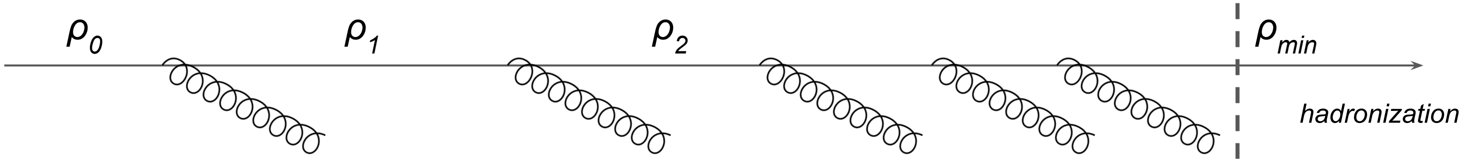
Dynamics of EW & Strong Interactions

Part 4 - Dr. Michele Pinamonti (INFN Trieste)
Lecture 4 - Trieste, 09/01/2023

MC simulation (continued)

Where we left

- Parton shower:
    - simulation of (QCD) radiation by partons from hard scattering
    - evolution to lower-and-lower virtuality / alternative evolution variable ρ (e.g. $p_\perp$):



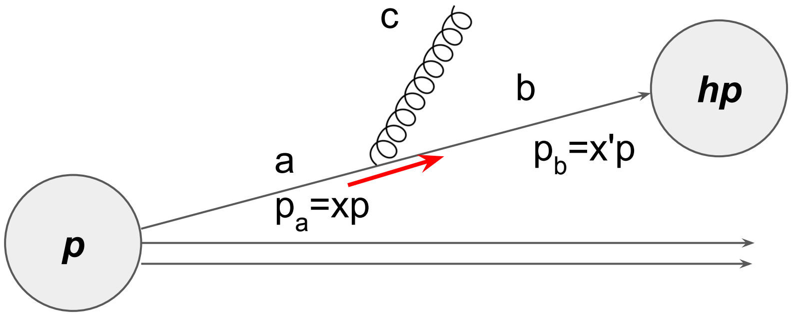
    - emission probability from real emission expression + Sudakov form factor:

$$d\mathcal{P}_{first}(\rho) = \frac{\alpha_s}{2\pi}\frac{d\rho}{\rho}\int_{z_{min}}^{z_{max}} \hat{P}_{a\to bc}(z)dz \cdot \underbrace{\Delta(\rho_0,\rho)}$$

$$\Delta(\rho_0,\rho) = \exp\left(-\sum_{b'}\int_{\rho}^{\rho_0} \frac{\alpha_s}{2\pi}\frac{d\rho'}{\rho'}d\rho' \int_{z_{min}}^{z_{max}} \hat{P}_{a\to b'c'}(z')dz'\right)$$

ISR parton shower - Forward evolution

- Idea:
  - pick parton in incoming hadron from PDF at certain low Q^2 and mom. fraction x
  - evolve event by event (with splitting + Sudakov) to obtain parton(s) at higher Q^2



Difference with FSR:
evolution to larger-and-larger scales ρ

- Disadvantages:
  - cannot "control" parton shower to get certain x' needed to initiate h.p. of interest (e.g. resonance production)
  - will need to evolve for all possible fluctuations, but at most one parton will enter h.p.

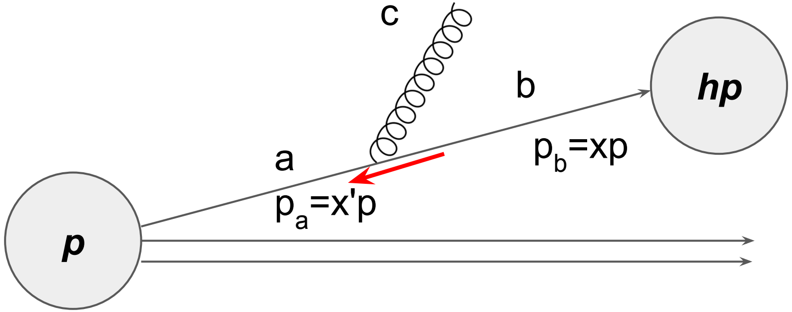
    ⇒ INEFFICIENT!

Backward evolution

- Change of paradigm:
    - consider PDF at large scale Q^2, giving distribution of partons *after* ISR
        - this implicitly means summing over all possible emissions from lower scale to collision scale Q^2

can write DGLAP equation as:

$$\frac{\mathrm{d}f_b(x,Q^2)}{\mathrm{d}(\ln Q^2)} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} f_a(x',Q^2)\frac{\alpha_s}{2\pi} P_{a\to bc}(z=\frac{x}{x'})$$

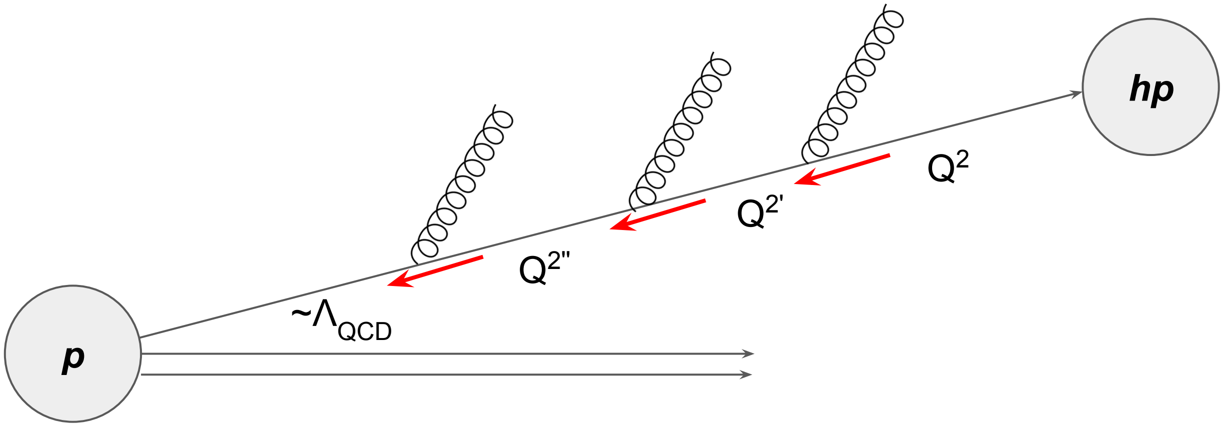


Now "evolution" to smaller-and-smaller scales ρ

-
    - then pick one exclusive ISR history
    - use $dP_{a\to bc}$ and apply Sudakov factors as for FSR
        - Sudakov factor again = $\exp(-\int dP)$

