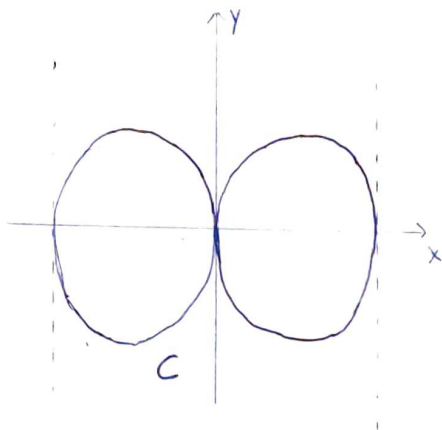③ $V(y^4 + x^4 - x^2) \subseteq \mathbb{A}^2$

C

$(0,0)$ IS A TACNODE

$x = 0$ DOUBLE TANGENT

$\widehat{\mathbb{A}^2} = \left\{ (x,y), (t_0 : t_1) \mid t_1 x - t_0 y = 0 \right\} \subseteq \mathbb{A}^2 \times \mathbb{P}^1$

BLOW-UP OF THE AFFINE PLANE AT THE ORIGIN

$\widehat{\mathbb{A}}_2 \cap V_0 = \left\{ (x, ux), (1 : u) \right\} \subseteq \mathbb{A}^2 \times \mathbb{P}^1$

$\widehat{\mathbb{A}}_2 \cap V_1 = \left\{ (vy, y), (v : 1) \right\} \subseteq \mathbb{A}^2 \times \mathbb{P}^1$ } CHARTS $\cong \mathbb{A}^2$
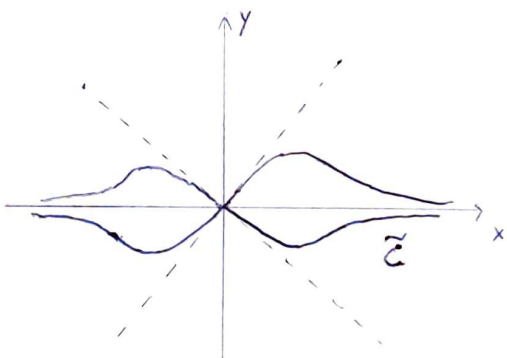
CONSIDER $p_1 : \widehat{\mathbb{A}^2} \longrightarrow \mathbb{A}^2$

$p_1^{-1}(C) \cap V_1 = \left\{ (vy, y), (v : 1) \mid y^2 (y^2 + v^4 y^2 - v^2) = 0 \right\}$

$V(y^2) = E$ IS THE EXCEPTIONAL DIVISOR. $V(y^2 + v^4 y^2 - v^2)$ IS A CURVE IN $\mathbb{A}^2 (v,y)$

WITH A NODE.   RELABEL $v := x$:
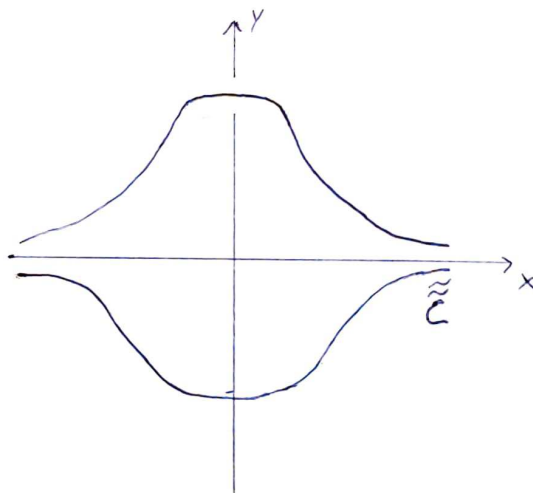
$V(y^2 + x^4 y^2 - x^2) \subseteq \mathbb{A}^2$

