Image Processing for Physicists

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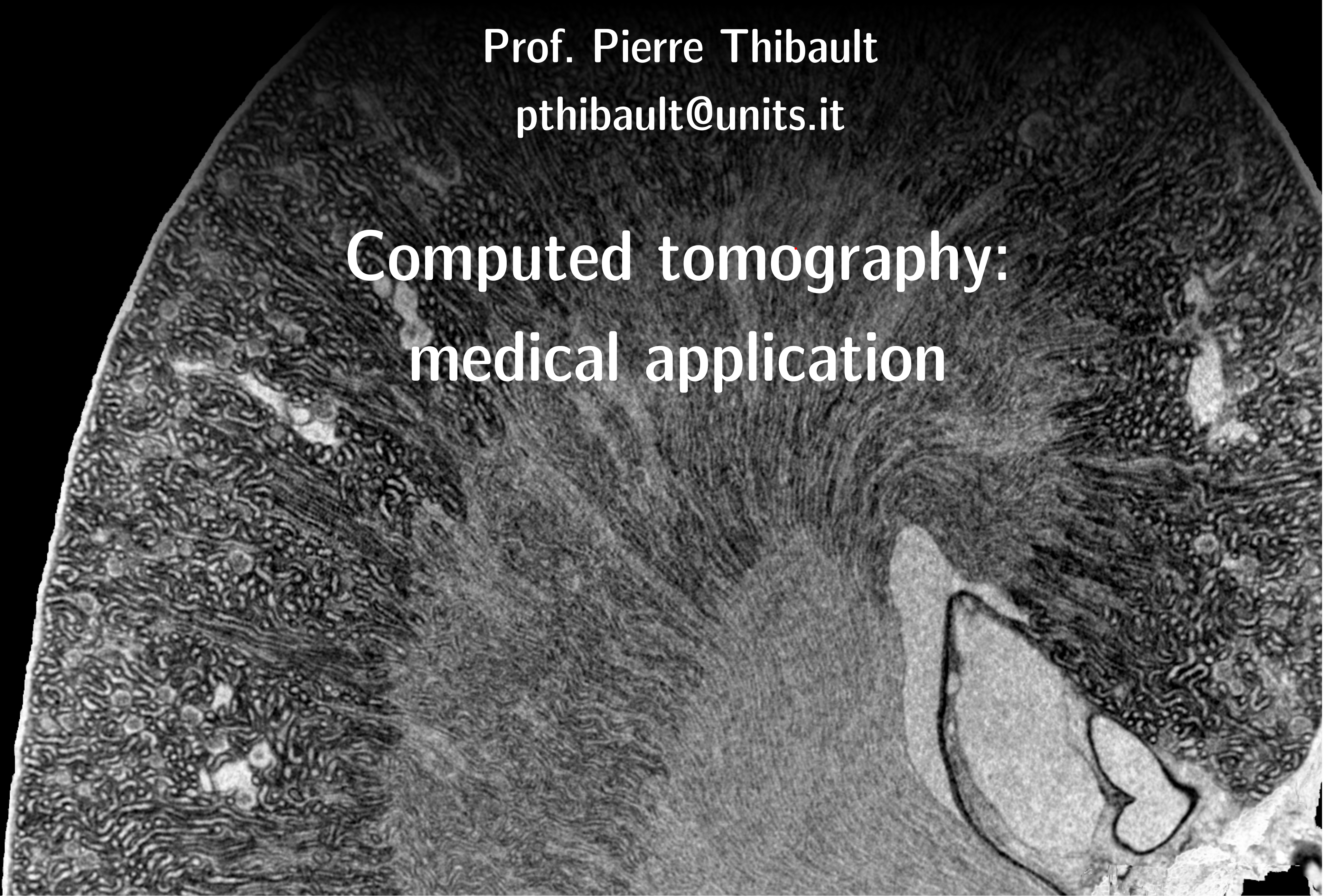
Computed tomography: medical application



Overview

- X-ray interaction with matter
- Medical CT systems
- Image quality:
  - Artifacts
  - Dose
- Iterative techniques

X-ray interaction with matter

- Photons interact with charged particles
- X-rays: mainly with atomic electrons
- Two main interaction types:
  - Scattering → photon "bouncing" on atoms
  - Absorption → photon energy is converted inside atom
    ↳ excited state → relax → ionization, Auger / fluorescence

medical imaging:
combined effect: attenuation

X-ray interaction with matter

Atomic cross-sections

Total cross-section of an atom:

$$\sigma_{tot} = \sigma_{pe} + \sigma_{coh} + \sigma_{incoh}$$

σ_{pe}: photo-electric $\rightarrow$ absorption

σ_{coh}: coherent scattering (Thompson scattering)

σ_{incoh}: incoherent scattering (Compton scattering)

