AG 3 - sixth assignment

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1. Prove that the locus $\Sigma \subseteq \mathbb{P}(\mathbb{C}[x_0, x_1, x_2]_5)$ of reducible homogeneous polynomials of degree 5 is closed and determine its irreducible components with their dimensions.

2. Let $X \subset \mathbb{P}^n_{\mathbb{C}}$ be a projective variety, and $F \in \mathbb{C}[x_0, \dots, x_n]_d$ a homogeneous polynomial of degree $d \geq 1$. Prove that if X is not just a point, then $X \cap V_P(F) \neq \emptyset$.

3. Prove that the blow-up $\widehat{\mathbb{A}}^2_{\mathbb{C}}$ of $\mathbb{A}^2_{\mathbb{C}}$ in the origin is not isomorphic to an affine variety.

4. Let $X \subseteq \mathbb{A}^n$ and $Y \subseteq \mathbb{A}^m$ be two affine varieties, and let $\varphi = (f_1, \dots, f_m) : X \to Y$ be a morphism. Prove that the differential $d_p f = (d_p f_1, \dots, d_p f_m)$ maps $t_p X$ to $t_{\varphi(p)} Y$.

5. Let $X \subset \mathbb{P}^4_{\mathbb{C}}$ be a hypersurface of degree $d \geq 2$. Prove that if X contains a projective plane $L \cong \mathbb{P}^2_{\mathbb{C}}$, then X has singular points.