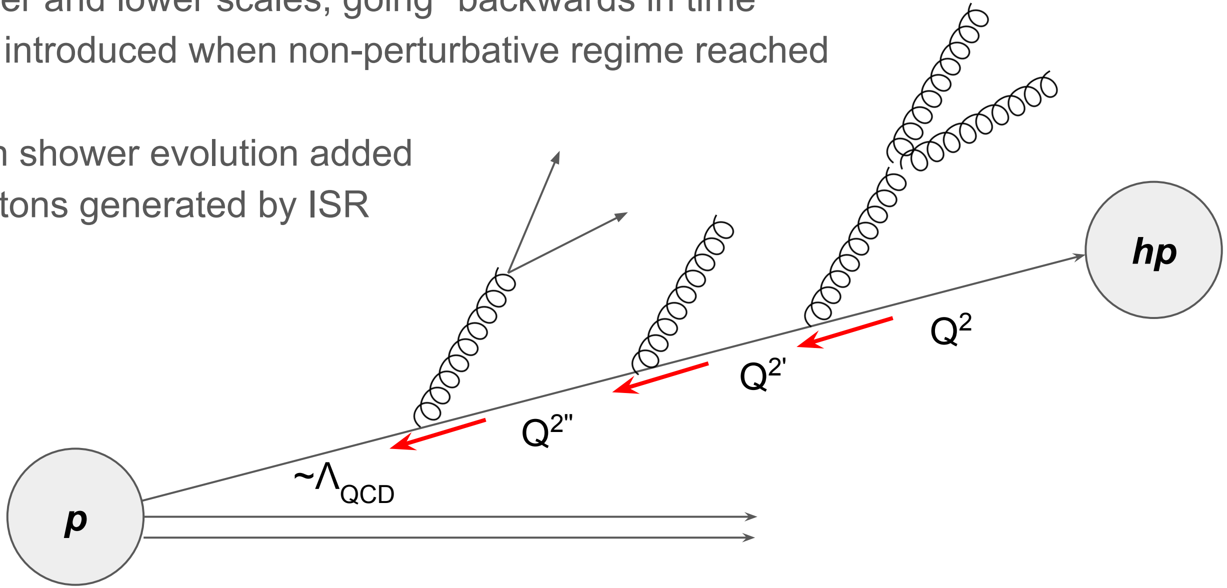
Backward evolution

- Procedure:
  - hard scattering selected, with PDF at final scale Q^2
  - with ***hp*** as upper maxim scale, succession of ISR branching simulated at lower and lower scales, going "backwards in time"
  - cutoff introduced when non-perturbative regime reached



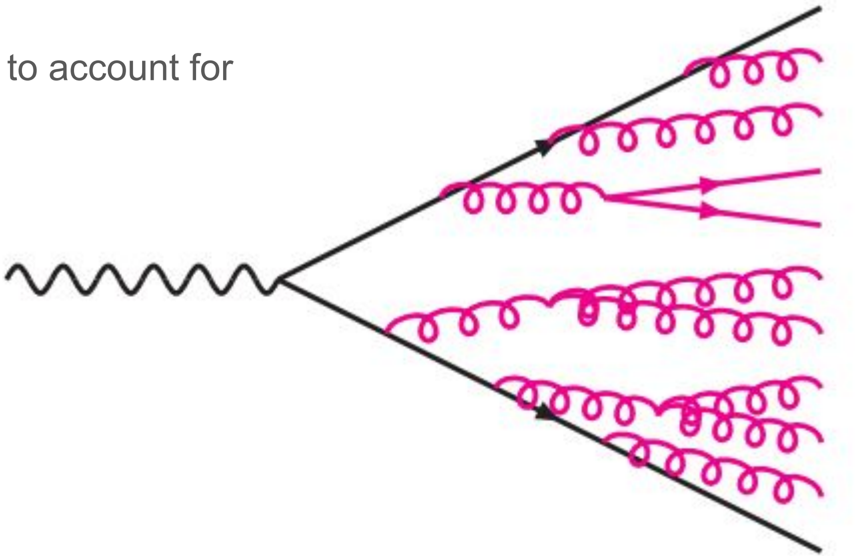
Backward evolution

- Procedure:
  - hard scattering selected, with PDF at final scale Q^2
  - with ***hp*** as upper maxim scale, succession of ISR branching simulated at lower and lower scales, going "backwards in time"
  - cutoff introduced when non-perturbative regime reached
  - parton shower evolution added to partons generated by ISR



