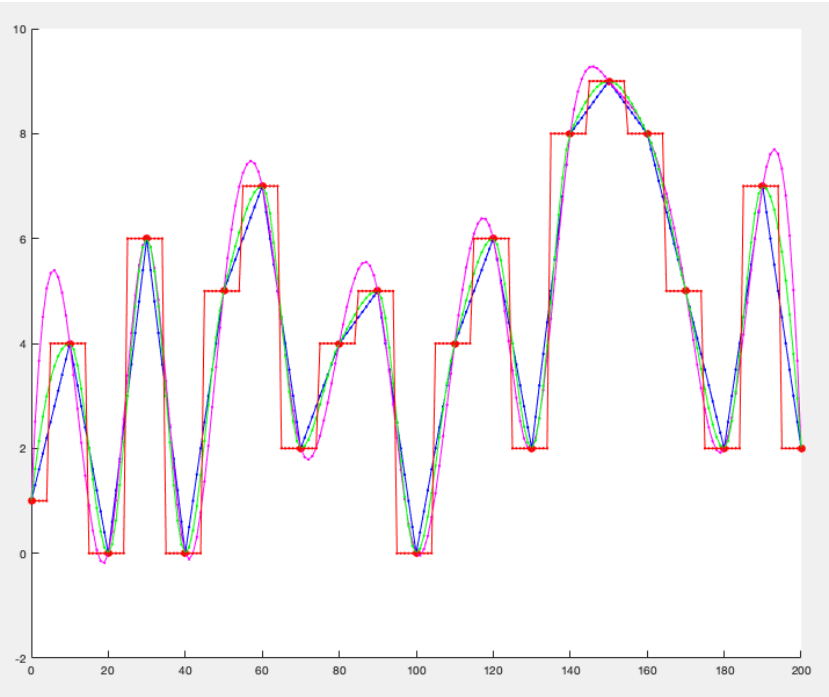
L'interpolazione in MATLAB

Eugenio G. Omodeo



Trieste, 30.11.2021

Che significa *interpolare* ?

Interpolation *is a method by which we can use data points to estimate the unknown value of a function at a given location. This is a common problem in the geosciences; for example, if we have a collection of radiocarbon ages through a sediment core, how can we estimate the age of the sediment at any given depth? MATLAB ha inbuilt function to perform interpolation in any number* [1] *of dimensions, they are easy to use but it is important to appreciate that the various options you can select from strongly influence your results.*

(Heslop 2012, pag.91)

[1] Dim. 1, 2, 3, n

Che significa *interpolare* ?

```
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 interp1 1-D interpolation (table lookup)

    Vq = interp1(X,V,Xq) interpolates to find Vq, the values of the
    underlying function V=F(X) at the query points Xq.

    X must be a vector. The length of X is equal to N.
    If V is a vector, V must have length N, and Vq is the same size as Xq.
    If V is an array of size [N,D1,D2,...,Dk], then the interpolation is
    performed for each D1-by-D2-by-...-Dk value in V(i,:,:,...,:). If Xq
    is a vector of length M, then Vq has size [M,D1,D2,...,Dk]. If Xq is
    an array of size [M1,M2,...,Mj], then Vq is of size
    [M1,M2,...,Mj,D1,D2,...,Dk].

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    Vq = interp1(X,V,Xq,METHOD) specifies the interpolation method.
    The available methods are:

      'linear'   - (default) linear interpolation
      'nearest'  - nearest neighbor interpolation
      'next'     - next neighbor interpolation
      'previous' - previous neighbor interpolation
      'spline'   - piecewise cubic spline interpolation (SPLINE)
      'pchip'    - shape-preserving piecewise cubic interpolation
      'cubic'    - same as 'pchip'
      'v5cubic'  - the cubic interpolation from MATLAB 5, which does not
                   extrapolate and uses 'spline' if X is not equally
                   spaced.
      'makima'   - modified Akima cubic interpolation

    Vq = interp1(X,V,Xq,METHOD,'extrap') uses the interpolation algorithm
    specified by METHOD to perform extrapolation for elements of Xq outside
    the interval spanned by X.
```

Esempio di interpolazione

Esempio: Che valore y' aspettarci per $x' = 41$?

Evidenziati in rosso, pochi dati misurati; come possiamo avanzare ipotesi circa i dati mancanti?

