Dynamics of EW & Strong Interactions

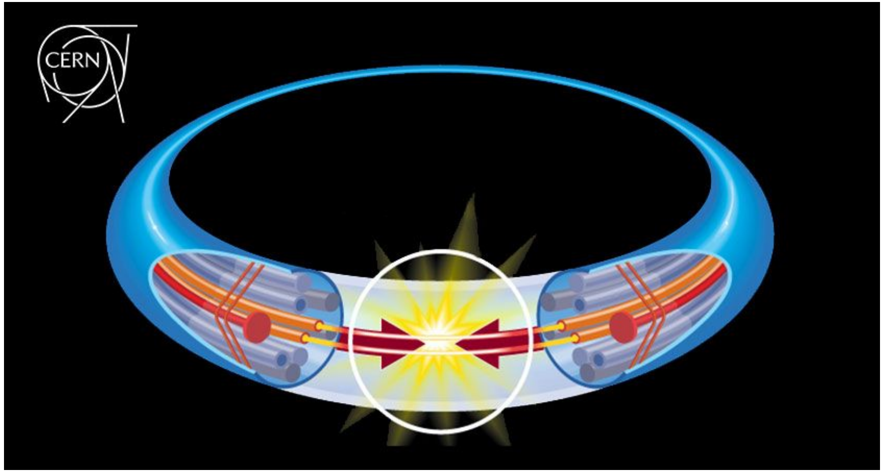
Part 4 - Dr. Michele Pinamonti (INFN Trieste)

Lecture 1 - Trieste, 12/12/2022

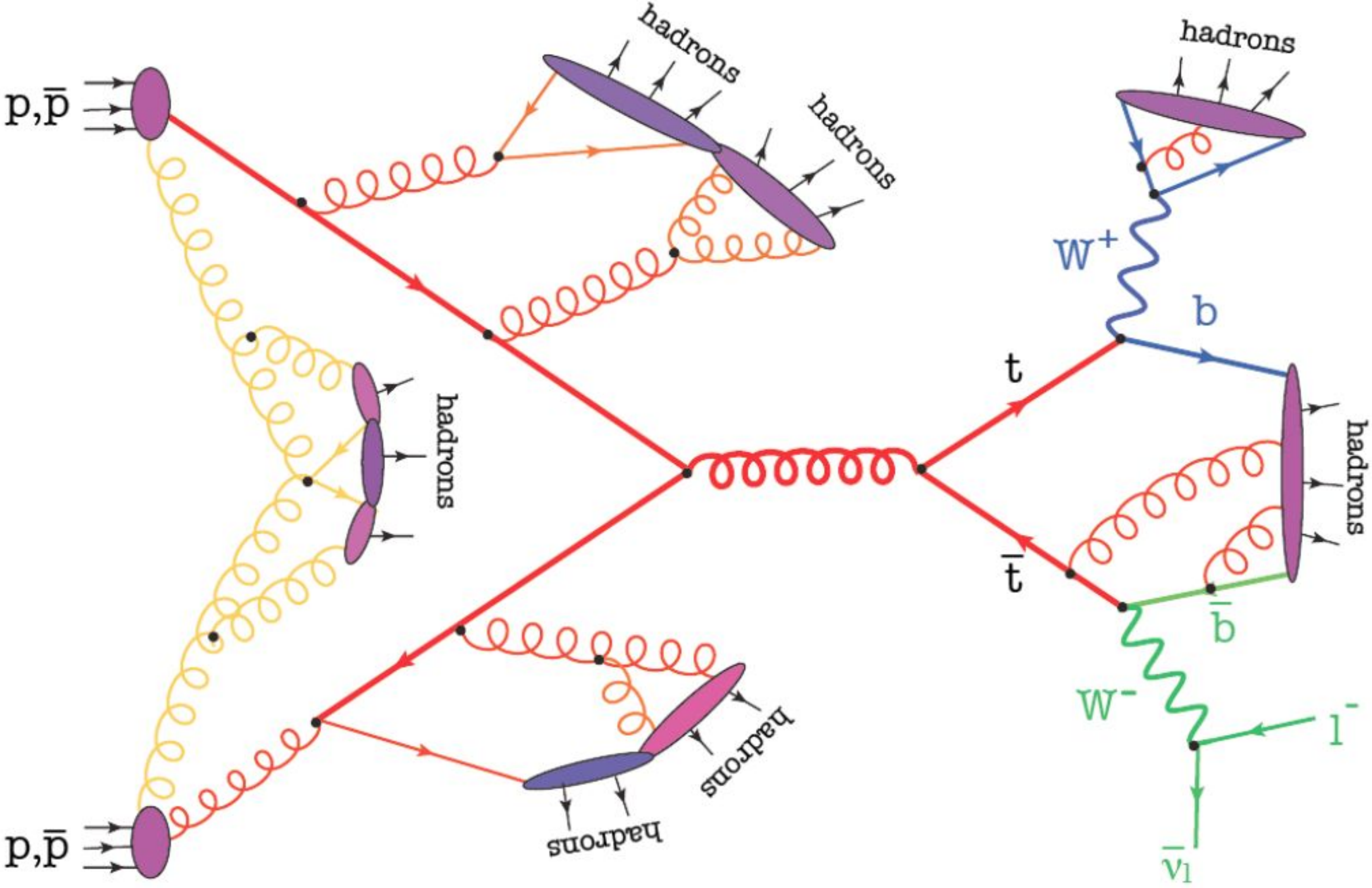
Introduction

- Main goal of these lectures:
  - fill gap between theory and "real world" (actually collider experiments)

