Dynamics of EW & Strong Interactions

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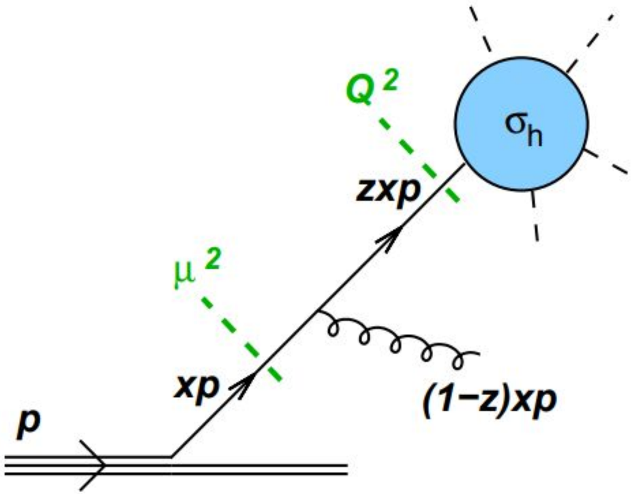
Lecture 3 - Trieste, 20/12/2022

Initial State Radiation (continued)

Collinear cutoff and PDFs



- For kt → 0 QCD becomes non-perturbative
- Cut out divergent region and add non-perturbative quark distribution is proton (PDF)

$$\sigma_0 = \int dx \, \sigma_h(xp) \, q(x, \mu^2)$$

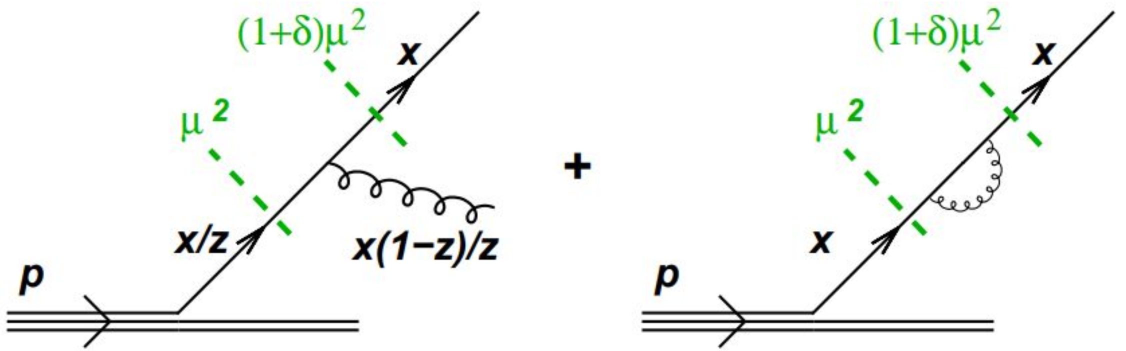
proton momentum fraction carried by quark

factorisation scale

$$\sigma_1 \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_{\mu^2}^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{finite (large)}} \underbrace{\int \frac{dx\, dz}{1-z} \left[\sigma_h(zxp) - \sigma_h(xp)\right] q(x, \mu^2)}_{\text{finite}}$$

DGLAP evolution

- DGLAP (Dokshizer Gribov Lipatov Altarelli Parisi) equations regulate PDF evolution to different scales



(change of strategy: fixing momentum of parton entering hard process)

$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 dz\, P_{qq}(z)\,\frac{q(x/z,\mu^2)}{z}$$

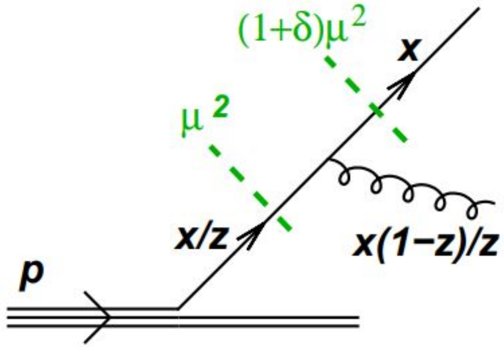
plus notation

splitting function

$$= \frac{\alpha_s}{2\pi}\int_x^1 dz\, p_{qq}(z)\,\frac{q(x/z,\mu^2)}{z} - \frac{\alpha_s}{2\pi}\int_0^1 dz\, p_{qq}(z)\, q(x,\mu^2)$$