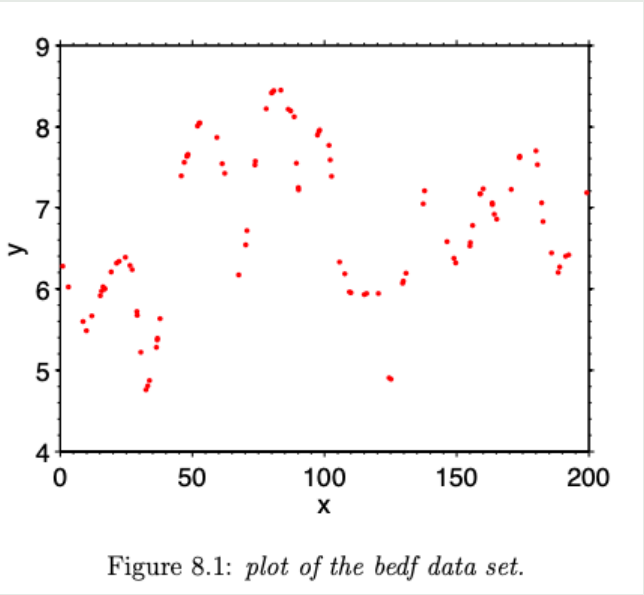

Figure 8.1: *plot of the bedf data set.*

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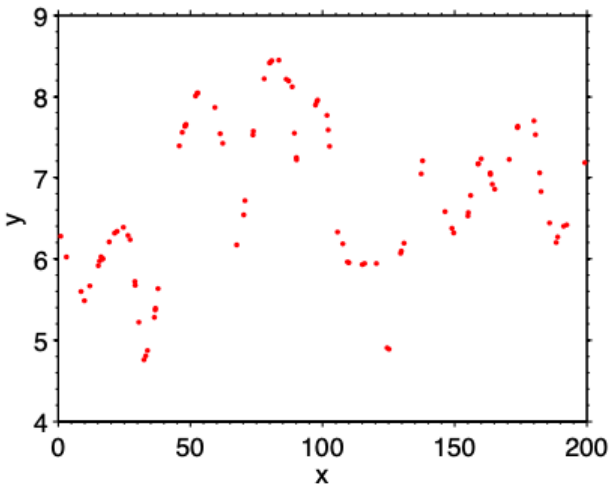
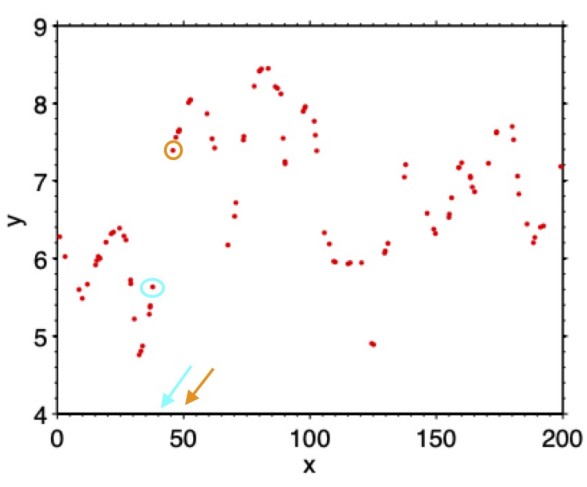


Figure 8.1: *plot of the bedf data set.*



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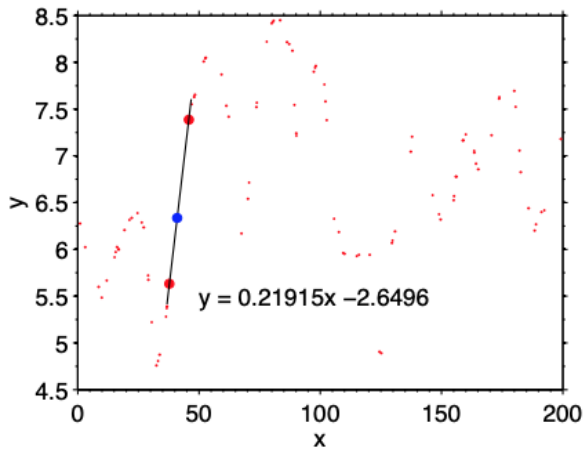

Figure 8.2: *If we are interested in finding the value of y' at $x = 41$, we simply determine the straight line between the two data points which span the neighborhood of interest (large red symbols). Once we have the equation; $y = 0.21915x -$ 26496 we can estimate that when $x' = 41$, $y' = 6.34$ (blue circle).*