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
& + i \bar{\psi} \not{D} \psi + h.c. \\
& + \bar{\psi}_i y_{ij} \psi_j \phi + h.c. \\
& + |D_\mu \phi|^2 - V(\phi)
\end{aligned}
$$



- Focusing on QCD effects in first part
  - moving to EW boson experimental signatures in the second part


p,p̄
hadrons
hadrons
hadrons
W$^+$
b
t
hadrons
hadrons
t̄
b̄
W$^-$
l$^-$
hadrons
hadrons
p,p̄
$\bar{\nu}_l$

QCD recap

QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}_{\mathrm{quarks}} + \mathcal{L}_{\mathrm{gluons}} + \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{ghost}}$$

QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \boxed{\mathcal{L}_{\text{quarks}}} + \mathcal{L}_{\text{gluons}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{ghost}}$$

$$\mathcal{L}_{\text{quarks}} = \sum_{q \in \{u,d,s,c,b,t\}} \bar{q}_a \left(i\gamma^\mu (\mathcal{D}_\mu)_{ab} - m_q \right) q_b$$

$$(\mathcal{D}_\mu)_{ab} = \partial_\mu \delta_{ab} + i g_s \mathcal{T}^A_{ab} \mathcal{A}^A_\mu$$

flavours

3 colours

8 gluon fields

QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \boxed{\mathcal{L}_{\text{quarks}}} + \mathcal{L}_{\text{gluons}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{ghost}}$$

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$$(\mathcal{D}_\mu)_{ab} = \partial_\mu \delta_{ab} + i g_s \boxed{\mathcal{T}^A_{ab}} \mathcal{A}^A_\mu$$

SU(3) generators: $\mathcal{T}^A = \lambda^A/2$

Gell-Mann Matrices:

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix},$$

QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{quarks}} + \mathcal{L}_{\text{gluons}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{ghost}}$$

$$\mathcal{L}_{\text{gluons}} = -\frac{1}{4}\mathcal{G}^{A}_{\mu\nu}\mathcal{G}^{\mu\nu}_{A}$$

gluon field tensor

$$\mathcal{G}^{A}_{\mu\nu} = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\mu - g_s f^{ABC}\mathcal{A}^B_\mu \mathcal{A}^C_\nu$$

generators' commutation rules:

$$[\mathcal{T}^A, \mathcal{T}^B] = i f^{ABC}\mathcal{T}^C$$

structure constants of SU(3)

$$f^{123} = 1$$

$$f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} = \frac{1}{2}$$

$$f^{458} = f^{678} = \frac{\sqrt{3}}{2},$$

QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{quarks}} + \mathcal{L}_{\text{gluons}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{ghost}}$$

Local gauge invariance i.e. invariance under simultaneous redefinition of q and g fields
⇒ explicit choice of gauge needed to define g field propagator

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{2\xi}\left(\partial^{\mu}\mathcal{A}_{\mu}^{A}\right)^{2}$$

A possible choice of gauge: *generalisation of the covariant Lorentz gauge* $\partial^{\mu}\mathcal{A}_{\mu}^{A} = 0$

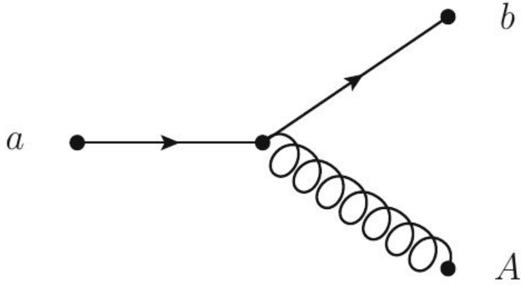
$$\mathcal{L}_{\text{ghost}} = \partial_{\mu}\eta^{A\dagger}\left(\mathcal{D}_{AB}^{\mu}\eta^{B}\right)$$

Ghosts: *complex scalar fields following Fermi-Dirac statistics; do not have a physical meaning, but should be considered as a mathematical trick to cancel nonphysical degrees of freedom*

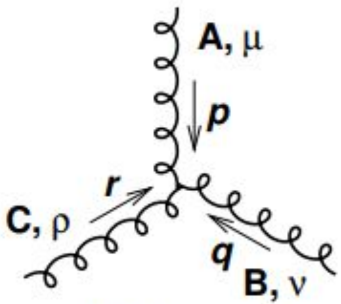
Feynman rules

$$\delta^{ab}\frac{i}{\not p - m} = \delta^{ab}\frac{i(\not p + m)}{p^2 - m^2}$$

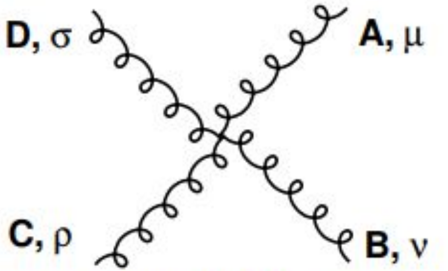
$$\delta^{AB}[-g_s^{\mu\nu} + (1-\lambda)\frac{p^\mu p^\nu}{p^2}]\frac{i}{p^2}$$



$$-ig_s\mathcal{T}^A_{ab}\gamma^\mu$$

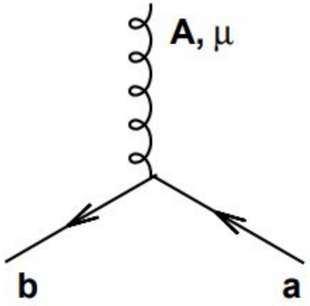


$$-g_s f^{ABC}[(p-q)^\rho g^{\mu\nu} + (q-r)^\mu g^{\nu\rho} + (r-p)^\nu g^{\rho\mu}]$$

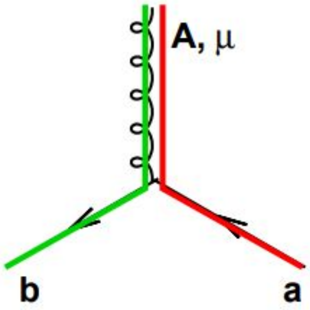


$$-ig_s^2 f^{XAC} f^{XBD}[g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\gamma}] + (C,\gamma) \leftrightarrow (D,\rho) + (B,\nu) \leftrightarrow (C,\gamma)$$

What do Feynman rules mean physically?



