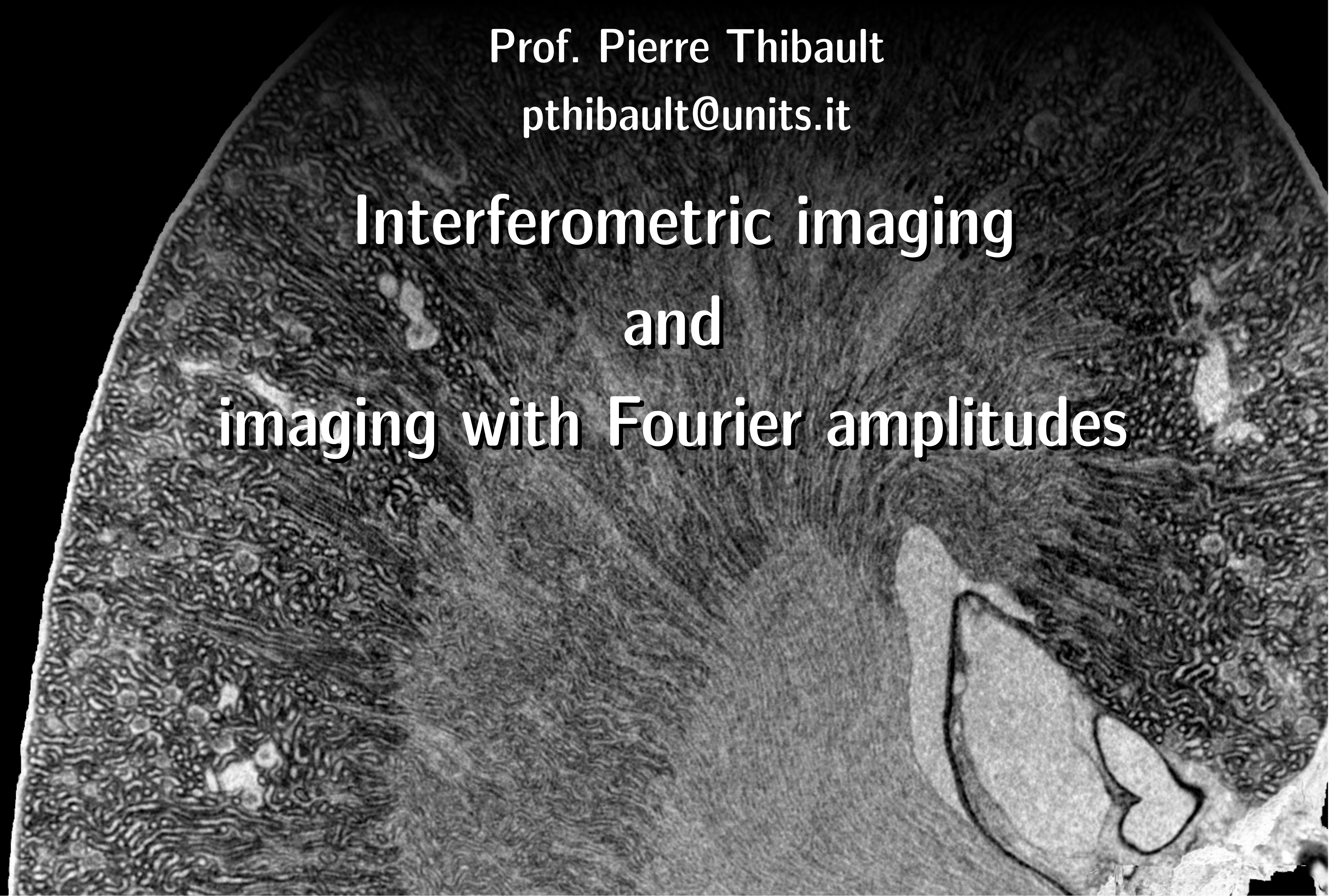
Image Processing for Physicists

Prof. Pierre Thibault

pthibault@units.it

Interferometric imaging
and
imaging with Fourier amplitudes



Overview

- The phase problem
- Holography: on/off-axis
- Grating interferometric imaging
- Imaging using far-field amplitude measurements
  - Fourier transform holography
  - Coherent diffraction imaging
  - Ptychography

Wave propagation

$$\psi(\vec{r}; z) = \mathcal{F}^{-1}\left\{\mathcal{F}\{\psi(\vec{r}; z=0)\}\exp\left(-i\pi \underbrace{u^2\lambda z}_{\text{unitless}}\right)\right\}$$

a

same if

$$\frac{\lambda_0 z_0}{a_0^2} = \frac{\lambda_1 z_1}{a_1^2}$$

$$\frac{a^2}{\lambda z} = f$$

"Fresnel number"

$f \ll 1$: far-field

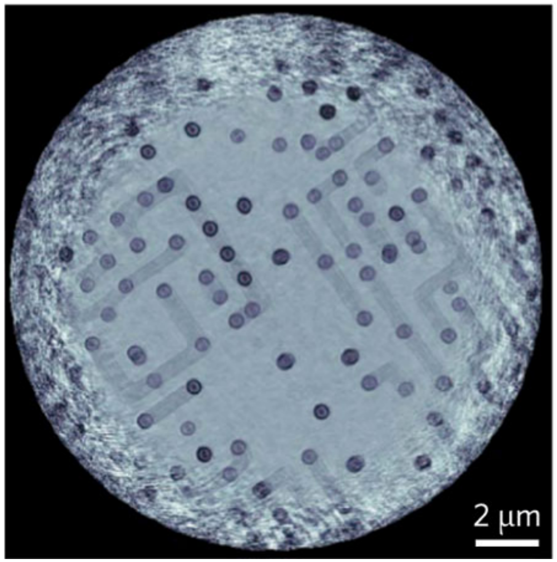
$f \gg 1$: near-field

a : here size of an aperture, but can be any characteristic length of interest

$f = 1 \rightarrow a = \sqrt{\lambda z}$ characteristic length of oscillations caused by propagation

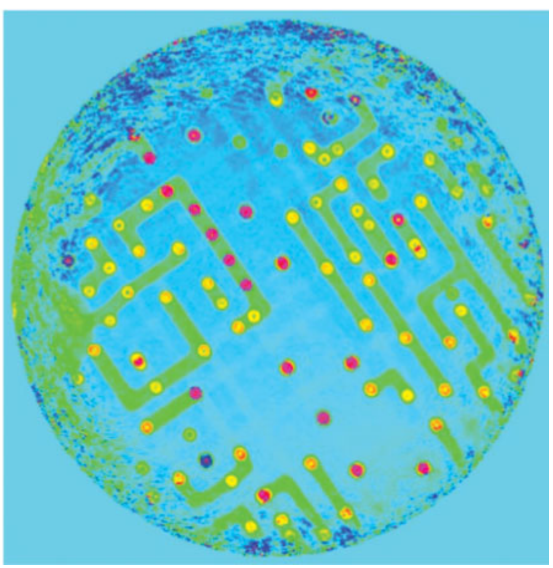
Complex-valued images

X-ray transmission image



Amplitude

attenuation of the wave

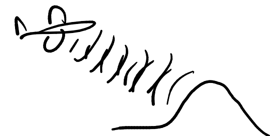


Phase

delay in wave phase (refraction)

synthetic aperture radar

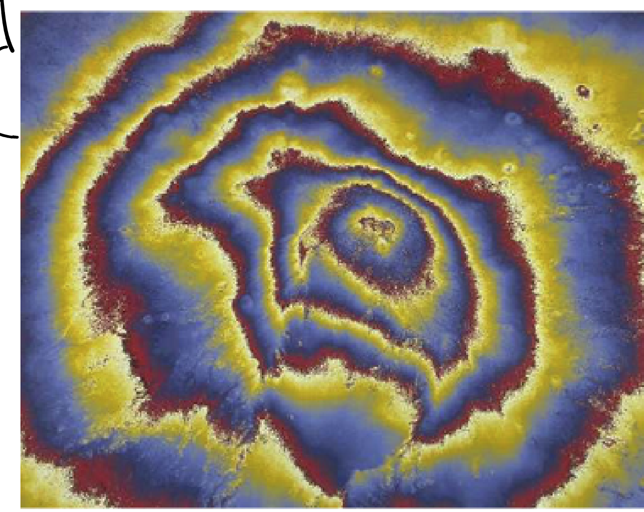
SAR



phase unwrapping

raw measurement

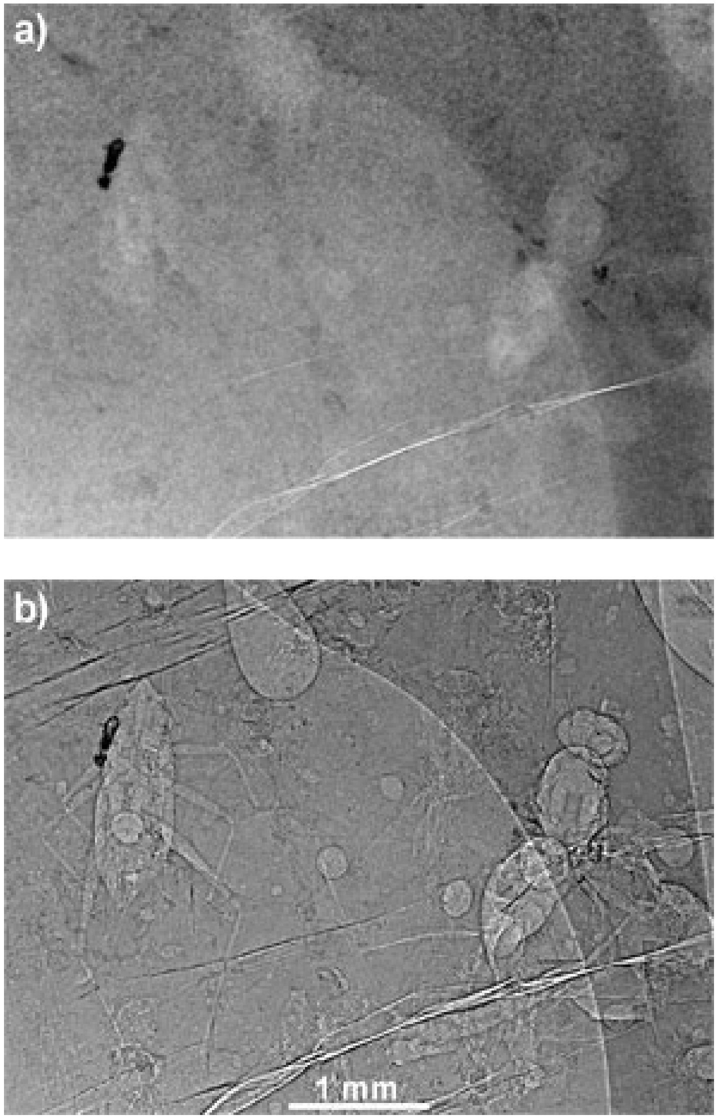
phase



Etna

Phase-contrast

Hard X-ray propagation-based phase contrast



Source: www.esrf.eu/news/general/amber/amber/

"contact mode" images

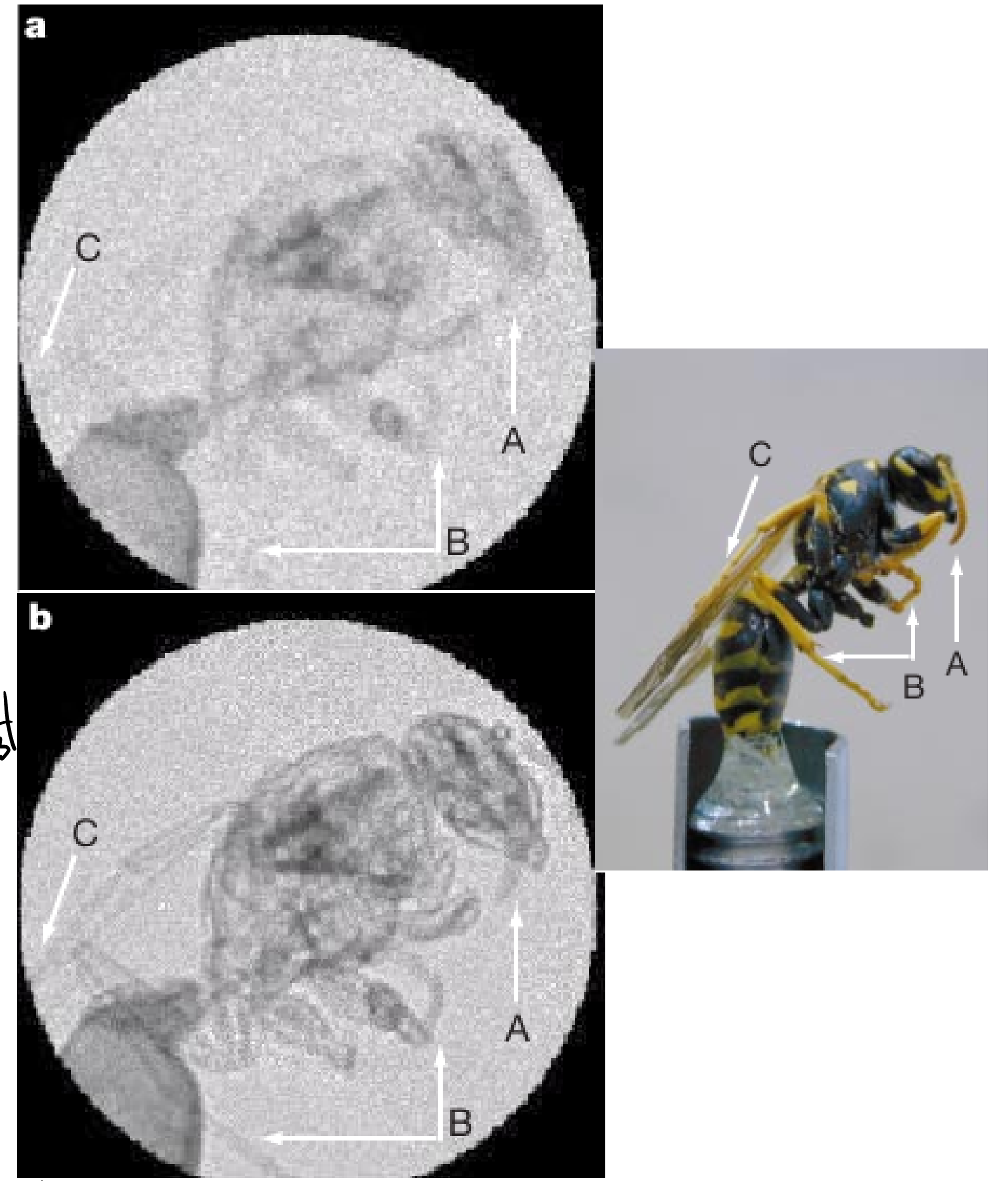
$f \gg 1$

propagation based phase contrast

= Laplace filter

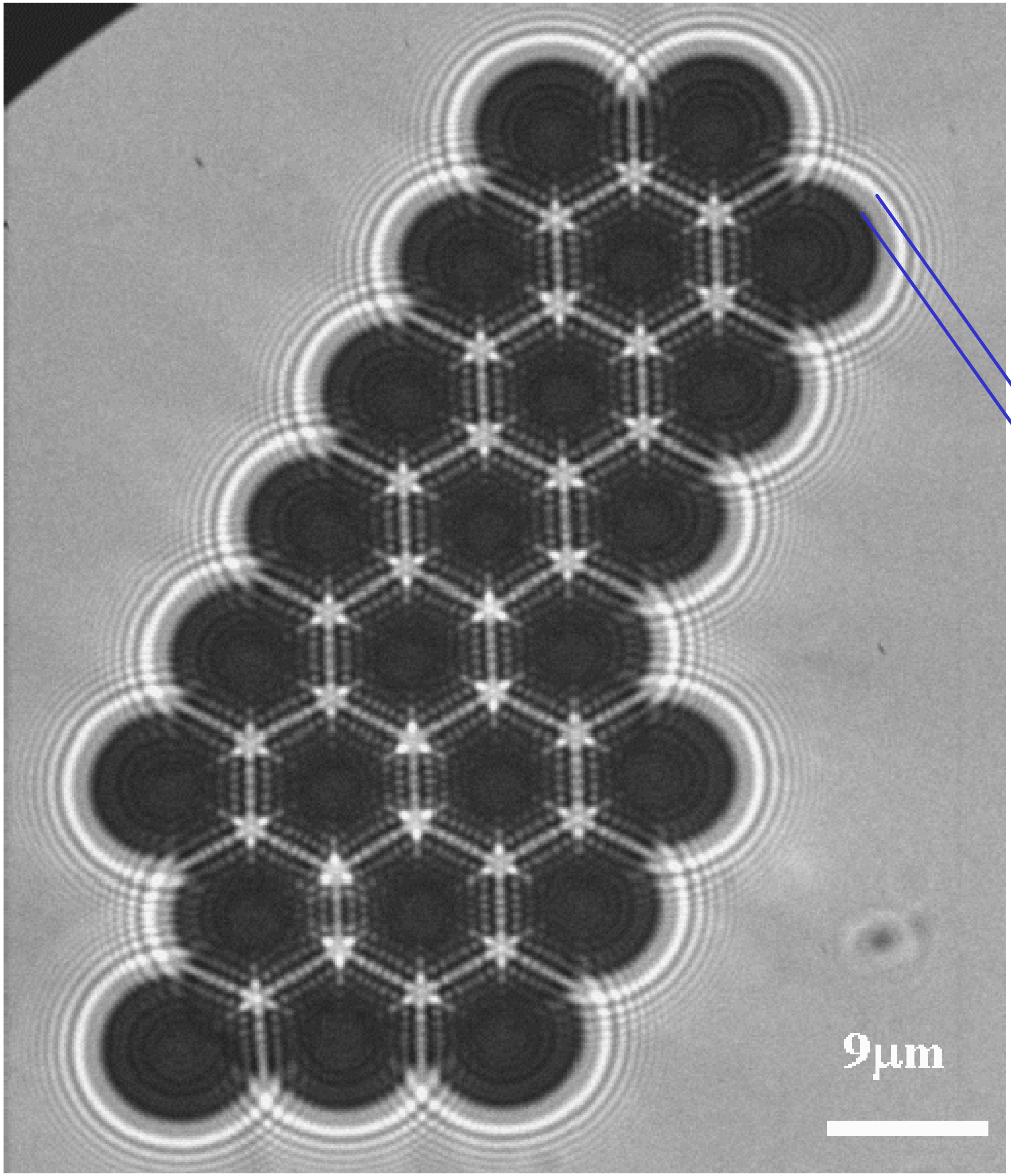
edge enhancement

Neutron phase contrast



Source: Allman et al. Nature **408** (2000).

In-line holography



"Deep Fresnel" regime

$\sqrt{\lambda z}$

Source: Mayo et al. Opt Express **11** (2003).

In-line holography

Measure $I(\vec{r}) = |\psi(\vec{r};z)|^2$

If * the illumination is a plane monochromatic wave

** the transmission of the imaged object is weak:

$$\psi(\vec{r};z=0) = A(1+\varepsilon(\vec{r}))$$

small perturbation of plane incident wave

$$I(\vec{r}) = |A(1+\varepsilon(\vec{r};z))|^2 = |A^2|\Big(\underbrace{1}_{\text{uniform}} + \boxed{\underbrace{\varepsilon(\vec{r};z) + \varepsilon^*(\vec{r};z)}_{\text{propagated}}} + \underbrace{|\varepsilon(\vec{r};z)|^2}_{\text{negligible}}\Big)$$

$$\varepsilon^*(\vec{r};z) \to \varepsilon(\vec{r};-z)$$

discards the phase part

superposition of two images propagated by z and $-z$

"twin image problem"

The phase problem

We can measure only the squared amplitude of a wave

(e.m. / matter)