Initial state radiation and factorization scale

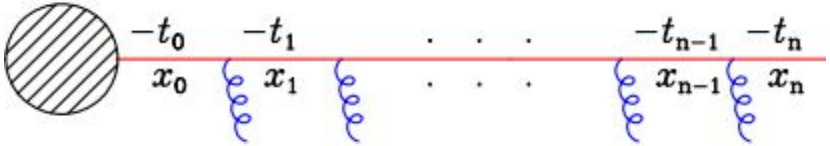
- Important to keep in mind:



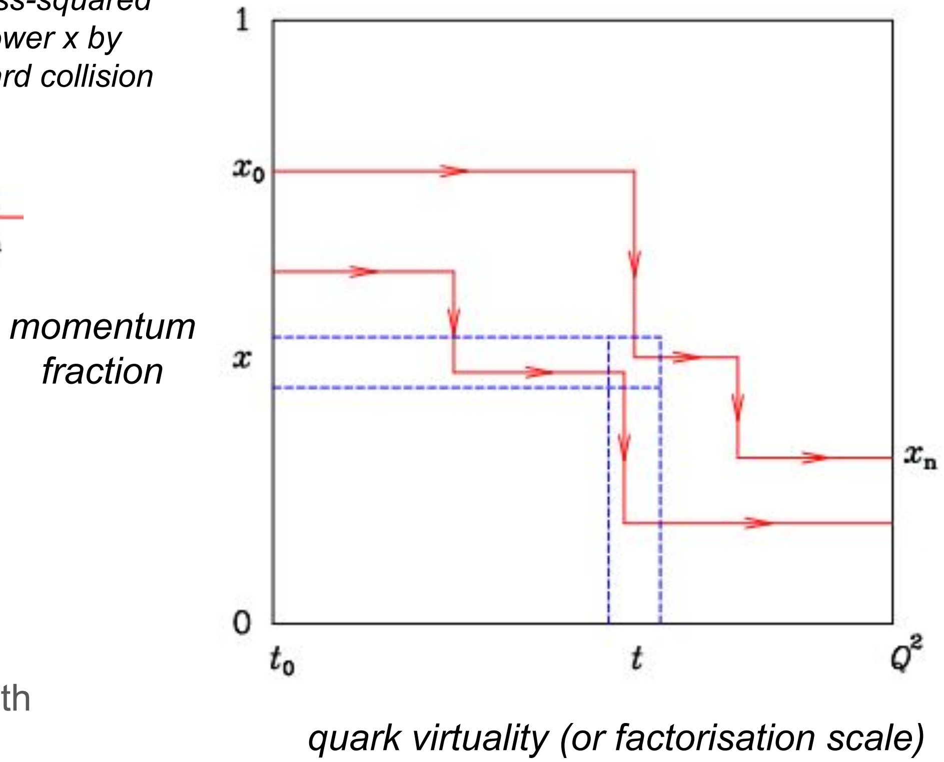
- changing factorization scale can affect ISR calculation / simulation!

Pictorial representation of DGLAP evolution

Incoming quark from proton, initially with low virtual mass-squared $-t_0$ and with $x=x_0$, moves to more virtual masses and lower x by successive small-angle emissions, and finally enters hard collision with virtual mass-squared $q^2 = -Q^2$

