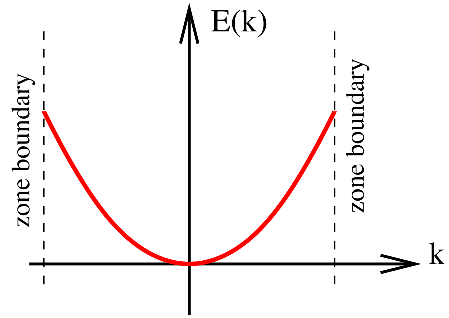
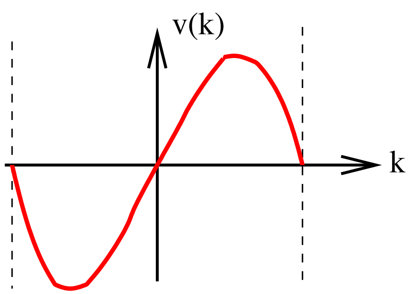
Motion in a uniform E field



$$\hbar \frac{d\mathbf{k}}{dt} = -e\mathbf{E}$$

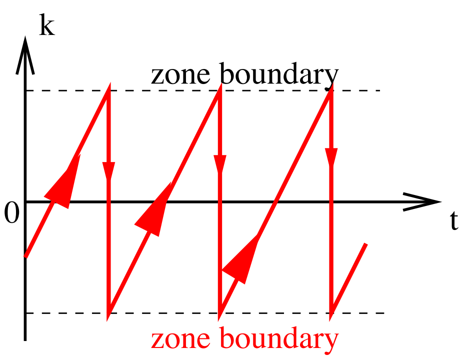


$$\mathbf{k}(t) = \mathbf{k}(0) - \frac{e\mathbf{E}}{\hbar} t$$

without collisions or for $t << \tau$

with collisions **k** saturates at

$$\mathbf{k}_{avg} = -\frac{e\mathbf{E}}{\hbar} t_{avg} = -\frac{e\mathbf{E}}{\hbar} \tau$$



without collisions or for $t << \tau$

<u>electron velocity oscillates → electron motion is oscillatory</u>

<u>Bloch oscillations</u>

But: if the band is filled an applied electric field cannot change k

→ no current is induced by an applied electric field

Motion in a uniform H field (i)

velocity $$\mathbf{v}_n = \dot{\mathbf{r}} = \frac{1}{\hbar}\frac{\partial E_n}{\partial \mathbf{k}}$$ ←

equation of motion $$\hbar\dot{\mathbf{k}} = -e\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_n \times \mathbf{H}\right)$$

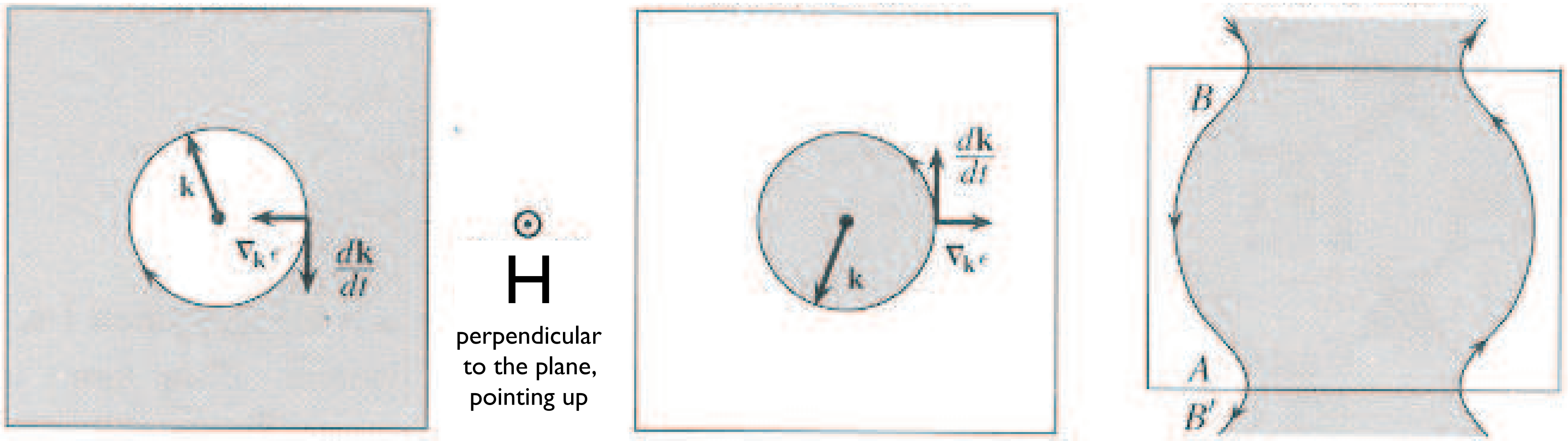
⇨ $$\hbar\frac{d\mathbf{k}}{dt} = -\frac{e}{\hbar c}\frac{\partial E_n}{\partial \mathbf{k}} \times \mathbf{H}$$

⇨ $\mathbf{k}$ evolves $\perp$ to $\dfrac{\partial E_n}{\partial \mathbf{k}}$ and $\mathbf{H}$:

electrons in a static magnetic field move on a curve of <u>constant energy</u> on a plane normal to **H**