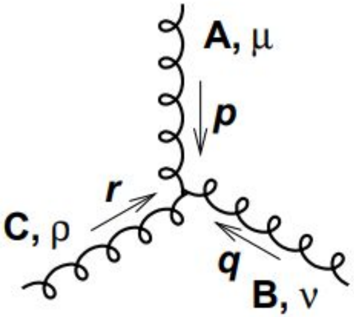
$\bar{\psi}_b(-ig_s t^A_{ba}\gamma^\mu)\psi_a$



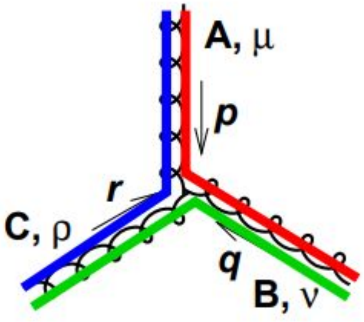
$$\underbrace{\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}}_{\bar{\psi}_b} \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{t^1_{ab}} \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{\psi_a}$$

A gluon emission **repaints** the quark colour.
A gluon itself carries colour and anti-colour.

What do Feynman rules mean physically?



$$-g_s f^{ABC}[(p-q)^\rho g^{\mu\nu} \\ +(q-r)^\mu g^{\nu\rho} \\ +(r-p)^\nu g^{\rho\mu}]$$



A gluon emission also repaints the gluon colours.

Because a gluon carries colour + anti-colour, it emits ∼ twice as strongly as a quark (just has colour)

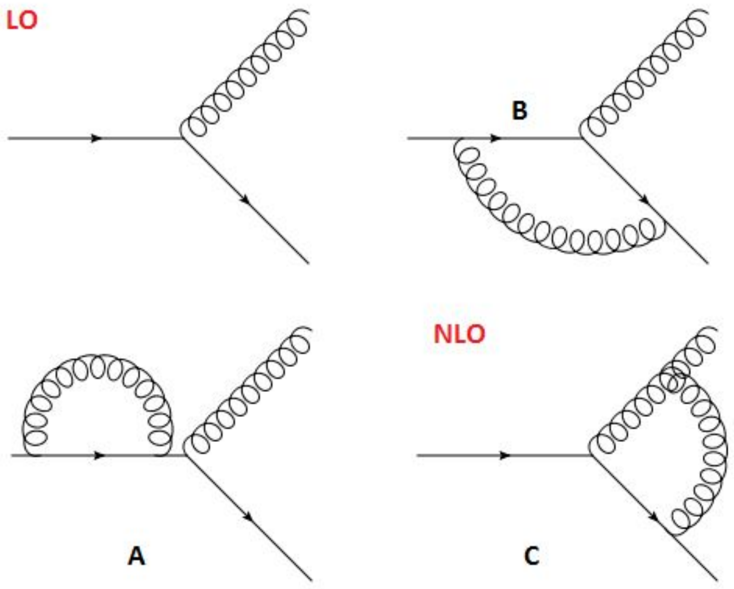
Perturbative QCD

- Quantitative predictions:
  - Lattice gauge theory (LGT)
  - Perturbative QCD (pQCD)
    - valid if $\alpha_s \ll 1$
- Perturbation theory:
  - quantity calculable as (converging) series:

$$\alpha_s + \underbrace{\alpha_s^2}_{\text{small}} + \underbrace{\alpha_s^3}_{\text{smaller}} + \underbrace{\cdots}_{\text{negligible?}}$$

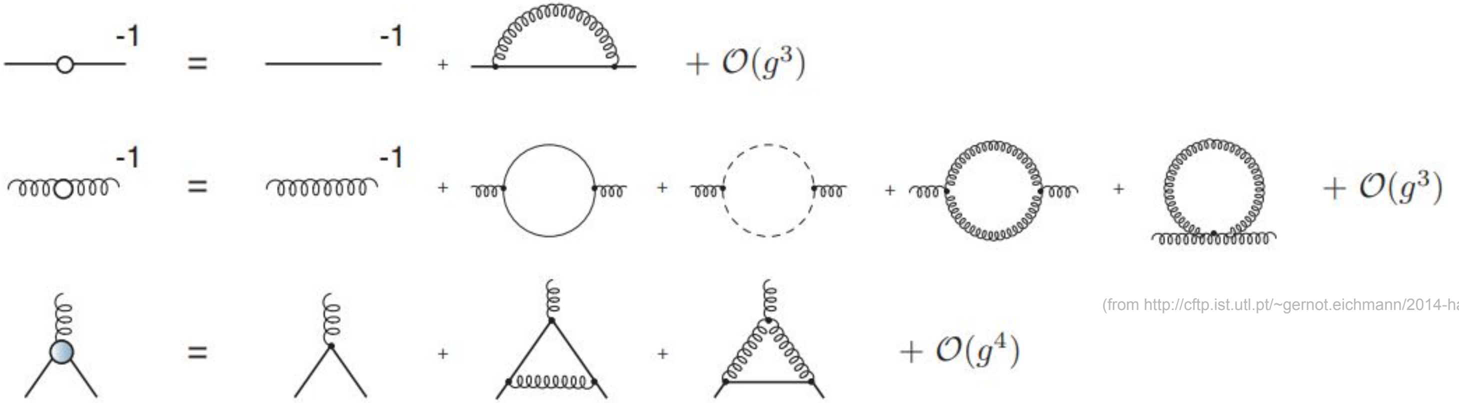
coupling constant

$$\alpha_S = g_s^2/4\pi$$



Renormalization and UV divergences

- Remember renormalization idea:
  - infinites in "bare" quantities absorbed in redefinition of "renormalized" quantities