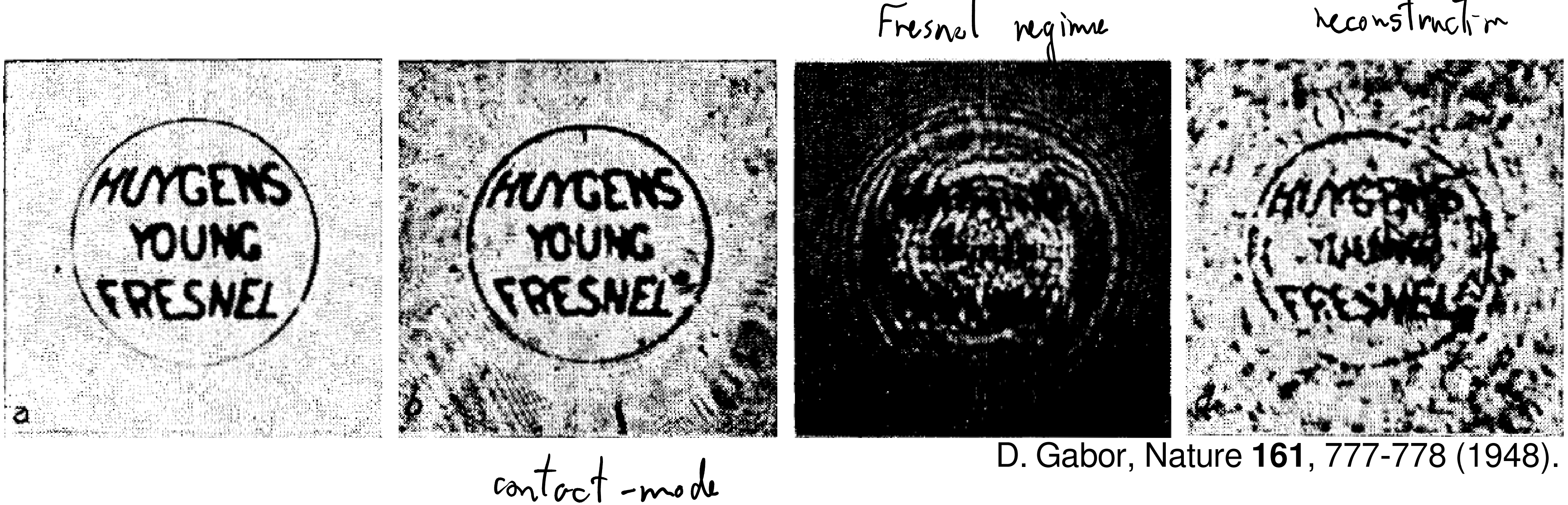
$$I = |\psi|^2$$

Sometimes: - phase is the quantity of interest

- phase is auxiliary quantity required to extract the sought information

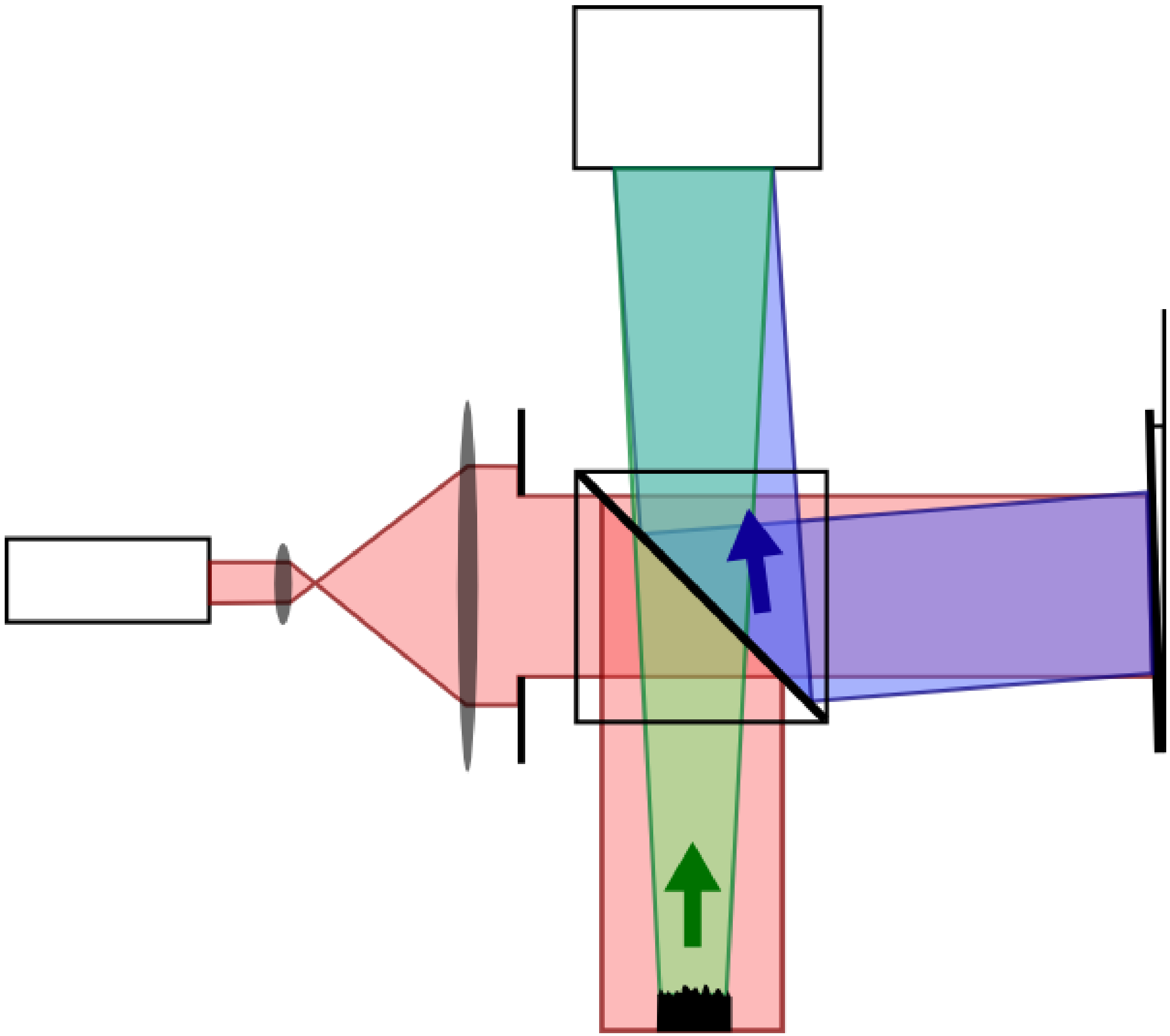
(e.g. through propagation)

In-line holography



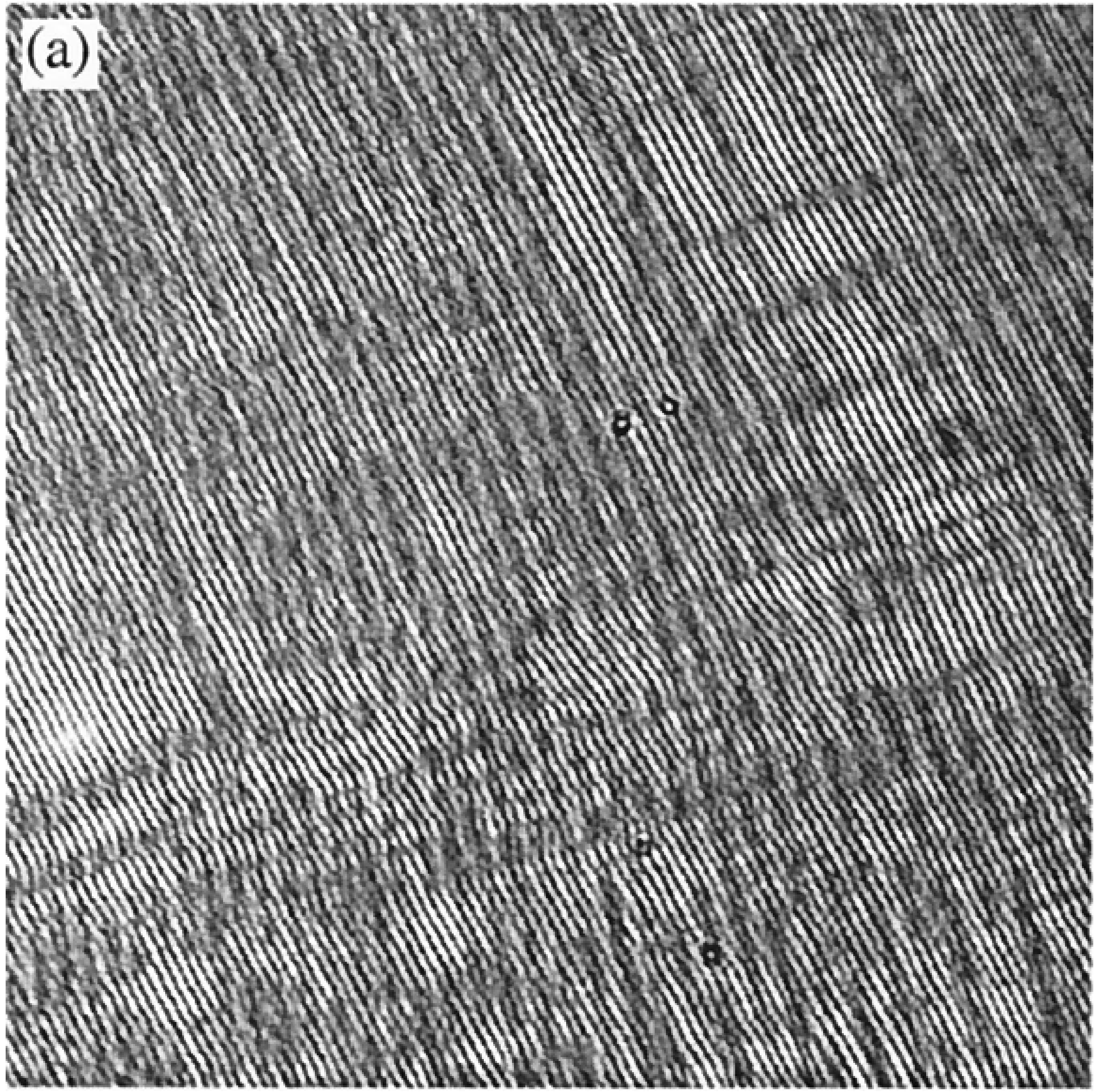
D. Gabor, Nature **161**, 777-778 (1948).

Fringe interferometry



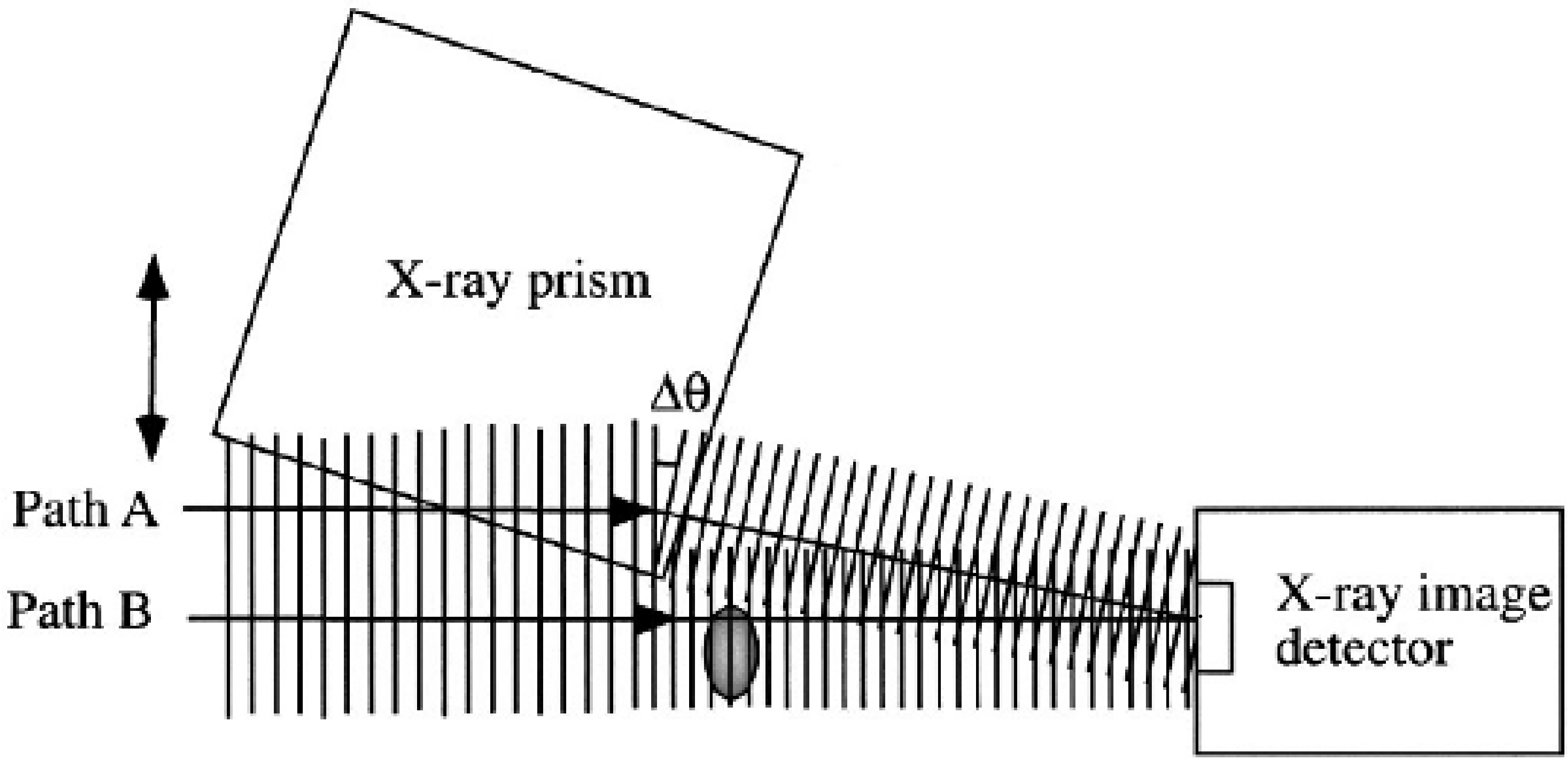
Twyman-Green interferometer

Fringe interferometry



Source: Cuche et al. Appl. Opt. **39**, 4070 (2000)

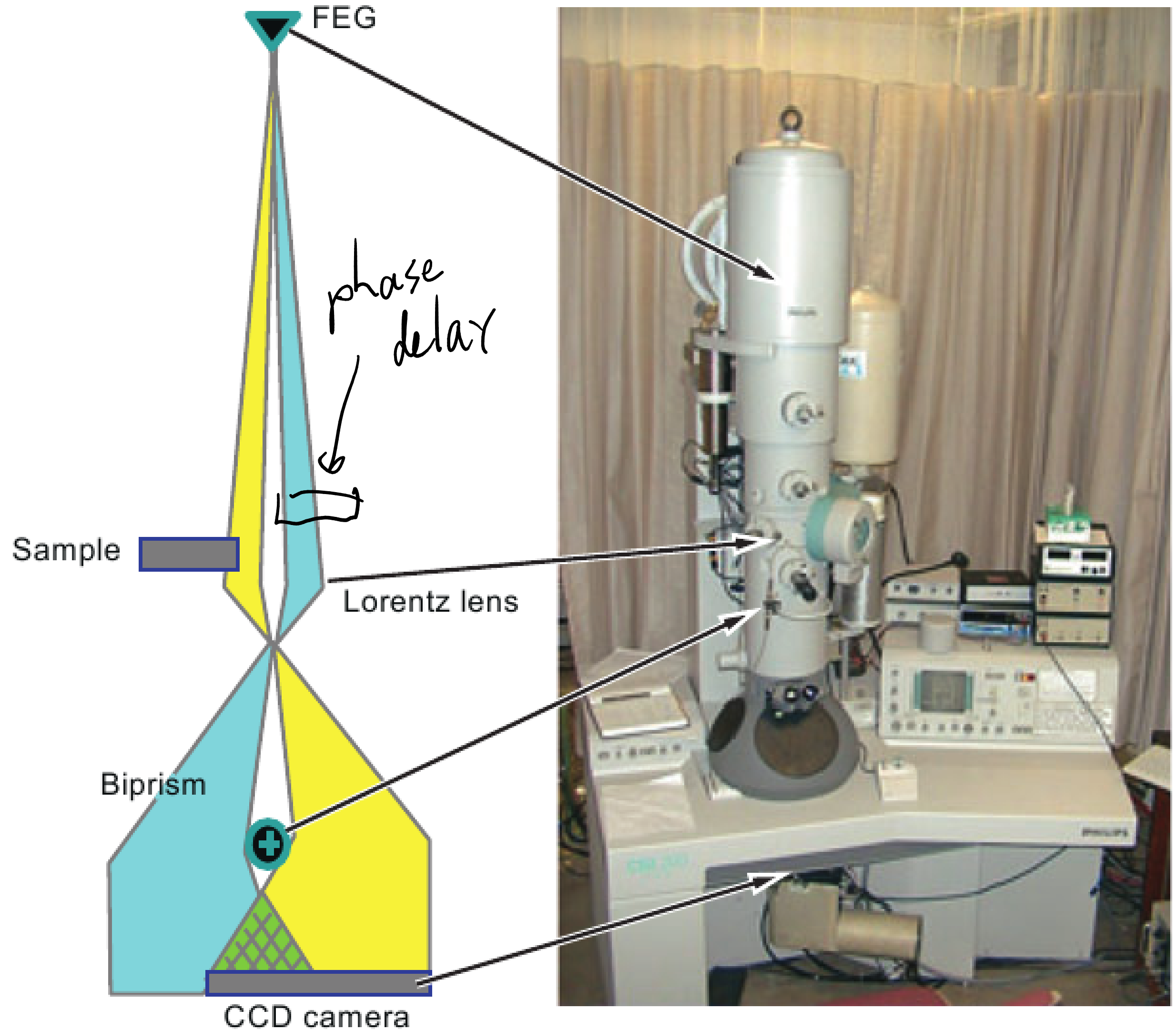
Off-axis X-ray holography



Source: Y. Kohmura, J. Appl. Phys. **96**, 1781-1784 (2004)

Off-axis electron holography

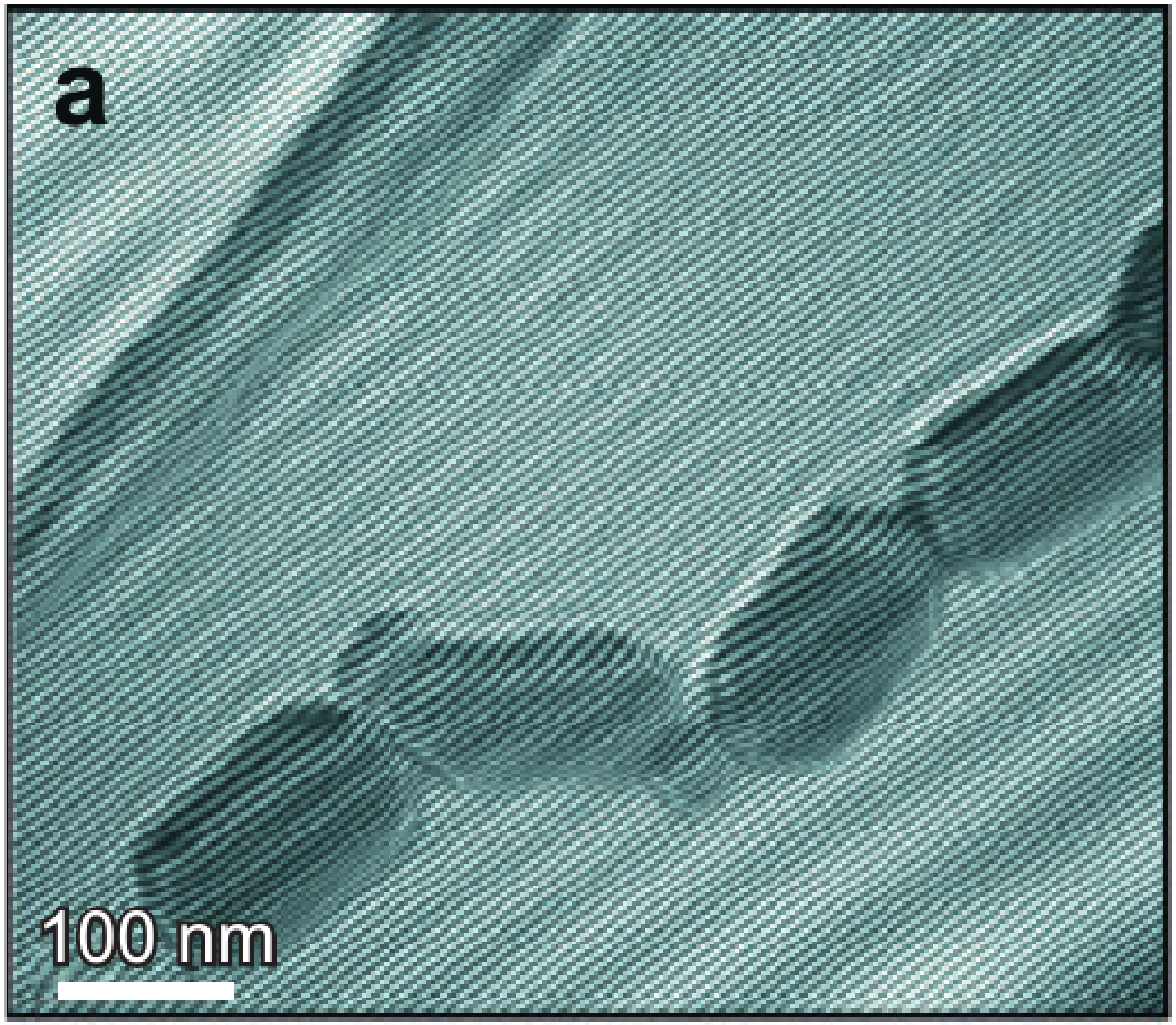
Electron microscopy



Hologram **Philips CM200 FEG**

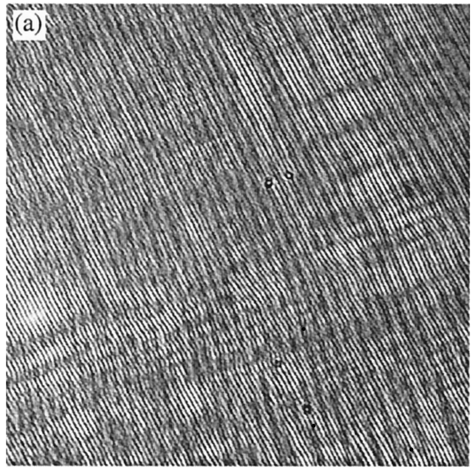
Source: M. R. McCartney, Ann. Rev. Mat. Sci. **37** 729-767 (2007)