(an electron on the Fermi surface will move in a curve on the <u>Fermi surface</u>)

Motion in a uniform H field (ii)



(a) (b) (c)

hole-like orbit

clockwise motion, as expected for a positively charged particle

electron-like orbit

anticlockwise motion, as expected for a negatively charged particle

open orbit

Motion in a uniform H field (iii)

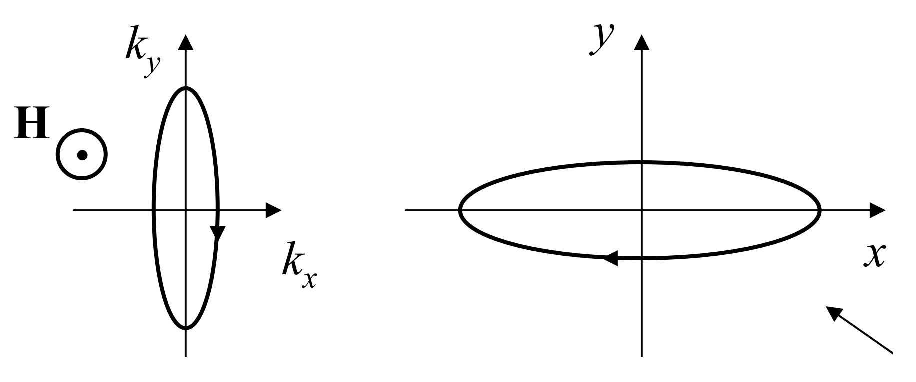
real space orbit vs *k*-space orbit

From the eqs. of motions it follows:

$$\frac{d\mathbf{k}}{dt} = -\frac{e}{\hbar c}\frac{d\mathbf{r}_\perp}{dt} \times \mathbf{H} = -\frac{eH}{\hbar c}\frac{d\mathbf{r}_\perp}{dt} \times \hat{\mathbf{H}}$$

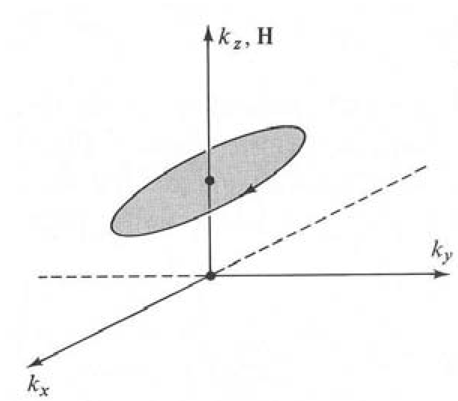
(where $\mathbf{r}_\perp$ is the projection of $\mathbf{r}$ on a plane $\perp\mathbf{H}$, and $\hat{\mathbf{H}} = \mathbf{H}/H$)

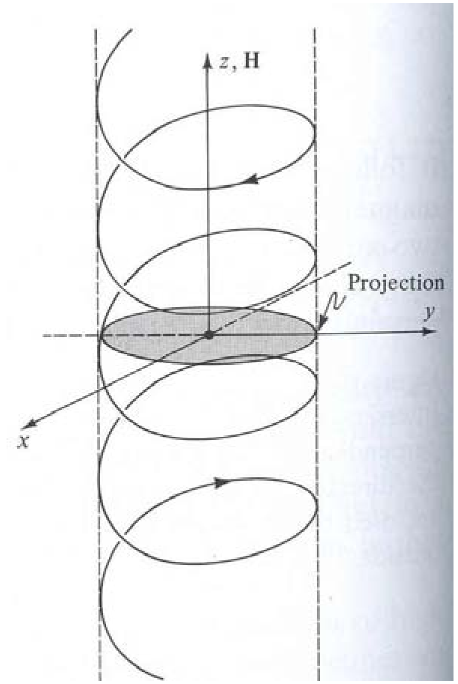
i.e. $\mathbf{r}$ and $\mathbf{k}$ evolve following orbits $\perp$ one to the other:



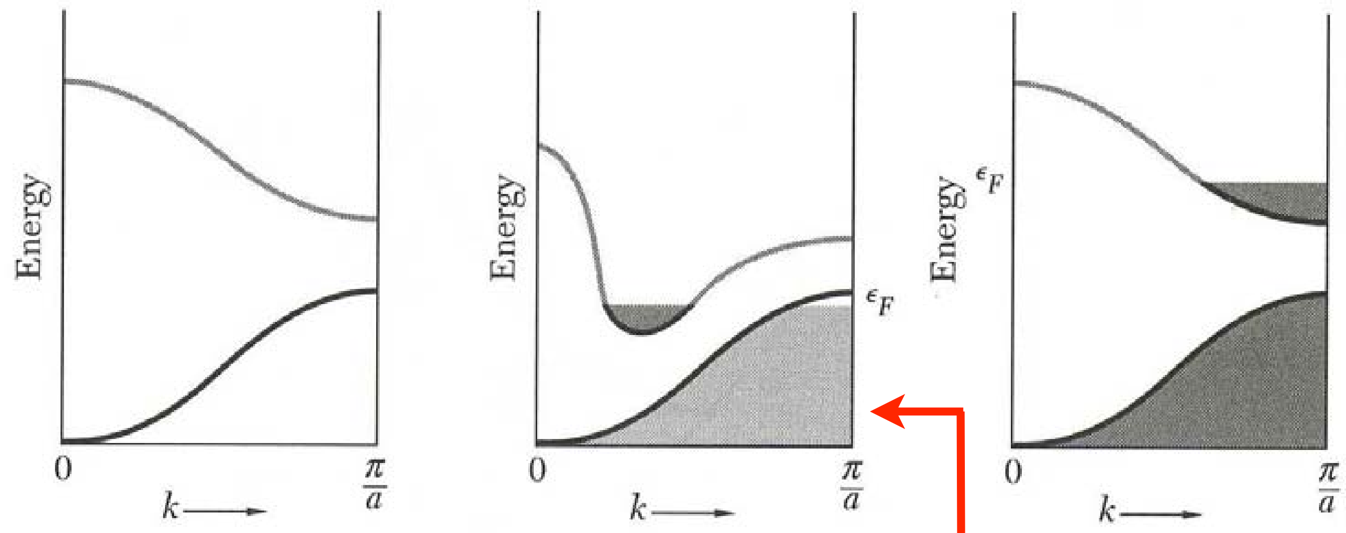
Motion in a uniform H field (iv)

3D: the <u>projection of the real space orbit</u> in a plane perpendicular to the field is the *k*-space orbit rotated through 90^0 about the field direction and scaled by the factor $l_H^2 = \frac{\hbar c}{eH}$





metals and insulators



insulator or semiconductor

metal or semimetal

metal

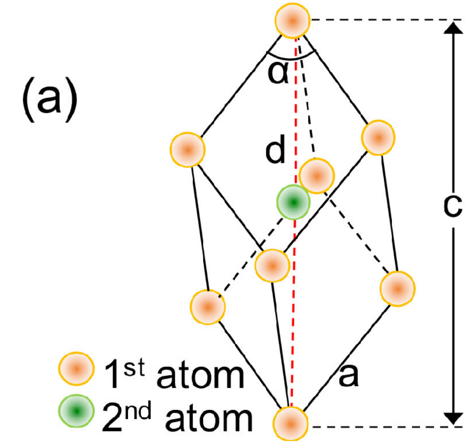
<u>a certain number of bands are completely filled</u>, all other remains empty

a configuration with a band gap can arise only if number of electrons per primitive cell is <u>even</u>

<u>some bands are partly filled</u>

(this is case for an <u>odd</u> number of el.; could be also with an <u>even</u> number of electrons but in presence of a band crossing)

An example of semi-metal

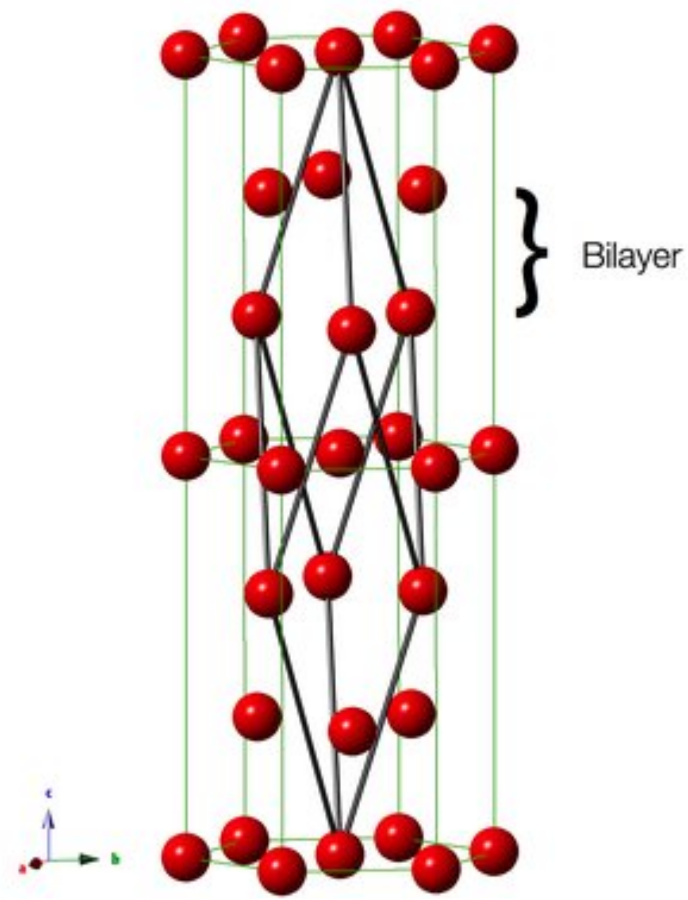


Bi Z=83, group VA ; structure: RHL

two atoms per unit cell => 10 valence electrons per unit cell
=> insulator OR metal ?

Bi has:

- the **highest Hall coefficient**, $R_H = -1/(nec)$, is several orders of magnitude higher than expected with that *n*.
- the **second lowest thermal conductivity** (after Hg)
- a **high electrical resistance** (or low electrical conductivity)
(look for instance at Tab 1.2 and 1.6 of A&M)



Why?

