8 gennaio 2019

$$f(x) = \begin{cases} \int_0^x \frac{1}{(t+1)^2 (t+2)} & x \geq 0 \\ \int_0^x \frac{t+1}{(t-\sqrt{2})^2} dt & x \leq 0 \end{cases}$$

$$\lim_{x \to +\infty} f(x) = ?$$

$$\frac{1}{(t+1)^2 (t+2)} = \frac{A}{t+1} + \frac{B}{(t+1)^2} + \frac{C}{t+2}$$

$$C = \frac{1}{(t+1)^2}\Big|_{t=-2} = 1, \quad B = \frac{1}{t+2}\Big|_{t=-1} = 1$$

$$\frac{1}{(t+1)(t+2)} = A + \frac{B}{t+1} + C\,\frac{t+1}{t+2}$$

$$\downarrow t \to +\infty$$

$$0 = A + 0 + C \qquad A = -1$$

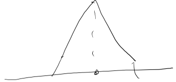
$$A = \frac{1}{1!}\left(\frac{d}{dx}\right)^1 \frac{1}{x+2}\Big|_{x=-1} = \frac{-1}{(x+2)^2}\Big|_{x=-1} = -1$$

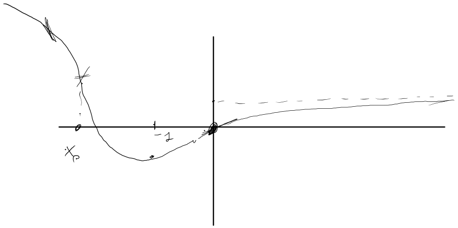
$$\int_0^x \frac{1}{(t+1)^2 (t+2)} = \int_0^x \frac{-1}{t+1} + \int_0^x \frac{1}{t+2} + \int_0^x \frac{1}{(t+1)^2}$$

$$= -\lg(x+1) + \lg(x+2) - \lg 2 + \left.\frac{-1}{t+1}\right]_0^x$$

$$= \lg\frac{x+2}{x+1} - \lg 2 - \frac{1}{x+1} + 1 \xrightarrow[x \to +\infty]{} 1 - \lg 2 > 0$$

$$\downarrow 0 \qquad\qquad \downarrow 0$$





$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\int_0^x \frac{t+1}{(t-\sqrt2)^2}\,dt = -\lim_{x\to-\infty}\int_x^0 \frac{t+1}{(t-\sqrt2)^2} = +\infty$$

$$f'(x) = \begin{cases} \dfrac{1}{(x+1)^2(x+2)} & x>0 \\[2mm] \dfrac{x+1}{(x-\sqrt2)^2} & x<0 \end{cases}$$

$$f'_d(0) = \lim_{x\to0^+}\frac{f(x)}{x} \overset{H}{=} \lim_{x\to0^+} f'(x) = \lim_{x\to0^+}\frac{1}{(x+1)^2(x+2)} = \frac12$$

$$f'_s(0) = \lim_{x\to0^-}\frac{f(x)}{x} \overset{H}{=} \lim_{x\to0^-} f'(x) = \lim_{x\to0^-}\frac{x+1}{(x-\sqrt2)^2} = \frac12$$

$$f'(0) = \frac12$$

$f'(-1) = 0$ $\qquad f'(x)<0$ per $x<-1$, $f'(x)>0$ per $x>-1$

per $x<0$

$$f''(x) = \left(\frac{x+1}{(x-\sqrt2)^2}\right)' = \frac{(x-\sqrt2)^2-(x+1)\,2(x-\sqrt2)}{(x-\sqrt2)^4} =$$

$$= \frac{1}{(x-\sqrt2)^3}\left[x-\sqrt2-(2x+2)\right] = \frac{1}{(x-\sqrt2)^3}\left(-x-2-\sqrt2\right) = 0$$

$$x_0 = -2-\sqrt2 < -1 \qquad = \underbrace{\frac{-1}{(x-\sqrt2)^3}}_{>0}(x-x_0) = \begin{cases} >0 & x>x_0 \\ <0 & x<x_0 \end{cases}$$