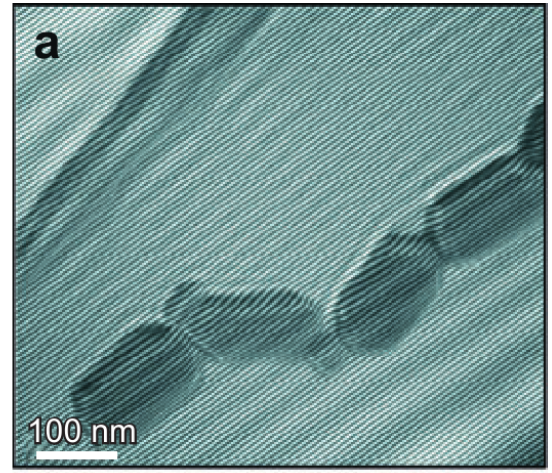
Off-axis electron holography



Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

Fringe interferometry





$$\Psi = \underset{\text{object}}{\Psi_o} + \underset{\text{reference}}{\Psi_r}$$

$$\Psi_r(\vec r) = A e^{i\vec k\cdot\vec r}$$ ← tilted plane wave

$$\Psi_o(\vec r) = A\, a(\vec r)\, e^{i\varphi(\vec r)}$$

phase shift caused by refraction

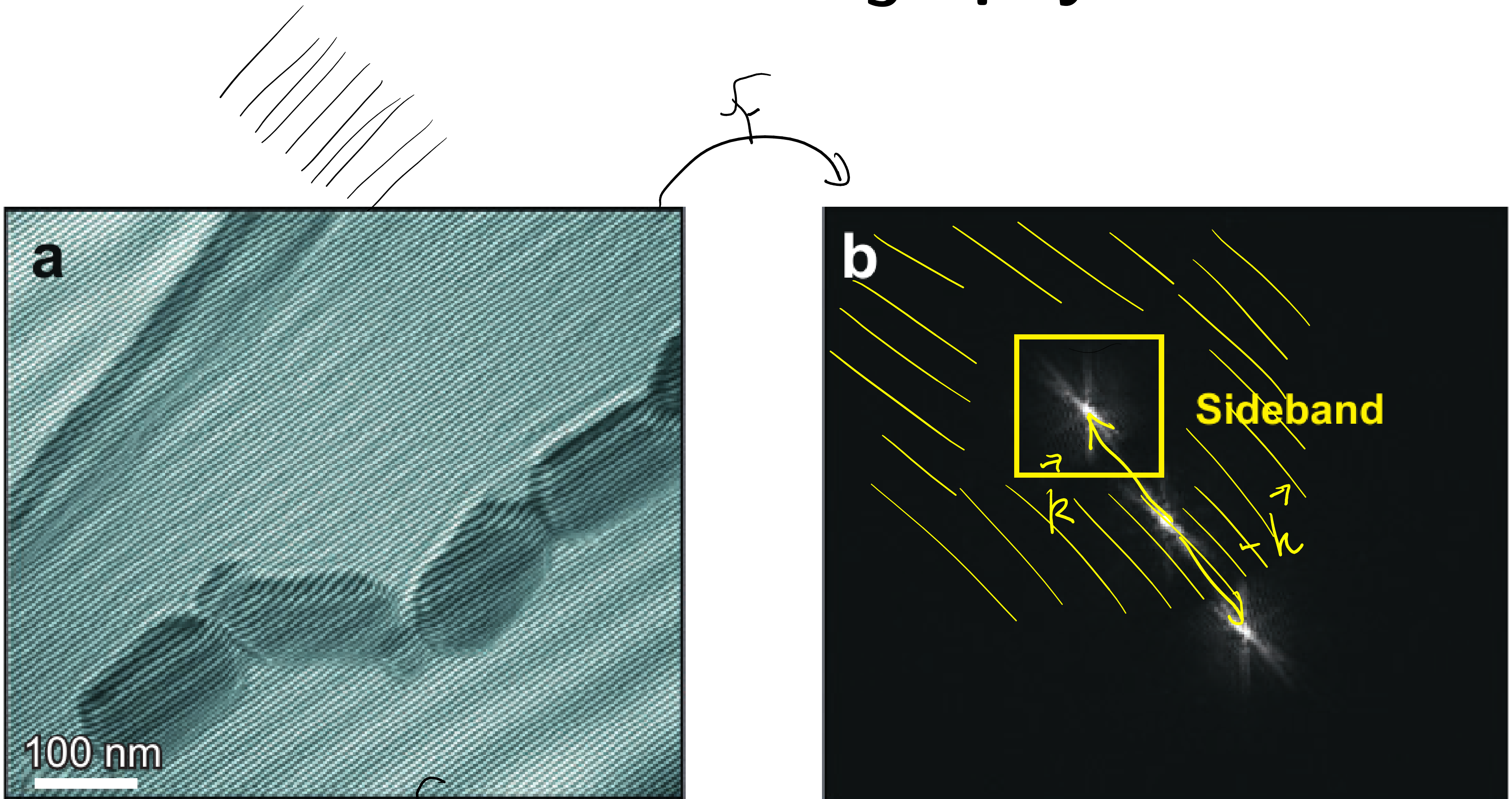
absorption

Measurement:

$$|\Psi(\vec r)|^2 = (\Psi_o + \Psi_r)(\Psi_o^* + \Psi_r^*)$$

$$= |A|^2 \left(a^2(r) + 1 + \underbrace{a(\vec r)\, e^{i(k\cdot r - \varphi)} + a(\vec r)\, e^{-i(\vec k\cdot\vec r - \varphi)}}_{2\, a(\vec r) \cos(\vec k\cdot\vec r - \varphi)} \right)$$

Off-axis holography



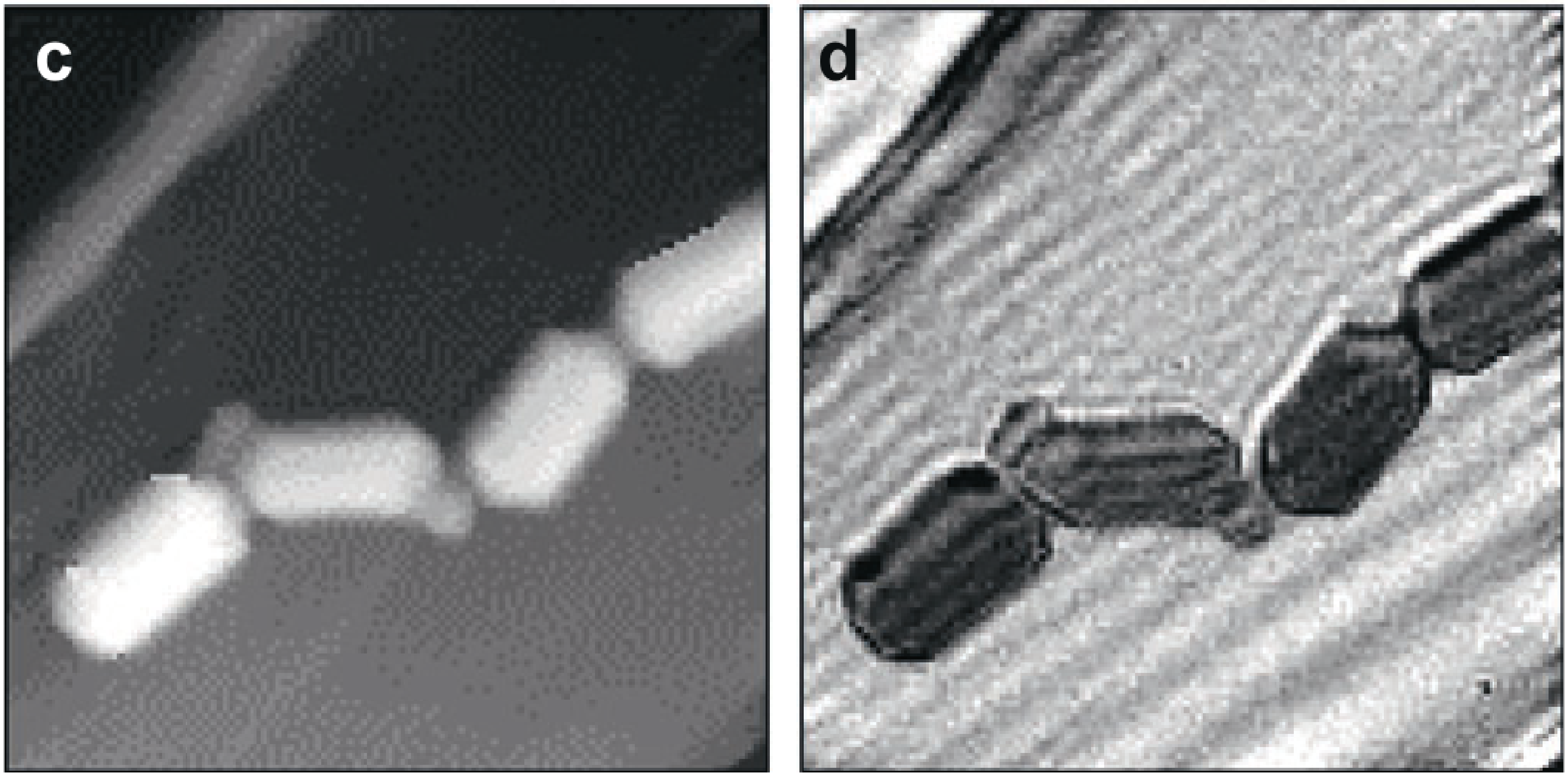
$$\mathcal{F}\{|\psi|^2\} = |A|^2 \left[\mathcal{F}\{a^2(r)+1\} + \mathcal{F}\{\underbrace{a(\vec r)e^{-i\varphi}}_{\psi_0^*} e^{i\vec k\cdot\vec r}\} + \mathcal{F}\{\underbrace{a(\vec r)e^{i\varphi}}_{\psi_0} e^{-i\vec k\cdot\vec r}\} \right]$$

$**$ $\quad$ $*$

$*$: $\underline{\mathcal{F}\{\psi_0\}}\left(\vec k - \frac{\vec k}{2\pi}\right)$

Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

Off-axis holography



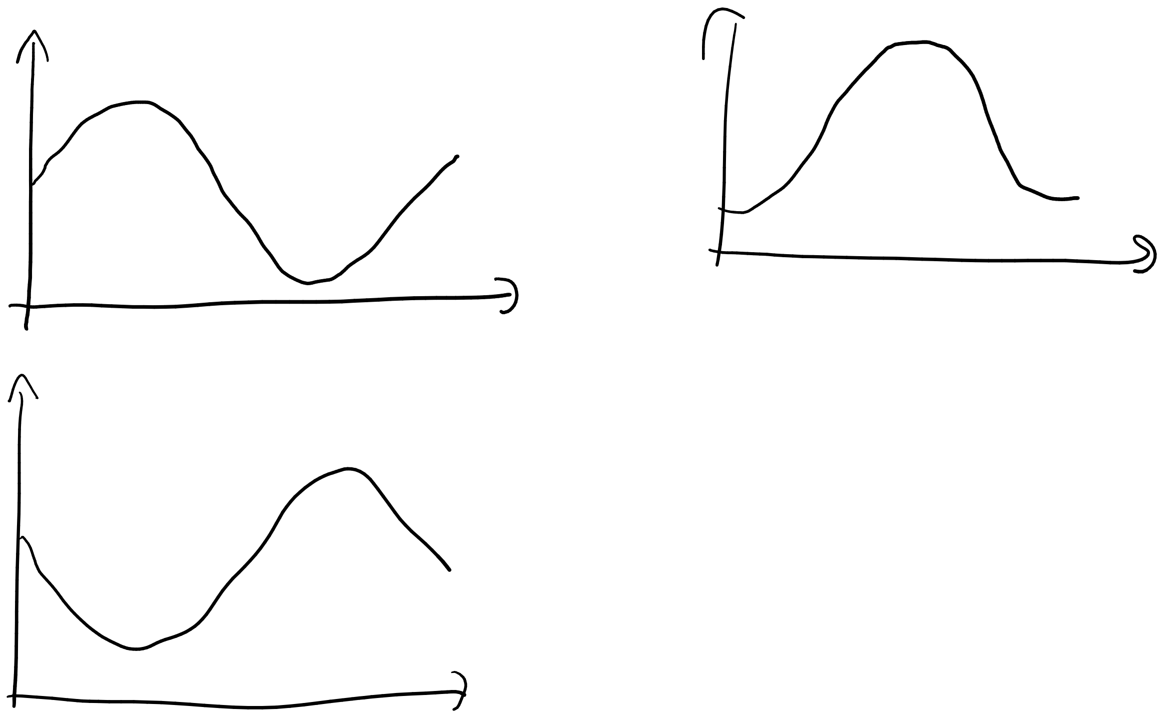
Reconstruction easy but comes at the cost of lower resolution (because of the cropping operation)

Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

Phase stepping

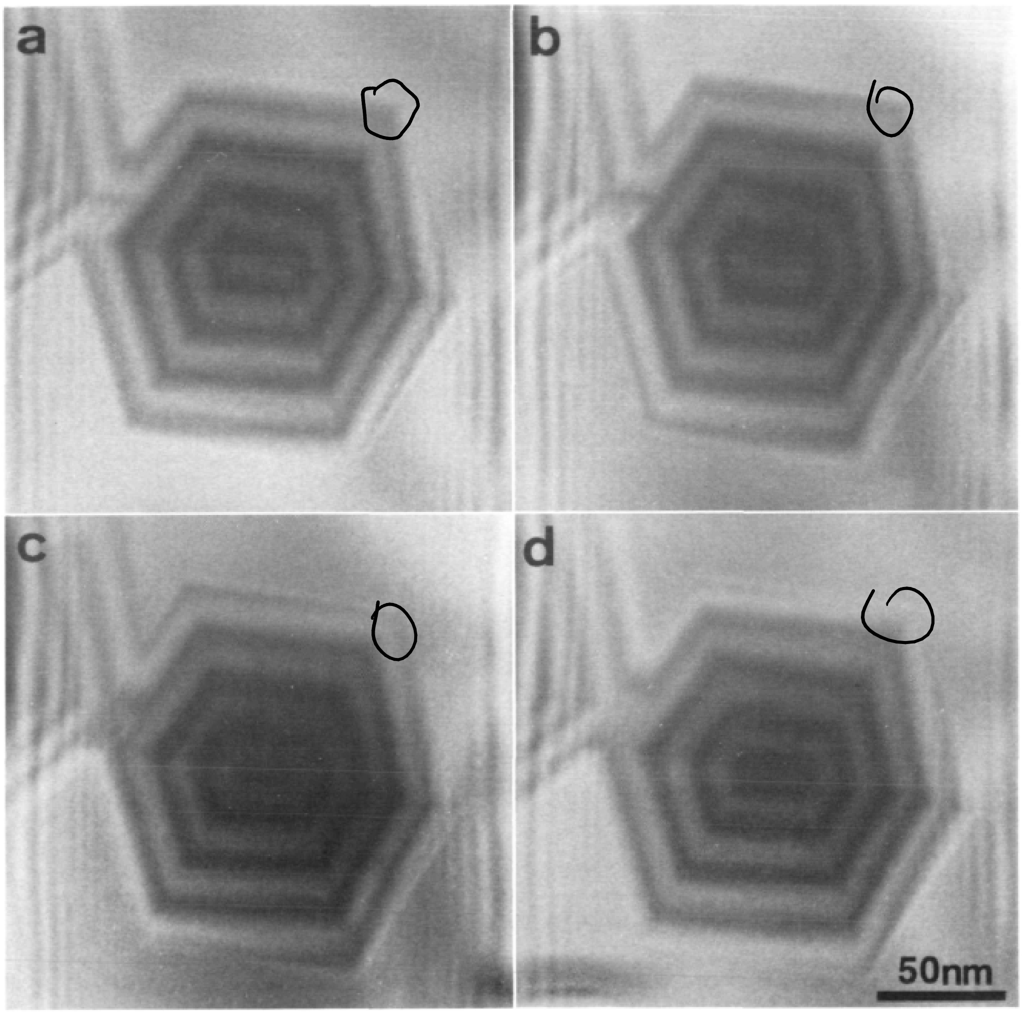
- Encoding phase **and** amplitude in a single image has a price: resolution

  $\rightarrow$ Take more than one image, changing the reference in each.



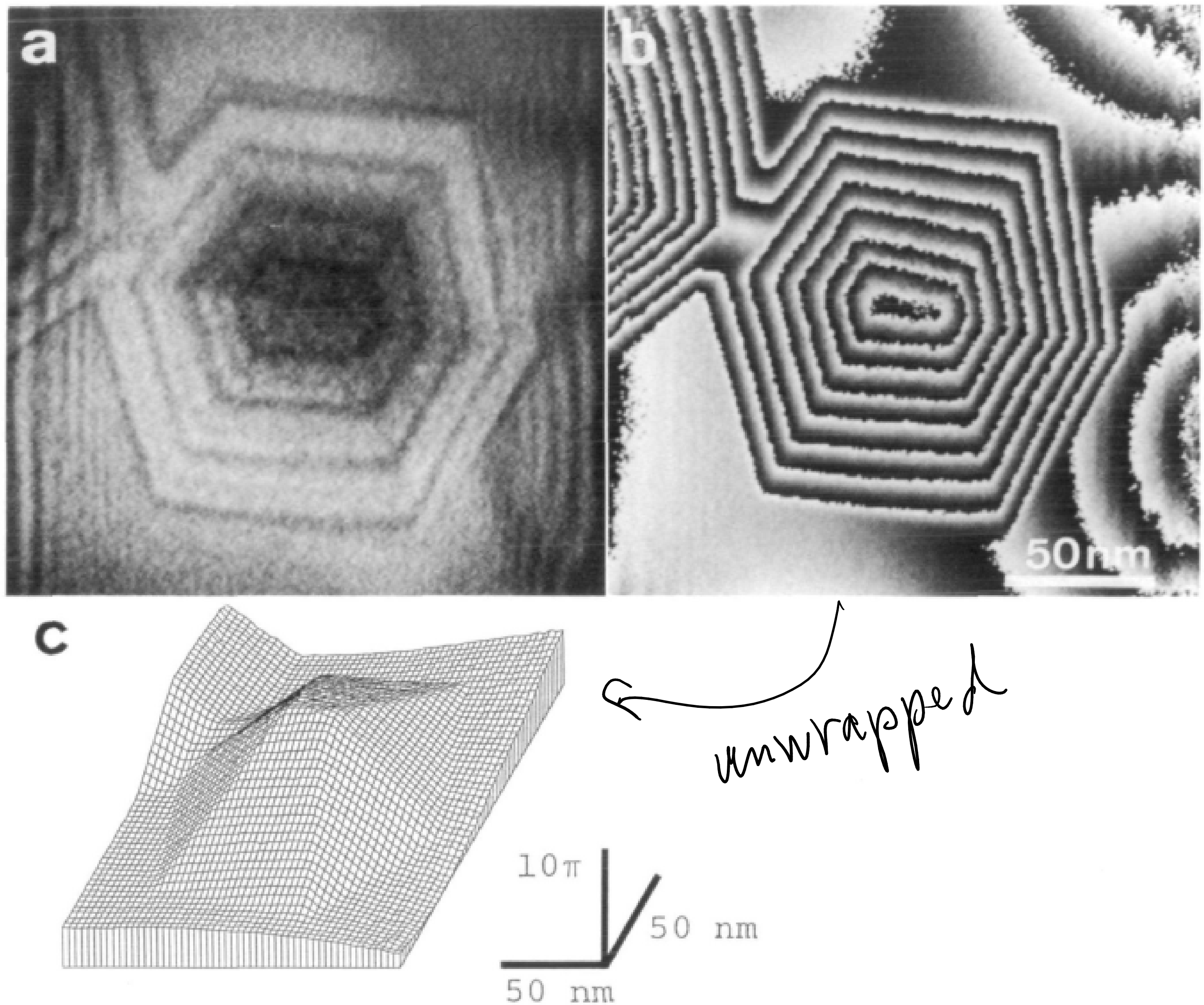
Fringe scanning

Electron microscopy



Source: K. Harada, J. Electron Microsc. **39** 470-476 (1990)

Fringe scanning



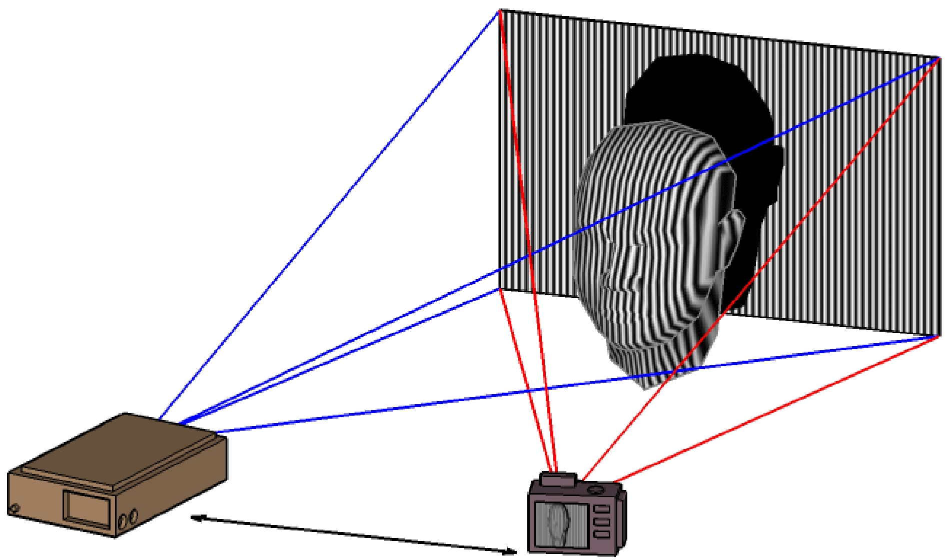
~16π

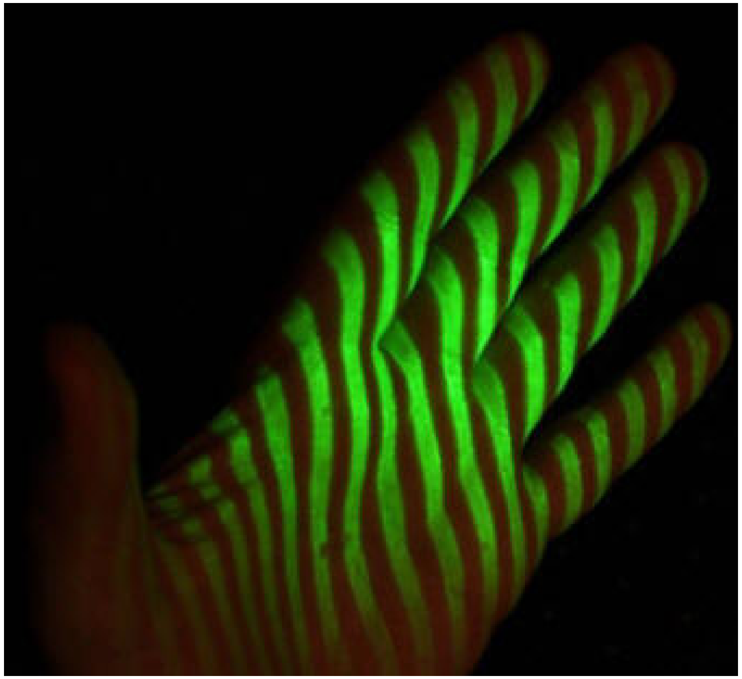
unwrapped

Source: K. Harada, J. Electron Microsc. **39** 470-476 (1990)

Structured light sensing

- Project a structured light pattern onto sample
- Distortions of light pattern allow reconstruction of sample shape





Phase unwrapping

- Phase is measured only in the interval [0, 2π)
- Physical phase shifts (which can be larger) are wrapped on this interval