Is the "effective" electron concentration *n* for some reason much lower than the calculated one?

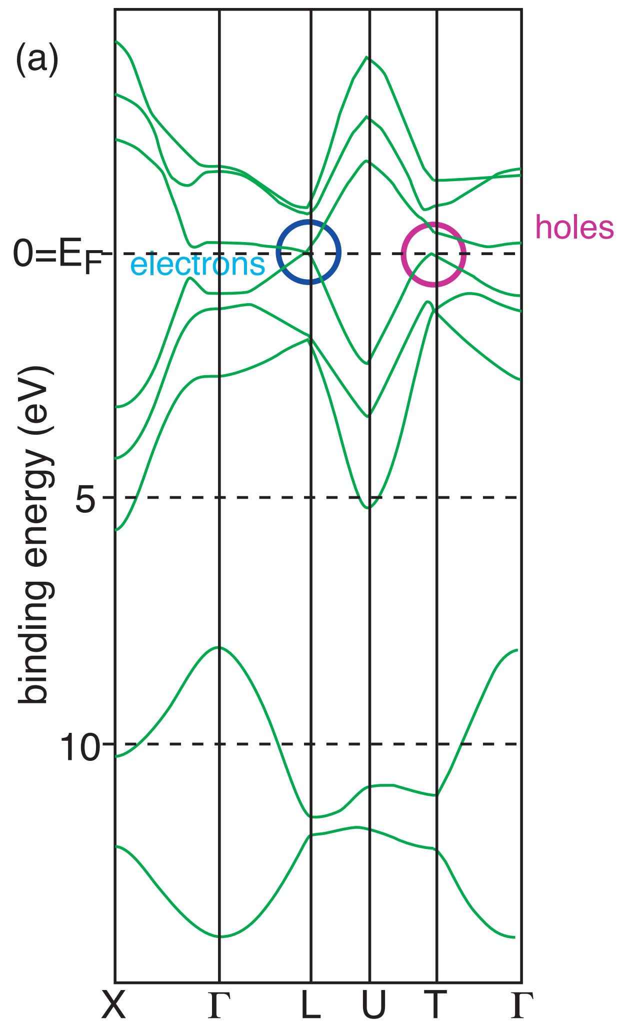
Bi Z=83, group VA ; rhombohedral structure (RHL)

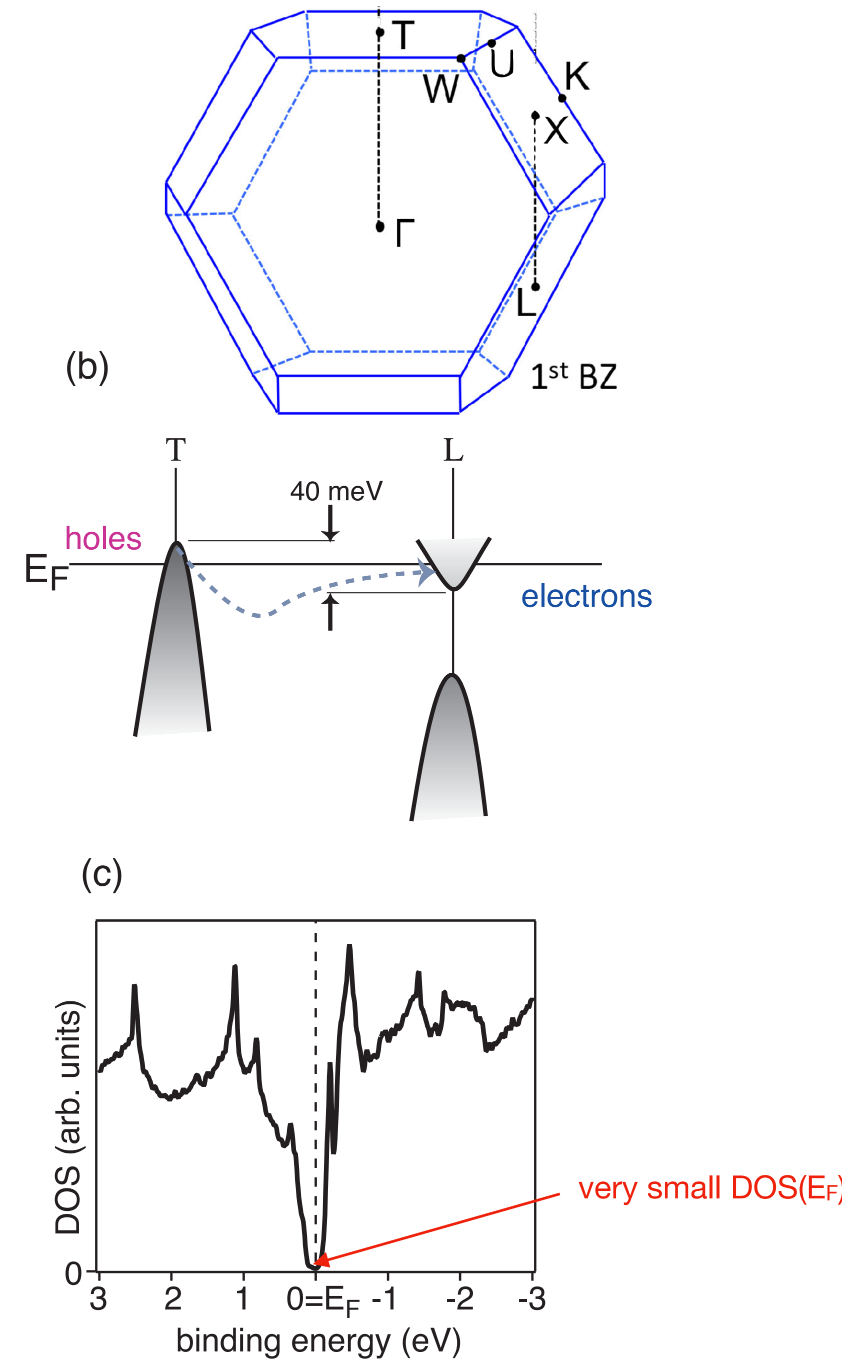
a nearly perfect "compensated semi-metal"