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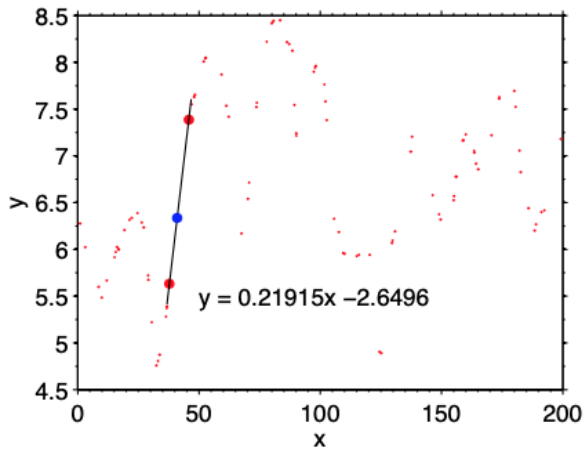

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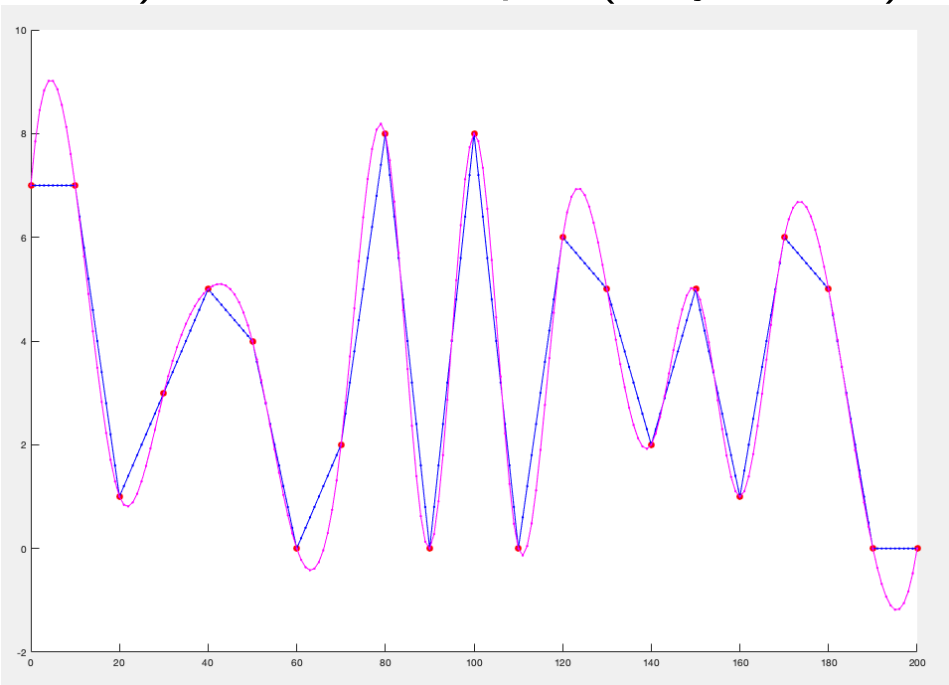
```
» y0=interp1(x,y,41,'linear')   % chiediamo, per x0 = 41, una stima della
                                % sua y0 desunta 'linearmente' dai dati (x, y)
y0 =
6.3357
```

Interpolazione: lineare *versus* tramite *spline*

```
» close all,    x=0:10:200;          y=floor(rand(1,21)*10);
  x0=[0:1:200]';
  figure,       hold on,             plot(x,y,'or','markerfacecolor','r'),
  y0=interp1(x,y,x0,'linear');       y1=interp1(x,y,x0,'spline');
» plot(x0,y0,'.b-'),                 plot(x0,y1,'.m-')
```



Esercizio

Completate la fig. con le interpolazioni 'pchip' e 'nearest'

Un'osservazione dall'Heslop

When performing interpolation with the `interp1` *function, we don't need to limit ourselves to determining single values. If we supply the function with a vector of* x' *values, it will perform the interpolation for each one. This is useful for a lot of time series analysis techniques which assume that you have equally spaced data. If your measured data isn't equally spaced, then you simply interpolate it to produce a suitable new data series.*