(from http://cftp.ist.utl.pt/~gernot.eichmann/2014-hadron-physics/hadron-part-3.pdf)

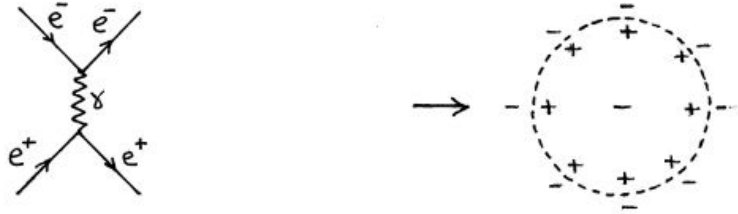
$$\int \frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m^2}$$

  - renormalization removes "ultra-violet" (UV) divergences:
  - divergent part (separated with "cut-off", μ_R) absorbed by redefinition of quantity

$k \rightarrow \infty \;\Rightarrow\; \int \rightarrow \infty$

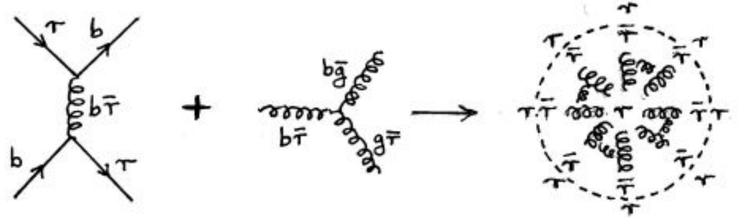
Running coupling

- Renormalization in QFT implies dependence of coupling constant on energy:
  - e.g. electric charge depends on momentum with which is probed
  - dependence via renormalization group equation



- $\alpha_S = g_s^2/4\pi$ dependence on Q^2 is:

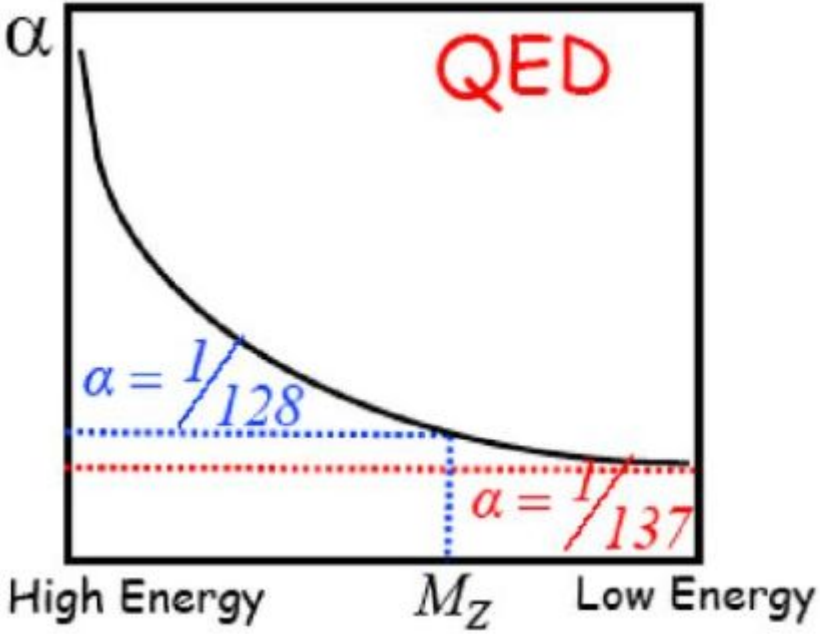
$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \beta_0\, \alpha_s(\mu^2) \log \frac{Q^2}{\mu^2}} = \frac{1}{\beta_0 \log \frac{Q^2}{\Lambda^2_{\text{QCD}}}}$$

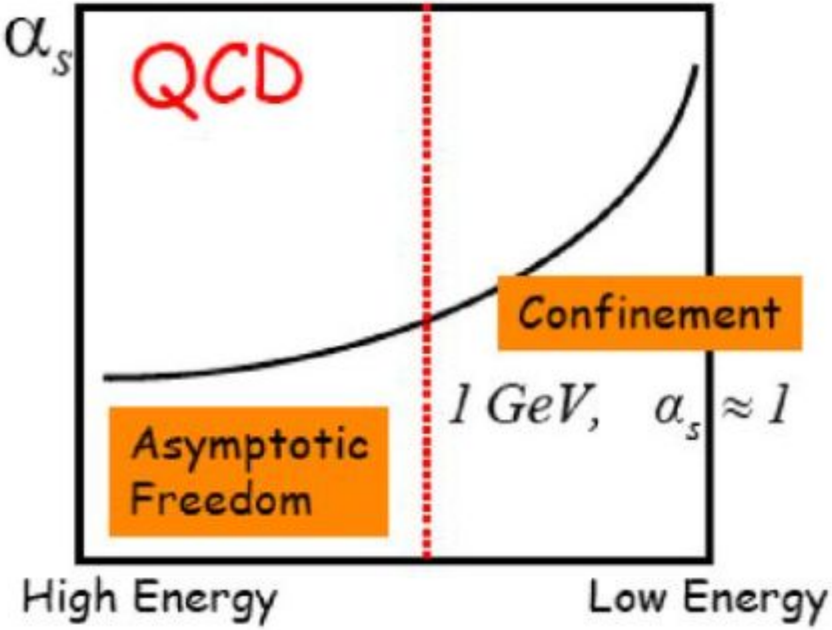


- $\beta_0 < 0$ in QED
- $\beta_0 > 0$ in QCD
  (due to gluon self-interaction)

$$\beta_0 = (11N_c - 2N_f)/12\pi \qquad \Lambda^2_{\text{QCD}} = \mu^2 \exp\left[\frac{-1}{\beta_0\, \alpha_s(\mu^2)}\right]$$