with small electron and hole pockets;

low carrier density;

small Fermi surface





Adapted from:

Online note to accompany the book "Solid State Physics - An Introduction", Wiley, by Philip Hofmann

Figure 1: Electronic structure of Bismuth. (a) Bulk band dispersion in different directions of the Brillouin zone (b) Schematic band structure of the bands crossing the Fermi energy. (c) Density of states.

Bi Z=83, group VA ; structure: RHL

The effect of the presence of both holes and electrons on the Hall constant can be understood qualitatively from the expression for R_H:

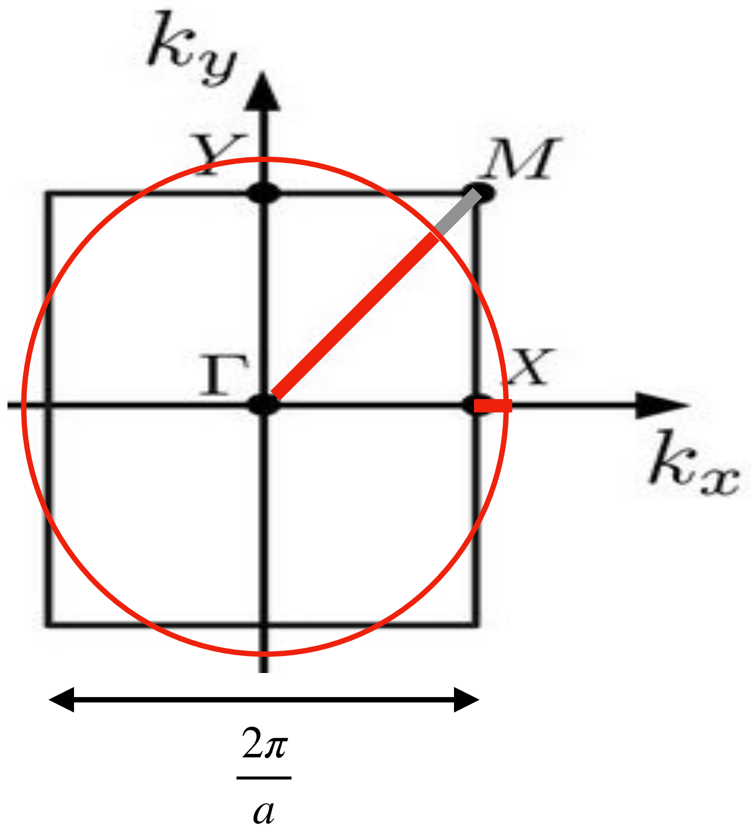
$$R_H = \frac{p\mu_h^2 - n\mu_e^2}{e(p\mu_h + n\mu_e)^2}$$