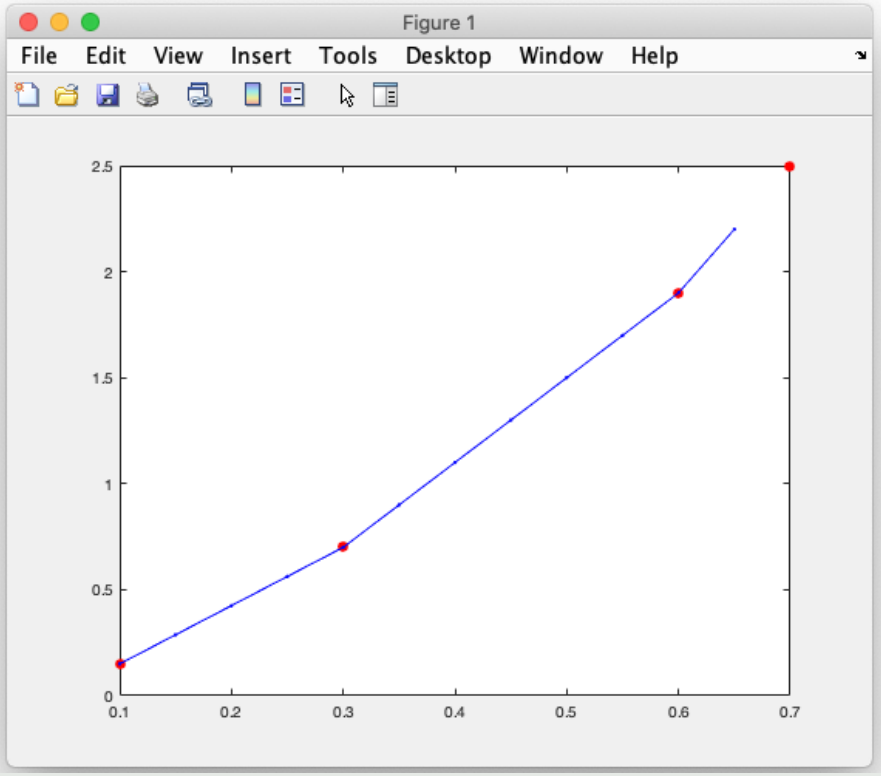
(Heslop 2012, pag.93)

Esempio motivante all'estrapolazione

Interpolare, fuori dalla finestra-dati, non si può...

Command Window

```
>> a=[0.1 0.3 0.6 0.7];  % Ascisse non equidistanti
b=[0.15 0.7 1.9 2.5];    % Corrispettive ordinate
figure
plot(a,b,'or','markerfacecolor','r')
x0=[0:0.05:0.8];
y0=interp1(a,b,x0,'linear');
hold on
plot(x0,y0,'.b-')
>>
```