Running coupling

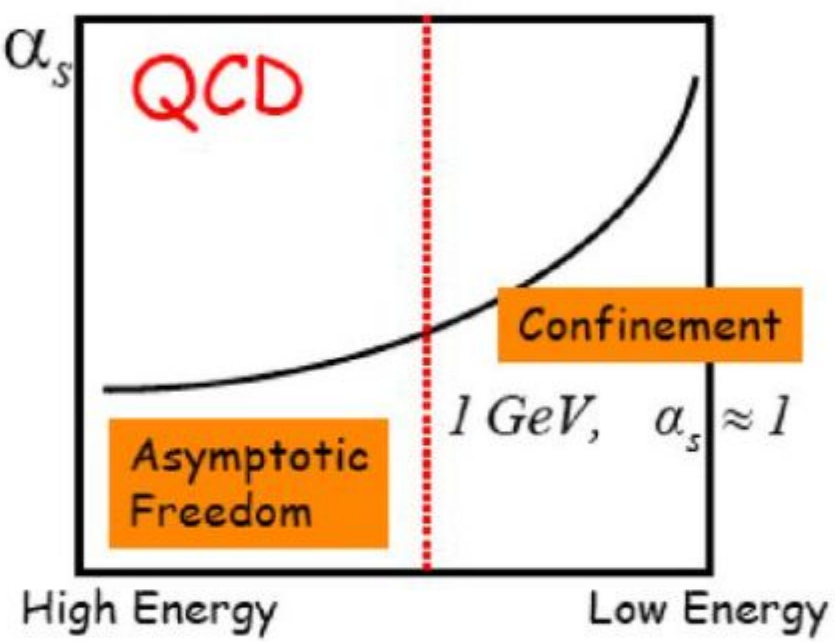




Perturbative vs. non-perturbative regimes

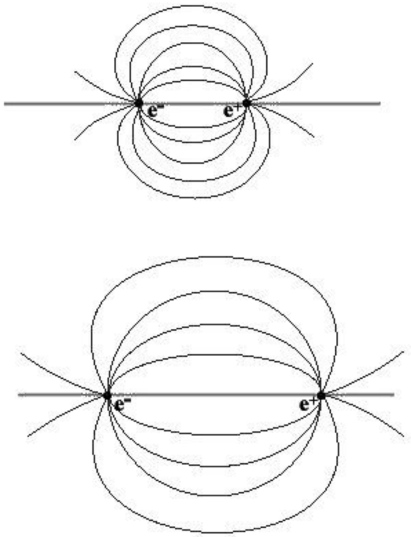
- High-energy / short distance:
  - small coupling $\alpha_S < 1$
  - perturbation theory valid

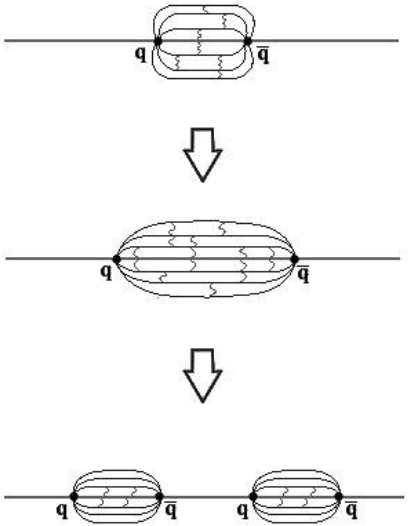
- Low-energy ($\lesssim$ 1 GeV) / long distance:
  - $\alpha_S > 1$
  - non-perturbative region
  - confinement