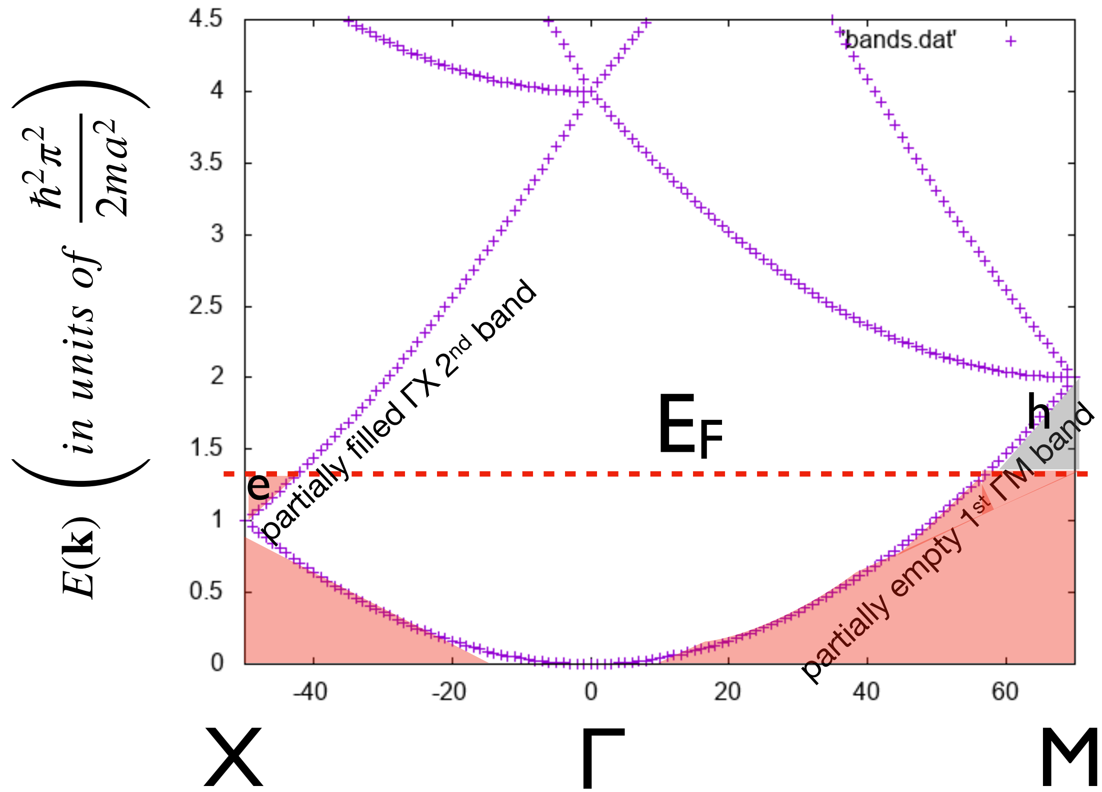
if n, p (here: n=p) are very small
=> small denominator => high R_H

No longer true if $p\mu_h^2 = n\mu_e^2$
since also the numerator vanishes

(see: Ashcroft-Mermin: problem 12.4
or
written test of 11/04/2007)