Quanto varrà $y0$ in corrispondenza ad

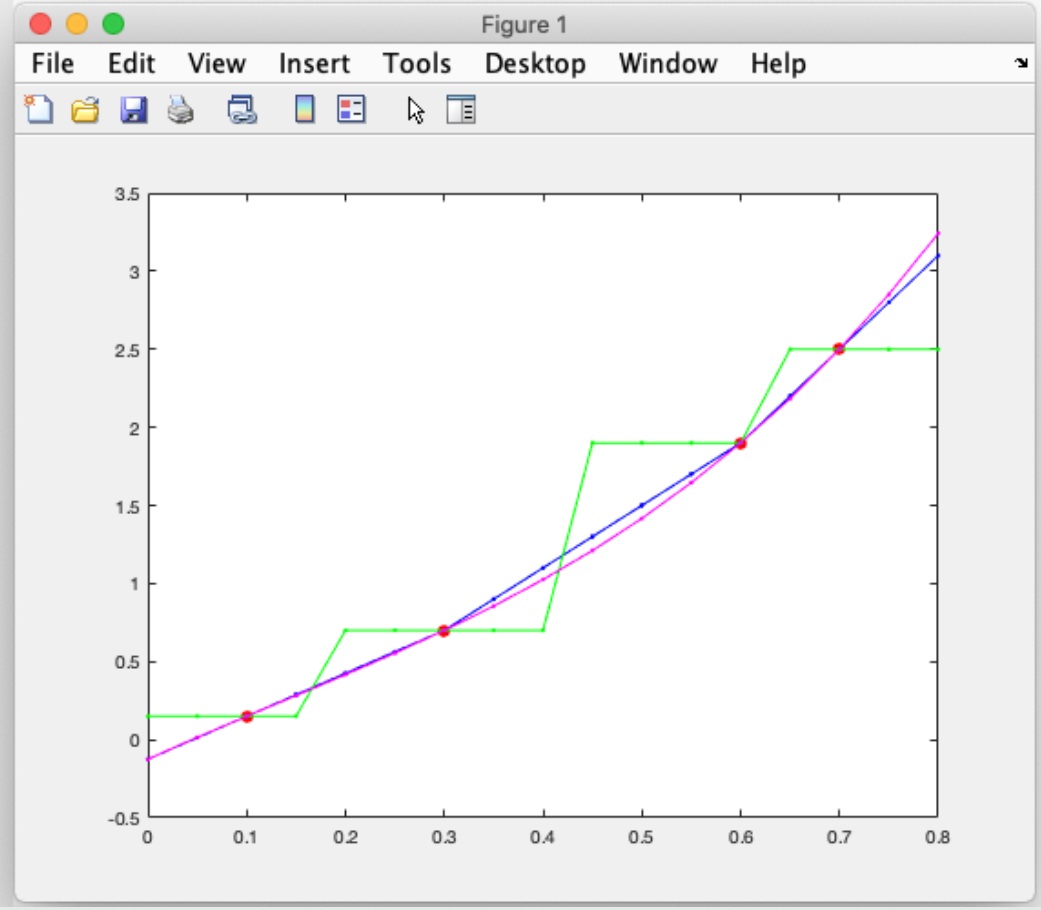
$$x0 = 0,\ 0.05,\ 0.7,\ 0.75,\ 0.8\ ?$$

Esempio motivante all'estrapolazione

...e allora estrapoliamo

Command Window

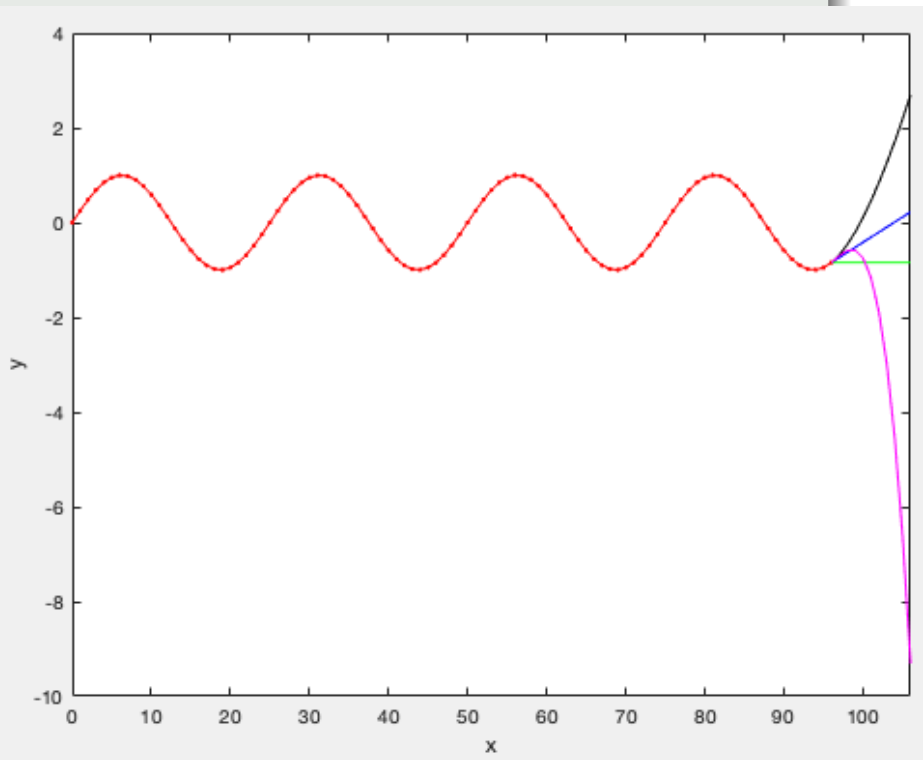
```
>> %...e allora estrapoliamo
y0=interp1(a,b,x0,'linear','extrap');
plot(x0,y0,'.b-')
y1=interp1(a,b,x0,'nearest','extrap');
plot(x0,y1,'.g-')
y2=interp1(a,b,x0,'spline','extrap');
plot(x0,y2,'.m-')
>>
```