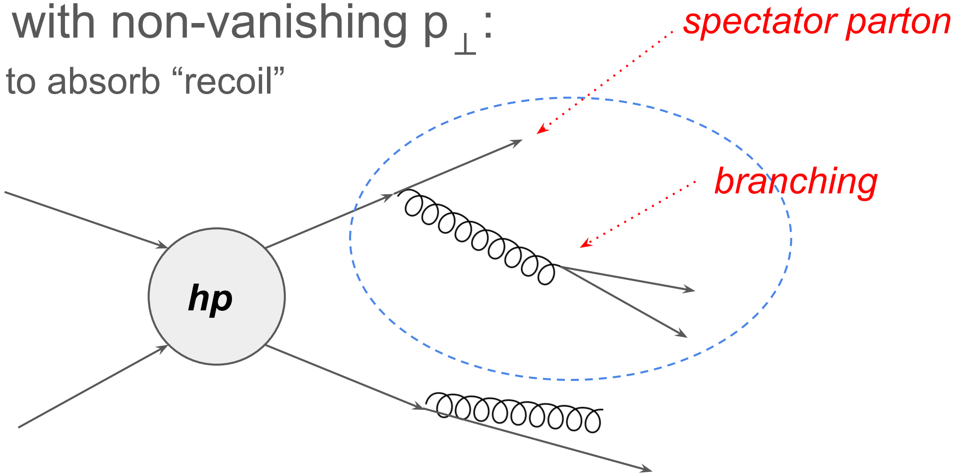
Additional details in PS

- 4-momentum conservation at each vertex
- Color flow
- Each vertex is LO
  - using "effective α_S" ($>\alpha_S$) at earache vertex to account for missing higher orders
- Large-angle radiation not modelled correctly
- Interference not necessarily included
  - but some effects are,
    e.g. angular ordering / coherence



Energy-momentum conservation and recoil

- For 1 → 2 branching with non-vanishing $p_\perp$:
  - need other parton(s) to absorb "recoil"

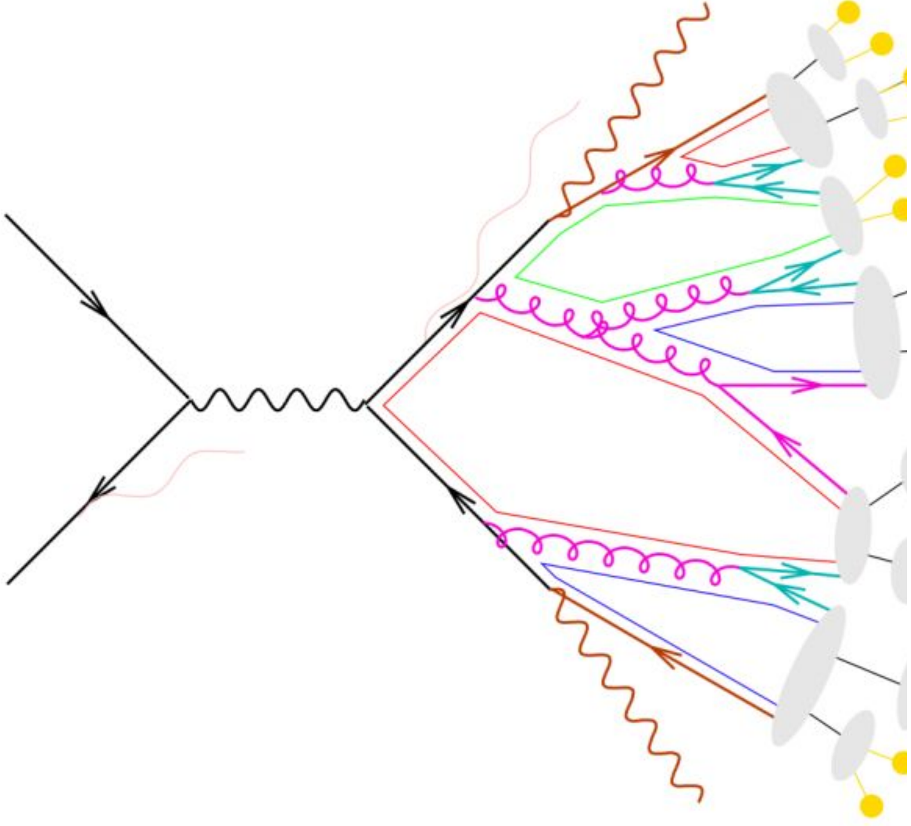


  - Different choices:
    - *global* vs. *local* recoil
    - colour-connected third parton to absorb recoil ("dipole")
    - don't distinguish emitter and spectator, do 2 → 3 splitting instead ("antenna")