The 2D empty square lattice model



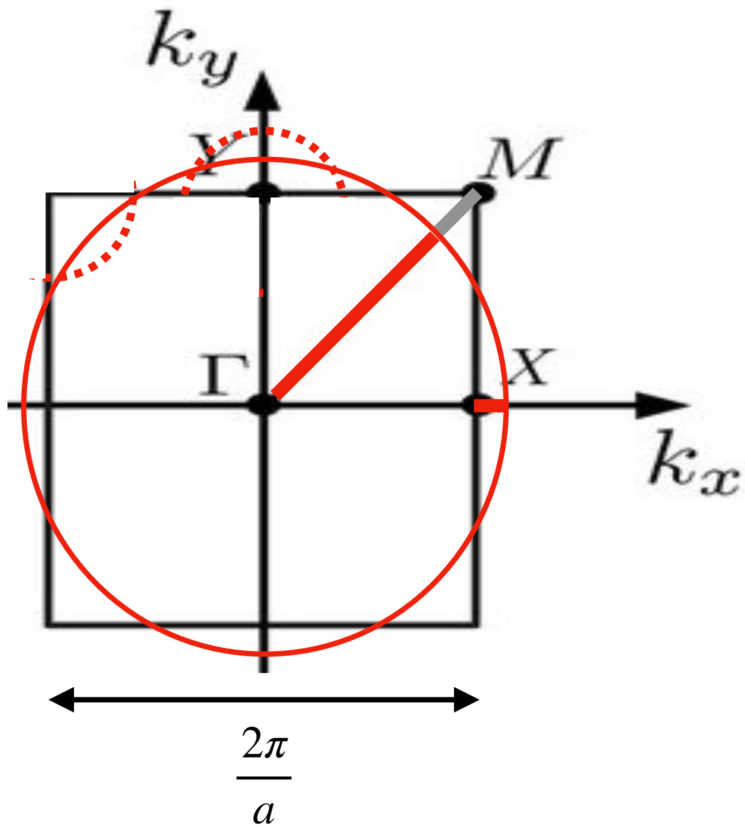


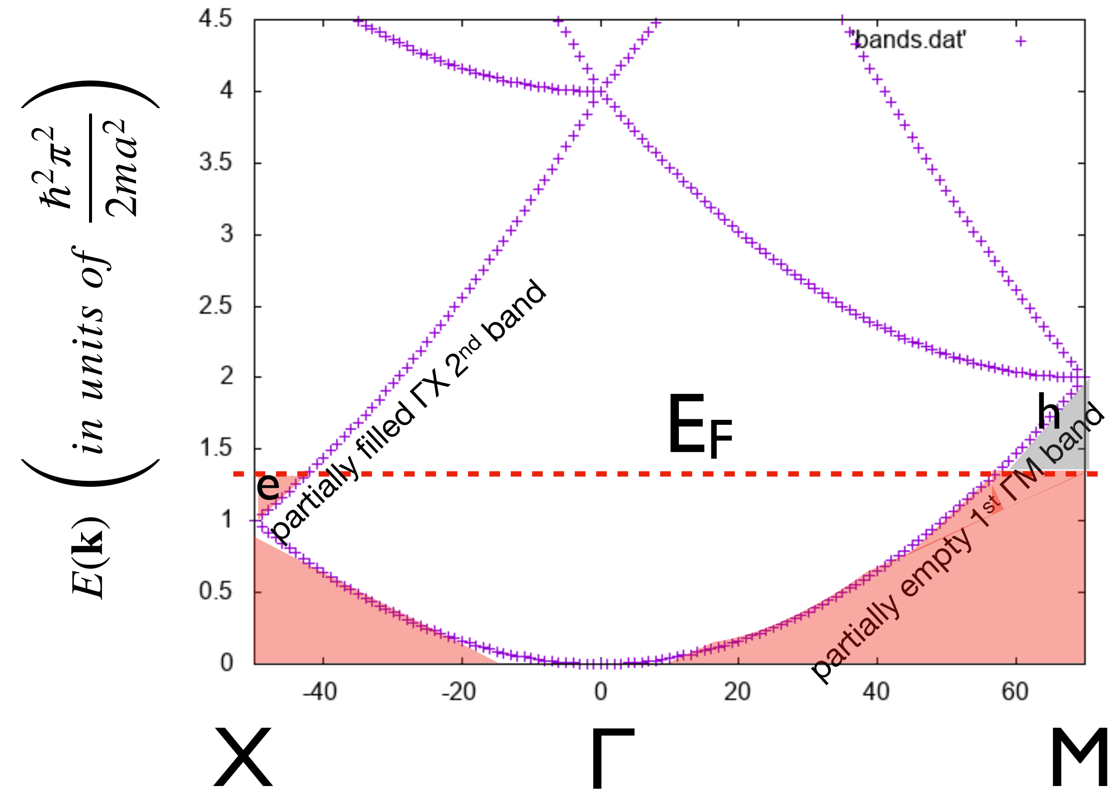
$Z = 2$ e/cell

$$\overline{\Gamma X}^2 = \frac{\pi^2}{a^2} < k_F^2 = 2\pi n = \frac{4\pi}{a^2} < \overline{\Gamma M}^2 = \frac{2\pi^2}{a^2}$$

$$E_F = \frac{4}{\pi}\frac{\hbar^2\pi^2}{2ma^2}$$

The 2D empty square lattice model





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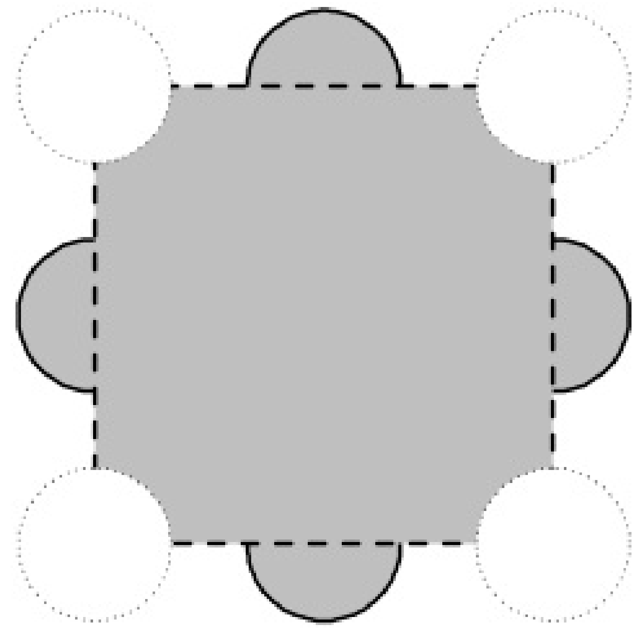
$$E_F = \frac{4}{\pi} \frac{\hbar^2\pi^2}{2ma^2}$$

The 3D empty square lattice model

written test of January 16, 2012 - problem n. 3

(qualitative picture!)

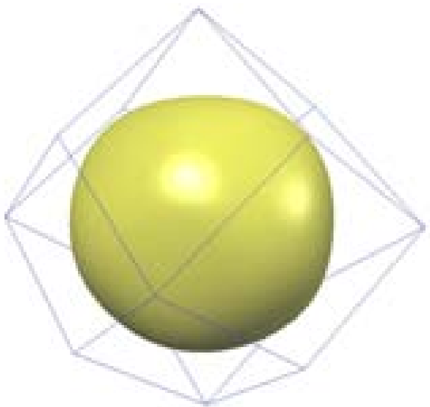


$Z = 2$ e/cell

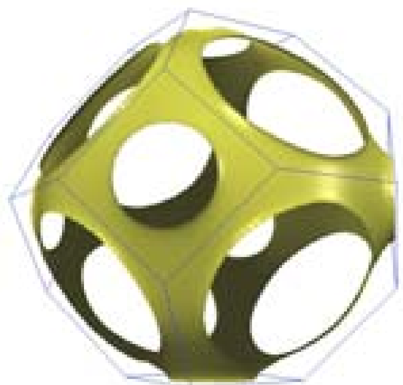
$$k_F^3 = 2\pi^2 n = \frac{6\pi^2}{a^3} \quad \Rightarrow \quad V_{Fermi\ sphere} = \left(\frac{2\pi}{a}\right)^3 = V_{1st\ Bz}$$