Estrapolazioni: Da usarsi con grande cautela

Esempio: Prolunghiamo una curva sinuisodale

```
>> x=0:1:96; % selection of x data sites
y=sin(2*pi*x/25); %sine function with a period of 25 units
plot(x,y,'.r-') %plot the data
set(gca,'xlim',[0 106]) %extend the graphic window
xlabel('x'), ylabel('y'),
hold on
>> x0=[96:1:110]'; %x locations for extrapolation
y0=interp1(x,y,x0,'linear','extrap'); %linear extrapolation
plot(x0,y0,'b') %add linear extrapolation to the figure
y0=interp1(x,y,x0,'nn','extrap'); %nn extrapolation
plot(x0,y0,'g') %add nn extrapolation to the figure
y0=interp1(x,y,x0,'spline','extrap'); %cubic spline extrapo
>> plot(x0,y0,'k') %add spline extrapolation to the figure
y0=interp1(x,y,x0,'pchip','extrap'); %pchip extrapolation
plot(x0,y0,'m') %add pchip extrapolation to the figure
>>
```



Estrapolazione / Extrapolation: –I

Estrapolazione

Da Wikipedia, l'enciclopedia libera.

In matematica, il termine **estrapolazione** indica il processo che permette di calcolare il valore di informazioni esterne ad un insieme discreto di dati noti. In sostanza, dato un piano cartesiano sul quale siano stati tracciati i punti (x_i, y_i) corrispondenti all'insieme di valori noti, si vuole trovare il valore y_w corrispondente ad un valore x_w maggiore (o minore) di ciascun x_i. L'estrapolazione è simile al processo di interpolazione, che costruisce nuovi punti all'interno di un insieme di punti noti, ma i suoi risultati hanno spesso minor significato e sono soggetti ad un abbondante grado di incertezza.

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Metodi di estrapolazione [modifica | modifica wikitesto]

La scelta di quale metodo di estrapolazione applicare deve essere determinata da una conoscenza a priori del processo che ha creato i dati esistenti. È fondamentale sapere se ad esempio i dati possono essere considerati continui, periodici, alternati ecc.

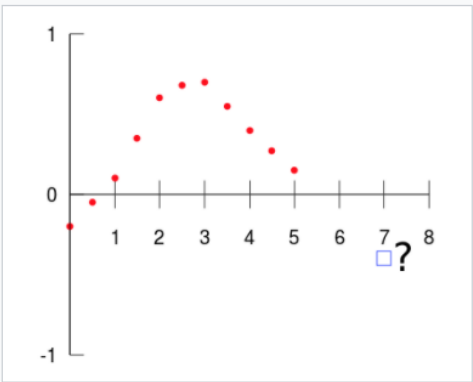
Extrapolation

From Wikipedia, the free encyclopedia

For the journal of speculative fiction, see Extrapolation (journal). For the John McLaughlin album, see Extrapolation (album).

In mathematics, **extrapolation** is a type of estimation, beyond the original observation range, of the value of a variable on the basis of its relationship with another variable. It is similar to interpolation, which produces estimates between known observations, but extrapolation is subject to greater uncertainty and a higher risk of producing meaningless results. Extrapolation may also mean extension of a method, assuming similar methods will be applicable. Extrapolation may also apply to human experience to project, extend, or expand known experience into an area not known or previously experienced so as to arrive at a (usually conjectural) knowledge of the unknown [1] (e.g. a driver extrapolates road conditions beyond his sight while driving). The extrapolation method can be applied in the interior reconstruction problem.

Contents [hide]



Example illustration of the extrapolation problem, consisting of assigning a meaningful value at the blue box, at $x = 7$, given the red data points.

Estrapolazione / Extrapolation: –IV

Methods [edit]

A sound choice of which extrapolation method to apply relies on *a prior knowledge* of the process that created the existing data points. Some experts have proposed the use of causal forces in the evaluation of extrapolation methods.[
Crucial questions are, for example, if the data can be assumed to be continuous, smooth, possibly periodic etc.

French curve [edit]

French curve extrapolation is a method suitable for any distribution that has a tendency to be exponential, but with accelerating or decelerating factors.[3] This method has been used successfully in providing forecast projections of the growth of HIV/AIDS in the UK since 1987 and variant CJD in the UK for a number of years. Another study has shown that extrapolation can produce the same quality of forecasting results as more complex forecasting strategies.[4]