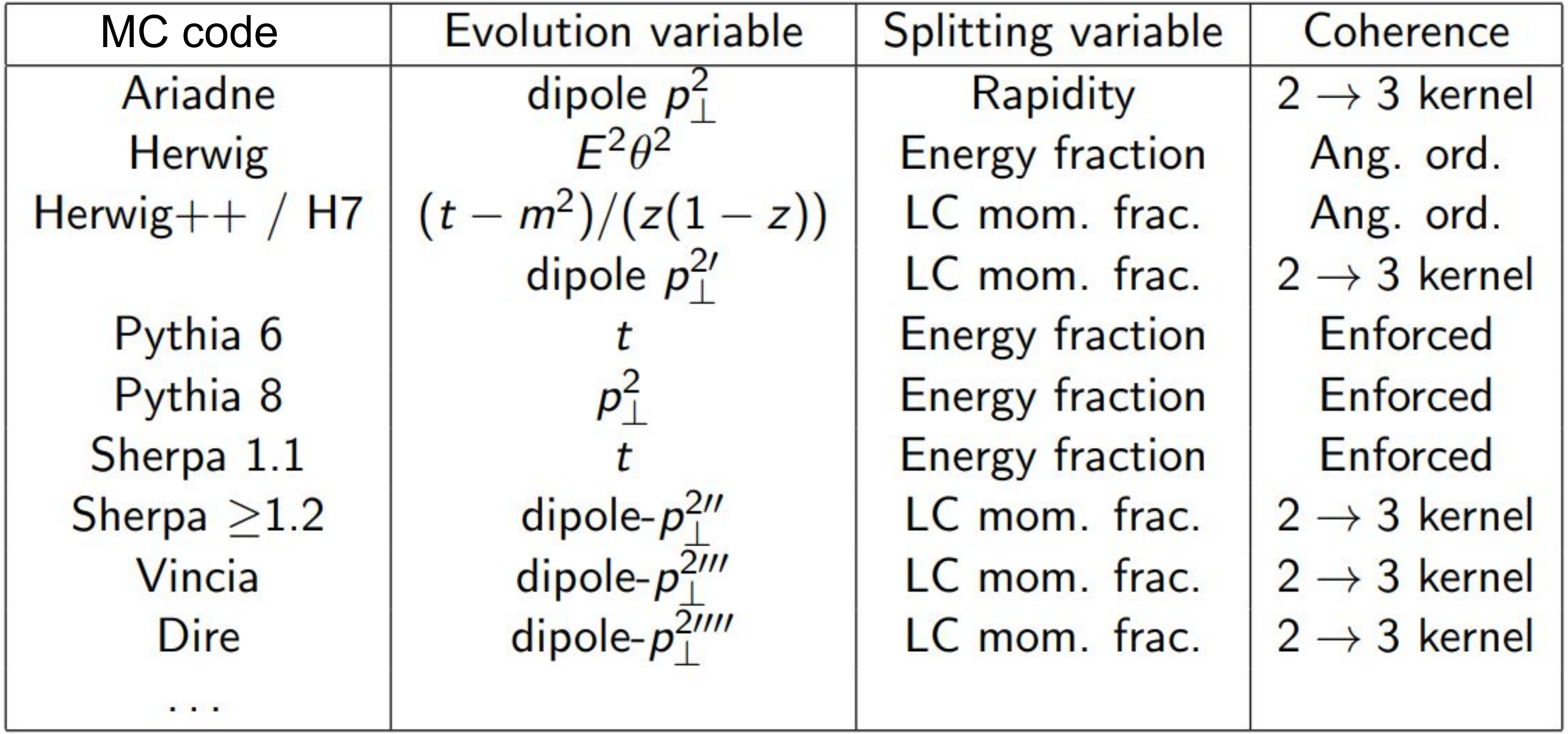
Colour flow and coherence

- Angular ordering implies "color coherence"
  - color-connected partons produced "closer" to each other

- Small caveat:
  - possible "color reconnection" (see later)

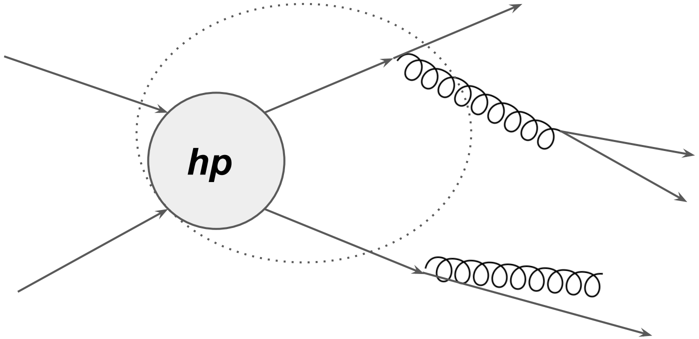


Different PS algorithms



MC code	Evolution variable	Splitting variable	Coherence
Ariadne	dipole $p_\perp^2$	Rapidity	$2 \rightarrow 3$ kernel
Herwig	$E^2\theta^2$	Energy fraction	Ang. ord.
Herwig++ / H7	$(t - m^2)/(z(1-z))$	LC mom. frac.	Ang. ord.
	dipole $p_\perp^{2\prime}$	LC mom. frac.	$2 \rightarrow 3$ kernel
Pythia 6	t	Energy fraction	Enforced
Pythia 8	$p_\perp^2$	Energy fraction	Enforced
Sherpa 1.1	t	Energy fraction	Enforced
Sherpa $\geq$1.2	dipole-$p_\perp^{2\prime\prime}$	LC mom. frac.	$2 \rightarrow 3$ kernel
Vincia	dipole-$p_\perp^{2\prime\prime\prime}$	LC mom. frac.	$2 \rightarrow 3$ kernel
Dire	dipole-$p_\perp^{2\prime\prime\prime\prime}$	LC mom. frac.	$2 \rightarrow 3$ kernel
...			

Matrix Element (ME) generators

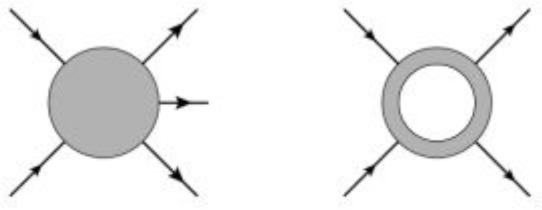


- Event generation simple at LO in 2 → 2 hard processes
  - general-purpose MC simulation programs (Pythia, Herwig...) already fully able to do the job

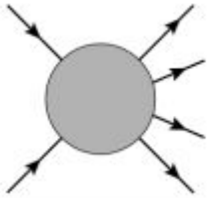
- More tricky when moving to $\mathcal{O}(\alpha_S^3)$:
  - 2 → 3 processes
  - virtual corrections to 2 → 2 process
  - (Note: remember the cancellation of IR divergences!)

Matrix Element (ME) generators

- Two approaches in MC simulation programs:
    - NLO (or NNLO, ...) MC simulation - full description of real and virtual emission in ME



    - "Multi-leg" MC simulation
        - no loop diagrams
        - only real emission of additional partons
        - need to apply kinematic cuts: well-separated and high-energy partons to avoid IR div.



MEs vs. PSs

- ME - Matrix Elements:

  \+ systematic expansion in α_s ('exact')

  \+ can include additional partons (at Born level)

  \+ flexible phase space cuts (can generate what we want ⇒ efficient)

  – loop calculations very tough (R-V cancellation becomes difficult)

  – failing in collinear regions (at Born level) ⇒ unpredictive jet/event structure

  – no easy match to hadronization

- PS - Parton showers:

  – approximate description, not precise prediction of well-separated jets

  – main topology not predetermined ⇒ inefficient for exclusive states

  \+ process-generic / universal ⇒ simple multi-parton

  \+ Sudakov form factors ⇒ sensible jet/event structure

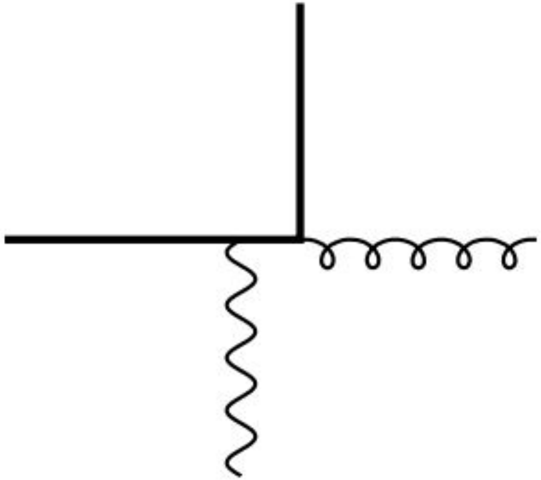
  \+ easy match to hadronization

Combining ME with PS

- Want to take advantage of both ME and PS approach
- To be useful in the real life, ME generators need to be interfaced with parton shower generators:
  - generated events need to be "showered", to produce soft and collinear radiation

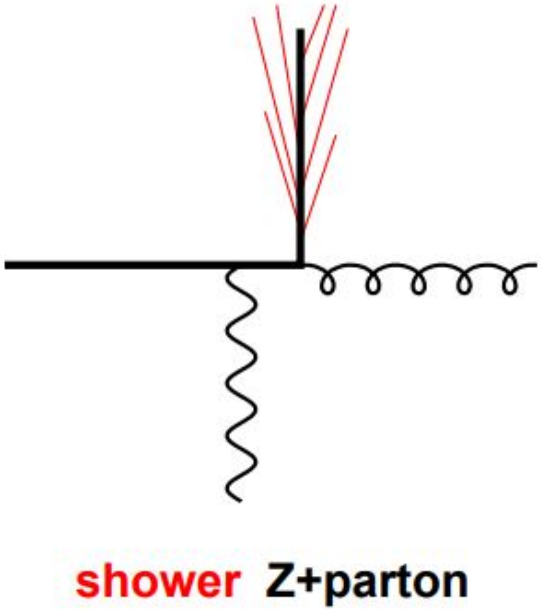
- Complication:
  - possible double-counting of additional parton emission with $\mathcal{O}(\alpha_S^3)$ MEs:
    - 2 → 3 events can be obtained by "showering" 2 → 2 events or via 2 → 3 at ME level
    - additional parton can have same kinematics ⇒ double counting!