  $\rightarrow$ Any multiple of 2π is possible

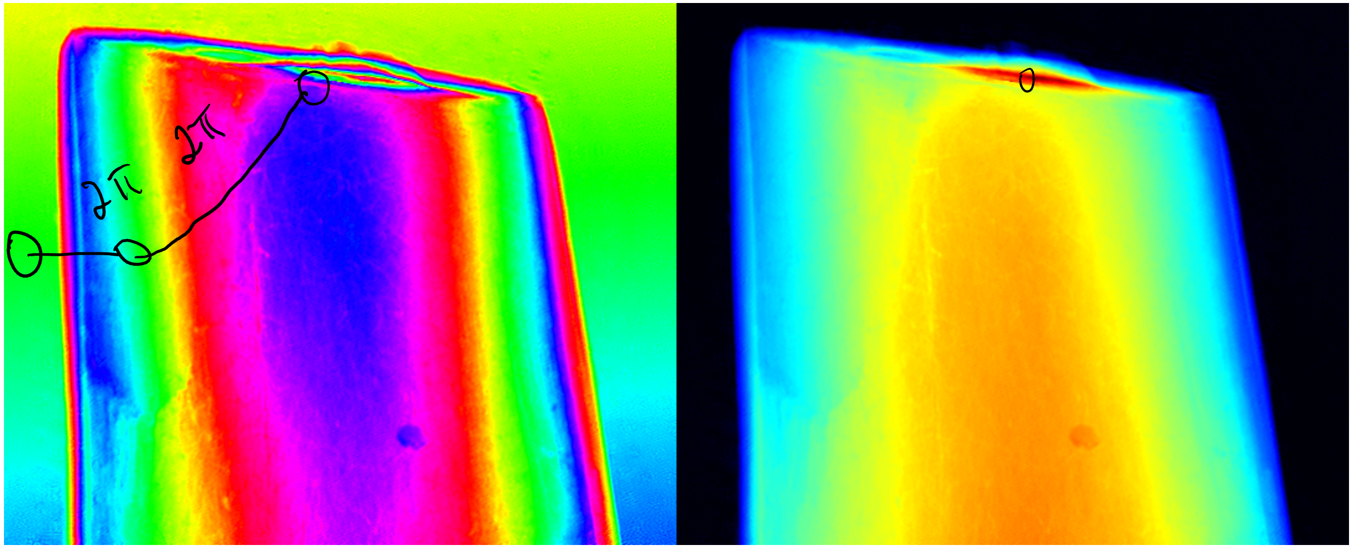
- Unwrapping: use correlations in the image to guess the total phase shift.
- Main difficulties:
  - aliasing: phase shifts are too rapid for the image sampling
  - noise: produces local singularities (vortices)
- Many strategies exist

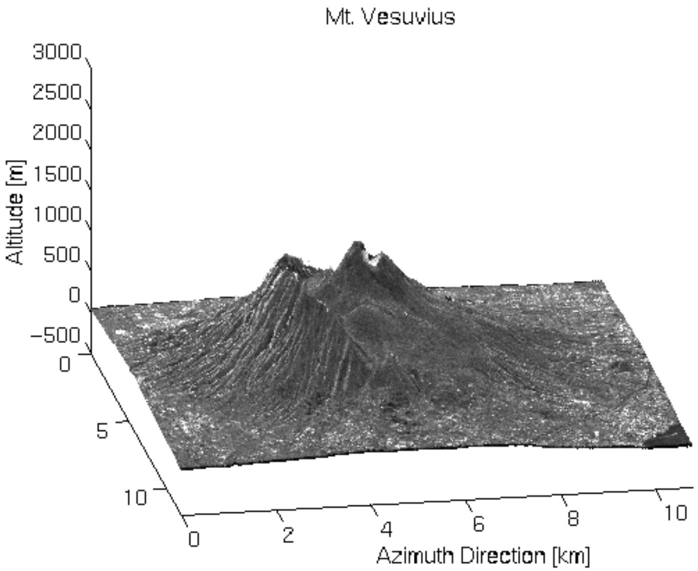
path following methods → unwrap phase pixel by pixel

identify phase vortices and connect them

Complex-valued images

Phase unwrapping

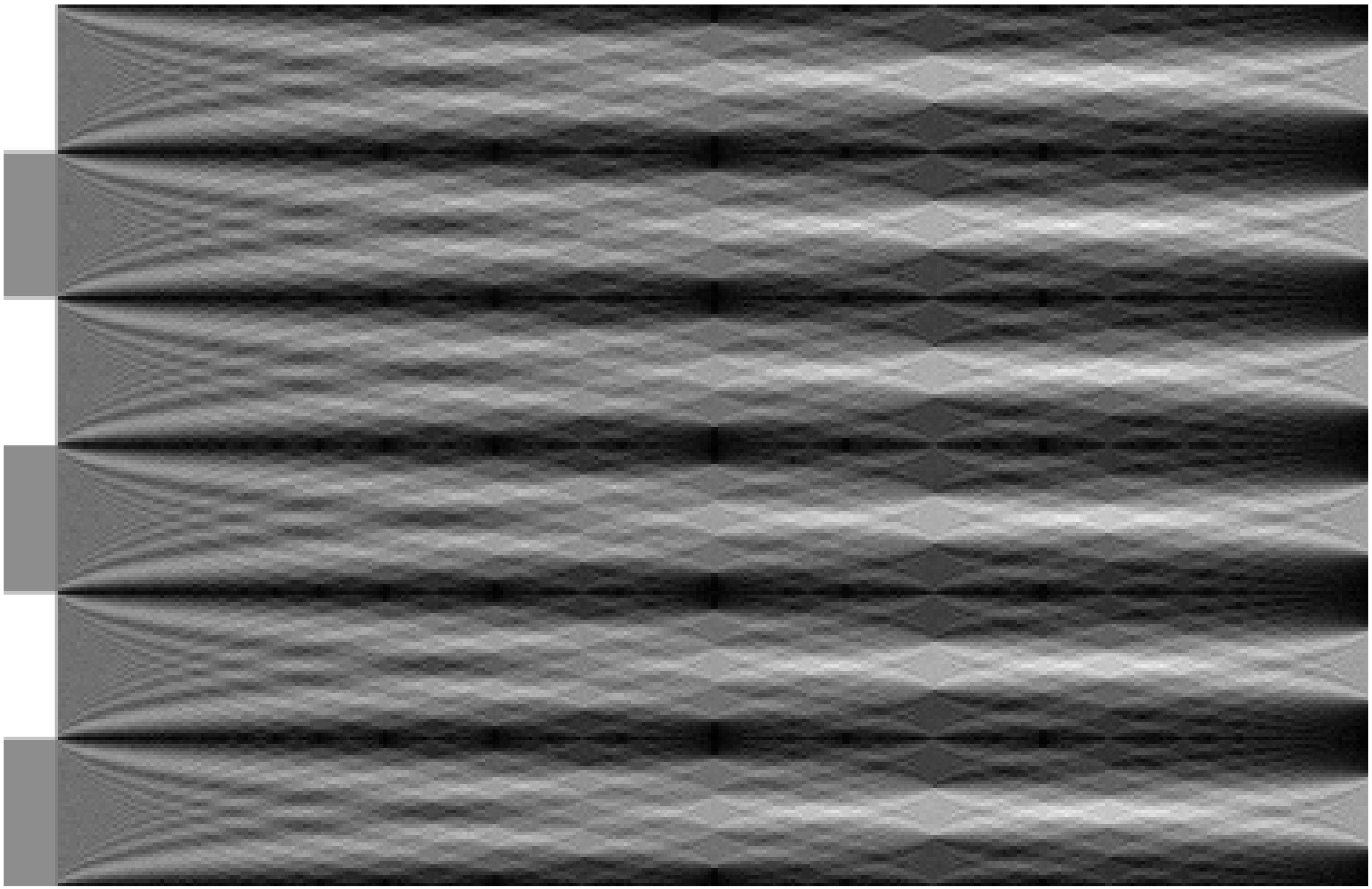




Source: http://earth.esa.int/workshops/ers97/program-details/speeches/rocca-et-al/

Grating interferometry

Diffraction from a grating

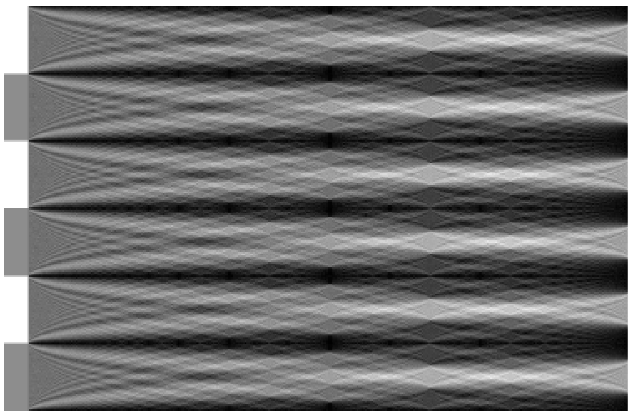


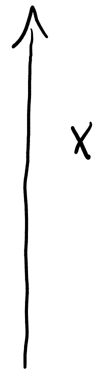
phase grating

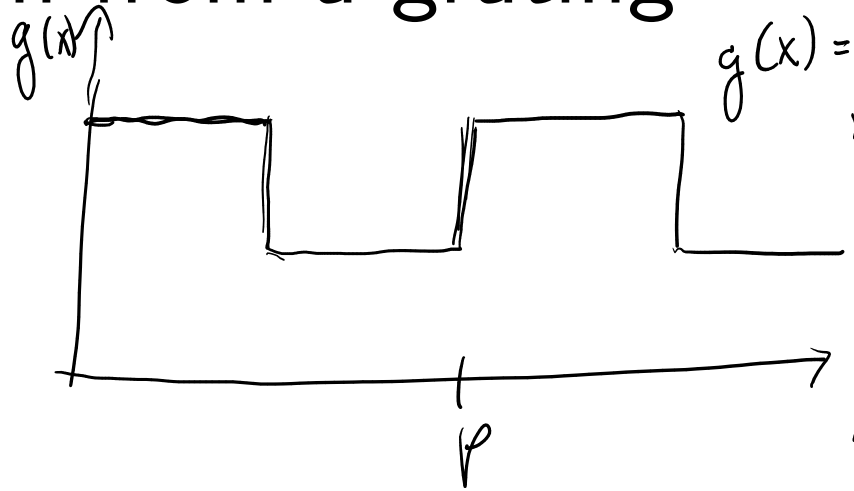
"Talbot carpet"

Grating interferometry

Diffraction from a grating

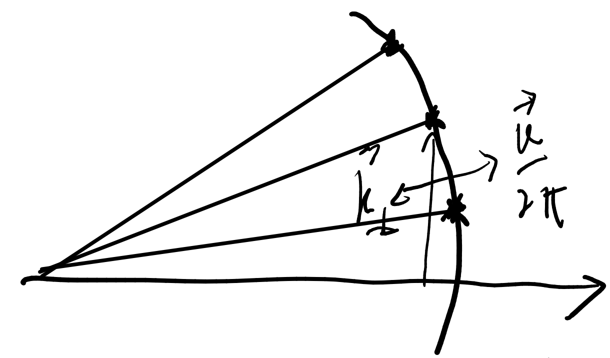


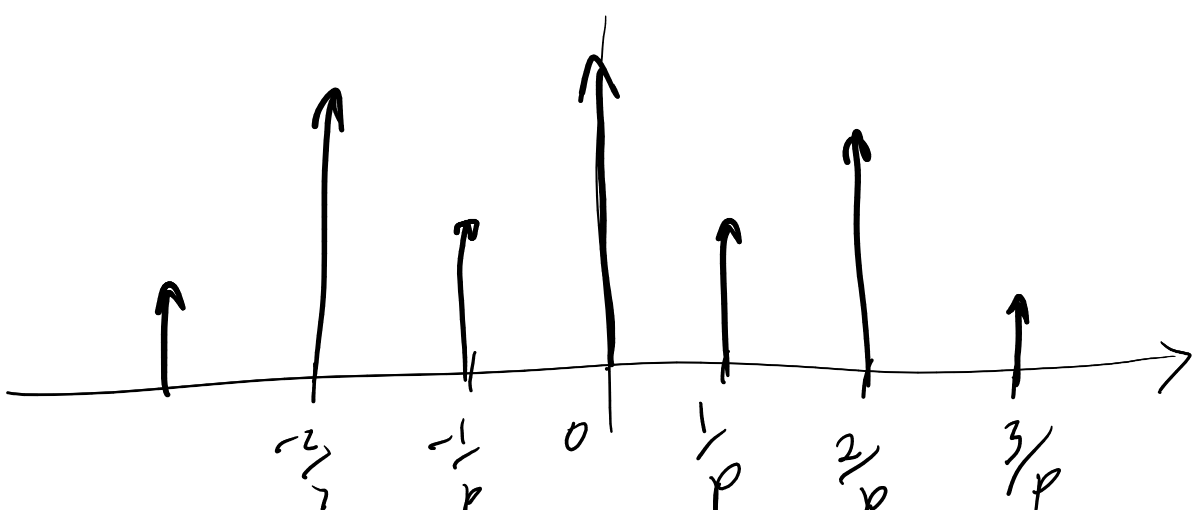




$$g(x) = \sum_{n=-\infty}^{\infty} g_n e^{2\pi i x n/p}$$

$$G(u) = \sum_{n=-\infty}^{\infty} g_n \, \delta(u - n/p)$$

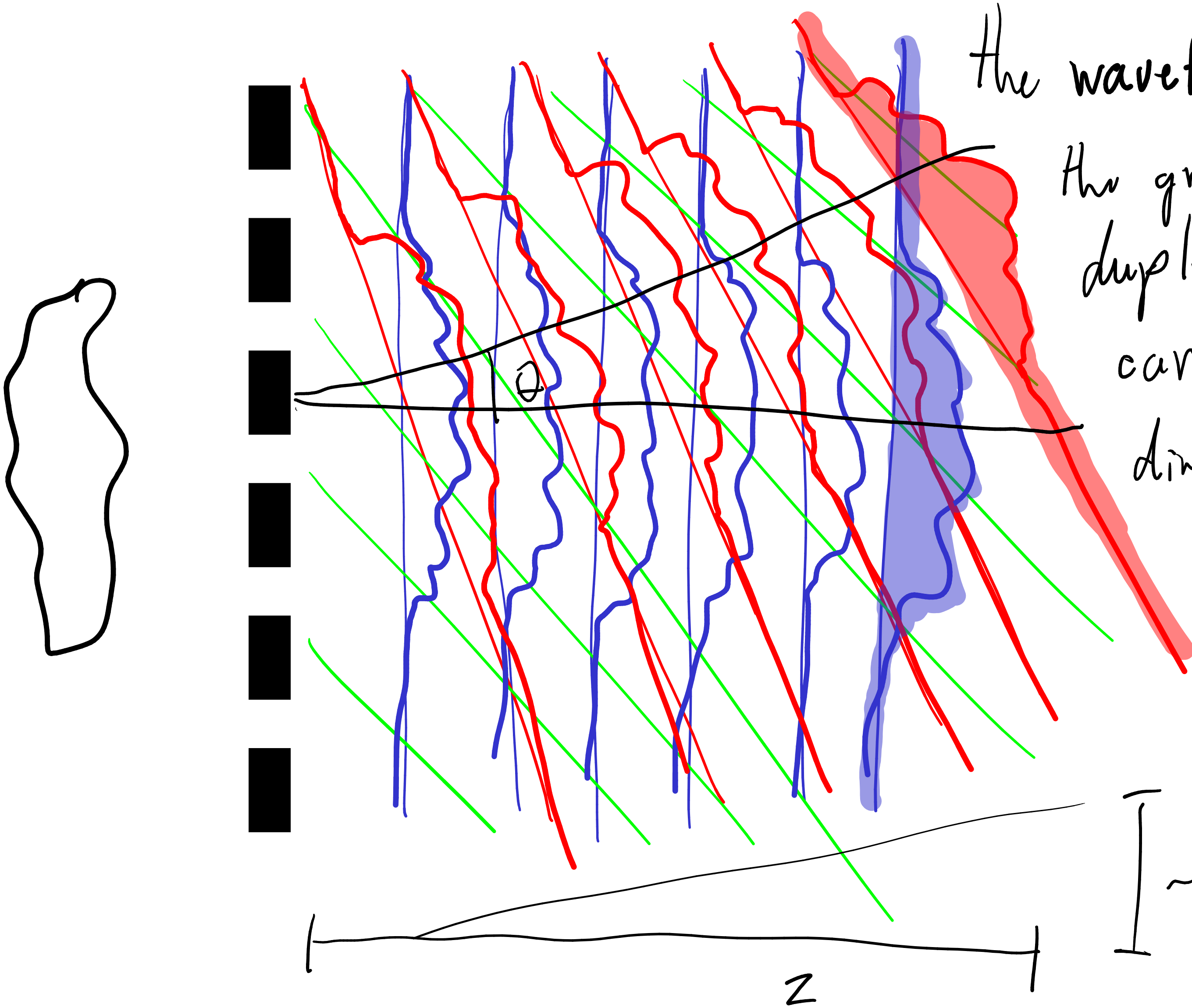




⇒ only a discrete set of plane waves have non-zero amplitude passed the grating

Grating interferometry

Observing the interference between two (slightly offset) copies of the same sample.



the wavefield passed the grating is duplicated and carried in different directions!

$$\sin\theta = \frac{k_\perp}{k} = \frac{\lambda}{p}$$

$$\sim \sin\theta \, z = \frac{\lambda z}{p}$$

Grating interferometry

Observing the interference between two (slightly offset) copies of the same sample.

e.g. if only orders ± 1 are relevant

$$\psi(\vec{r};z) = \psi_0\left(\vec{r} + \frac{\lambda z}{p}\hat{x}\right)e^{2\pi i x/p}$$

$$+ \psi_0\left(\vec{r} - \frac{\lambda z}{p}\hat{x}\right)e^{-2\pi i x/p}$$

$$\psi_0 = a e^{i\varphi}$$

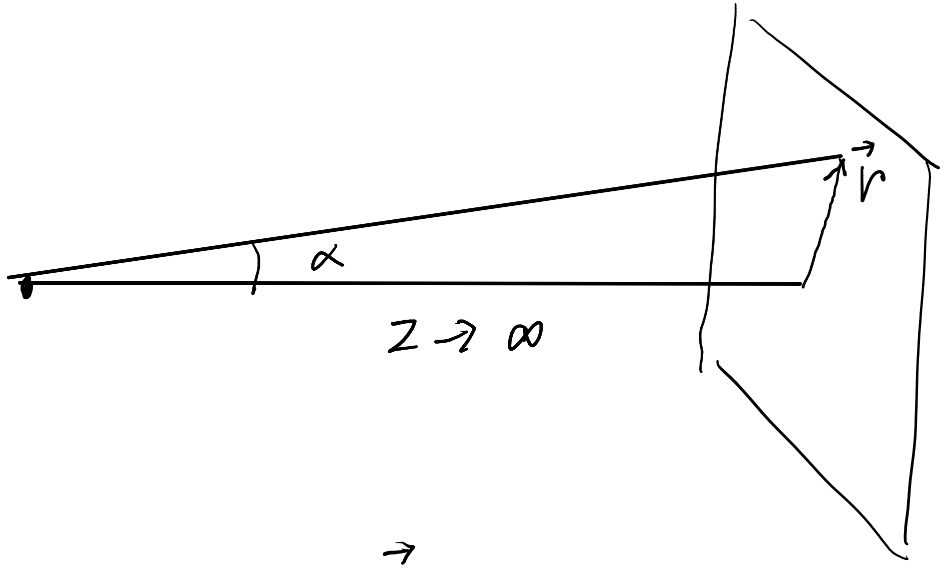
$$I = |\psi(\vec{r};z)|^2 = 2a^2 + 2a\left(\vec{r} + \frac{z\lambda}{p}\hat{x}\right)a\left(\vec{r} - \frac{\lambda z}{p}\hat{x}\right)$$

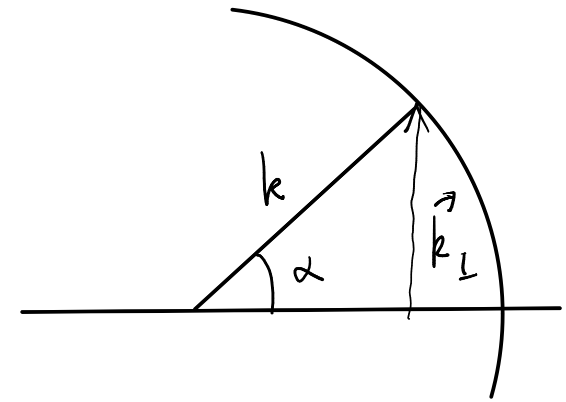
$$\cos\Bigg[\underbrace{\varphi\left(\vec{r} + \frac{\lambda z}{p}\hat{x}\right) - \varphi\left(\vec{r} - \frac{z\lambda}{p}\hat{x}\right)}_{\sim 2\frac{z\lambda}{p}\frac{\partial}{\partial x}\varphi} + \frac{4\pi x}{p}\Bigg]$$

"differential phase-contrast"

Far-field diffraction

The Fraunhofer regime



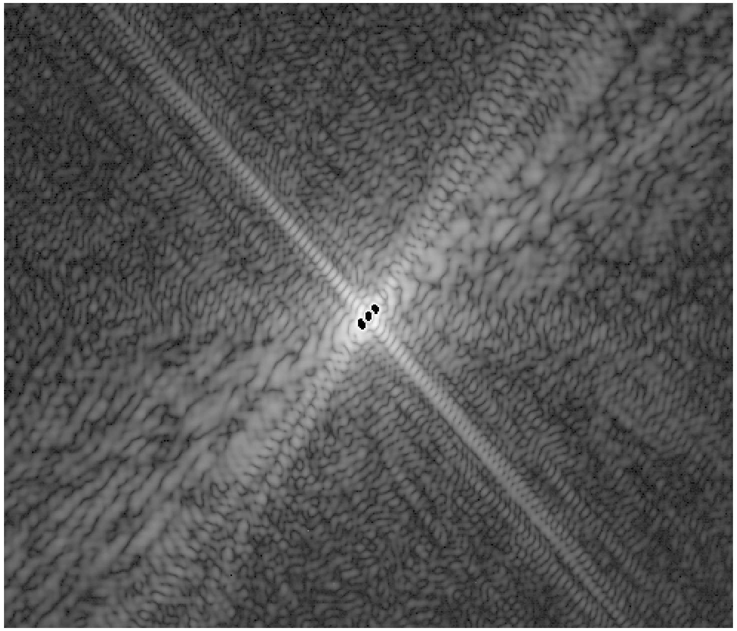


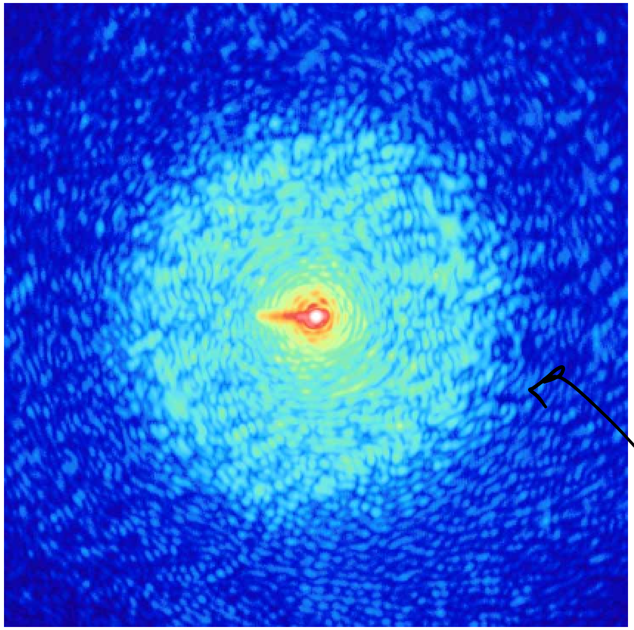
$$\frac{\vec{r}}{z} \simeq \frac{\vec{k}_\perp}{k} = \frac{2\pi\vec{u}}{2\pi/\lambda} = \lambda\vec{u}$$

$$\psi(\vec{r}, z) = \frac{-2\pi i}{\lambda z} \exp\left(\frac{i\pi r^2}{\lambda z}\right) \int d^2r' \, \psi(\vec{r}\,') \exp\Big(-2\pi i \vec{r}\,' \cdot \underbrace{\vec{r}/\lambda z}_{\vec{u}}\Big)$$

