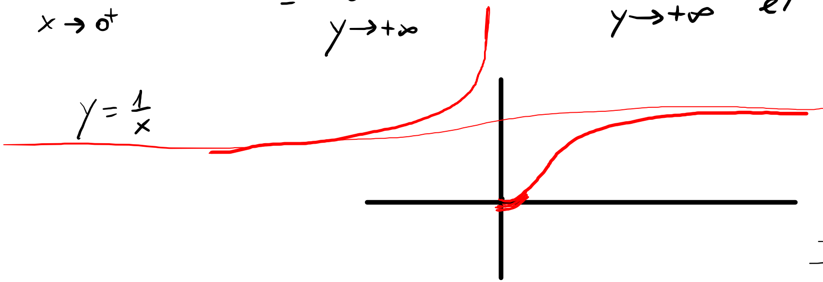
$$\frac{+\infty}{+\infty}$$

$$\lim_{x\to+\infty} \frac{x^\varepsilon}{\lg x} = \lim_{x\to+\infty} \frac{\varepsilon x^{\varepsilon-\cancel{1}}}{x^{-\cancel{1}}} = \lim_{x\to+\infty} \varepsilon x^\varepsilon = +\infty$$

Per qualsiasi $\varepsilon > 0$ la funzione x^ε è molto più grande di $\lg x$ per $x \gg 1$.

$$\lim_{x\to 0^+}\frac{e^{-\frac{1}{x}}}{x^n}=0 \qquad \frac{0}{0}$$

$$\lim_{x\to 0^+} e^{-\frac{1}{x}} = \lim_{y\to +\infty} e^{-y} = \lim_{y\to +\infty}\frac{1}{e^y}=0$$



$$\frac{e^{-\frac{1}{x}}}{x^n}$$

$$\frac{\left(e^{-\frac{1}{x}}\right)'}{(x^n)'} = \frac{e^{-\frac{1}{x}}\left(-\frac{1}{x}\right)'}{n\,x^{n-1}} = \frac{e^{-\frac{1}{x}}\,x^{-2}}{n\,x^{n-1}} =$$

$$\frac{e^{-1/x}}{n x^{n-1}\,x^2} = \frac{e^{-\frac{1}{x}}}{n\,x^{n+1}}$$

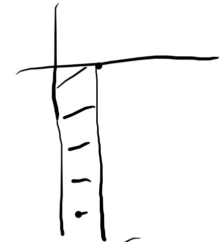
$$\lim_{x\to 0^+}\frac{e^{-1/x}}{x^n} = \lim_{y\to +\infty}\frac{y^n}{e^y} = 0$$

$$y=\frac{1}{x} \qquad \lim_{x\to 0^+}\frac{1}{x}=+\infty$$

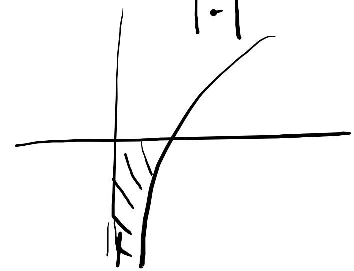
$x > 0$

$$\lim_{x \to 0^+} x^x \qquad 0^0$$

$$= \lim_{x \to 0^+} e^{\lg(x^x)} = \lim_{x \to 0^+} e^{x \lg x}$$



$$= e^0 = 1$$



$$\lim_{x \to 0^+} x \lg x = \qquad 0 \cdot -\infty$$

$$= \lim_{x \to 0^+} \frac{\lg x}{\frac{1}{x}} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \to 0^+}(-x) = 0$$

Derivate di ordine superiore

<u>Def</u> Sia $f : I \to \mathbb{R}$ e supponiamo che f' sia definita ovunque in I. Sia $x_0 \in I$. Supponiamo che esista

$$(f')'(x_0)$$

Questa la chiamiamo derivata seconda di f in x_0

e la denotiamo $f''(x_0) = \frac{d^2}{dx^2} f(x_0) = \left(\frac{d}{dx}\right)^2 f(x_0)$

$$\parallel$$

$$f^{(2)}(x_0)$$

$$f'(x_0) = f^{(1)}(x_0)$$

<u>Def</u> ~~Deri~~ Deriv

Sia $f : I \to \mathbb{R}$ e supponiamo per $n \geq 3$ di avere definito $f^{(n-1)}$ e supponiamo si definita su tutto I.

Allora, se in un punto $x_0 \in I$ esiste

$$\left(f^{(n-1)}\right)'(x_0)$$

allora la chiamiamo derivata di ordine n di f nel punto x_0 e la denotiamo con $f^{(n)}(x_0) = \frac{d^n}{dx^n} f(x_0) =$

$$= \left(\frac{d}{dx}\right)^n f(x_0)$$

f', f'', f''' . . .