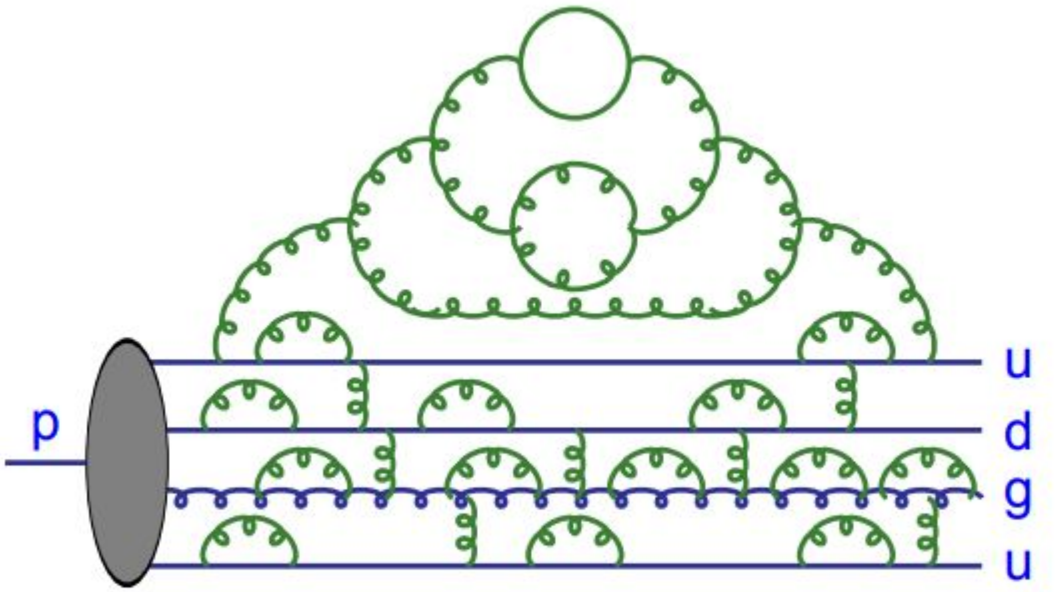


- At $t = t_0$ the x distribution is the PDF at $\mu^2 = t_0$
    - distribution of initial lines
- PDF at a different scale $\mu^2 = t$ is the distribution of paths
    - obtainable by evolving each path with proper branching probabilities



Pictorial representation of ISR

- Parton cascades in p are continuously born and recombined
  - hard scattering inhibits full recombination of the cascade



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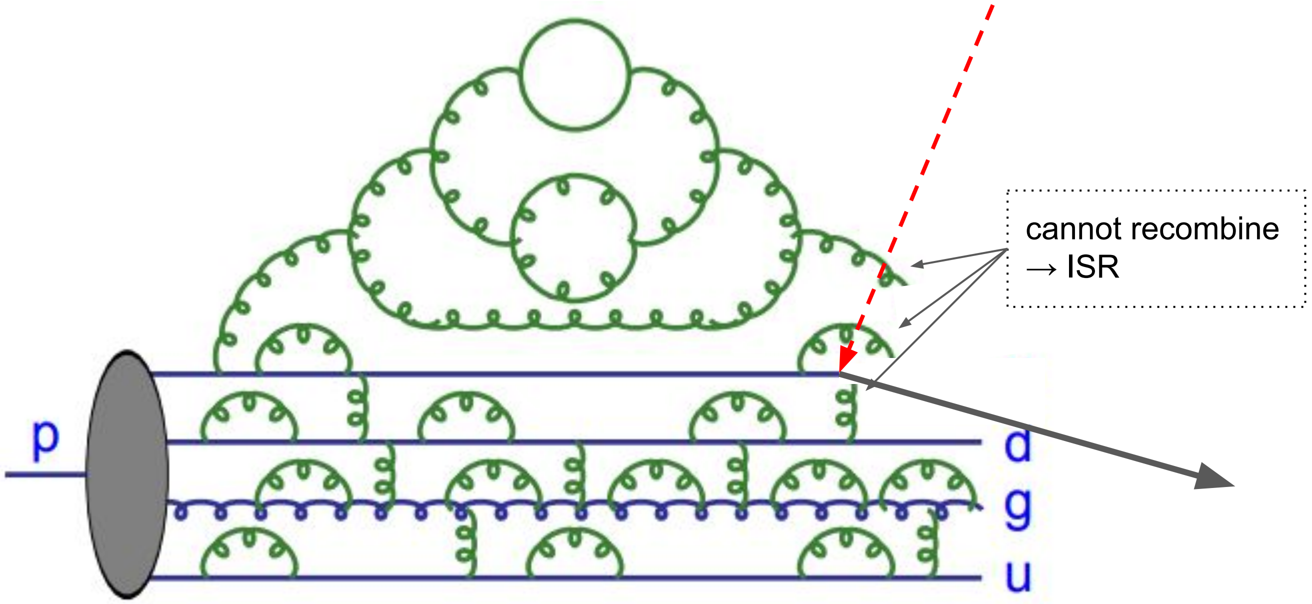
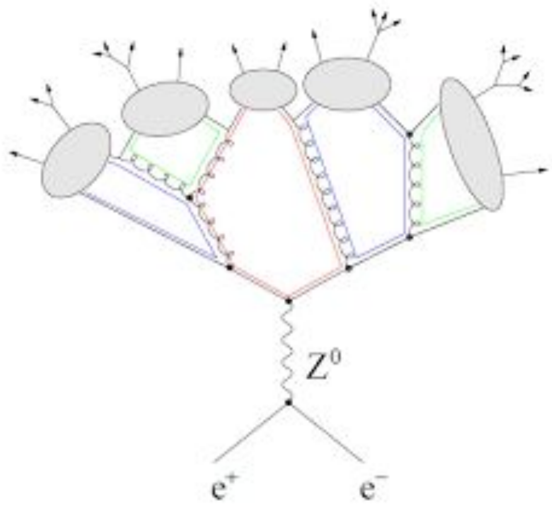