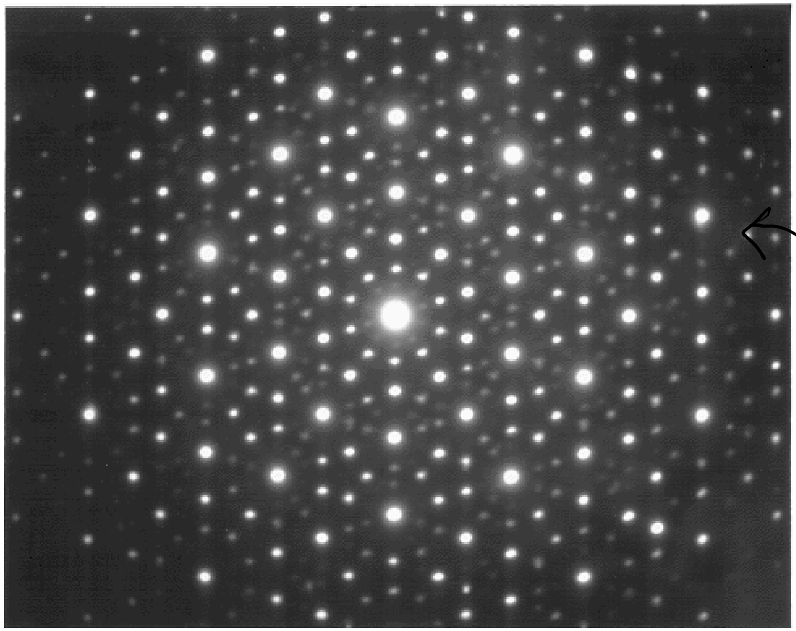
$$|\psi(r, z\to\infty)|^2 \propto |\mathcal{F}\psi|^2 = I(\vec{u})$$

Diffraction patterns

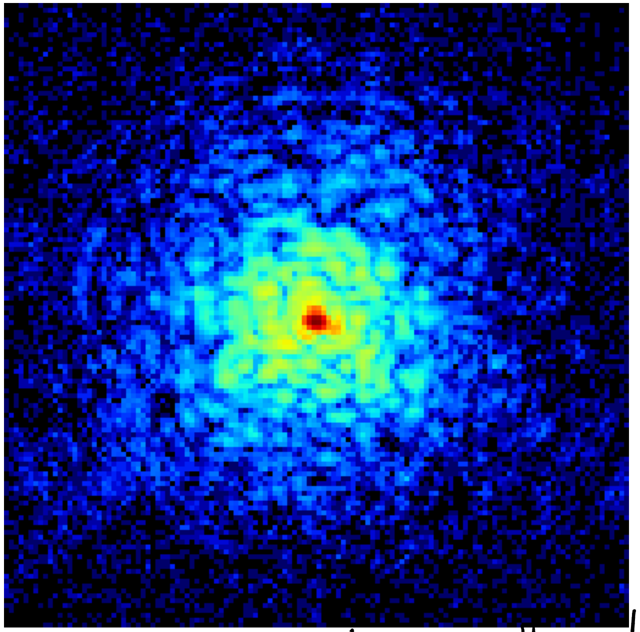




speckles



Bragg peaks



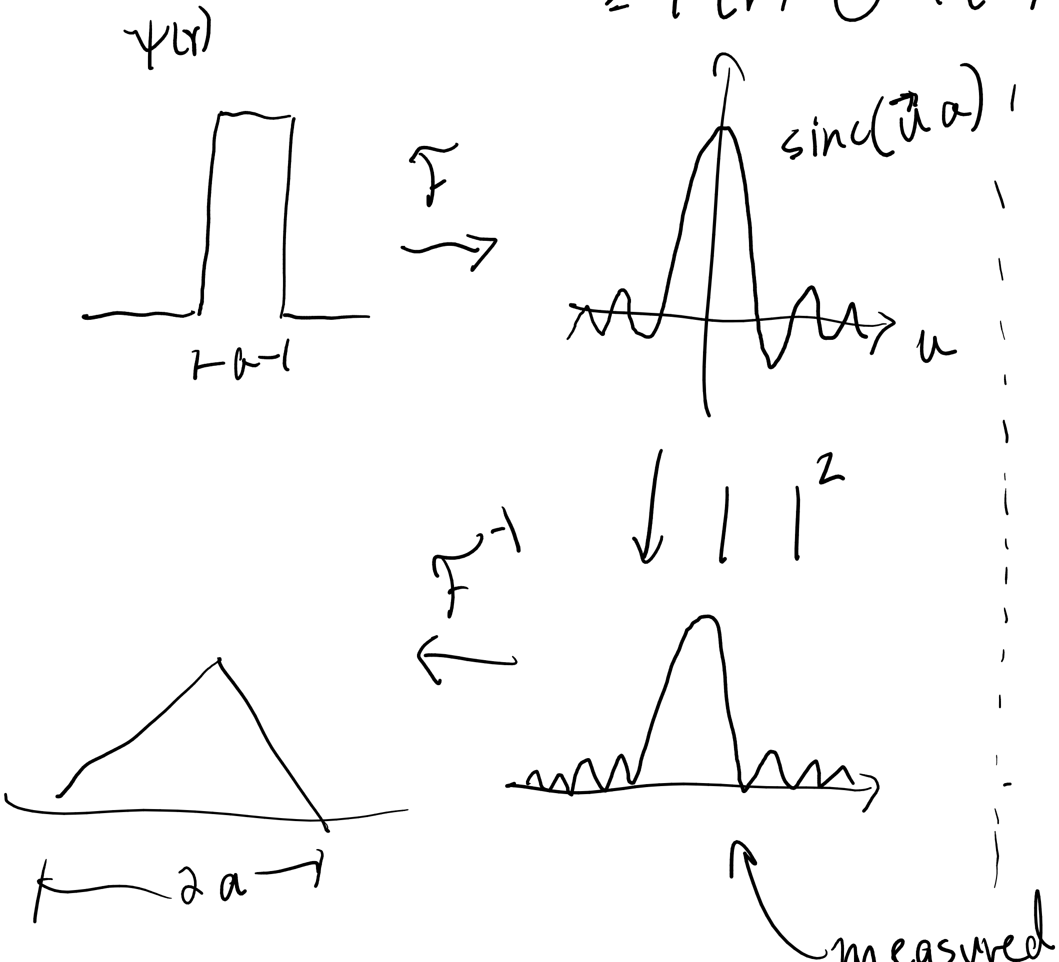
Question: can we reconstruct the objects that generated these?

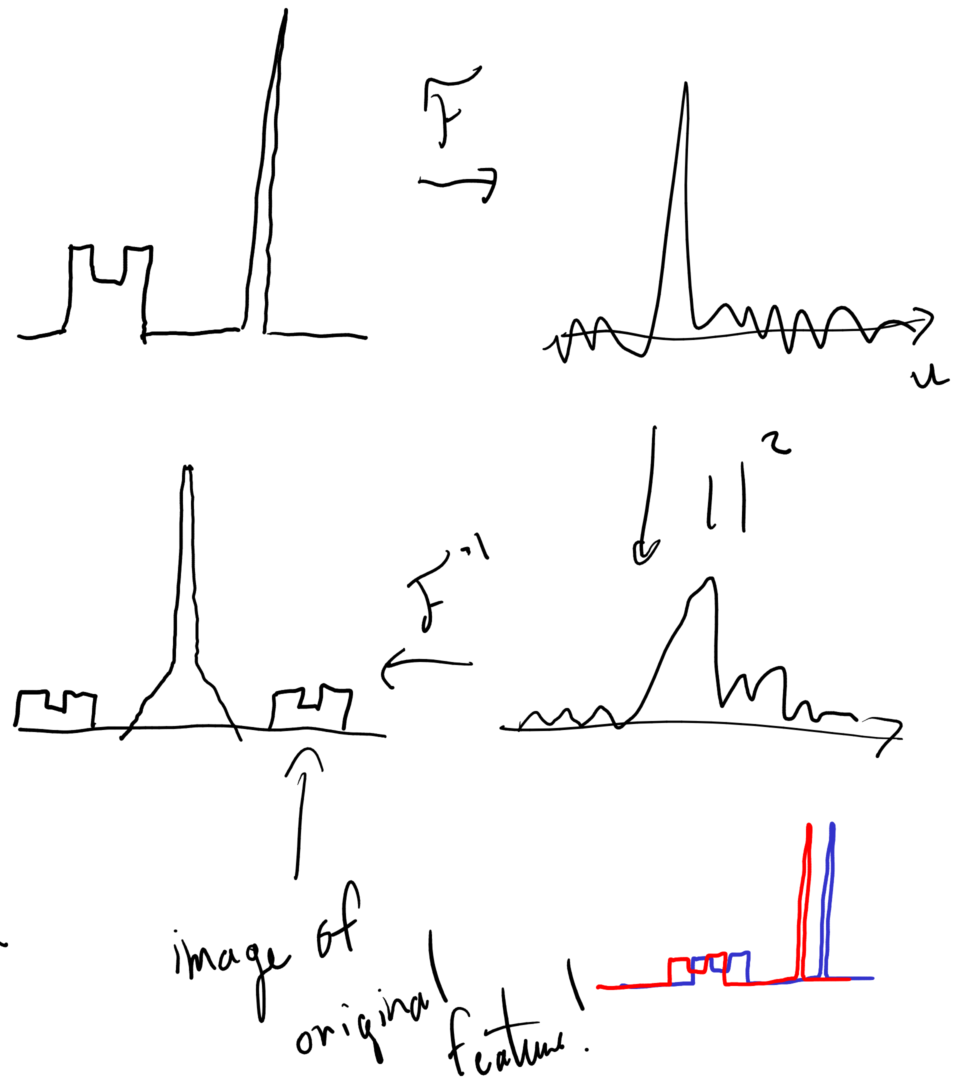
Diffraction and autocorrelation

$$\mathcal{F}^{-1}\{I(\vec{u})\}$$

$$= \mathcal{F}^{-1}\{\psi(\vec{u}) \cdot \psi^*(\vec{u})\}$$

$$= \psi(\vec{r}) \circledast \psi(\vec{r}) \leftarrow \text{autocorrelation}$$





Fourier transform holography

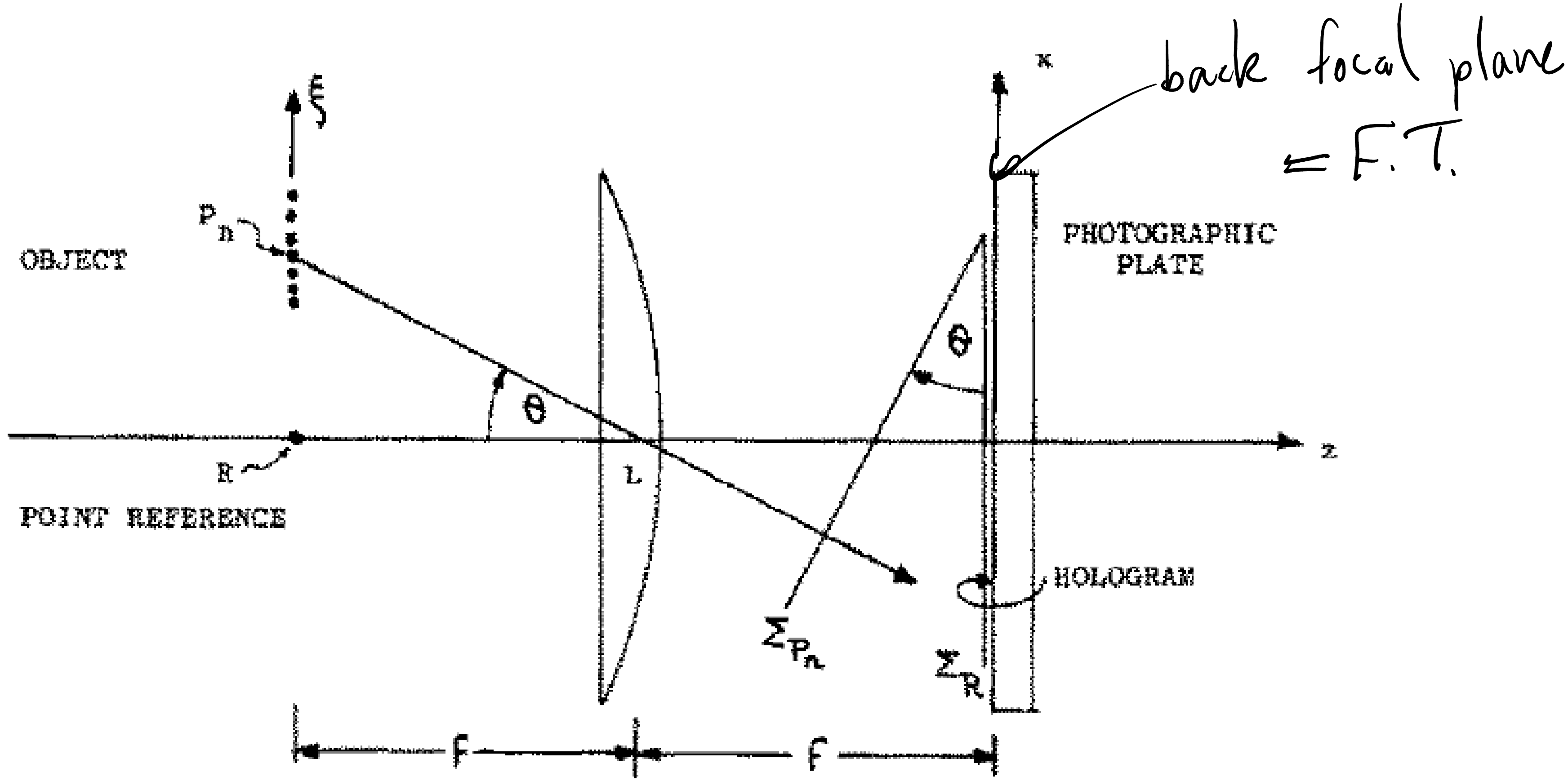
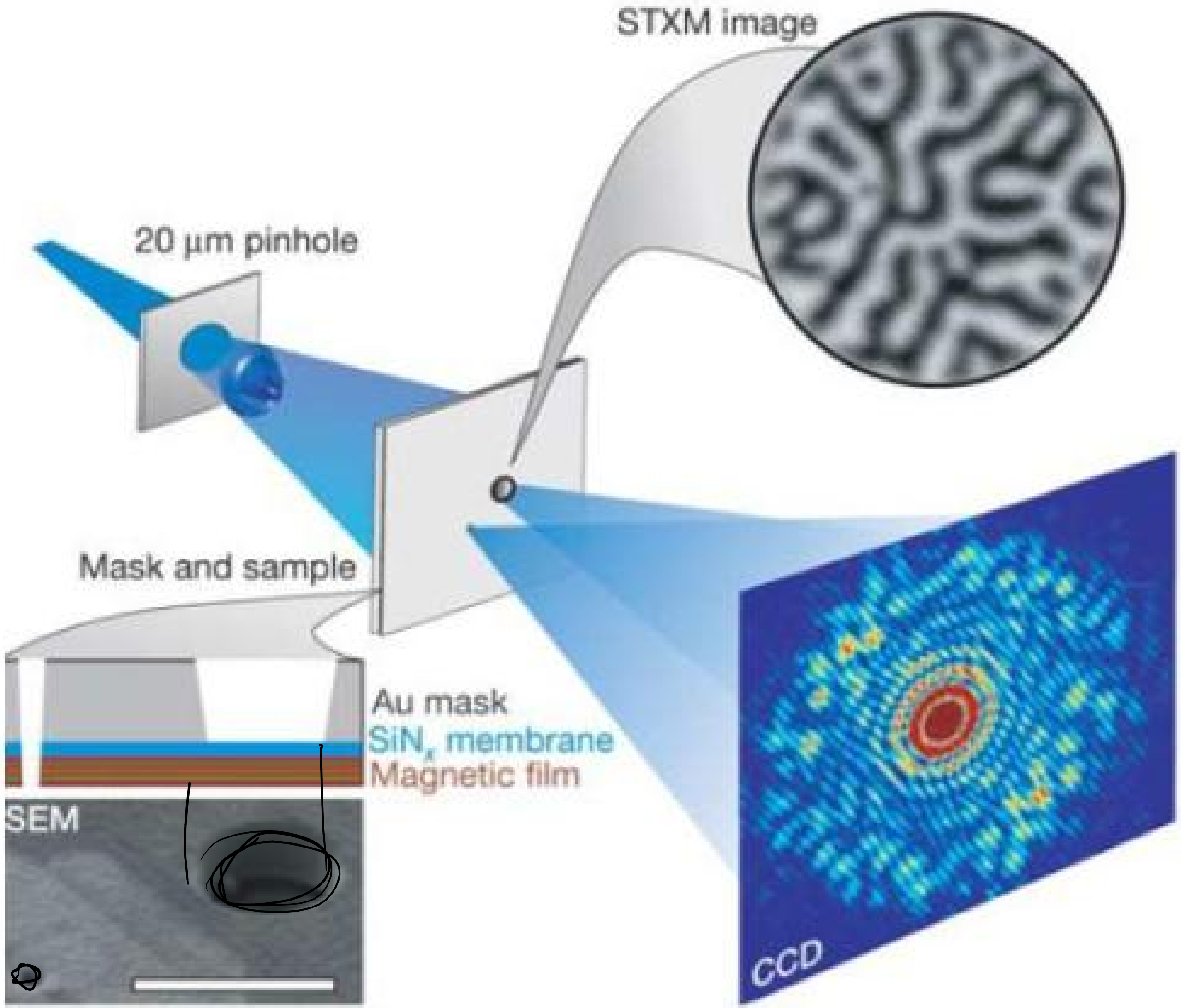


Fig. 1. Recording of a Fourier-transform hologram with a lens L. Σ_R = reference wavefront.

Source: G. Stroke, Appl. Phys. Lett. **6**, 201-203 (1965).

Fourier transform holography



Source: S. Eisebitt et al., Nature **432**, 885-888 (2004).

