Foglio 3

Exercise 2 Let $X \subseteq \mathbb{P}^n, Y \subseteq \mathbb{P}^m$ be qp varieties, and let $\{U_j\}_j$ be an open cover of X. Prove that a map $\phi : X \to Y$ is a morphism if and only if every restriction $\phi\big|_{U_i}$ is a morphism.

Proof. We check the two conditions that give rise to morphisms:

1. Continuity
2. Preserving regularity

For the first proprerty no check is necessary since continuity is well known to be a local proprety. For the second property we recall a Proposition seen during lecture: in the hypotesis of the exercise, the following are equivalent

1. ϕ is a morphism
2. ϕ is locally given by regular functions
3. ϕ is locally given by homogeneos polynomials of the same degee

This suffices to conclude the proof since if ϕ is locally given by regular functions, i.e. there exists U_j' open coper of X and maps $\phi_j : U_j' \to Y$ such that the restriction of ϕ to U_j' coincides with ϕ_j, then a similar cover can be obtained for each U_j simply by taking the same regular functions and restricting them to all non-empty intersections of the form $U_i \cap U_j'$. The opposite implication follows from the fact that open sets in U_j are again open in X. ■