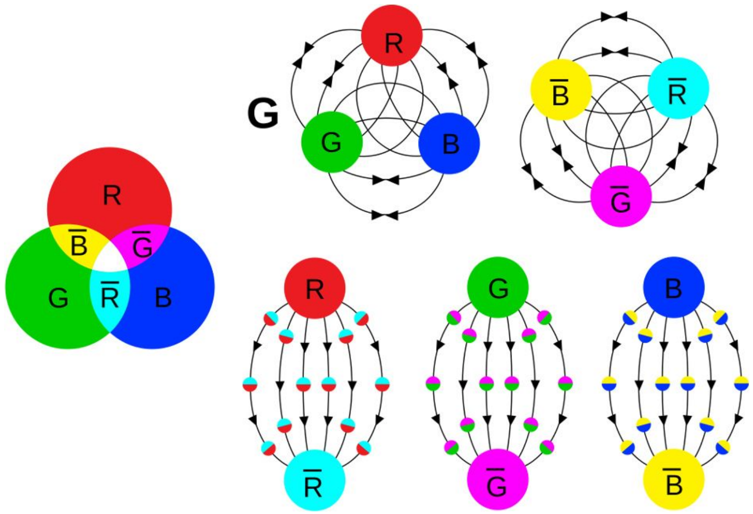


Confinement

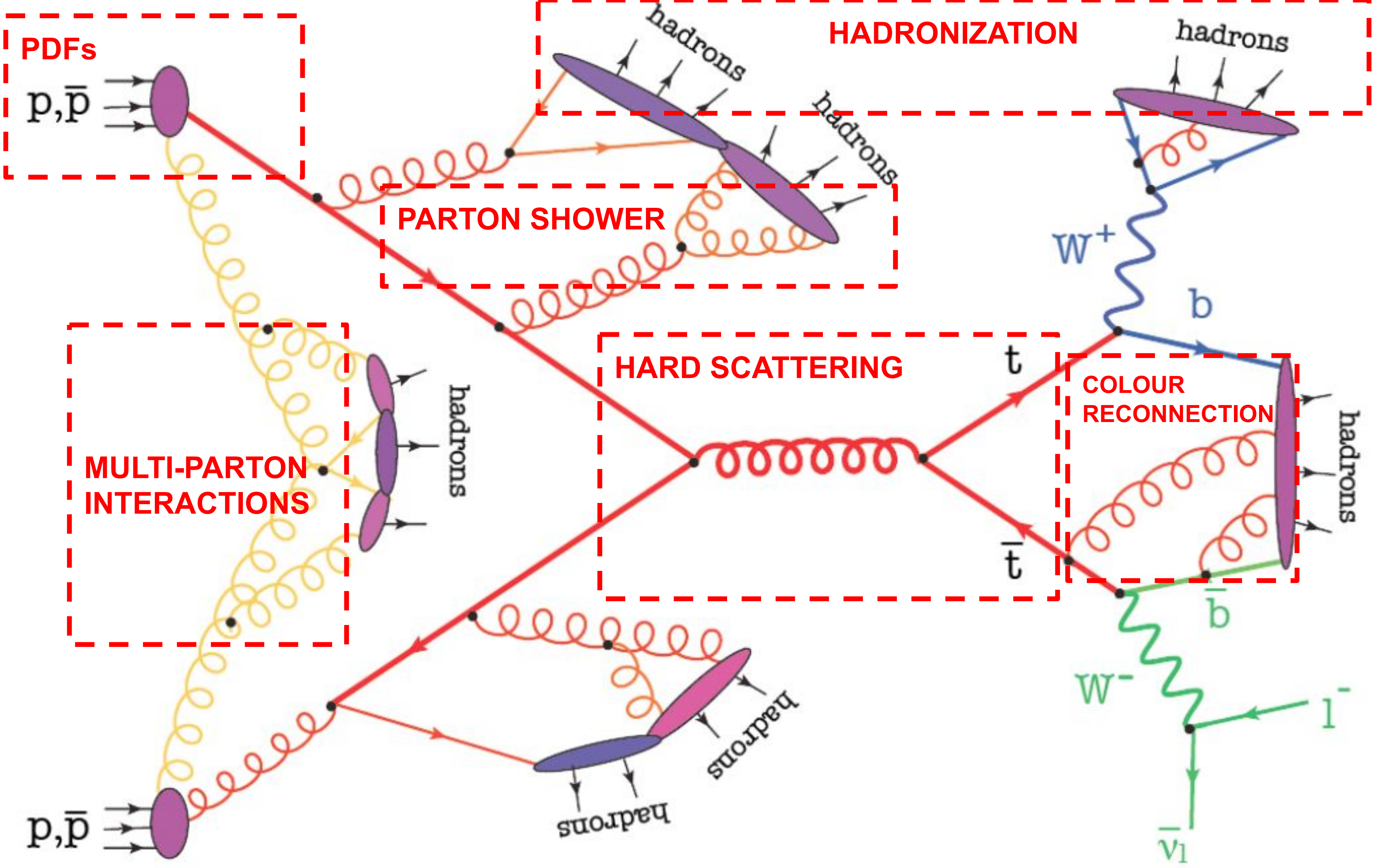
- Long distance ⇒ QCD interaction stronger and stronger
- In macroscopic world only colour-neutral states can exist







QCD at colliders

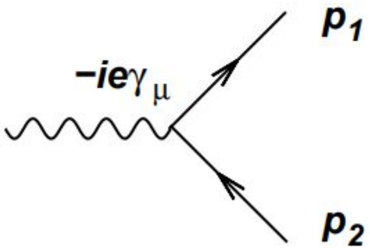


Infrared Divergences in QCD

IR divergence - soft gluon emission

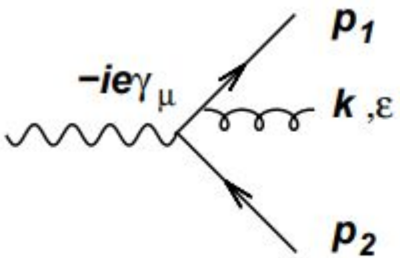
- Consider the process $e^+e^- \to \gamma^* \to q\bar{q}$

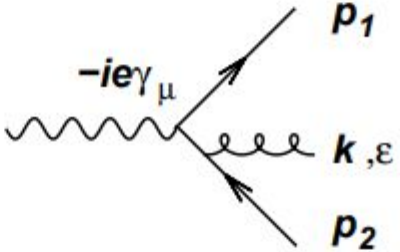
*: ignoring photon polarization



$$\mathcal{M}_{q\bar{q}} = -\bar{u}(p_1) i e_q \gamma_\mu v(p_2)$$

- Emit a gluon:





$$\mathcal{M}_{q\bar{q}g} = \bar{u}(p_1) i g_s \not{\epsilon} t^A \frac{i}{\not{p}_1 + \not{k}} i e_q \gamma_\mu v(p_2) + \bar{u}(p_1) i e_q \gamma_\mu \frac{i}{\not{p}_2 + \not{k}} i g_s \not{\epsilon} t^A v(p_2)$$