Fourier transform holography

$$\psi(\vec{r}) = \psi_r(\vec{r}) + \psi_o(\vec{r})$$

F.T. ↓

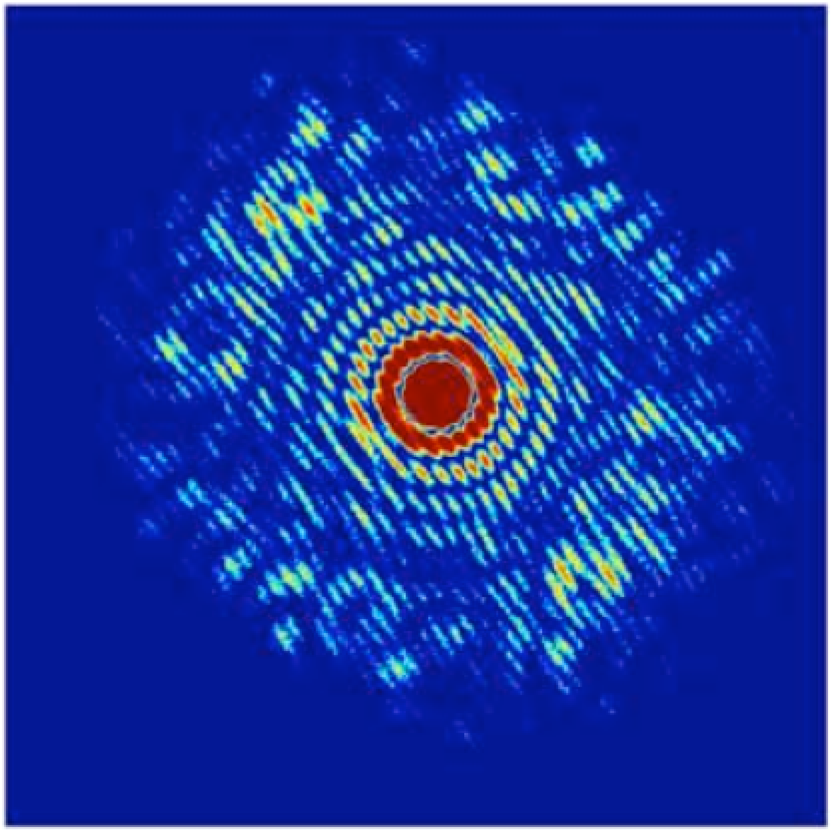
$$\tilde{\psi}(\vec{u}) = \tilde{\psi}_r(\vec{u}) + \tilde{\psi}_o(\vec{u})$$

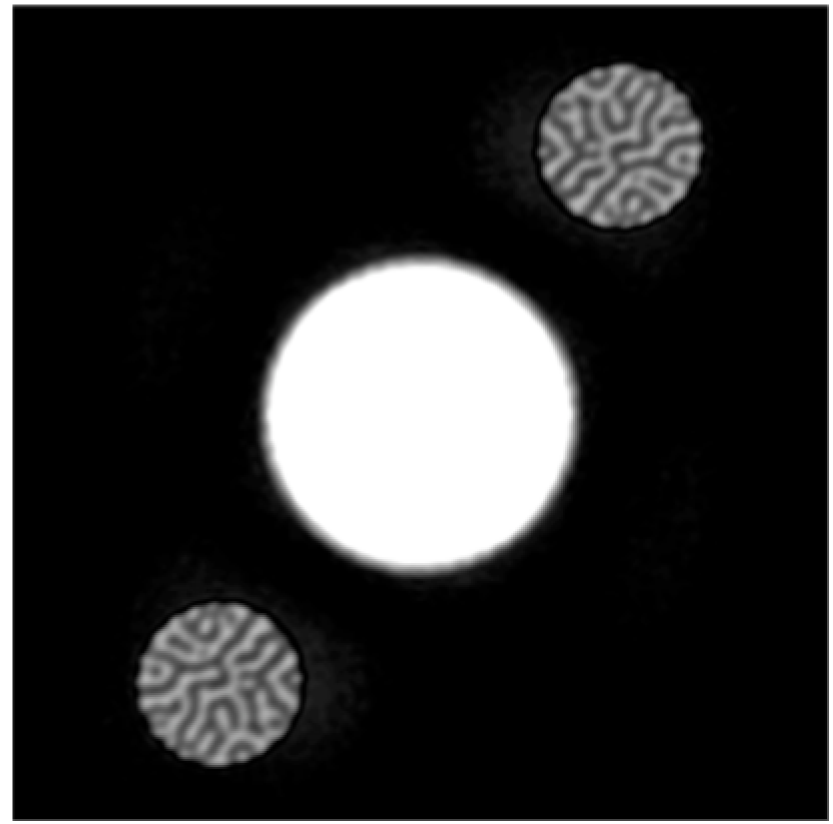
$$I(\vec{u}) = |\psi_r(\vec{u})|^2 + |\psi_o(\vec{u})|^2 + \psi_r(\vec{u})\psi_o^*(\vec{u}) + \text{c.c.}$$

$$\mathcal{F}^{-1}\{I(\vec{u})\} = \psi_r \circledast \psi_r + \psi_o \circledast \psi_o + \underbrace{\psi_r \circledast \psi_o^* + \psi_o \circledast \psi_r^*}_{\text{cross-correlations!}}$$

Fourier transform holography

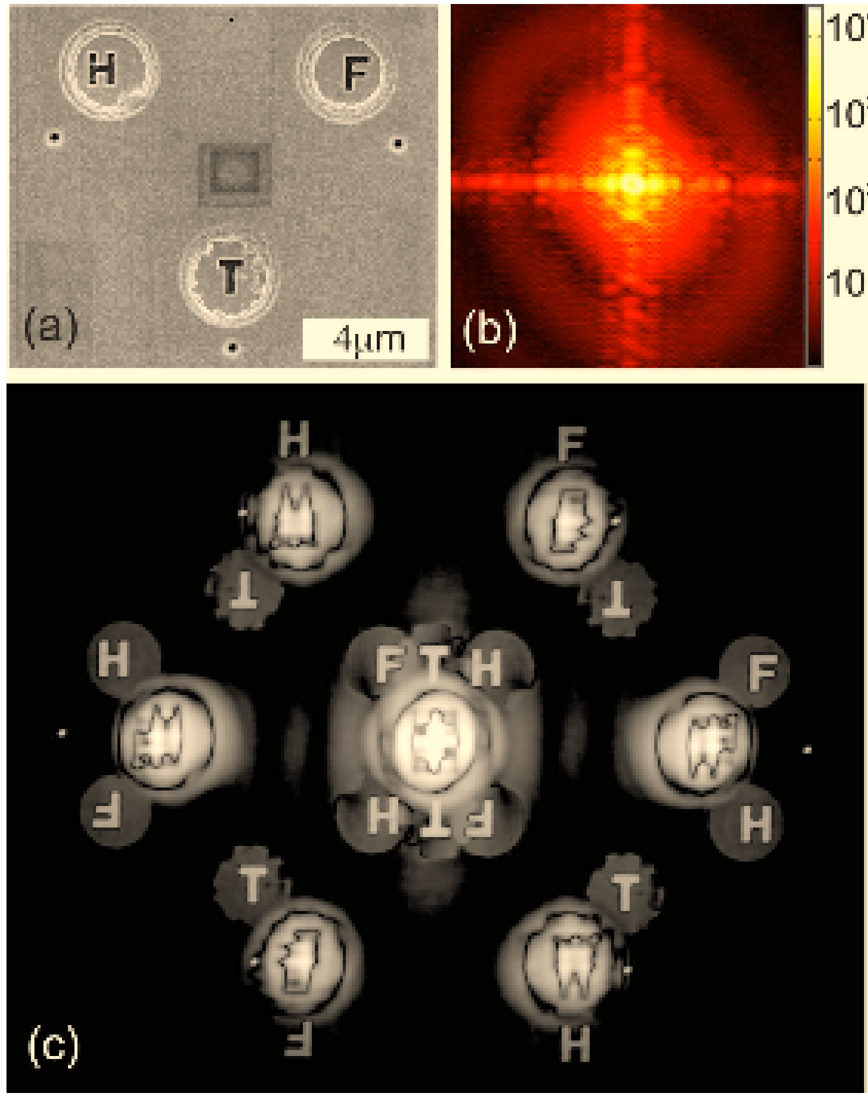
image of sample!





Fourier transform holography

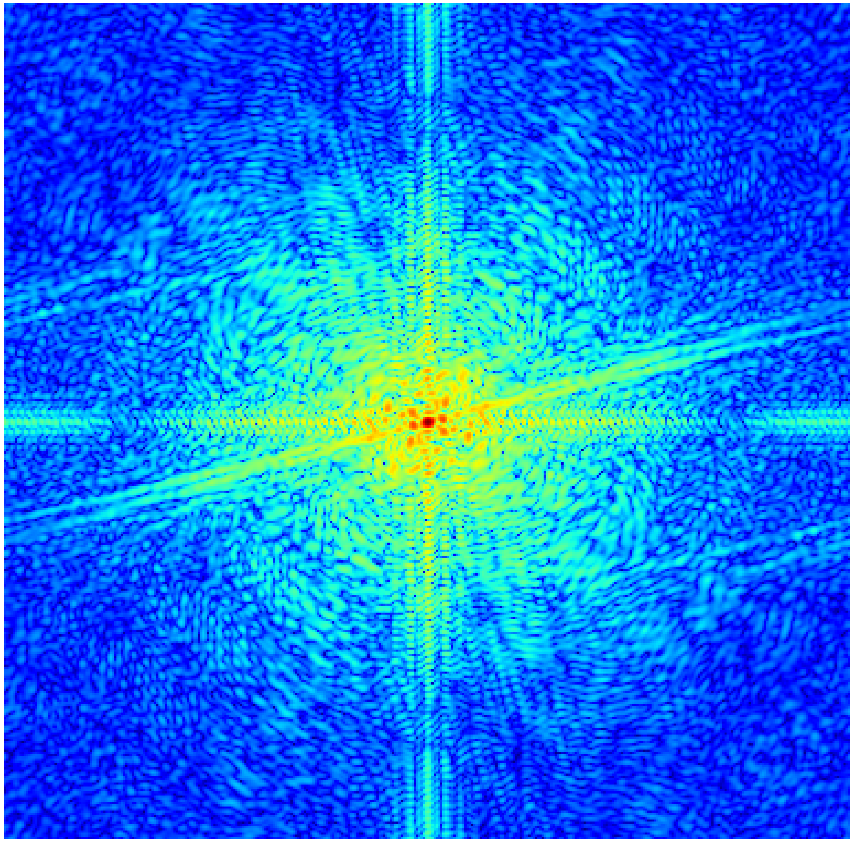
Multiple references



Source: W. Schlotter et al., Opt.. Lett. **21**, 3110-3112 (2006).

Coherent diffractive imaging

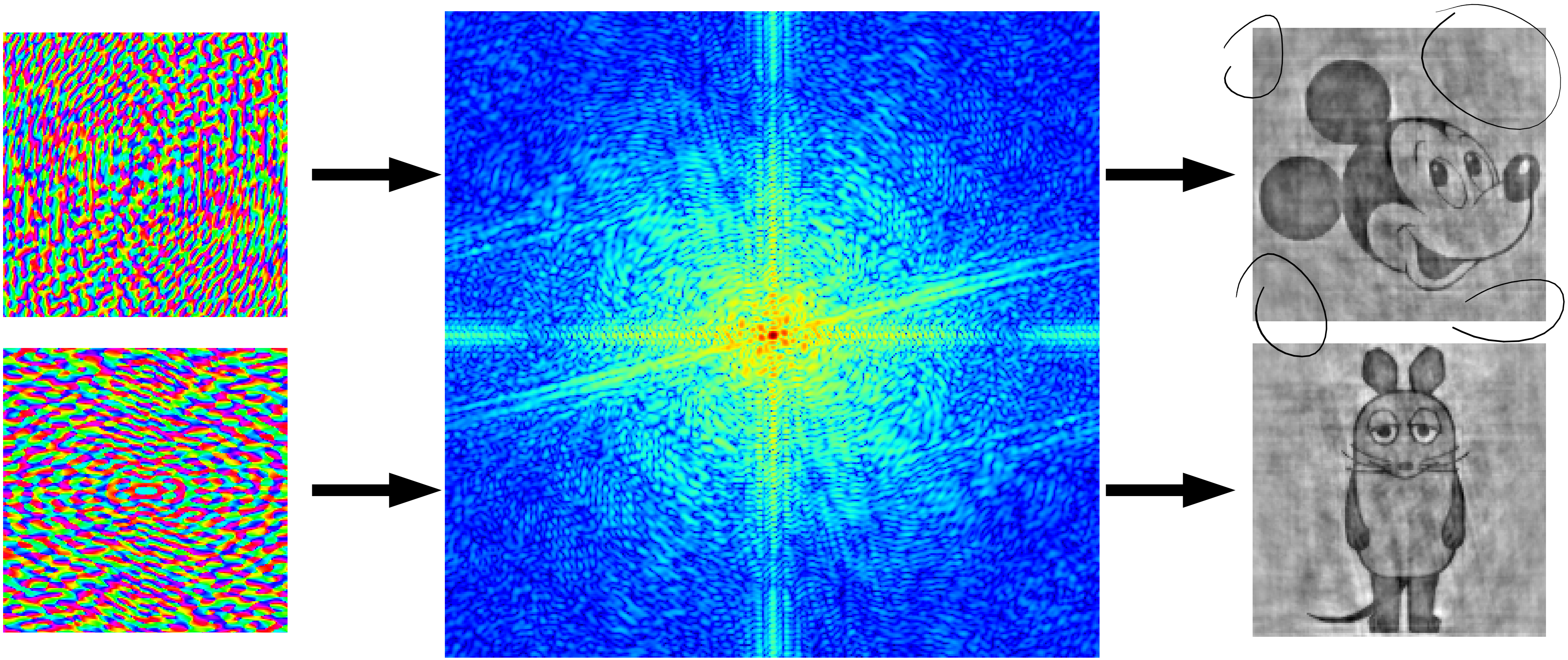
Diffraction pattern of an isolated sample



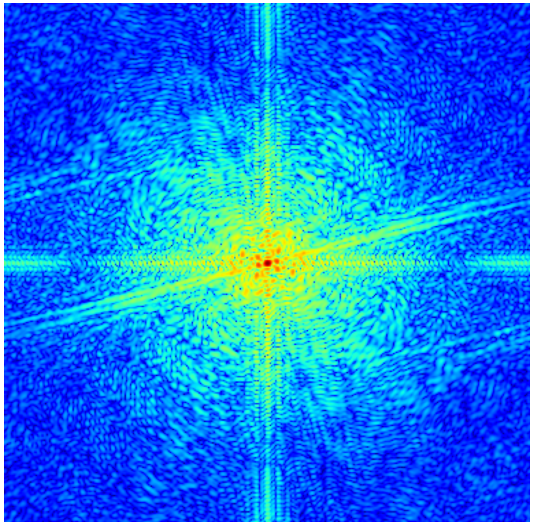
The phase problem

$e^{i\varphi(\vec{u})}$

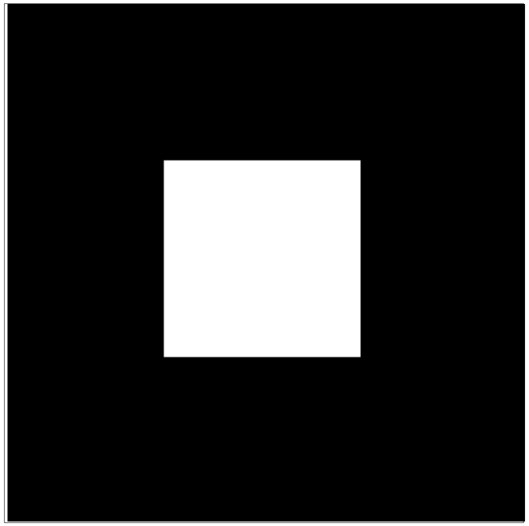
$\mathcal{F}^{-1}\left\{\sqrt{I(\vec{u})} \cdot e^{i\varphi(\vec{u})}\right\}$



Coherent diffractive imaging

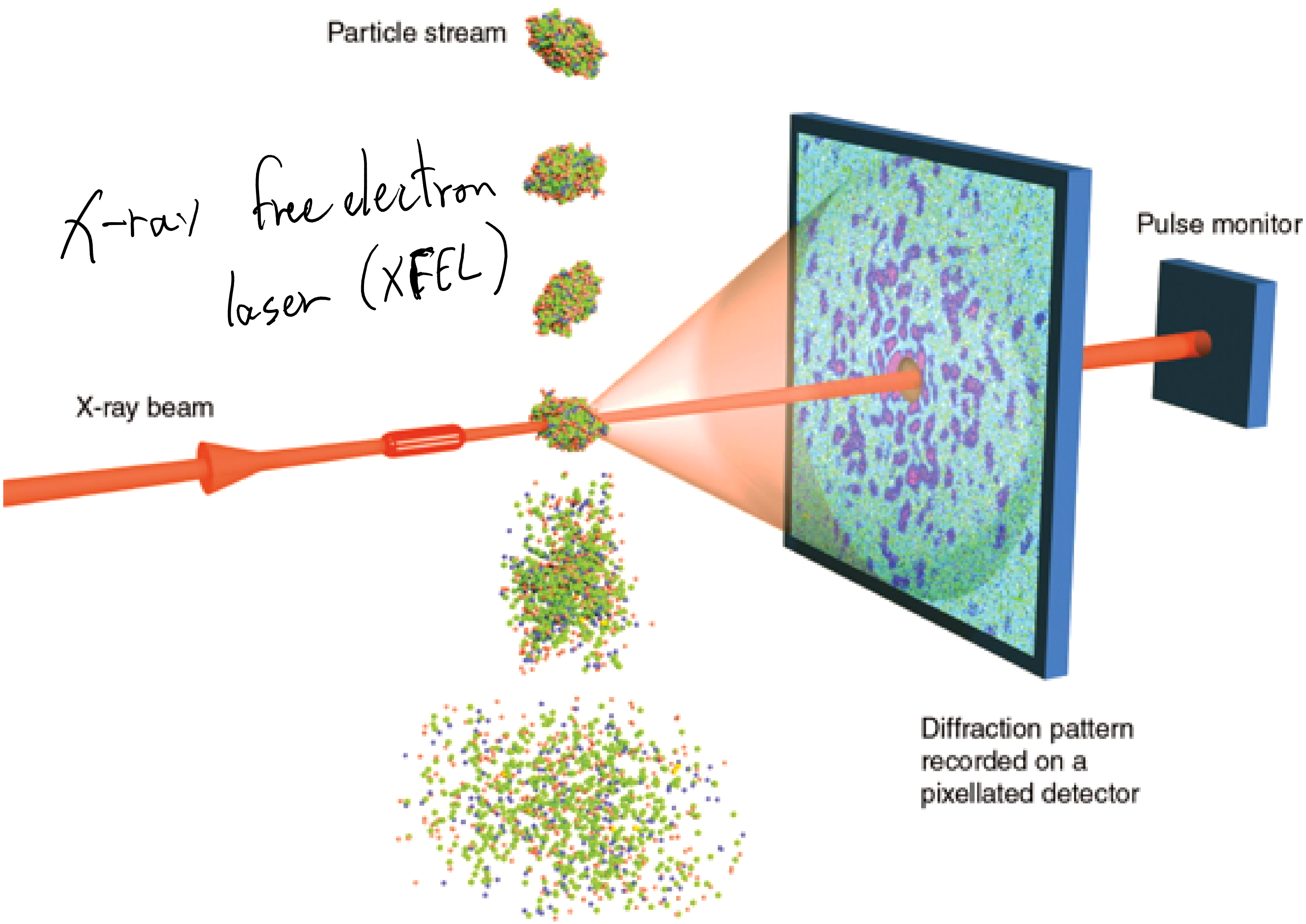


1. Solution has to be consistent with the measured Fourier amplitudes



2. Solution is isolated

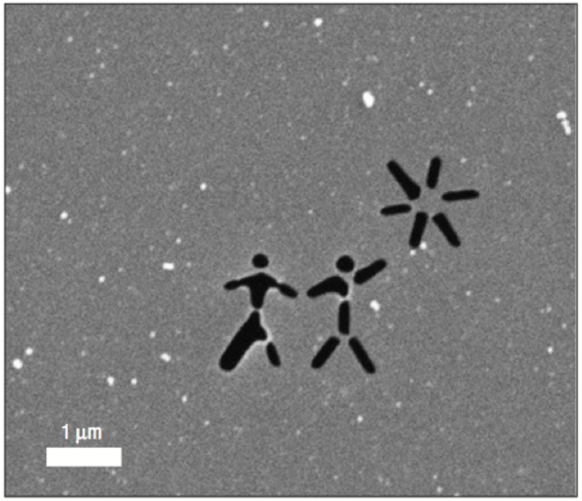
Radiation damage limits on radiation

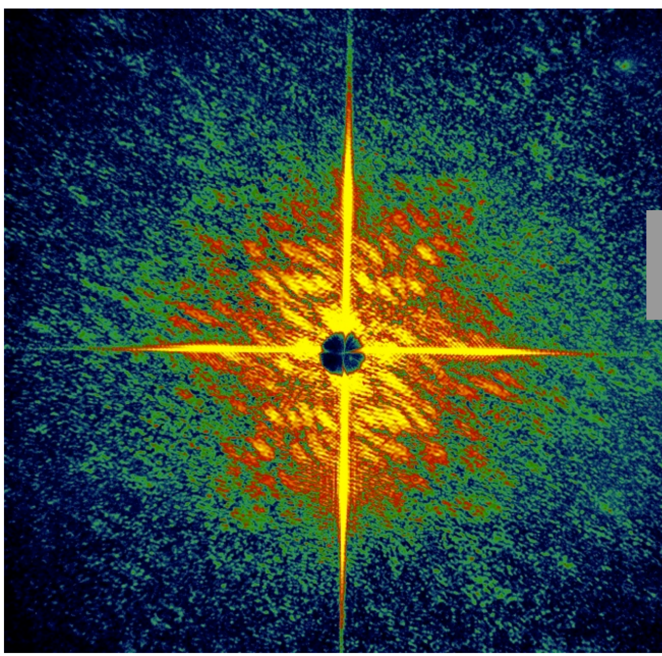


R. Neutze *et al*, Nature **406**, 752 (2000)

K. J. Gaffney *et al*, Science **316**, 1444 (2007)

“Diffraction before destruction”