$* \ (e^x)^{(n)} = e^x$

$(e^x)' = e^x$

supponendo per induzione che $(e^x)^{(n-1)} = e^x$

ho $(e^x)^{(n)} = \left((e^x)^{(n-1)} \right)' = (e^x)' = e^x$

$\Rightarrow *$ vera $\forall n$.

$(e^x)^{(n)}(0) = e^0 = 1$

$f^{(0)} = f$

0 $(\sin x)^{(0)} = \sin x$ $\qquad$ $(\sin x)^{(4)} = (-\cos x)' = \sin x$

1 $(\sin x)^{(1)} = \cos x$ $\qquad$ $(\sin x)^{(5)} = \cos x$

0 $(\sin x)^{(2)} = (\cos x)' = -\sin x$ $\qquad$ $(\sin x)^{(6)} = -\sin x$

-1 $(\sin x)^{(3)} = (-\sin x)' = -\cos x$ $\qquad$ $(\sin x)^{(7)} = -\cos x$

se $n = q4 + r$ dove $r \in \{0, 1, 2, 3\}$

$$\sin^{(n)} x = \sin^{(q4+r)}(x) = \left(\frac{d}{dx}\right)^{q\cdot 4+r} \sin x =$$

$$= \left(\frac{d}{dx}\right)^{r} \overbrace{\left(\frac{d}{dx}\right)^{4} \cdots \left(\frac{d}{dx}\right)^{4}}^{q \text{ volte}} \sin x = \left(\frac{d}{dx}\right)^{r} \sin x = \sin^{(r)}(x)$$

1 $(\cos x)^{(0)} = \cos x$ $\qquad$ $\cos^{(4)} x = \cos x$

0 $(\cos x)^{(1)} = -\sin x$

-1 $\cos^{(2)} x = -\cos x$ $\qquad$ $\cos^{(4\cdot q+r)}(x) = \cos^{(r)}(x)$

0 $\cos^{(3)} x = \sin x$

$a_1, \dots, a_n$

$$\underbrace{a_1 \cdots a_n}_{\text{prodotto}} = \prod_{j=1}^{n} a_j$$

$$\sum_{j=1}^{0} a_j = 0$$

$$\prod_{j=1}^{0} a_j = 0$$

Lemma Consideriamo x^a $a \in \mathbb{R}$, $x > 0$

Allora $$(x^a)^{(n)} = \prod_{j=1}^{n} (a-j+1)\, x^{a-n} \qquad \forall\, n \in \mathbb{N}$$

Dim Per induzione

Per $n=1$, $$(x^a)^{(1)} = (x^a)' = a\, x^{a-1}$$

$$\prod_{j=1}^{1} (a-j+1)\, x^{a-1} = (a-\cancel{1}\cancel{+1})\, x^{a-1} \quad ✓$$

$n \Rightarrow n+1$

$$(x^a)^{(n+1)} = \left((x^a)^{(n)} \right)' = \left(\prod_{j=1}^{n} (a-j+1)\, x^{a-n} \right)'$$

$$= \prod_{j=1}^{n} (a-j+1)\, (x^{a-n})' = \prod_{j=1}^{n} (a-j+1)\, \underbrace{(a-n)}_{\substack{\shortparallel \\ (a-j+1) \\ \text{ma } j=n+1}} x^{a-n-1}$$

$$(x^a)^{(n+1)} = \prod_{j=1}^{n+1} (a-j+1)\, x^{a-(n+1)}$$

Esercizio $$\left((1+x)^a \right)^{(n)} = \prod_{j=1}^{n} (a-j+1)\, (1+x)^{a-n}$$

$$\left(x^n\right)^{(n)} = \prod_{j=1}^{n} (n-j+1)\; x^{n-n} =$$

$$= \prod_{j=1}^{n} (n-j+1) = n!$$

$m \in \mathbb{N}$

$m < n \qquad m+1 \leq n$

$$\left(x^m\right)^{(n)} = \prod_{j=1}^{n} (m-j+1)\; x^{m-n} = 0$$

se $\quad m-n+1 < 0$

Lemma

1) $(f+g)^{(n)}(x) = f^{(n)}(x) + g^{(n)}(x)$

2) $(cf)^{(n)}(x) = c\, f^{(n)}(x)$

3) $n = m + k \qquad m, k \in \mathbb{N}$

$$f^{(n)}(x) = \left(f^{(k)}\right)^{(m)}(x)$$