Hadronization

Hadronization

- Evolution of parton shower to lower and lower energies brings to non-perturbative regime
    - transition between perturbative regime (quark and gluon showers)
      and non-perturbative regime (hadrons) regulated by "hadronization" process



Fragmentation function

- Similarly to PDFs, can cut the perturbative evolution of the shower and absorb divergent + non-perturbative part into a "fragmentation function" (FF)

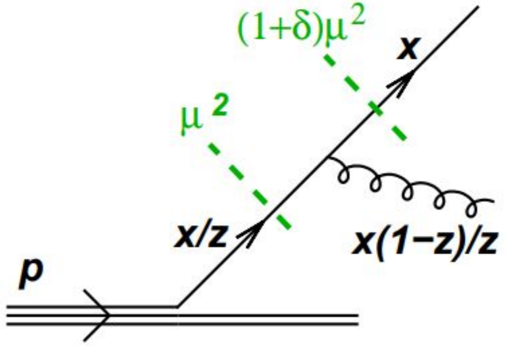
$$\frac{d\sigma}{dx}(e^+e^- \to H + X) = \frac{d\hat{\sigma}}{dx} \otimes D_{q\to H}(x)$$

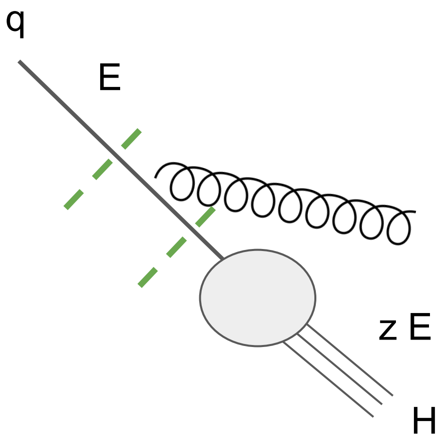
fragmentation function

perturbative
cross-section with cut-off

Fragmentation function

- Like for PDFs, distinction between parton shower and hadronization is arbitrary
  - ⇒ FF depend on cut-off value → factorization scale

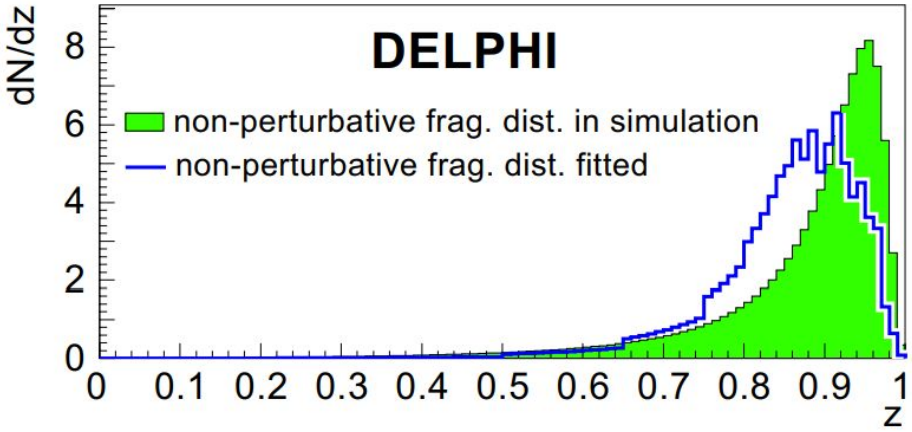




Fragmentation function

- Example of FF model:
  - Lund-Bowler (used in Pythia)

https://cds.cern.ch/record/1308131/files/CERN-PH-EP-2010-057.pdf



$$f(z) = c \cdot \frac{1}{z^{1+R_Q b m_Q^2}} z^{a_\alpha} \cdot \left(\frac{1-z}{z}\right)^{a_\beta} \cdot \exp\left(-\frac{b m_T^2}{z}\right)$$