ME+PS Double-counting

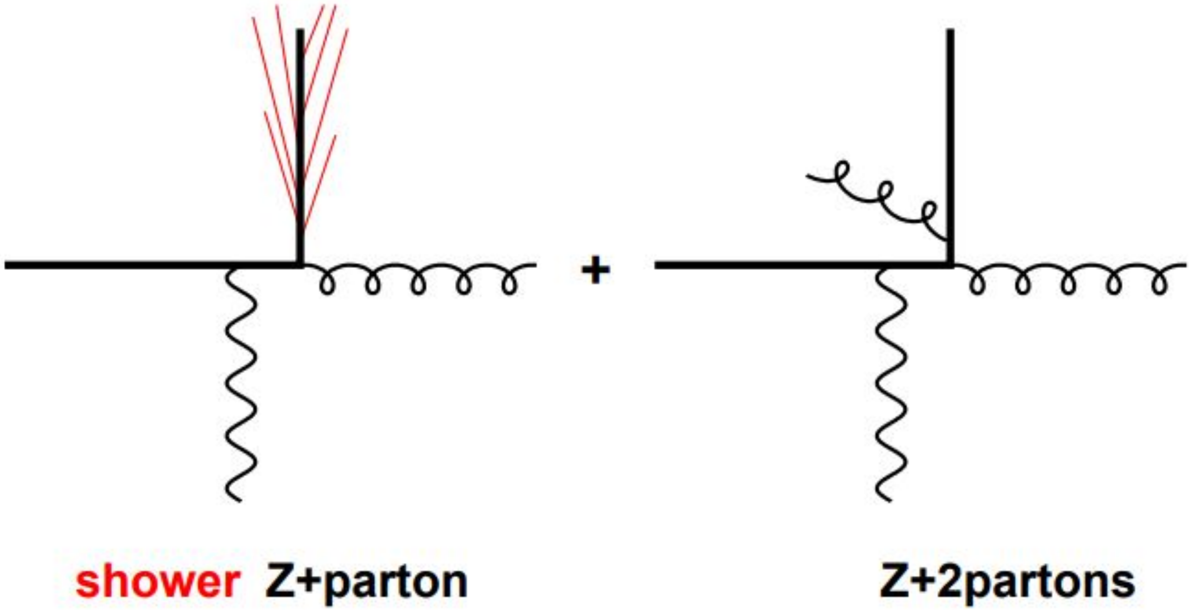


Z+parton

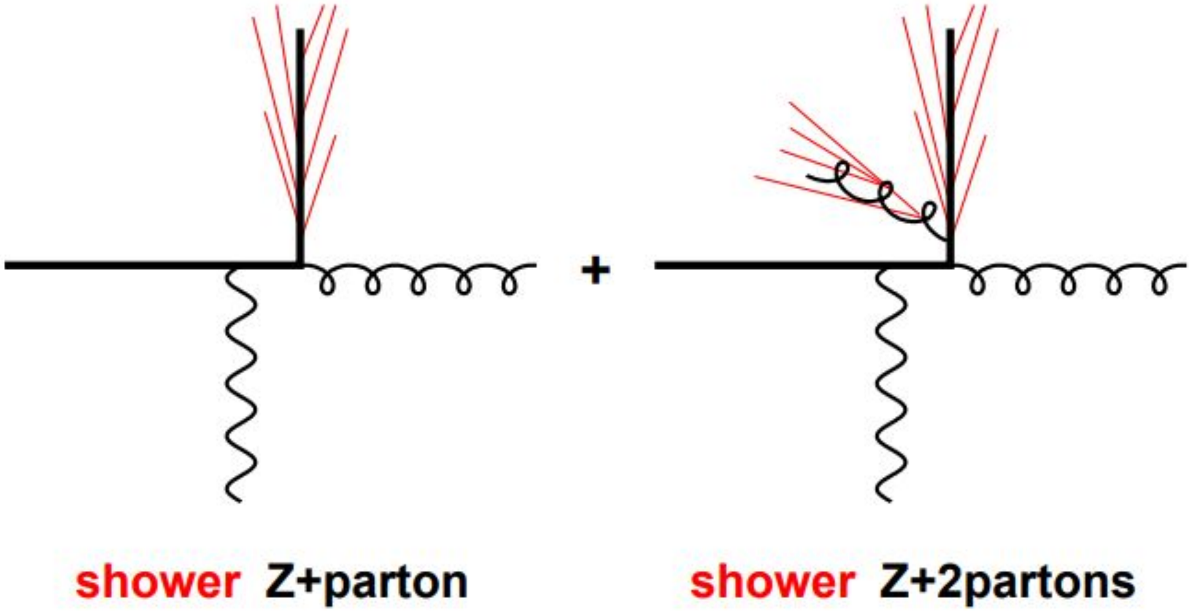
ME+PS Double-counting



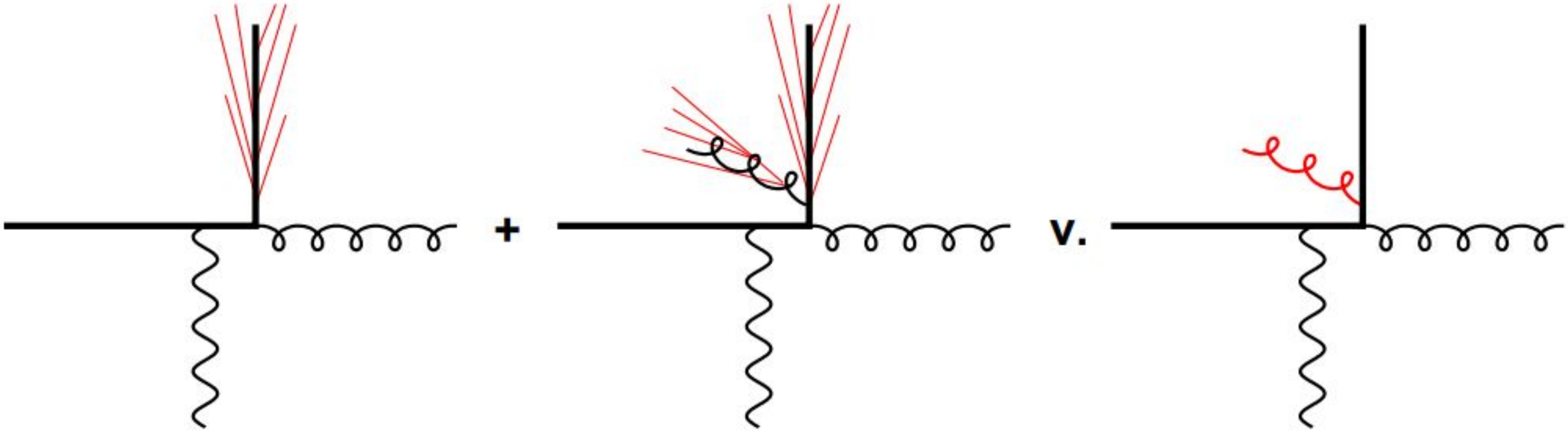
ME+PS Double-counting



ME+PS Double-counting



ME+PS Double-counting

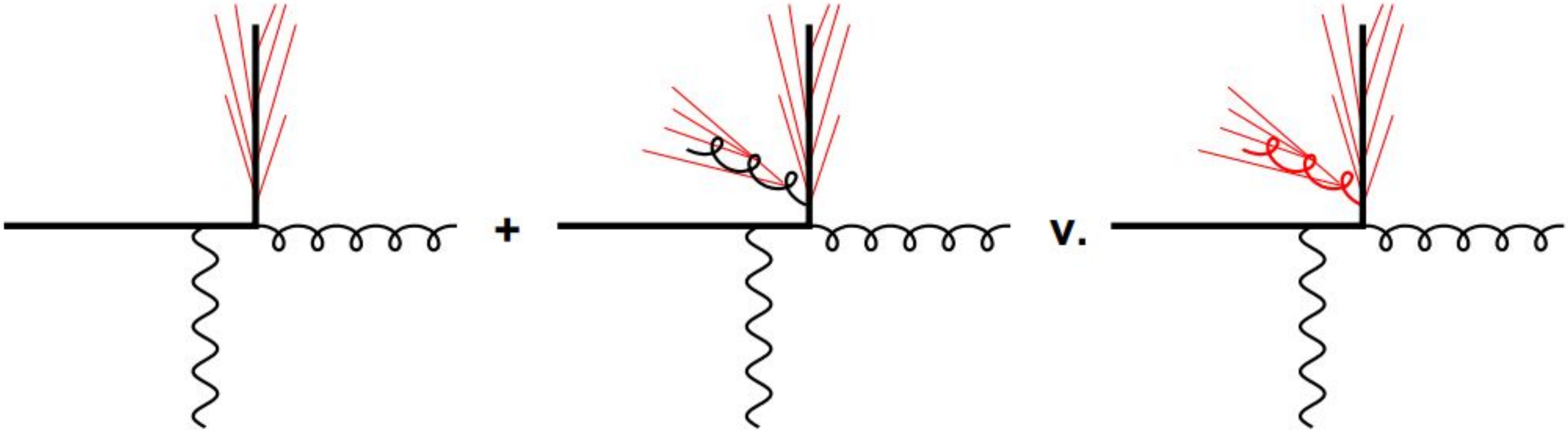


shower Z+parton

shower Z+2partons

shower of Z+parton
generates hard gluon

ME+PS Double-counting

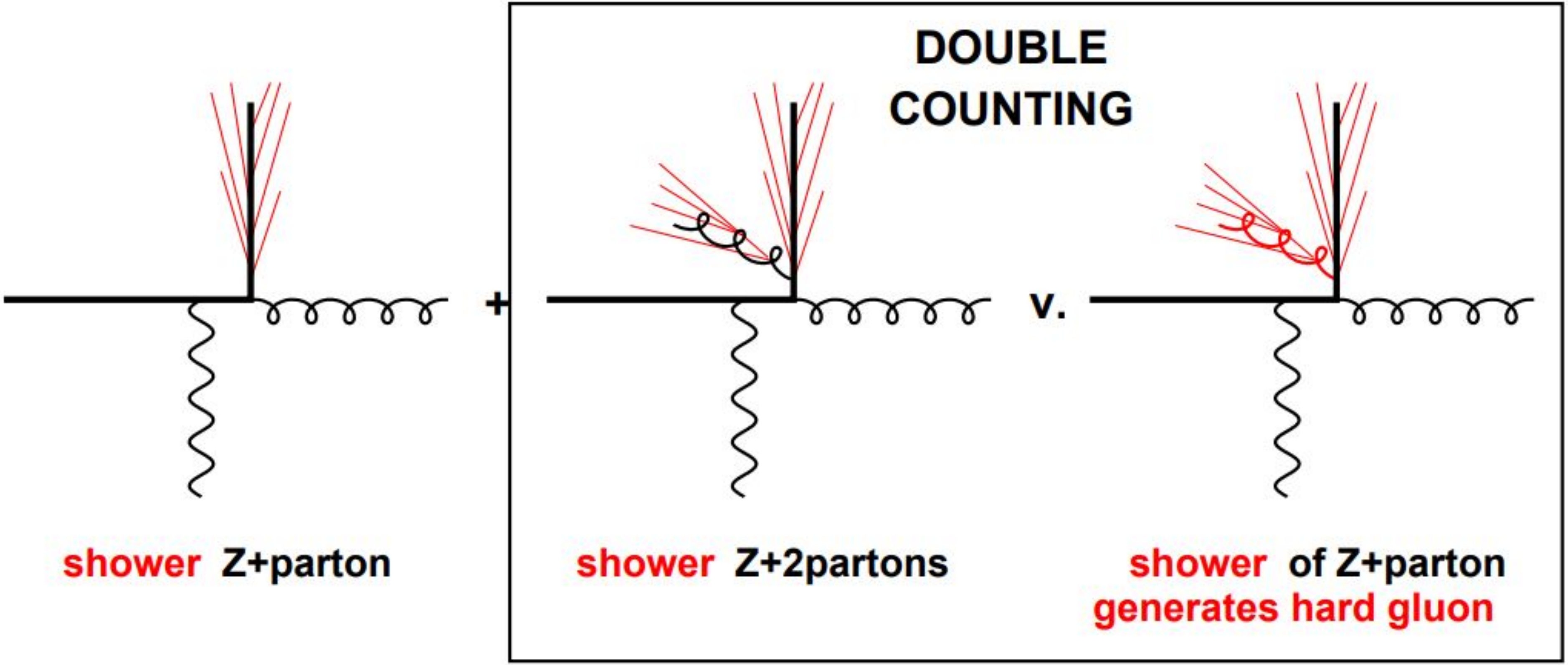


shower Z+parton

shower Z+2partons

shower of Z+parton
generates hard gluon

ME+PS Double-counting



DOUBLE COUNTING

shower Z+parton