$f''(x)$

$\exists$? retta asintotica a $-\infty$?

$$\lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{\int_0^x \frac{t+1}{(t-\sqrt{2})^2}\,dt}{x}$$

$$= \lim_{x \to -\infty} \frac{x+1}{(x-\sqrt{2})^2} = \lim_{x \to -\infty} \frac{x}{x^2} = 0 = m$$

$$\lim_{x \to -\infty} \left(f(x) - m x \right) = \lim_{x \to -\infty} f(x) = +\infty$$

$$f(x) = \int_0^{1+x} e^{t^2}\, dt \qquad P_5(x)$$

$$e^y = \sum_{j=0}^{n} \frac{y^j}{j!} + o(y^n)$$

$$e^{t^2} = \sum_{j=0}^{2} \frac{t^{2j}}{j!} + o(t^4)$$

$$f(x) = \int_0^{1+x} \sum_{j=0}^{2} \frac{t^{2j}}{j!}\, dt + \int_0^{1+x} o(t^4)\, dt$$

$$= \int_0^{1+x} \left(1 + t^2 + \frac{t^4}{2}\right) dt + \int_0^{1+x} o(t^4)\, dt$$

$$= (1+x) + \frac{(1+x)^3}{3} + \frac{(1+x)^5}{10} + \cancel{\int_0^{1+x} o(t^4)\, dt}$$

non e' $O(x^5)$

$$f(x) = \int_0^{1+x} e^{t^2}\,dt = \underbrace{\int_0^1 e^{t^2}\,dt}_{f(1)} + \int_1^{1+x} e^{t^2}\,dt$$

$$\int_1^{1+x} e^{t^2}\,dt \qquad t = s+1 \qquad dt = ds$$

$$= \int_0^x e^{(s+1)^2}\,ds = \int_0^x e^{(t+1)^2}\,dt$$

Basta calcolare il pol. di Mc-Laurin di ordine 4 di $e^{(t+1)^2}$.

$$e^{(t+1)^2} = e^{t^2+2t+1} = e\, e^{t^2}\, e^{2t} =$$

$$= e\left(1+t^2+\frac{t^4}{2}+o(t^4)\right)\left(1+2t+\frac{(2t)^2}{2}+\frac{(2t)^3}{6}+\frac{(2t)^4}{24}+o(t^4)\right)$$

$$= e\left(1+t^2+\frac{t^4}{2}+o(t^4)\right)\left(1+2t+2t^2+\frac{4}{3}t^3+\frac{16}{24}t^4+o(t^4)\right)$$

$\frac{2}{3}$

$$= e\left(1+t^2+\frac{t^4}{2}+2t+2t^2+\frac{4}{3}t^3+\frac{2}{3}t^4+\right.$$
$$\left.+2t^3+2t^4+o(t^4)\right)$$

$$= e\left(1+2t+t^2(1+2)+t^3\left(\frac{4}{3}+2\right)+t^4\left(\frac{1}{2}+\frac{2}{3}+2\right)+o(t^4)\right)$$

$$= \underbrace{e+2et+3et^2+e\left(\frac{4}{3}+2\right)t^3+e\left(\frac{1}{2}+\frac{2}{3}+2\right)t^4}_{q_4(t)} + o(t^4)$$

polinomio di McLaurin di ordine 4 di $e^{(t+1)^2}$

$$\int_0^x e^{(t+1)^2}\,dt = \boxed{\int_0^x q_4(t)\,dt} + \underbrace{\int_0^x o(t^4)\,dt}_{o(x^5)}$$

$$ex + ex^2 + ex^3 + \frac{e}{4}\left(\frac{4}{3}+2\right)x^4 + \frac{e}{5}\left(\frac{1}{2}+\frac{2}{3}+2\right)x^5 + o(x^5)$$

$f(0)$