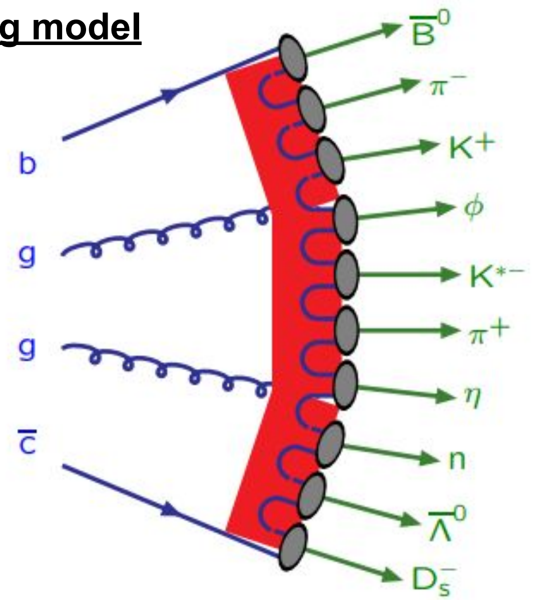
probability density of getting hadron with energy $z \cdot E$ from a "final-state" quark Q with energy E

Note:
measurement obtained from $e^+e^- \to b\bar{b}$ data at fixed CM energy (m_Z) ⇒ fixed b-quark energy, looking at energy distribution of B-hadrons
⇒ measuring FF($\mu = m_Z/2$)
and then evolving down to $\mu \sim \Lambda_{QCD}$

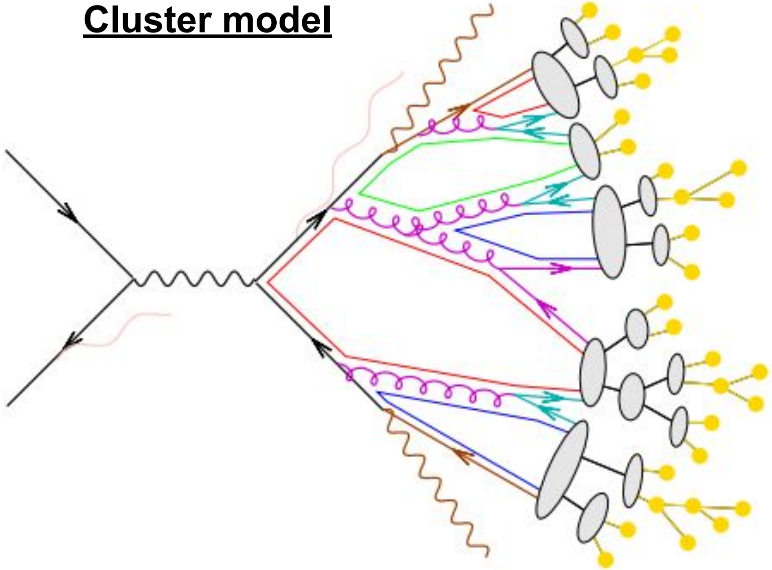
Hadronization models

- For detailed description of hadronization process particle-by-particle
  - via some empirical models: ideologically motivated models
    + "cookbook" recipes with free parameters tuned with experimental data

String model



Cluster model



MC simulation

The ideas behind MC generation

- Want to generate events in as much detail as Mother Nature
  - get average and fluctuations right
  - make random choices ~ as in (microscopic) nature

- Factorization is the key:
  - split complex process into pieces and assume independence / factorization:

    $\sigma_{\text{final state}} = \sigma_{hp}$ P(hard process→final state)

    with:

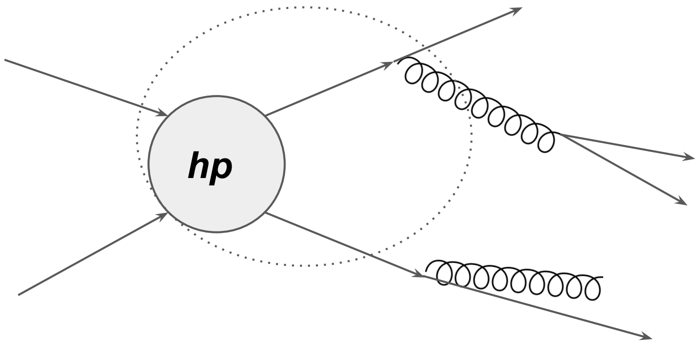
    P(hard process→final state) = $P_{ISR}\, P_{FSR}\, P_{had}\, P_{decay}$ ...

    $P_{FSR} = P_{i\to jk}(p_i,p_j,p_k) \cdot P_{k\to qm}(p_k,p_q,p_m) \cdot ...$

    ...