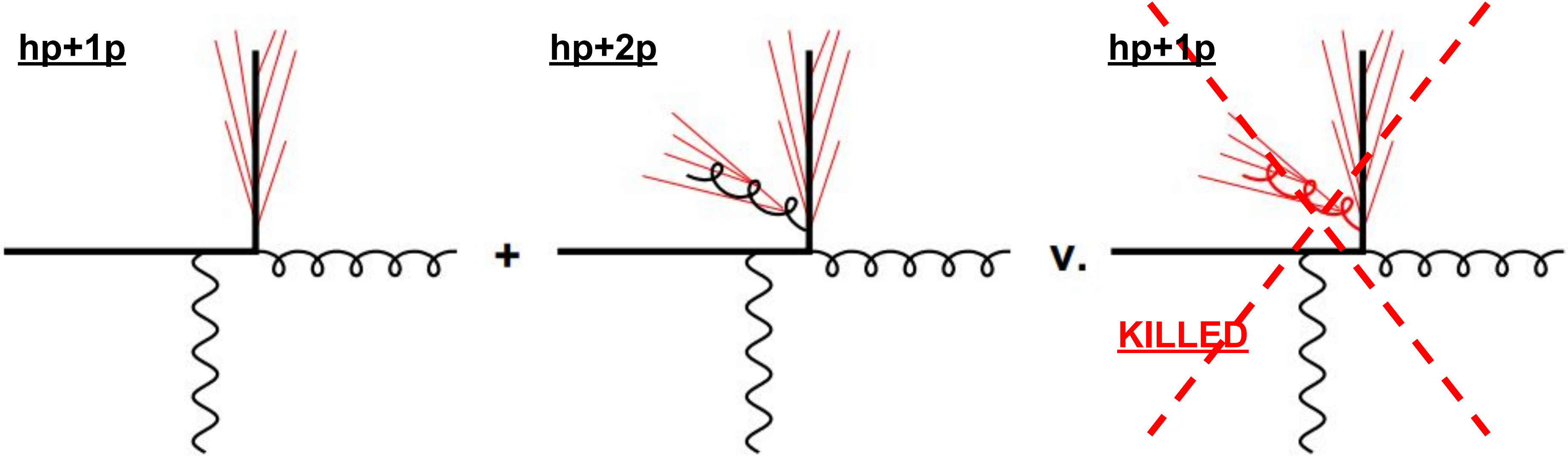
shower Z+2partons

shower of Z+parton generates hard gluon

Combining ME with PS - "MLM" matching

- Example of technique to remove double-counting
- Consider (LO) ME generation of:
  - hp+0 partons
  - hp+1 parton
  - hp+2 partons
  - ...
  - hp+N partons
- Allow each category of events to develop parton shower
- Kill all events where number of "jets" ≠ number of nominal partons
  - need to define "jets" with a certain jet algorithm
  - in last category will not kill events with too many jets, only those with too few jets