$\tilde{C}$

$p_1^{-1}(\tilde{C}) \cap V_0 = \left\{ (x, ux), (1 : u) \mid x^2 (x^4 u^2 + u^2 - 1) = 0 \right\}$
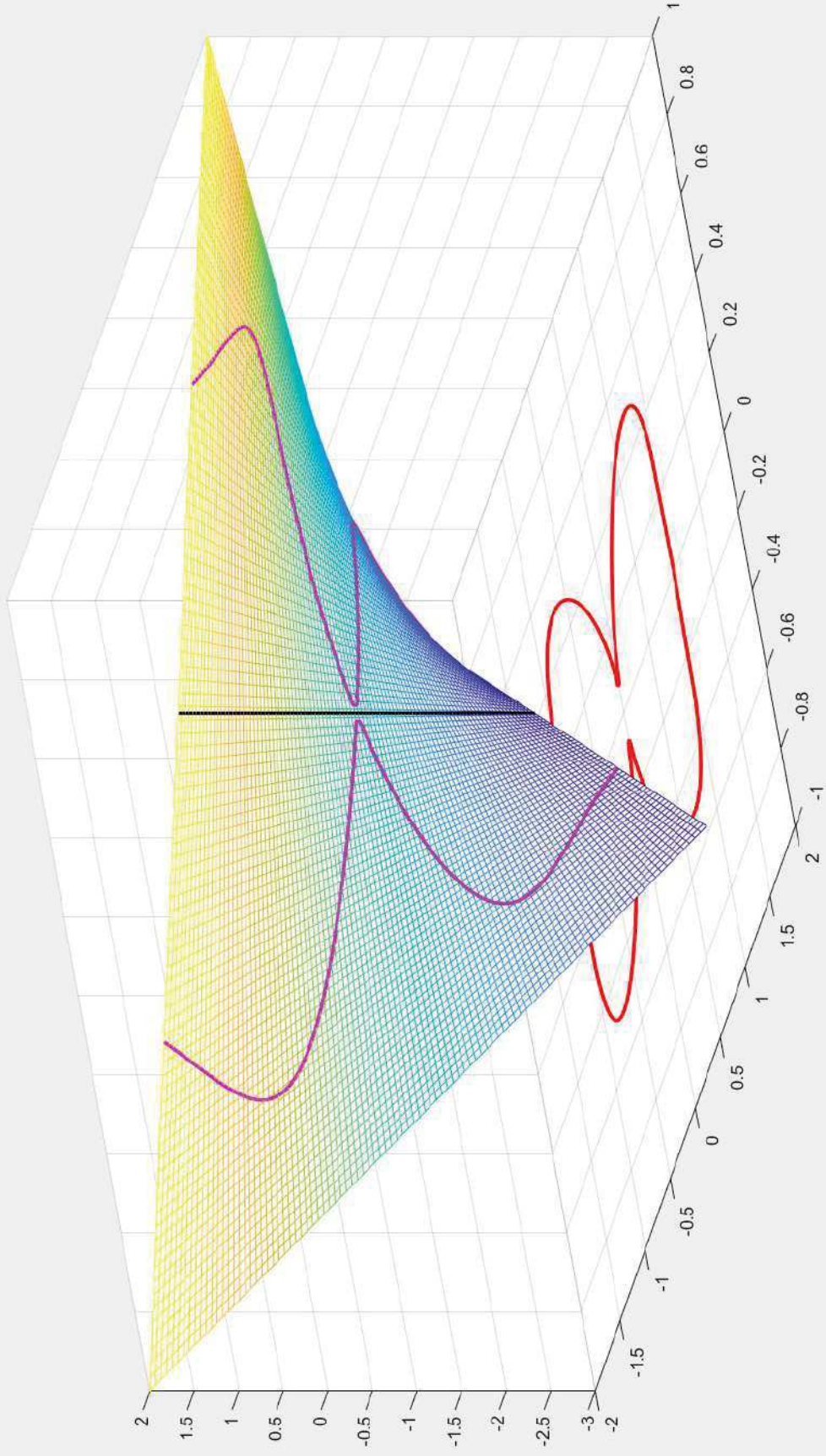
$V(x^2) = \tilde{E}$ IS THE EXCEPTIONAL DIVISOR