Parton shower

- Idea: factorize complex process 2 → n into a simple 2 → 2 + convolution with "parton shower"
    - approximation: tot cross-section not changed by parton shower
      i.e. $\sigma_{hp+0j} + \sigma_{hp+1j} + \sigma_{hp+2j} + ... = \sigma_{hp}$
        - valid if emissions are small-angle / soft !
- Parton-shower (PS) algorithms suitable for MC simulation
    - at present, most PS implementations are LO and use several approximations



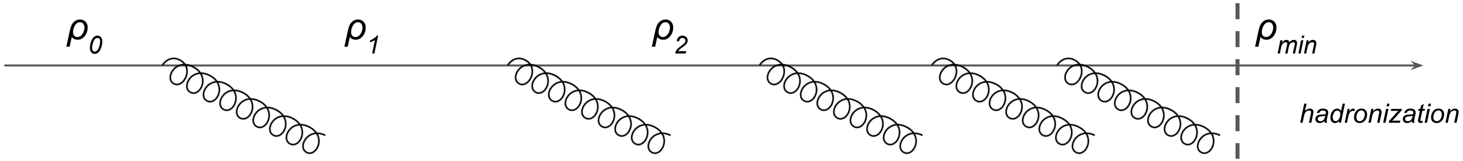
<u>Note</u>:
decision on what to include in "hp" arbitrary (hard gluon emission included or left to parton shower?)

PS evolution

- Remind FSR emission (real): $$d\sigma_{n+1} = d\sigma_n \frac{dt}{t} dz \frac{\alpha_s}{2\pi} \hat{P}_{ba}(z)$$

- Same equation for any "evolution variable" $\rho \propto \theta^2$:
  - $t = q^2 = z(1 - z)\theta^2 E^2$: virtuality of off-shell propagator
  - $p^2_\perp = z^2 (1 - z)^2 \theta^2 E^2$: gluon transverse momentum w.r.t. parent quark

$$\frac{d\theta^2}{\theta^2} = \frac{dq^2}{q^2} = \frac{dp_\perp^2}{p_\perp^2}$$



- In line with angular ordering, can consider system evolving to lower and lower ρ, by successive parton branching
  - "parton shower" process
  - stopping when $\rho \to \rho_{min} \leftrightarrow \Lambda_{QCD} \sim 1$ GeV

Emission probability

- What is the probability of emission / splitting for a final-state parton from a given hard process (*hp*)?
  - emission rate / probability density (?):

$$d\mathcal{P}_{em}(\rho, z) = d\mathcal{S} = d(\sigma_{hp+1j}/d\sigma_{hp}) = \frac{\alpha_S}{2\pi}\frac{d\rho}{\rho}\hat{P}_{ba}(z)dz$$

  - probability of emission in certain ranges of ρ and z:

$$\mathcal{P}_{naive} = \int d\mathcal{P}_{em} = \frac{\alpha_S}{2\pi}\int_{\rho_{min}}^{\rho_0}\frac{d\rho}{\rho}\int_{z_{min}}^{z_{max}}\hat{P}_{ba}(z)dz$$

constant-coupling approximation

e.g.
(with $p^2_\perp$ evolution)

$$= \frac{\alpha_S}{2\pi}\int_{p_{\perp,min}}^{p_{\perp,max}}\frac{dp_\perp^2}{p_\perp^2}\int_{z_{min}=p_\perp/E}^{z_{max}=1-p_\perp/E}\hat{P}_{ba}(z)dz$$

(see backup)

Sudakov Form Factors

- Probabilistic interpretation problematic:
  - $\mathcal{P}_{naive}$ > 1 close to divergence!

- Concept of Sudakov form factor:
  - probability of branching (**for the first time**) at certain evolution scale ρ
    = to probability of NOT having emitted before
    times probability of branching at ρ

$$d\mathcal{P}_{first}(\rho) = d\mathcal{P}(\rho) \cdot \mathcal{P}_{no-em}(\rho_0, \rho)$$

- Call $\Delta(\rho_1, \rho_2) := \mathcal{P}_{\mathrm{no-em}}(\rho_1, \rho_2)$ Sudakov form factor