3D Fermi Surface

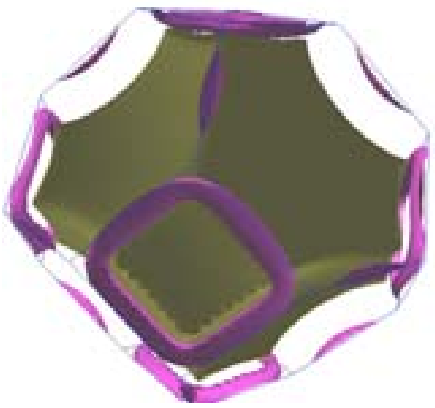
1 valence e^- **2 valence e^-** **3 valence e^-**



Na
BCC



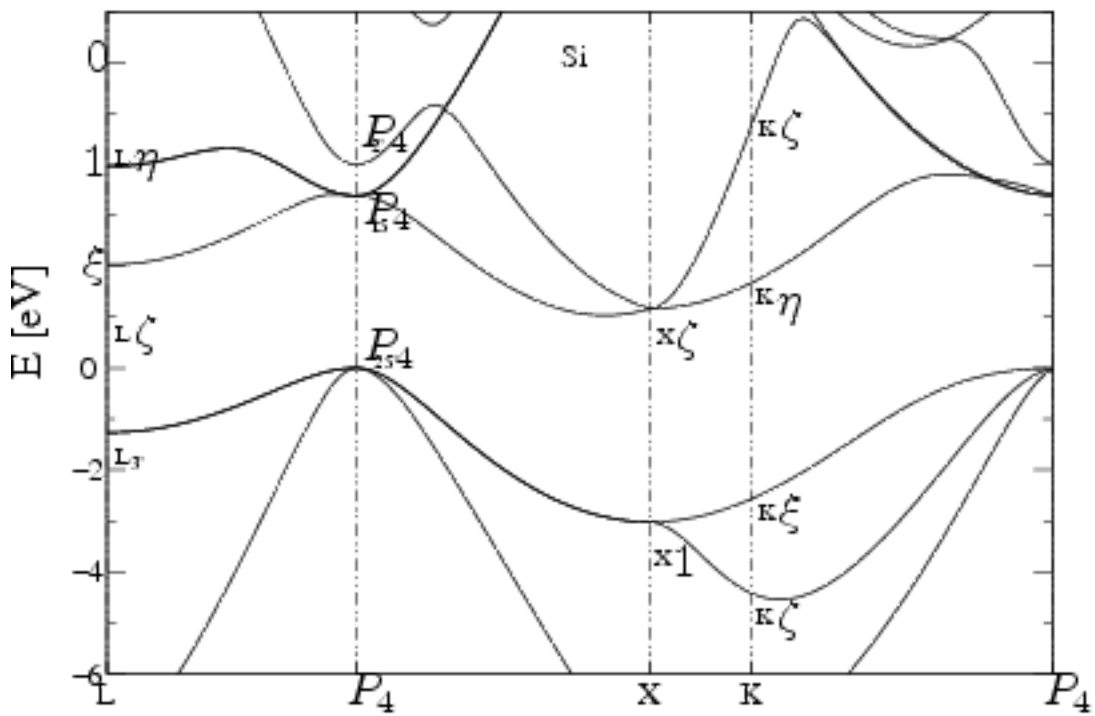
Ca
FCC



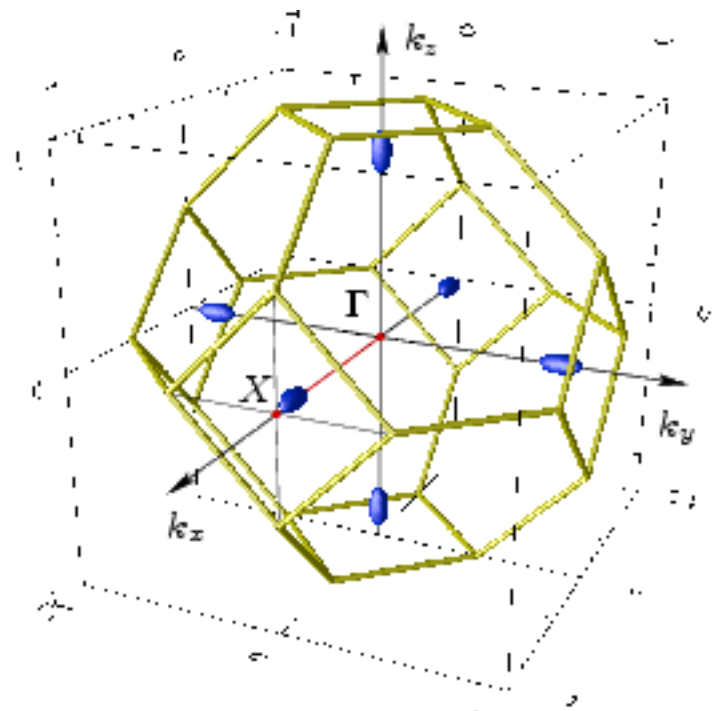
Al
FCC

web page: http://www.phys.ufl.edu/fermisurface/

Silicon bands and anisotropic effective masses



(a) Band diagram of silicon.



(b) First conduction band valleys.