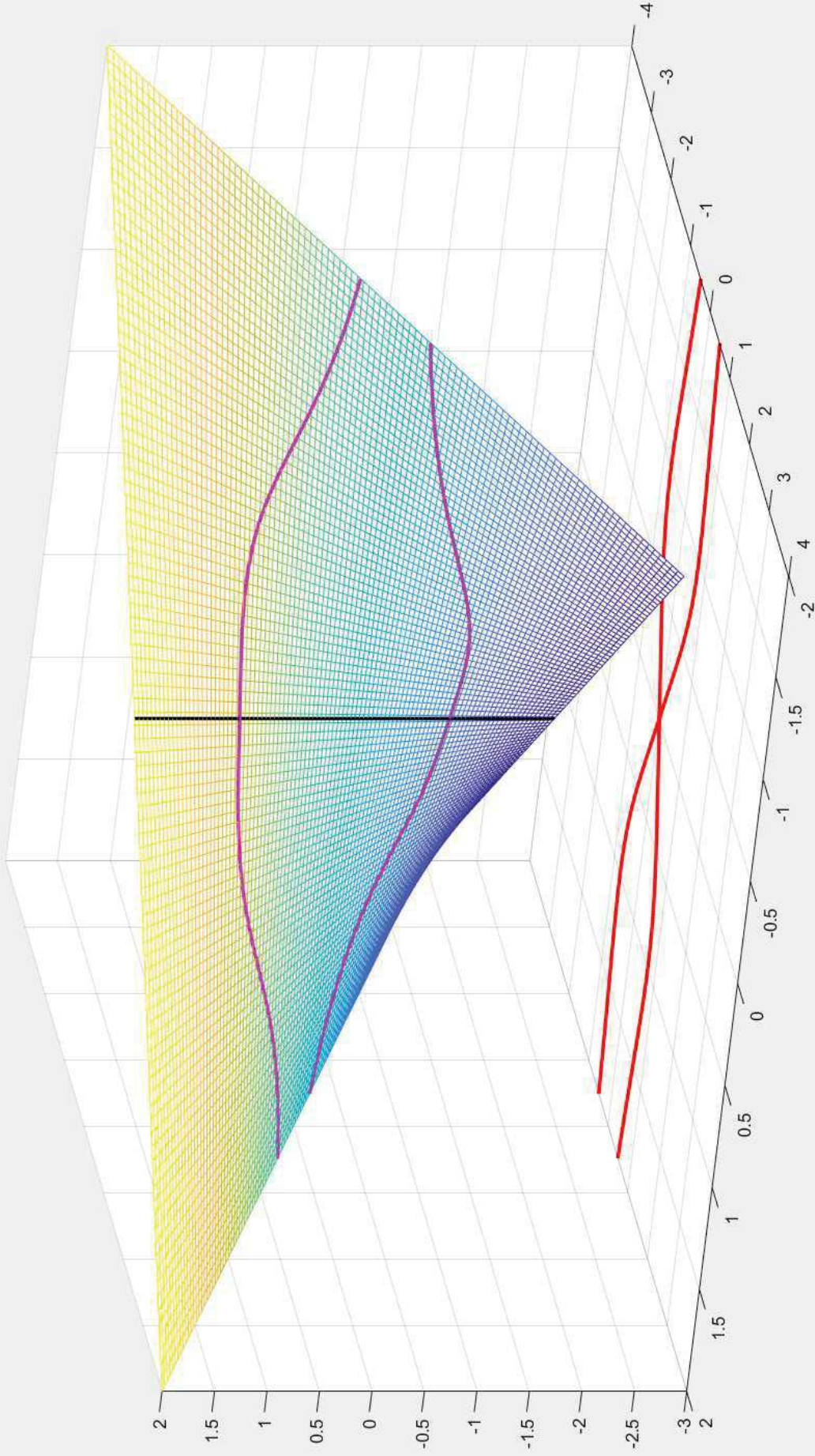$V(x^4 u^2 + u^2 - 1)$ IS A CURVE IN $\mathbb{A}^2 (x,u)$

WITHOUT SINGULARITIES.

$\tilde{\tilde{C}}$

Blow-up in the origin
Initial curve (1)
Initial curve (2)
Exceptional divisor
Strict transform

☐	Blow-up in the origin
▮	Initial curve (1)
▮	Initial curve (2)
▮	Exceptional divisor
▮	Strict transform

```matlab
clear
close all
clc

y=linspace(-1,1);
z=linspace(-2,2);
[Y,Z]=meshgrid(y,z);
X=Y.*Z;
mesh(X,Y,Z);   %plot blowup in the origin restricted to the chart V1
hold on
pause

t1=y;
t2=( t1.^4 - t1.^2 ).^(1/4);
t3=-( t1.^4 - t1.^2 ).^(1/4);

plot3(t1,t2,-3*ones(100,1),"r","LineWidth",2);
hold on
plot3(t1,t3,-3*ones(100,1),"r","LineWidth",2);
hold on  %plot the curve with the tacnode
pause
plot3(zeros(100,1),zeros(100,1),linspace(-2,2),"k","LineWidth",2);
hold on  %plot the exceptional divisor
pause

v=linspace(-2,2);
y1=sqrt( v.^2 ./ (1+v.^4) );
y2=-sqrt( v.^2 ./ (1+v.^4) );
plot3(v.*y1,y1,v,"m","LineWidth",2);
hold on
plot3(v.*y2,y2,v,"m","LineWidth",2);
%plot the strict transform, which has a node

legend("Blow-up in the origin","Initial curve (1)","Initial curve
(2)","Exceptional divisor","Strict transform");
pause

figure(2);

x=linspace(-2,2);
z=linspace(-2,2);
[X,Z]=meshgrid(x,z);
Y=X.*Z;
mesh(X,Y,Z); %plot blowup in the origin restricted to the chart V0
hold on
pause

t1=linspace(-2,2);
t2= sqrt(t1.^2 ./ (t1.^4+1)) ;
t3=- sqrt(t1.^2 ./ (t1.^4+1)) ;

plot3(t1,t2,-3*ones(100,1),"r","LineWidth",2);
```

```matlab
hold on
plot3(t1,t3,-3*ones(100,1),"r","LineWidth",2);
hold on %plot the curve with a node
pause
plot3(zeros(100,1),zeros(100,1),linspace(-2,2),"k","LineWidth",2);
hold on %plot the exceptional divisor
pause

x=linspace(-2,2);
u1=sqrt( 1 ./ (1+v.^4) );
u2=-sqrt( 1 ./ (1+v.^4) );
plot3(x,x.*u1,u1,"m","LineWidth",2);
hold on
plot3(x,x.*u2,u2,"m","LineWidth",2);
%plot the strict transform, which has no singularities

legend("Blow-up in the origin","Initial curve (1)","Initial curve
(2)","Exceptional divisor","Strict transform");
```