Sudakov Form Factors

- Probability of no emission:
  - it's multiplicative:

$$\mathcal{P}_{no-em}(\rho_0, \rho_{max}) = \mathcal{P}_{no-em}(\rho_0, \rho_1) \cdot \mathcal{P}_{no-em}(\rho_1, \rho_{max})$$

$$= \prod_{i=0}^{N} \mathcal{P}_{no-em}(\rho_i, \rho_{i+1})$$

$$= \prod_{i=0}^{N} [1 - \mathcal{P}_{em}(\rho_i, \rho_{i+1})]$$

$$= \lim_{N \to \infty} \prod_{i=0}^{N} [1 - d\mathcal{P}_{em}(\rho_i)]$$

Sudakov Form Factors

- We have: $$\mathcal{P}_{no-em}(\rho_0, \rho_{max}) = \lim_{N\to\infty} \prod_{i=0}^{N} [1 - d\mathcal{P}_{em}(\rho_i)]$$
  - take the log:

$$\log \mathcal{P}_{no-em}(\rho_0, \rho_{max}) = \lim_{N\to\infty} \sum_{i=0}^{N} \log[1 - d\mathcal{P}_{em}(\rho_i)]$$

$$= - \lim_{N\to\infty} \sum_{i=0}^{N} d\mathcal{P}_{em}(\rho_i)$$

$$= - \int_{\rho_{min}}^{\rho_0} \frac{d\mathcal{P}_{em}(\rho)}{d\rho} d\rho$$

$$\log(1 - x) = -x + \mathcal{O}(x^2)$$

$$d\mathcal{P}_{em}(\rho) = \frac{\alpha_S}{2\pi} \frac{d\rho}{\rho} \int_{z_0}^{z_{max}} \hat{P}_{ba}(z) dz$$

$\Rightarrow$

$$\mathcal{P}_{no-em}(\rho_0, \rho_{min}) = \exp\left[- \int_{\rho_{min}}^{\rho_0} \frac{d\mathcal{P}_{em}(\rho)}{d\rho} d\rho\right] = \exp\left[- \frac{\alpha_S}{2\pi} \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_{min}}^{z_{max}} \hat{P}_{ba}(z) dz\right]$$

Sudakov Form Factors

- Finally we can write:

$$d\mathcal{P}_{first}(\rho) = d\mathcal{P}(\rho) \cdot \Delta(\rho_0, \rho)$$
$$= \frac{\alpha_S}{2\pi} \frac{d\rho}{\rho} \int_{z_{min}}^{z_{max}} \hat{P}_{ba}(z) dz \Delta(\rho_0, \rho)$$

  - with:

$$\Delta(\rho_0, \rho) = \exp\left[- \int_{\rho}^{\rho_0} \frac{d\mathcal{P}_{em}(\rho')}{d\rho'} d\rho' \right]$$

$$= \exp\left[- \frac{\alpha_S}{2\pi} \int_{\rho}^{\rho_0} \frac{d\rho'}{\rho'} \int_{z_{min}}^{z_{max}} \hat{P}_{ba}(z) dz \right]$$

<u>Note</u>:

- *a → bc type of splitting fixed in dP_{em}*
- *not fixed in Sudakov factor: should sum over all possible types of splitting for parton a (need to take probability of not undergoing any splitting)*

Properties of Sudakov Form Factor

1. Unitarity - probability of first emission happening somewhere:

$$\int_{\rho_{\min}}^{\rho_0} \frac{\mathrm{d}\rho}{\rho} \int_{z_{\min}}^{z_{\max}} \mathrm{d}z \frac{\alpha_{\mathrm{s}}}{2\pi} P(z) \Delta(\rho_0, \rho) = \int_{\rho_{\min}}^{\rho_0} \mathrm{d}\rho \frac{\mathrm{d}}{\mathrm{d}\rho} \exp\left(-\int_{\rho}^{\rho_0} \frac{\mathrm{d}\rho'}{\rho'} \int_{z_{\min}}^{z_{\max}} \mathrm{d}z \frac{\alpha_{\mathrm{s}}}{2\pi} P(z)\right)$$

$$= \exp\left(-\int_{\rho_0}^{\rho_0} \frac{\mathrm{d}\rho}{\rho} \cdots\right) - \exp\left(-\int_{\rho_{\min}}^{\rho_0} \frac{\mathrm{d}\rho}{\rho} \cdots\right)$$

$$\xrightarrow{\rho_{\min} \to 0} \exp(-0) - \exp(-\infty) = 1$$

Properties of Sudakov Form Factor

2. Sudakov FF is an all-order expression:

$$\Delta(\rho_0, \rho_1) = e^{-\alpha_S F(\rho_0, \rho_1)}$$

$$= 1 - \alpha_S F(\rho_0, \rho_1) + \frac{1}{2}\alpha_S^2 F(\rho_0, \rho_1)^2 - \frac{1}{6}\alpha_S^3 F(\rho_0, \rho_1)^3 + \ldots$$

- Sudakov contains divergent terms of first order virtual correction, second order virtual correction, . . . all orders!