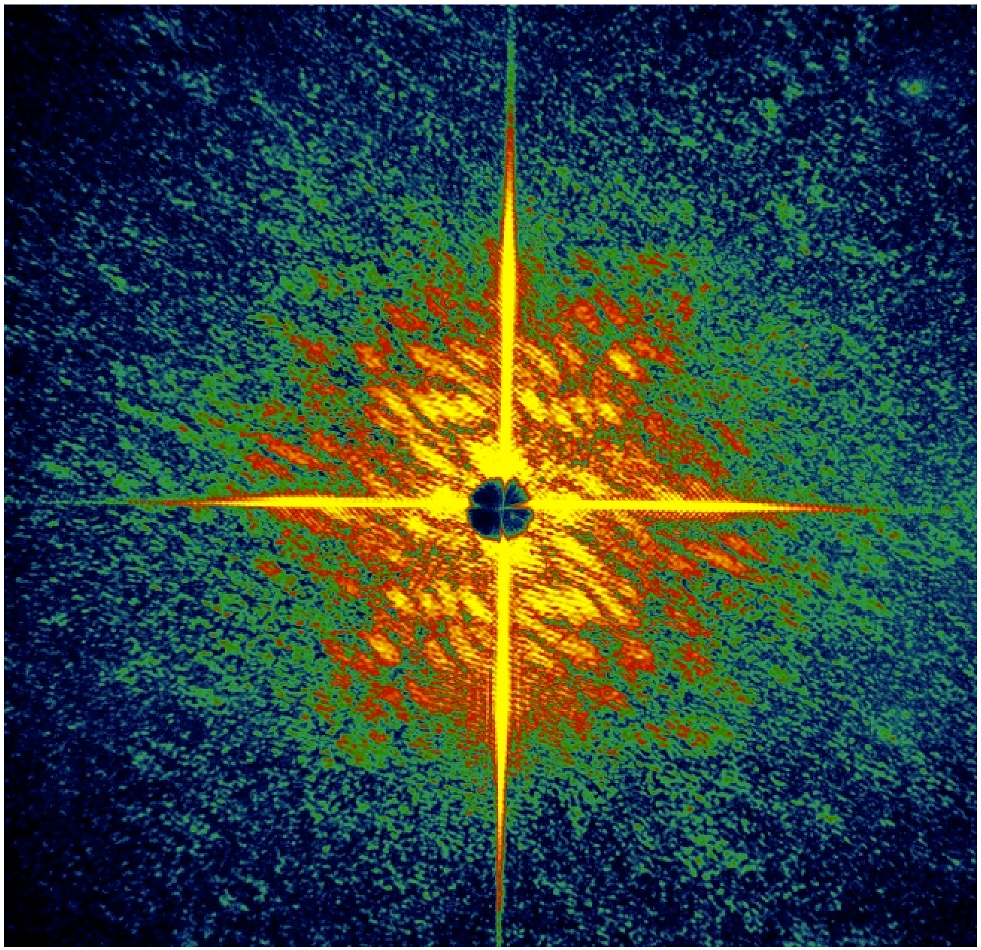


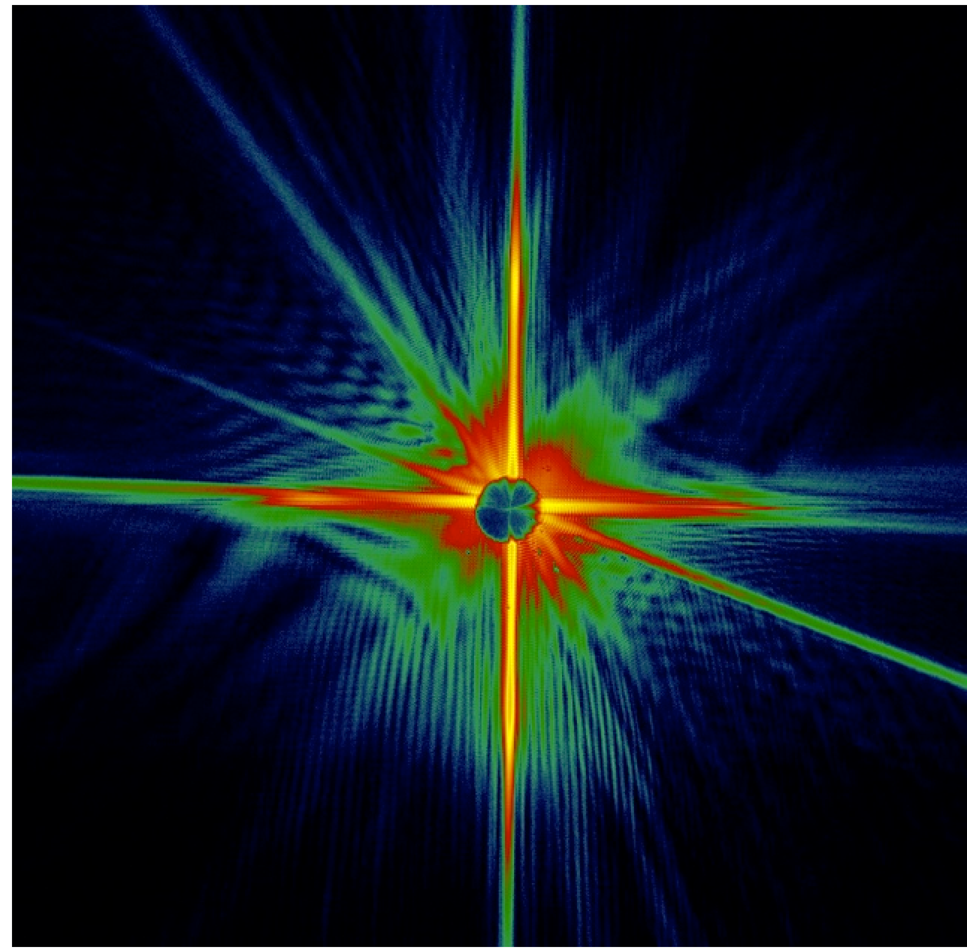


H. N. Chapman *et al*, Nat. Phys. **2**, 839 (2006)

"Diffraction before destruction"

The imaging pulse vaporized the sample





H. N. Chapman *et al*, Nat. Phys. **2**, 839 (2006)

Ptychography

- Scanning an isolated illumination on an extended specimen
- Measure full coherent diffraction pattern at each scan point
- Combine everything to get a reconstruction

Dynamische Theorie der Kristallstrukturanalyse durch Elektronenbeugung im inhomogenen Primärstrahlwellenfeld

1970

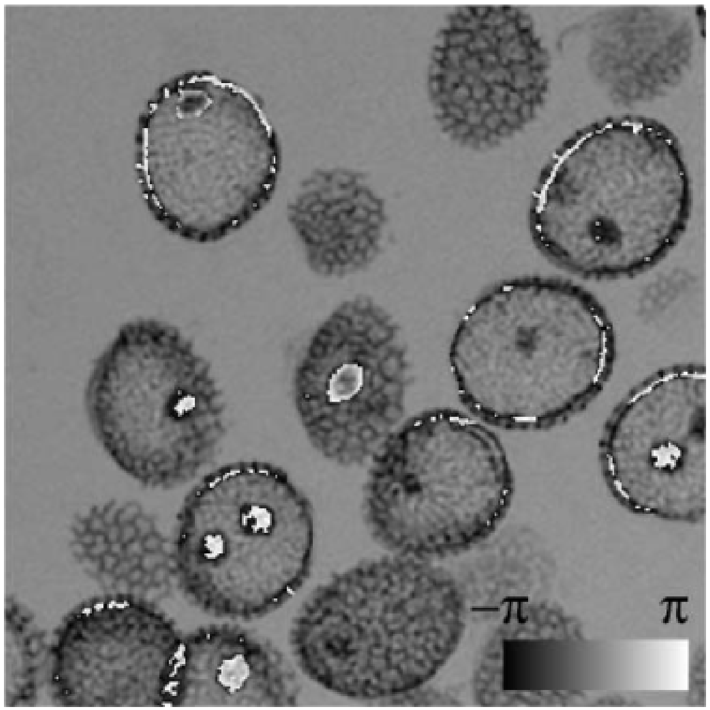
Von R. Hegerl und W. Hoppe

Some time ago a new principle was proposed for the registration of the complete information (amplitudes and phases) in a diffraction diagram, which does not – as does Holography – require the interference of the scattered waves with a single reference wave. The basis of the principle lies in the interference of neighbouring scattered waves which result when the object function $\varrho(x, y)$ is multiplied by a generalized primary wave function $p(x, y)$ in Fourier space (diffraction diagram) this is a convolution of the Fourier transforms of these functions. The above mentioned interferences necessary for the phase determination can be obtained by suitable choice of the shape of $p(x, y)$. To distinguish it from holography this procedure is designated "ptychography" ($\pi\tau\upsilon\xi$ = fold). The procedure is applicable to periodic and aperiodic structures. The relationships are simplest for plane lattices. In this paper the theory is extended to space lattices both with and without consideration of the dynamic theory. The resulting effects are demonstrated using a practical example.

Ptychography

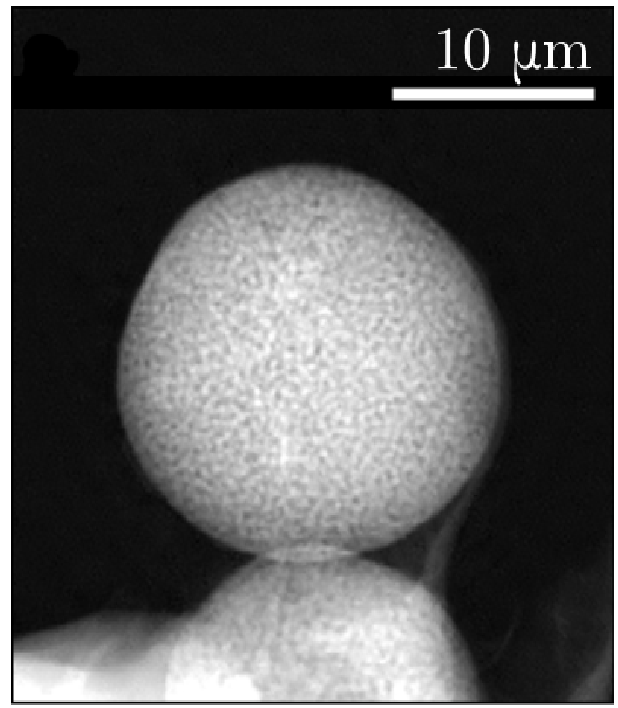
A few examples

Visible light



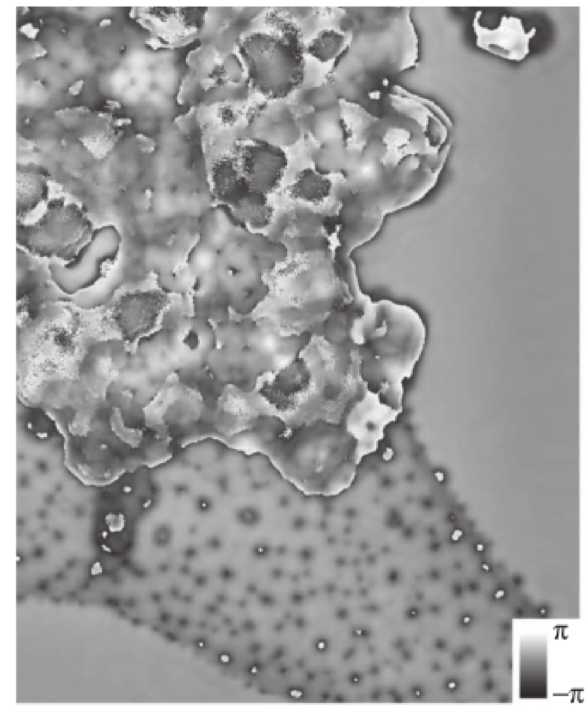
A. Maiden *et al.*, Opt. Lett. **35**, 2585-2587 (2010).

X-rays



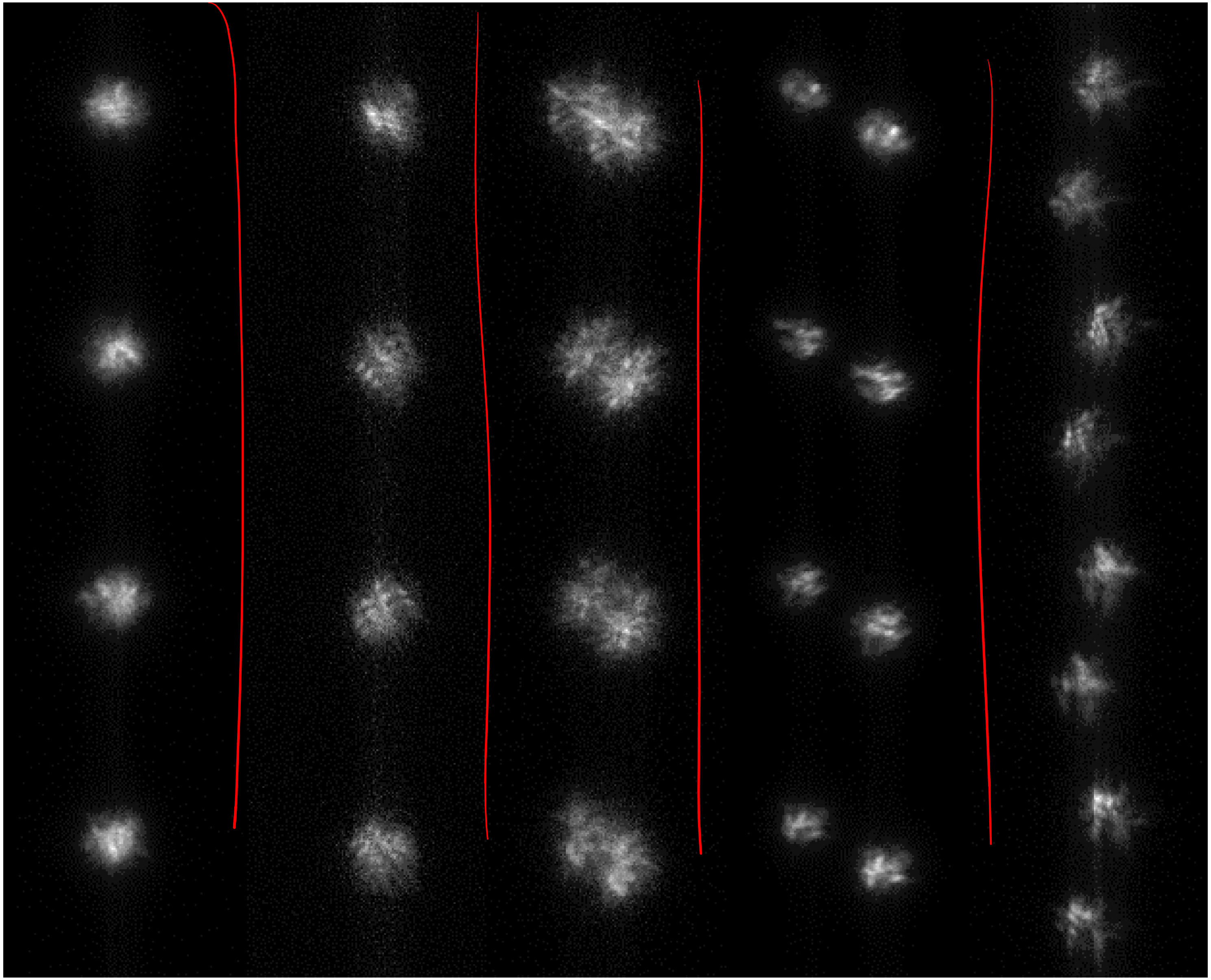
P. Thibault *et al.*, New J. Phys **14**, 063004 (2012).

electrons



M. Humphry *et al.*, Nat. Comm. **3**, 730 (2012).

Speckle imaging in astronomy



result of air turbulence

Source:http://www.cis.rit.edu/research/thesis/bs/2000/hoffmann/thesis.html

Speckle imaging in astronomy

Model

measure: $I(\vec{r}) = O * |P|^2$ "instantaneous PSF" keeps changing because of turbulence

$\downarrow \mathcal{F}$

$\tilde{I}(\vec{u}) = \tilde{O} \cdot P_A$ autocorrelation of P

$\downarrow |\,|^2$

$|\tilde{I}|^2 = |\tilde{O}|^2 \cdot |P_A|^2$

average over mutliple independent measurements

$\langle |\tilde{I}|^2 \rangle = |\tilde{O}|^2 \langle |P_A|^2 \rangle$ ← known quantity (from fluid dynamics)

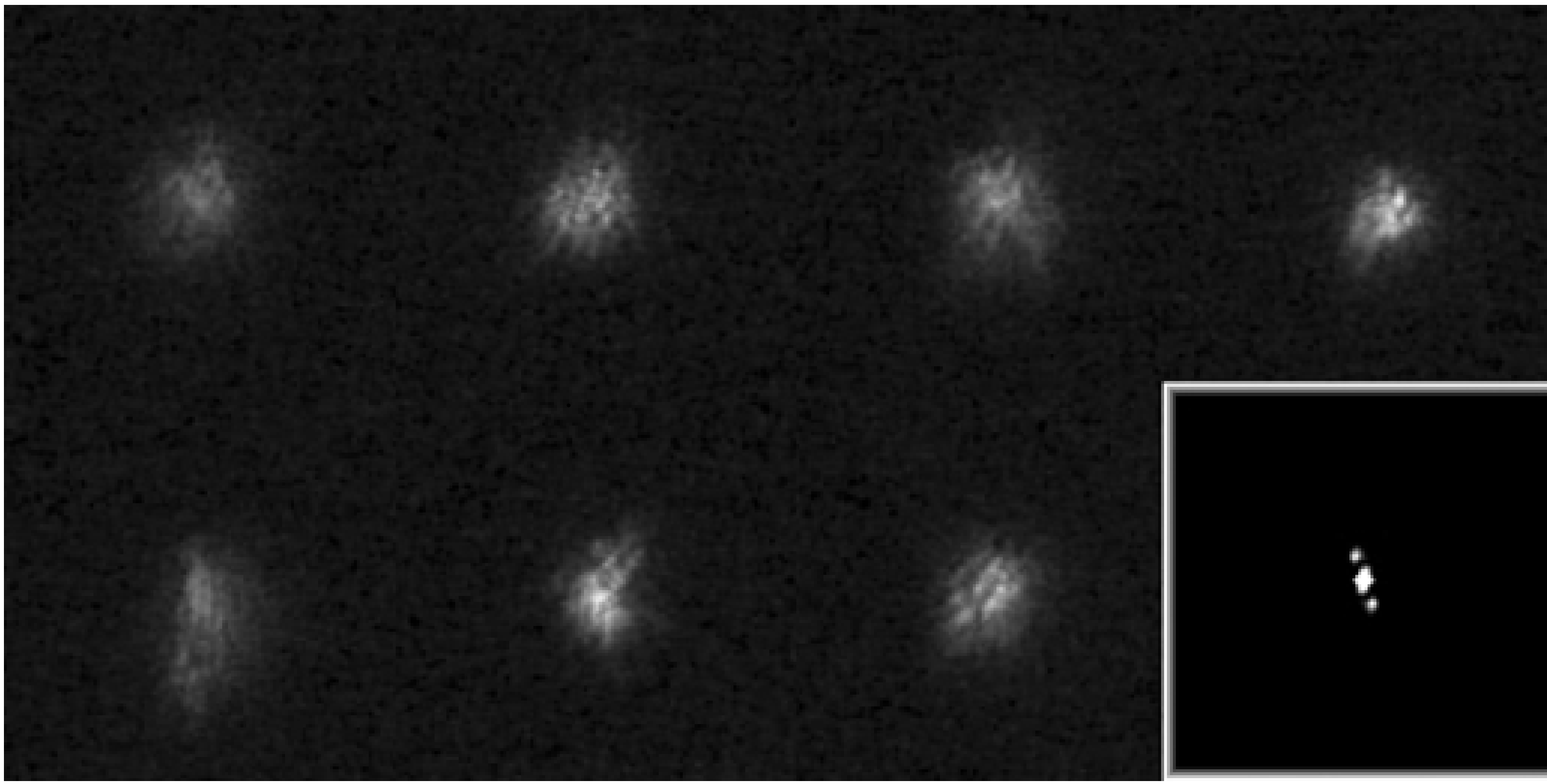
$$|\tilde{O}|^2 = \frac{\langle |\tilde{I}|^2 \rangle}{\langle |P_A|^2 \rangle}$$

← measmnt

← model

Speckle imaging in astronomy

Retrieval of the autocorrelation



autocorrelation

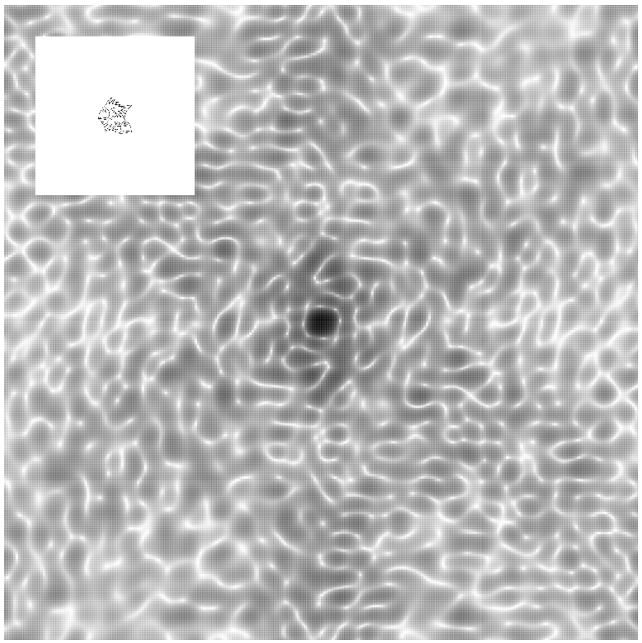
$|\tilde{O}|^2$

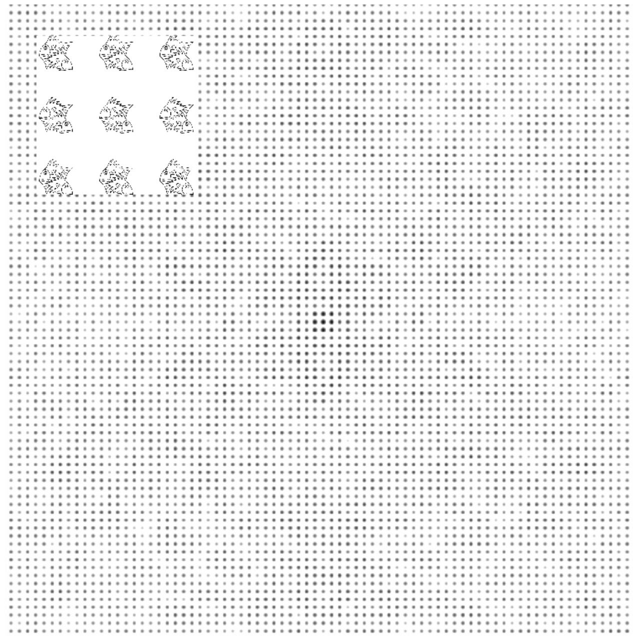
need phase-retrieval to recover O from $O \star O$

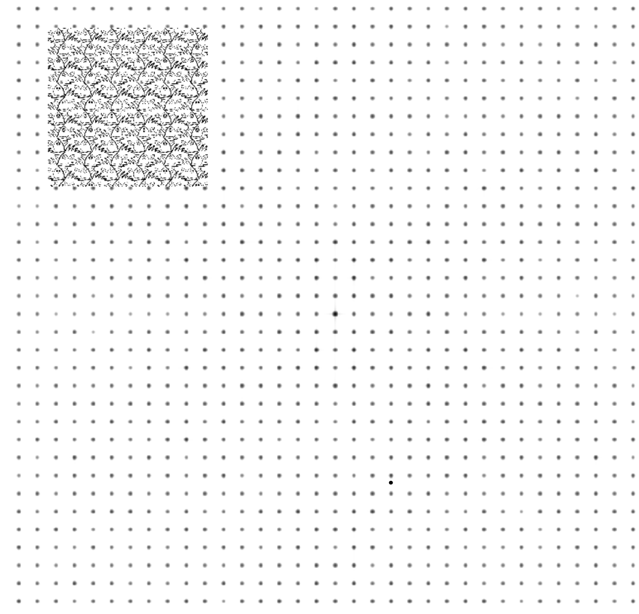
Source: http://www.astrosurf.com/hfosaf/uk/speckle10.htm