IR divergence - soft gluon emission

- Some rewriting:

$$\bar{u}(p_1) ig_s \not{\epsilon} t^A \frac{i}{\not{p}_1 + \not{k}} ie_q \gamma_\mu v(p_2) = -ig_s \bar{u}(p_1) \not{\epsilon} \frac{\not{p}_1 + \not{k}}{(p_1+k)^2} e_q \gamma_\mu t^A v(p_2)$$

Use $\not{A}\not{B} = 2A.B - \not{B}\not{A}$:

$$= -ig_s \bar{u}(p_1)[2\epsilon.(p_1+k) - (\not{p}_1 + \not{k})\not{\epsilon}] \frac{1}{(p_1+k)^2} e_q \gamma_\mu t^A v(p_2)$$

$$\bar{u}(p_1) = \bar{u}(0) \frac{\not{p}_1 + m}{\sqrt{2m(E+m)}}$$

Use $\bar{u}(p_1)\not{p}_1 = 0$ and $k \ll p_1$ (p_1, k massless)

- Make gluon soft:
  - $k \ll p_{1,2}$

$$\simeq -ig_s \bar{u}(p_1)[2\epsilon.p_1] \frac{1}{(p_1+k)^2} e_q \gamma_\mu t^A v(p_2)$$

$$= -ig_s \frac{p_1.\epsilon}{p_1.k} \underbrace{\bar{u}(p_1) e_q \gamma_\mu t^A v(p_2)}_{\text{pure QED spinor structure}}$$

$$\mathcal{M}_{q\bar{q}g} \simeq \bar{u}(p_1) ie_q \gamma_\mu t^A v(p_2)\, g_s \left(\frac{p_1.\epsilon}{p_1.k} + \frac{p_2.\epsilon}{p_2.k} \right)$$

$\not{p}v(p) = 0$,
$\not{p}\not{k} + \not{k}\not{p} = 2p.k$

IR divergence - soft gluon emission

- Get squared amplitude:

$$|M^2_{q\bar{q}g}| \simeq \sum_{A,\mathrm{pol}} \left| \bar{u}(p_1) i e_q \gamma_\mu t^A v(p_2)\, g_s \left(\frac{p_1.\epsilon}{p_1.k} + \frac{p_2.\epsilon}{p_2.k} \right) \right|^2$$

$$= -|M^2_{q\bar{q}}| C_F g_s^2 \left(\frac{p_1}{p_1.k} + \frac{p_2}{p_2.k} \right)^2 = |M^2_{q\bar{q}}| C_F g_s^2 \frac{2p_1.p_2}{(p_1.k)(p_2.k)}$$

sum on color comb. gives C_F

$$C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$

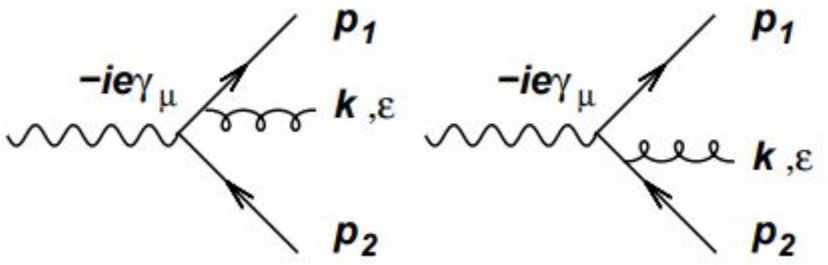
sum on polarizations

- Include phase space:

$$d\Phi_{q\bar{q}g} |M^2_{q\bar{q}g}| \simeq (d\Phi_{q\bar{q}} |M^2_{q\bar{q}}|) \frac{d^3\vec{k}}{2E(2\pi)^3} C_F g_s^2 \frac{2p_1.p_2}{(p_1.k)(p_2.k)}$$

IR divergence - soft gluon emission

- Factorization into qq and gluon emission:



$$d\Phi_{q\bar{q}g}|M^2_{q\bar{q}g}| \simeq (d\Phi_{q\bar{q}}|M^2_{q\bar{q}}|)\,\frac{d^3\vec{k}}{2E(2\pi)^3}C_F g_s^2\frac{2p_1.p_2}{(p_1.k)(p_2.k)}$$

$$\left(d^3\vec{k} = E^2 d\phi\, d\cos\theta \right)$$

$$\boxed{dS = EdE\, d\cos\theta\,\frac{d\phi}{2\pi}\cdot\frac{2\alpha_s C_F}{\pi}\frac{2p_1.p_2}{(2p_1.k)(2p_2.k)}}$$

$\theta \equiv \theta_{p_1 k}$

$\phi = \text{azimuth}$

IR divergence - soft gluon emission

- Rewrite in terms of E, θ (of emitted gluon, in CM frame):

$$\frac{2p_1.p_2}{(2p_1.k)(2p_2.k)} = \frac{1}{E^2(1-\cos^2\theta)}$$

$\left(\theta_{12} = \pi,\ \theta_{2k} = \pi - \theta_{1k} \Rightarrow \cos\theta_{2k} = -\cos\theta_{1k}\right)$

- Final expression for soft gluon emission:

$\left(d\cos\theta = -\sin\theta\ d\theta\right)$

$$dS = \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

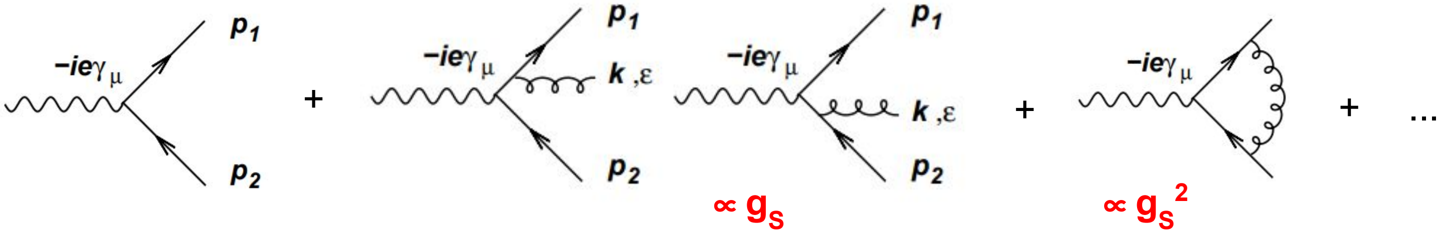
Soft divergence ($E \to 0$)

Collinear divergence ($\theta \to 0$)

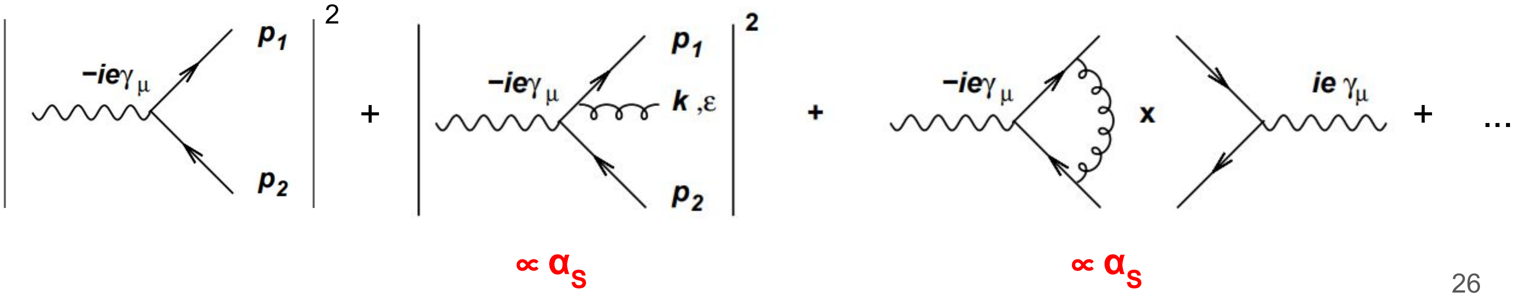
Real-virtual cancellation

- Is this a problem of the theory?
  - need to consider virtual / loop corrections as well!

$$\mathcal{M}_{q\bar{q}+X} =$$



$$\sigma_{q\bar{q}+X} =$$



Real-virtual cancellation

- The total cross-section *is / must be* finite
    - i.e. divergent integration in real part cancelled by divergent part in virtual contribution

$$\sigma_{tot} = \sigma_{q\bar{q}} \left(1 + \frac{2\alpha_s C_F}{\pi} \int \frac{dE}{E} \int \frac{d\theta}{\sin\theta} R(E/Q,\theta) - \frac{2\alpha_s C_F}{\pi} \int \frac{dE}{E} \int \frac{d\theta}{\sin\theta} V(E/Q,\theta) \right)$$

Real-virtual cancellation

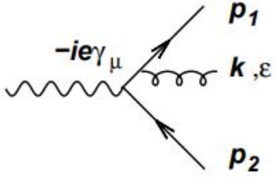
$$\sigma_{tot} = \sigma_{q\bar{q}} \left(1 + \frac{2\alpha_s C_F}{\pi} \int \frac{dE}{E} \int \frac{d\theta}{\sin\theta} \left(R(E/Q,\theta) - V(E/Q,\theta) \right) \right)$$

- R(E/Q, θ) and V(E/Q, θ) should cancel for soft and collinear divergences:

$$\lim_{E\to 0}(R - V) = 0\,, \qquad \lim_{\theta\to 0,\pi}(R - V) = 0$$

- Consequences:
  - emission of soft / collinear gluon cannot change total cross-section (contribution cancelled by virtual correction)
  - corrections to σ tot come from hard & large-angle gluon emission

Soft gluons don't matter...



- Physics reason:
  - soft gluons emitted on long timescale ~ 1/(Eθ) relative to collision (1/Q) ⇒ cannot influence cross section
  - transition to hadrons also on long time scale (~ $1/\Lambda_{QCD}$) ⇒ can also be ignored

- Correct renorm. scale for α_S: μ ~ Q ⇒ perturbation theory valid
  - dependence of total cross section on only hard gluons ⇒ ‘good behaviour’ of perturbation series

$$\sigma_{tot} = \sigma_{q\bar{q}} \left(1 + 1.045 \frac{\alpha_s(Q)}{\pi} + 0.94 \left(\frac{\alpha_s(Q)}{\pi} \right)^2 - 15 \left(\frac{\alpha_s(Q)}{\pi} \right)^3 + \cdots \right)$$

(for Q = M_Z)

End of Lecture 1

References

- Gavin Salam's lectures: https://gsalam.web.cern.ch/gsalam/teaching/PhD-courses.html
- LHC Physics - lectures at SUSSP65 Summer School https://www.taylorfrancis.com/books/edit/10.1201/b11865/lhc-physics-buttar-clark-binoth-glover
  - Perturbative QCD and the parton model - Keith Ellis
  - Monte Carlo tools - Torbjörn Sjöstrand
- Jet Physics at the LHC - The Strong Force beyond the TeV Scale - Klaus Rabbertz https://link.springer.com/book/10.1007/978-3-319-42115-5