Combining ME with PS - "MLM" matching



shower Z+parton

shower Z+2partons

shower of Z+parton
generates hard gluon

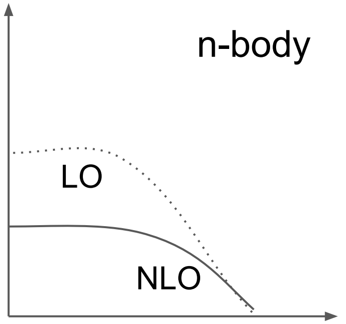
NLO matching

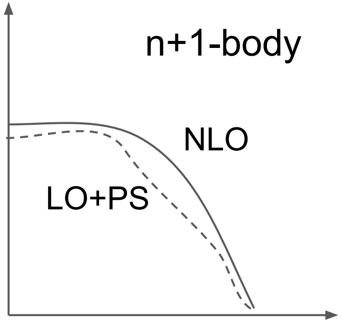
- Similar idea to MLM matching (or in general "vetoed" parton showers) but want to keep full NLO description (with real emission and virtual corrections) for ME part

- Two main methods:
  - MC@NLO
  - Powheg

NLO matching - MC@NLO method

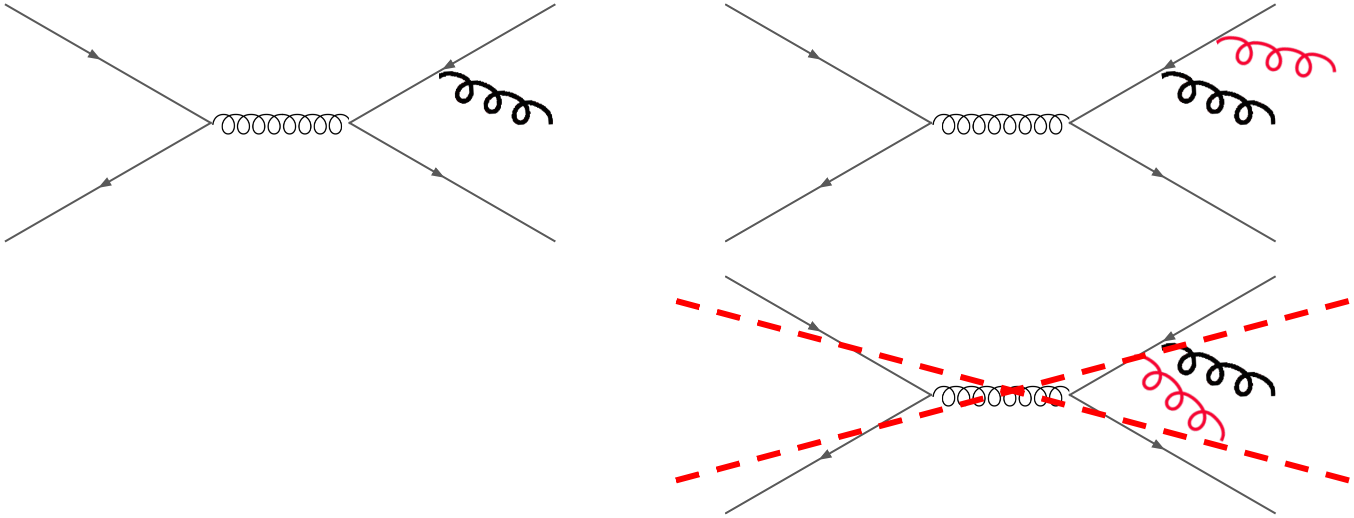
- Simplified receipt:
  - calculate NLO correction to n-body process
    - split into n-body and (n+1)-body phase-spaces
  - calculate analytically (no Sudakov)
    how first emission from n-body
    would populate (n+1)body phase-space
    → "shower expression" for n+1
  - subtract this shower expression for n+1
    from the full NLO in (n+1)-body phase-space
  - apply shower to both kind of events
    (double-counting avoided by subtraction *before* showering)
  - NB: total cross-section enforced to be the NLO one
  - side effect: when [n-body + shower] in (n+1) phase space > NLO
    ⇒ events with negative weights!





NLO matching - Powheg method

- Basic idea:
  - generate *first emission* (largest $p_\perp$, $p_\perp^0$) with NLO ME
  - *subsequent emissions* (i.e. from $p_\perp^0$ to 0) with PS
  - no negative weight events



Intermezzo - Resummation and "log terms"

- Neglecting Sudakovs, rate of one (gluon) emission is:

$$\mathcal{P}_{\mathrm{q}\to\mathrm{qg}} \approx \int \frac{\mathrm{d}Q^2}{Q^2} \int \mathrm{d}z \, \frac{\alpha_{\mathrm{s}}}{2\pi} \frac{4}{3} \frac{1+z^2}{1-z}$$
$$\approx \alpha_{\mathrm{s}} \ln\left(\frac{Q^2_{\mathrm{max}}}{Q^2_{\mathrm{min}}}\right) \frac{8}{3} \ln\left(\frac{1-z_{\mathrm{min}}}{1-z_{\mathrm{max}}}\right) \sim \alpha_{\mathrm{s}} \ln^2$$

  - rate of n gluon emissions:

$$\mathcal{P}_{\mathrm{q}\to\mathrm{q}n\mathrm{g}} \sim (\mathcal{P}_{\mathrm{q}\to\mathrm{qg}})^n \sim \alpha_{\mathrm{s}}^n \ln^{2n}$$

- "Resummation" means including all these log terms in a calculation (with n → ∞)
  - "next-to-leading log" (NLL) means including also sub-leading log terms: $\alpha_{\mathrm{s}}^n \ln^{2n-1}$

References

- Gavin Salam:
  - https://gsalam.web.cern.ch/gsalam/repository/talks/2009-Bautzen-lecture4.pdf
- Leif Gellersen:
  - https://indico.cern.ch/event/829653/contributions/3568527/attachments/1946887/3230236/ps.pdf