Crystallography

Diffraction by a crystal: Bragg peaks

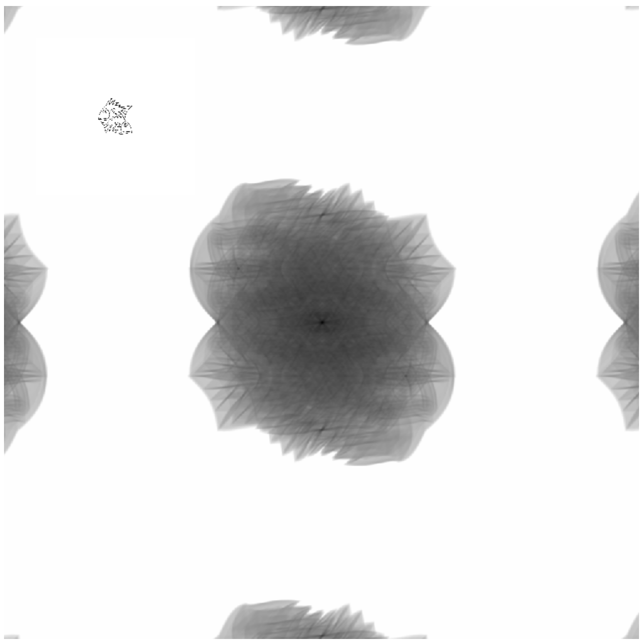


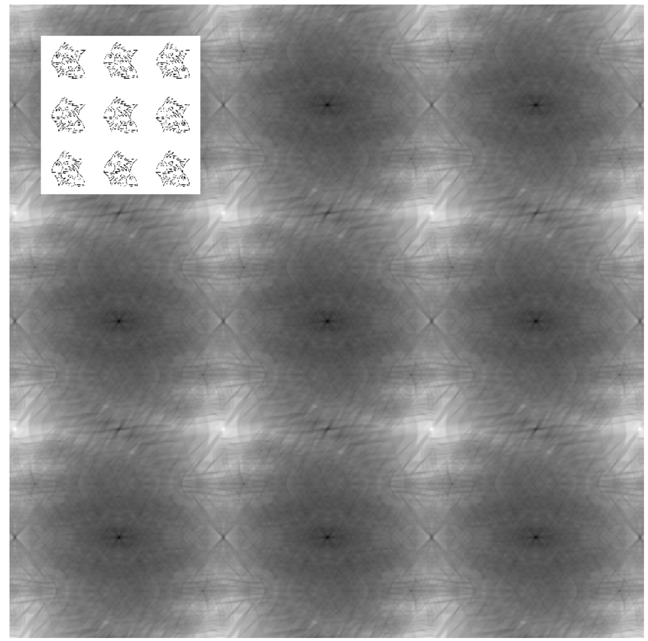


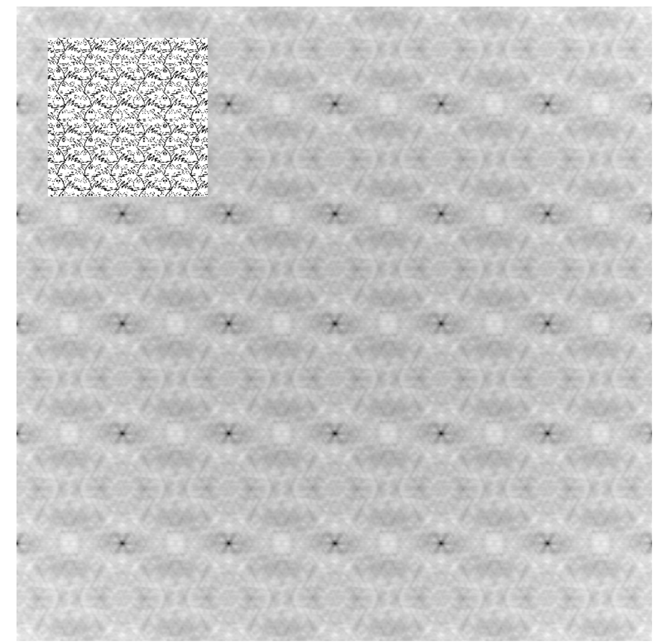


Crystallography

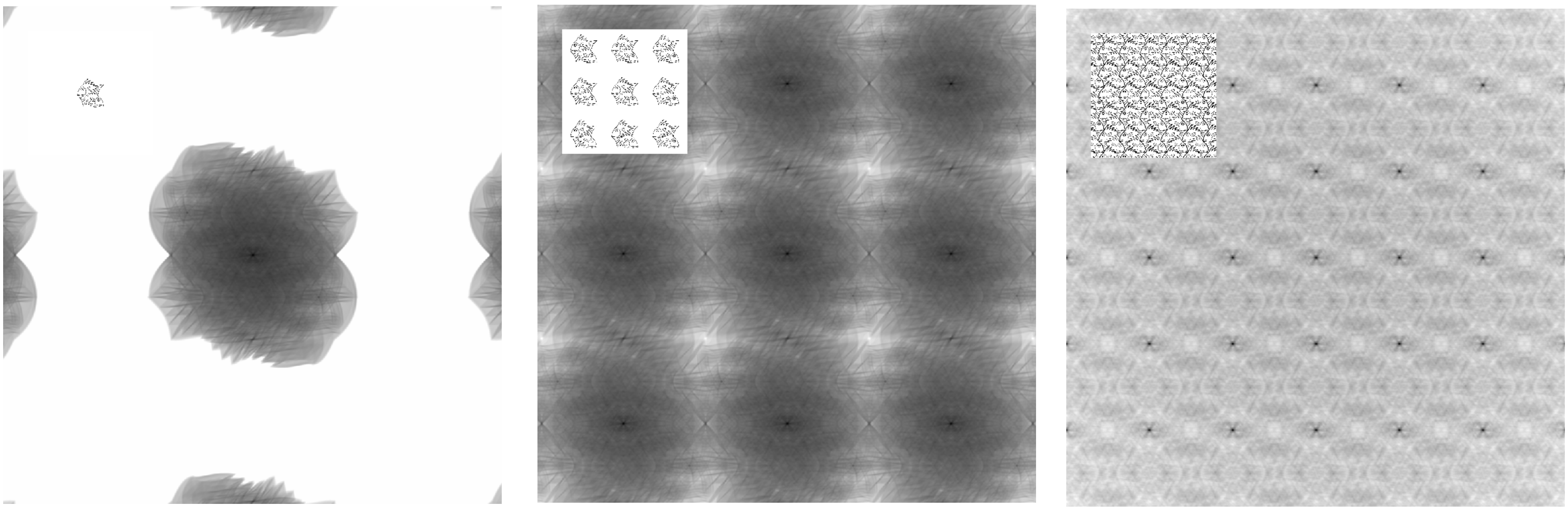
Fourier transform of intensity: autocorrelation





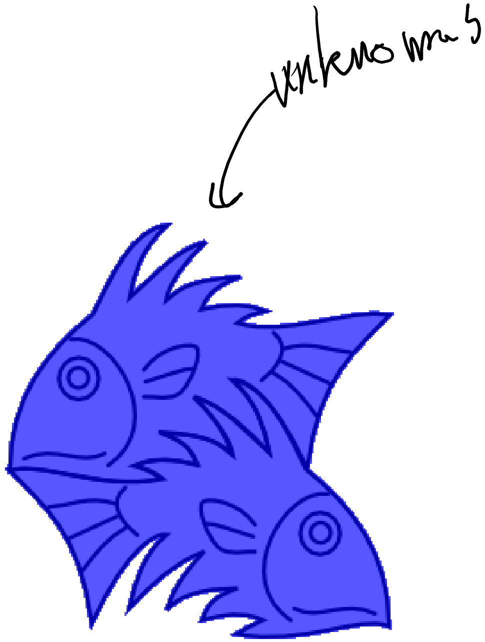


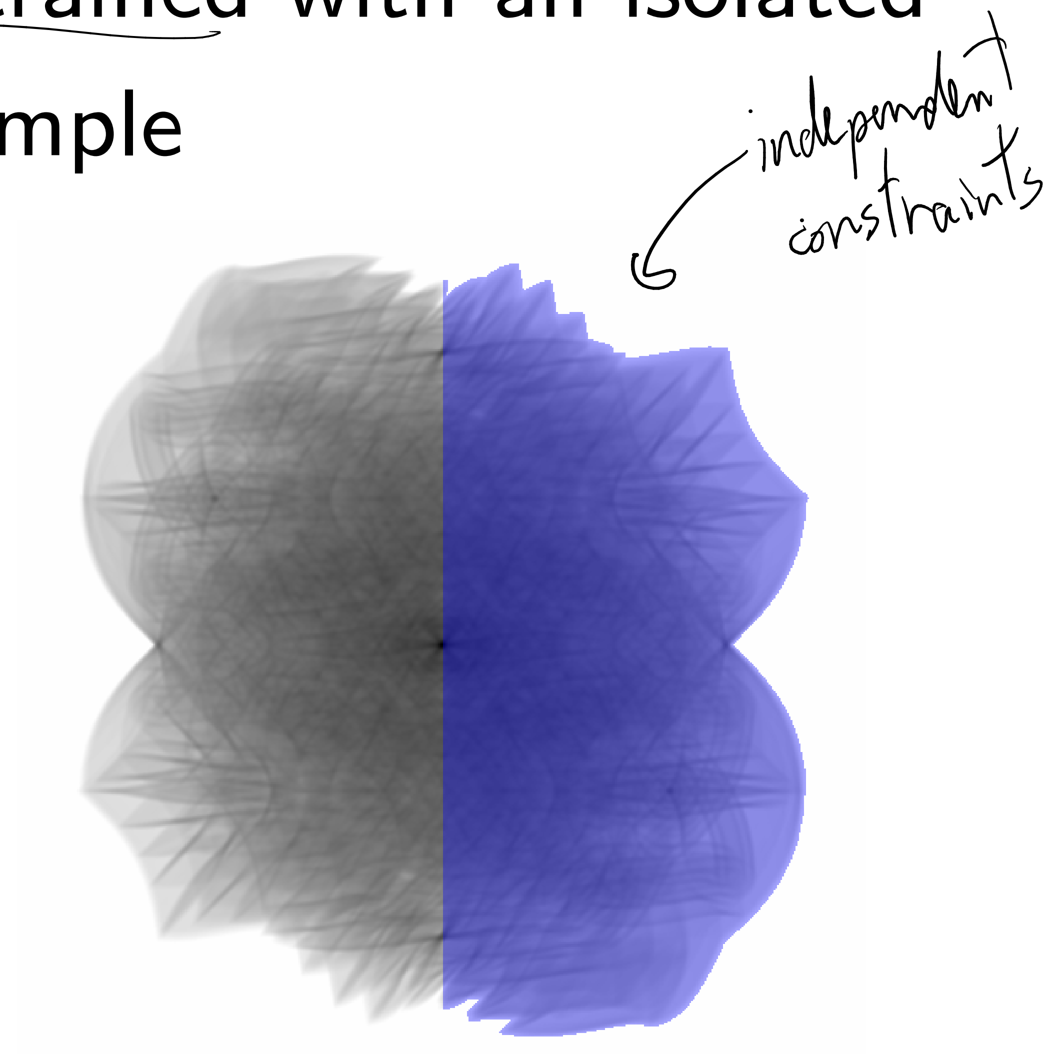
Crystallography



Crystallography

Problem is overconstrained with an isolated sample



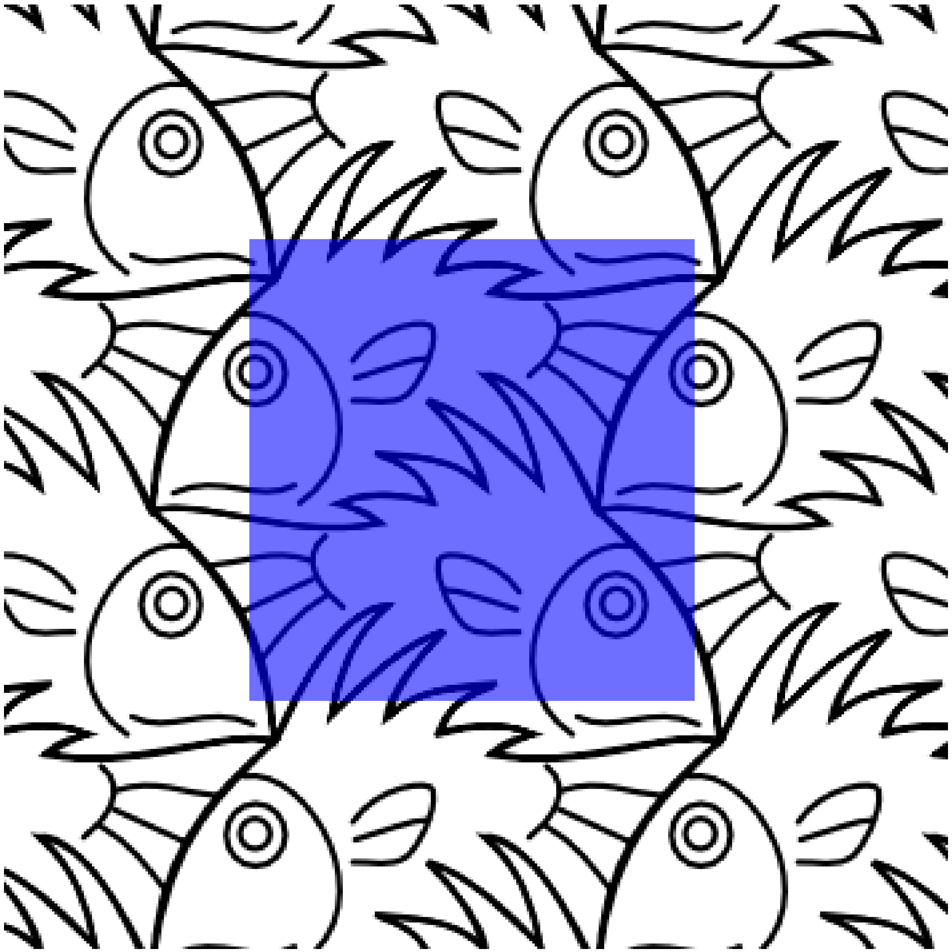


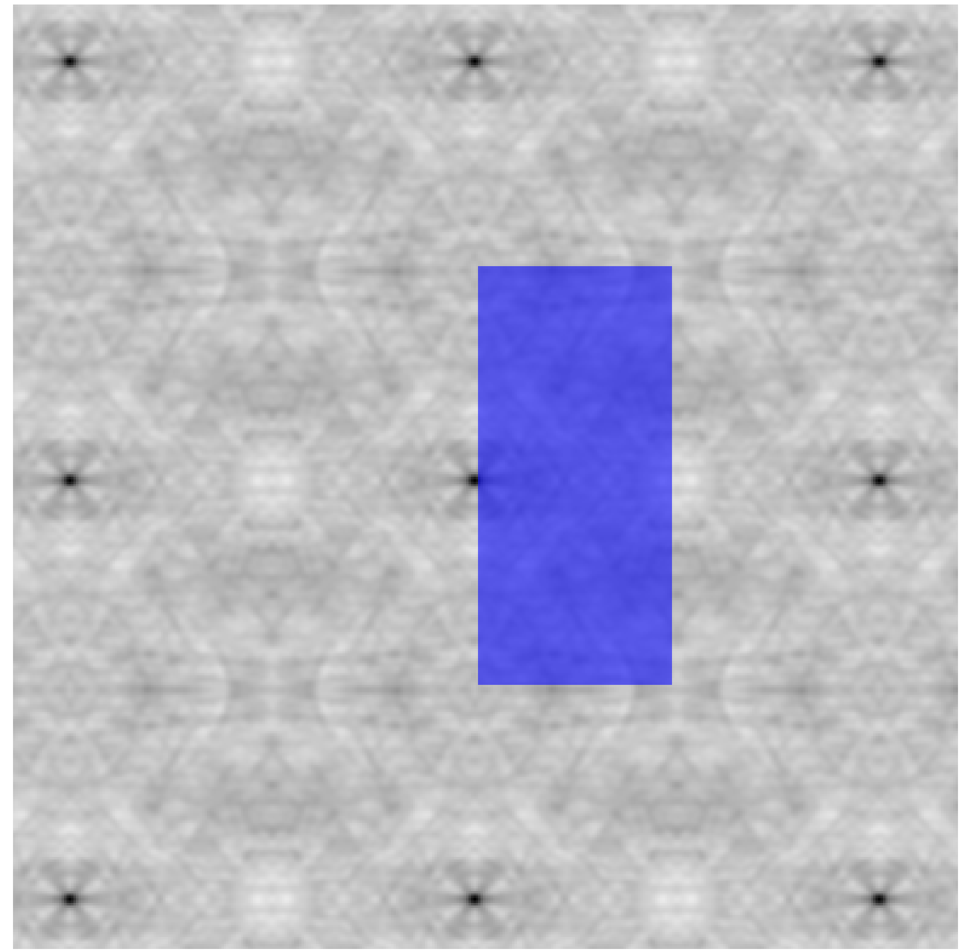
unknowns = N

constraints ≥ 2N

Crystallography

Problem is **under**constrained with a crystal

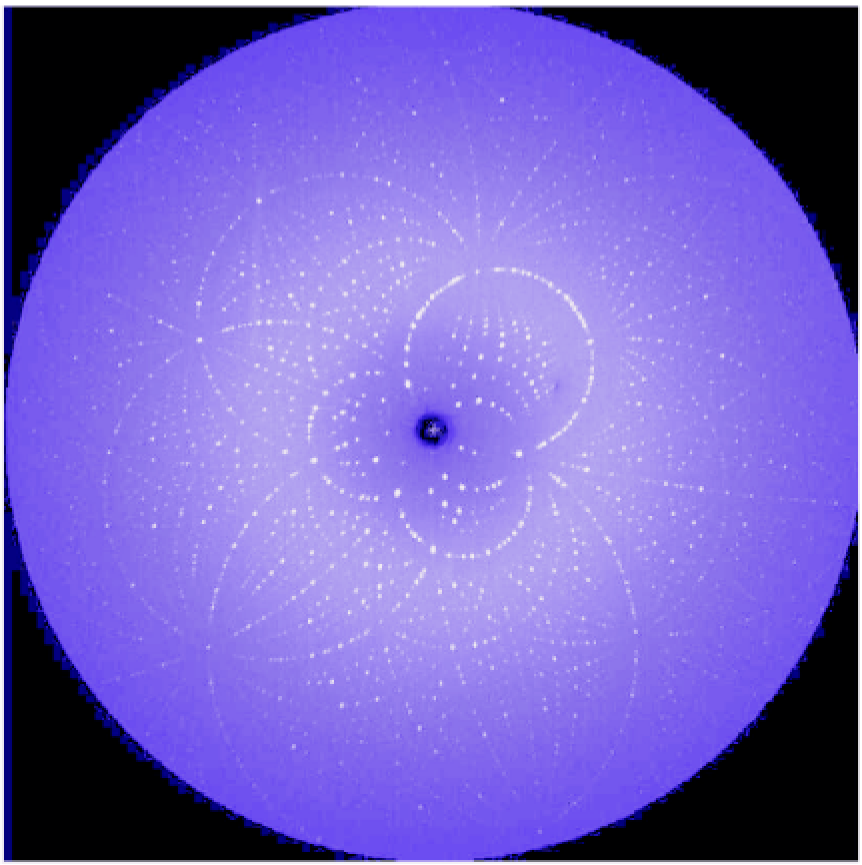




unknowns = N

constraints = N/2

Crystallography
Structure determination



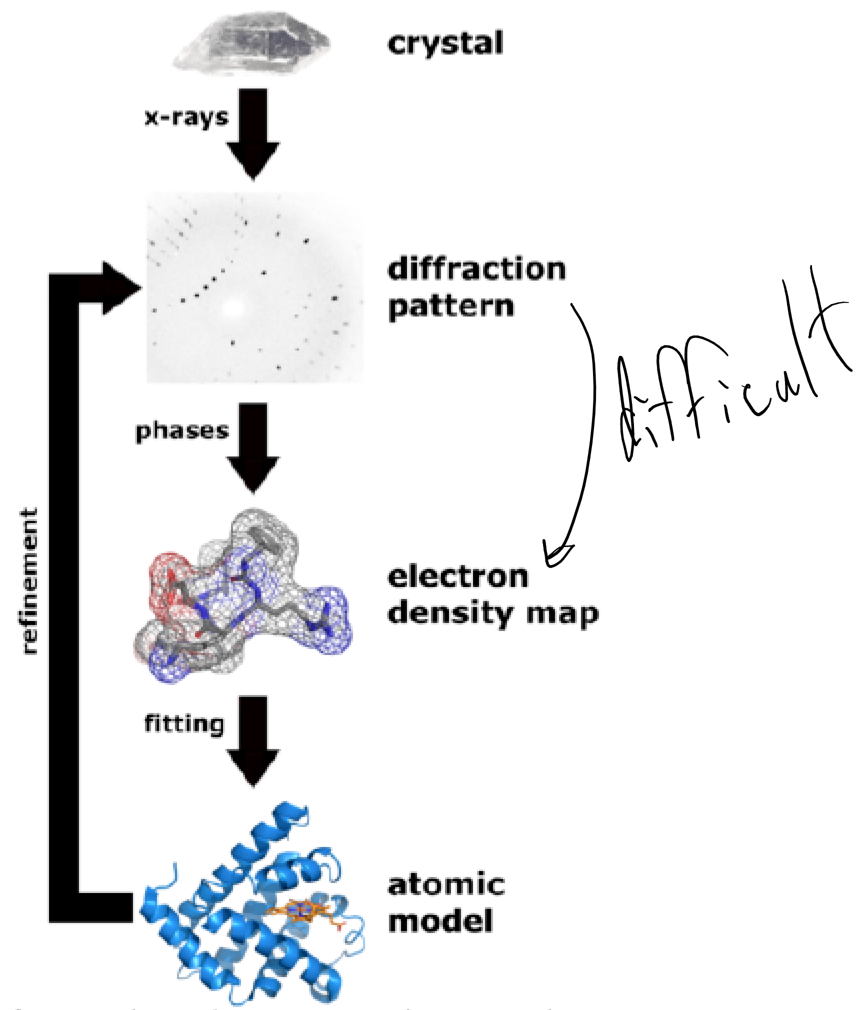


Image from Wikimedia courtesy Thomas Splettstoesser

Crystallography

Structure determination

- Hard problem: few measurements for the number of unknowns
- Luckily: crystals are made of atoms $\rightarrow$ strong constraint
- Also common: combining additional measurements (SAD, MAD, isomorphous replacement, ...)

Summary

Imaging from far-field amplitudes

- Used when image-forming lenses are unavailable (or unreliable) or to obtain more quantitative images.
- In general difficult because of the phase problem
- Solved with the help of additional information:
  - Strong *a priori* knowledge (e.g. CDI: support)
  - Multiple measurements (e.g. ptychography)