A (FSR) PS algorithm

- Given these properly defined probabilities, possible to build algorithms to "shower" final-state partons from hard scattering:
  - for each parton, with given initial evolution variable ρ_0, throw random number to determine at which value ρ_1 it branches for the first time
  - determine z from its probability distribution
  - continue evolving from ρ_1
  - evolve all other partons (also secondary)
  - stop at cut-off value ρ_{min}

A toy shower

```
#!/usr/bin/env python
# an oversimplified (QED-like) parton shower
# for Zuoz lectures (2016) by Gavin P. Salam
from random import random
from math import pi, exp, log, sqrt

ptHigh = 100.0
ptCut = 1.0
alphas = 0.12
CA=3

def main():
    for iev in range(0,10):
        print "\nEvent", iev
        event()
```

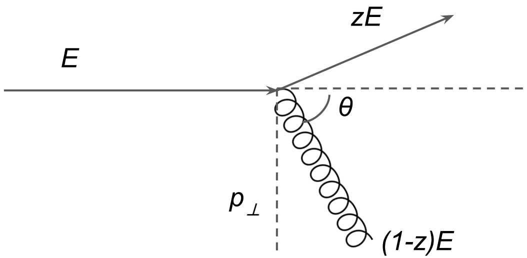
To run it simply copy-paste the text into a new file "myPS.py" and run from the terminal "python myPS.py"

```
def event():
    # start with maximum possible value of Sudakov
    sudakov = 1
    while (True):
        # scale it by a random number
        sudakov *= random()
        # deduce the corresponding pt
        pt = ptFromSudakov(sudakov)
        # if pt falls below the cutoff, event is finished
        if (pt < ptCut): break
        print " primary emission with pt = ", pt

def ptFromSudakov(sudakovValue):
    """Returns the pt value that solves the relation
    Sudakov = sudakovValue (for 0 < sudakovValue < 1)
    """
    norm = (2*CA/pi)
    # r = Sudakov = exp(-alphas * norm * L^2)
    # --> log(r) = -alphas * norm * L^2
    # --> L^2 = log(r)/(-alphas*norm)
    L2 = log(sudakovValue)/(-alphas * norm)
    pt = ptHigh * exp(-sqrt(L2))
    return pt

main()
```

Backup

Derivation of z_{min}, z_{max} (slide 19)

- We had this expression: $$\frac{\alpha_S}{2\pi}\int_{p_{\perp,min}}^{p_{\perp,max}}\frac{dp_\perp^2}{p_\perp^2}\int_{z_{min}=p_\perp/E}^{z_{max}=1-p_\perp/E}\hat{P}_{ba}(z)dz$$

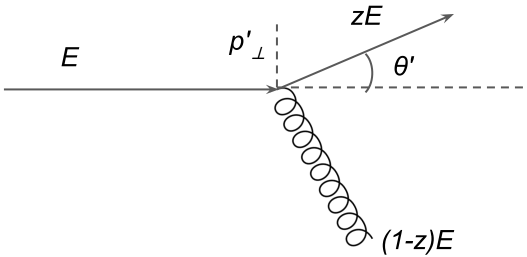


$p_\perp = (1\text{–}z)E\cos\theta$

$z = 1 – p_\perp/E\cos\theta$

which is *max* when $\cos\theta = 1$

$\Rightarrow z_{max} = 1 – p_\perp/E$



$p'_\perp = z\,E\cos\theta'$

$z = p_\perp/E\cos\theta'$

which is *min* when $\cos\theta' = 1$

$\Rightarrow z_{min} = p_\perp/E$

References

- Gavin Salam's lectures: https://gsalam.web.cern.ch/gsalam/teaching/PhD-courses.html
- LHC Physics - lectures at SUSSP65 Summer School
  https://www.taylorfrancis.com/books/edit/10.1201/b11865/lhc-physics-buttar-clark-binoth-glover
  - Perturbative QCD and the parton model - Keith Ellis
  - Monte Carlo tools - Torbjörn Sjöstrand
- Leif Gellersen - Madgraph School 2019:
  https://indico.cern.ch/event/829653/contributions/3568527/attachments/1946887/3230236/ps.pdf