σ: units of area

$$(\vec{p})^2 \rightarrow \left(\vec{p} + \frac{e\vec{A}}{c}\right)^2$$

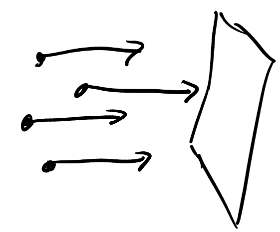
$$\underbrace{p^2 + A^2 \frac{e^2}{c^2}}_{\text{scattering}}$$

$$\underbrace{+2\vec{p}\cdot\vec{A}\,\frac{e}{c}}_{\text{photo electric}}$$

X-ray interaction with matter

Beer-Lambert law & linear attenuation

Φ : photon flux (# photons/s/area)



N atoms in volume $dV = A\,dz$

photons/s taken out of the incident beam : $\Phi\sigma$

$\Phi \sigma N$ (N atoms)

out going flux: $\Phi_{out} = \Phi - \Phi \frac{N\sigma}{A}$

$$\Phi(z+dz) = \Phi(z) - \Phi(z)\frac{N\sigma}{A}$$

$$\Phi(z) + \frac{d\Phi}{dz}dz = \Phi(z)\left[1 - \underbrace{\frac{N}{dV}}_{\text{number density } n}\sigma\,dz\right] = \Phi(z)(1 - n\sigma\,dz)$$

Beer-Lambert law

$$\Rightarrow \frac{d\Phi}{dz} = -\Phi n\sigma \quad \Rightarrow \Phi(z) = \Phi_0 \exp(-n\sigma z)$$

$$= \Phi_0 \exp(-\mu z)$$

μ: linear attenuation coefficient

mass density:

$\rho = n \cdot m_A$

$\mu = n\sigma$

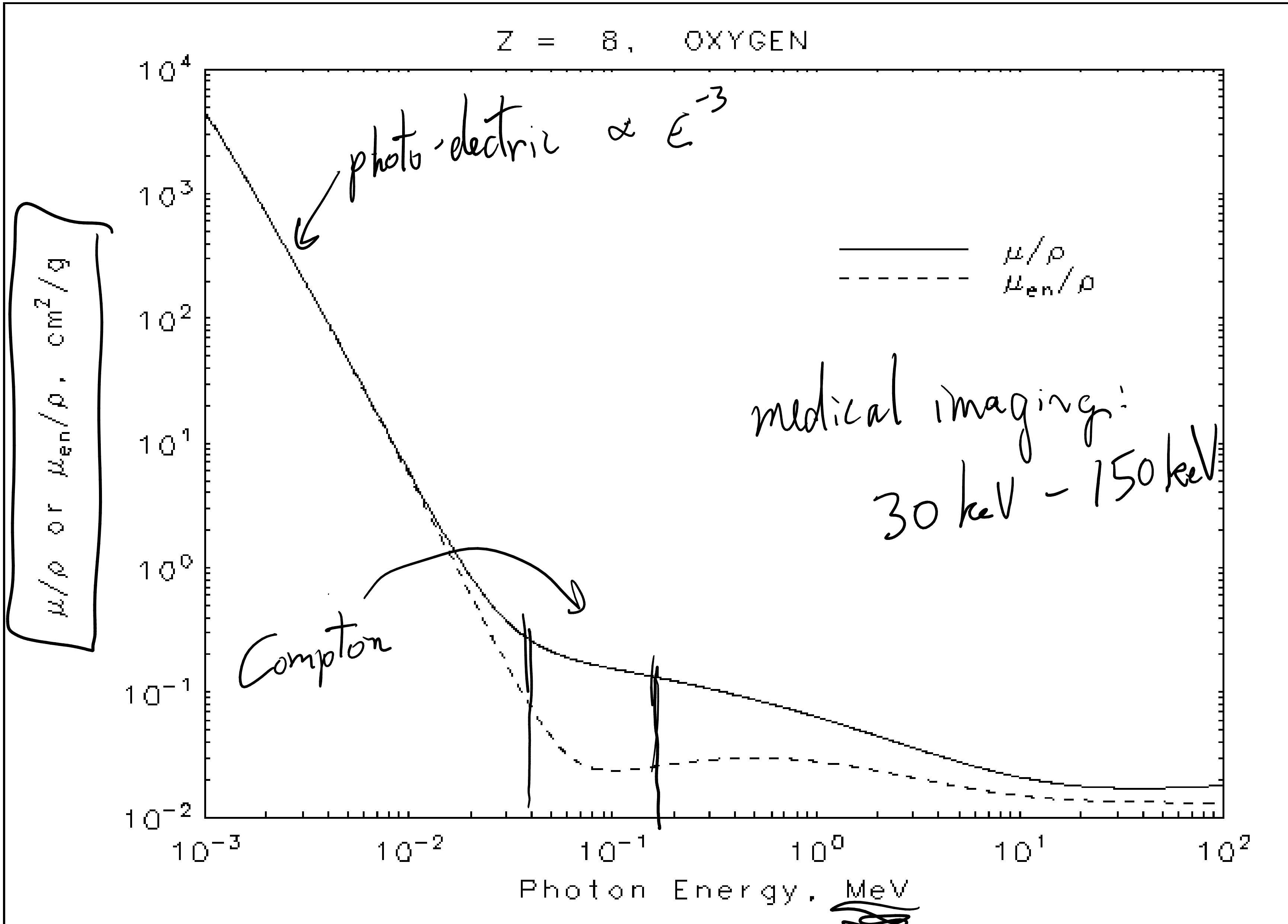
$= \rho \frac{\sigma}{m_A}$

$= \rho\left(\frac{\sigma}{Z}\right)\left(\frac{Z}{m_A}\right)$

X-ray interaction with matter

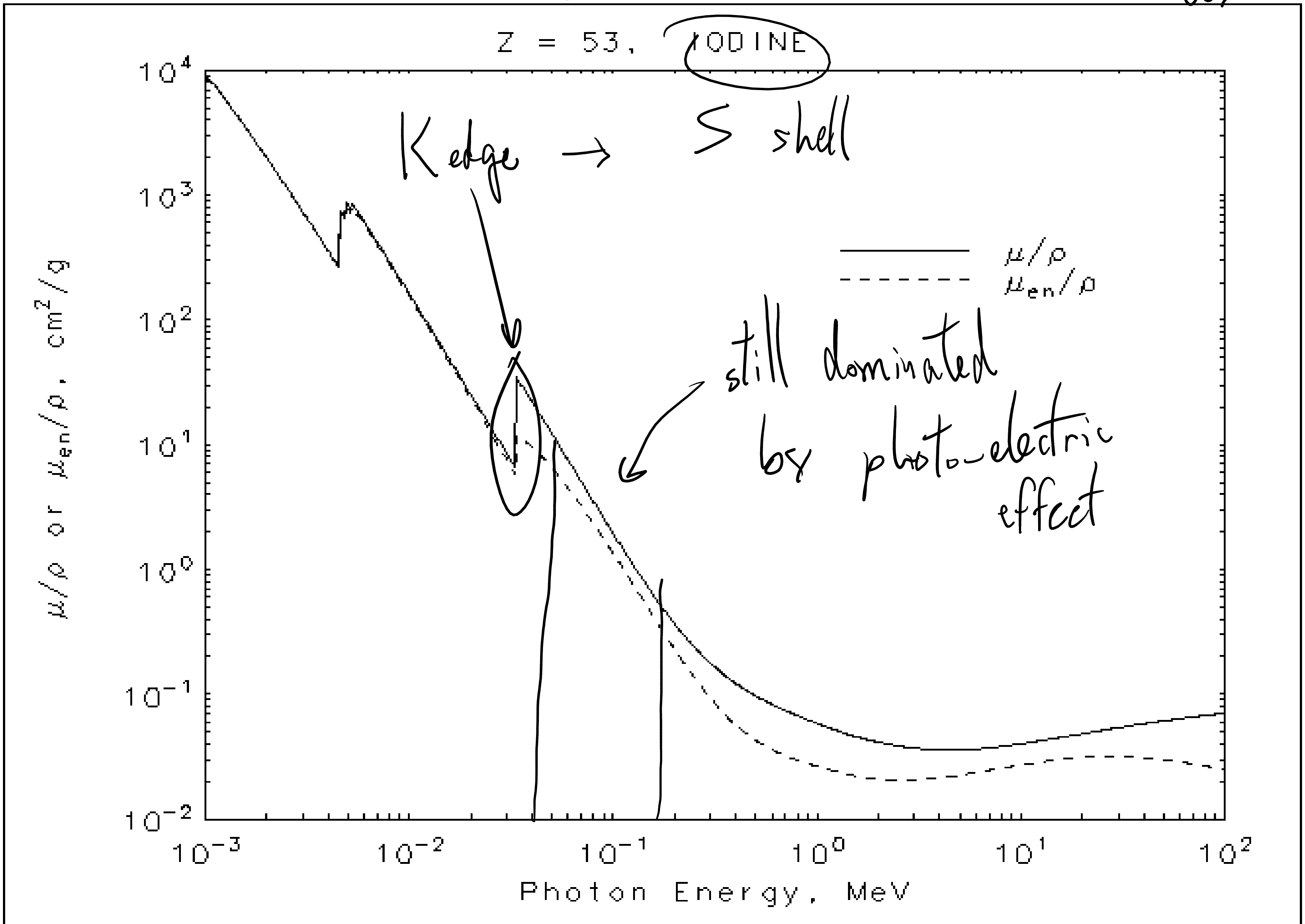
Energy dependence

NIST



X-ray interaction with matter
Energy dependence

iodine often used as contrast agent



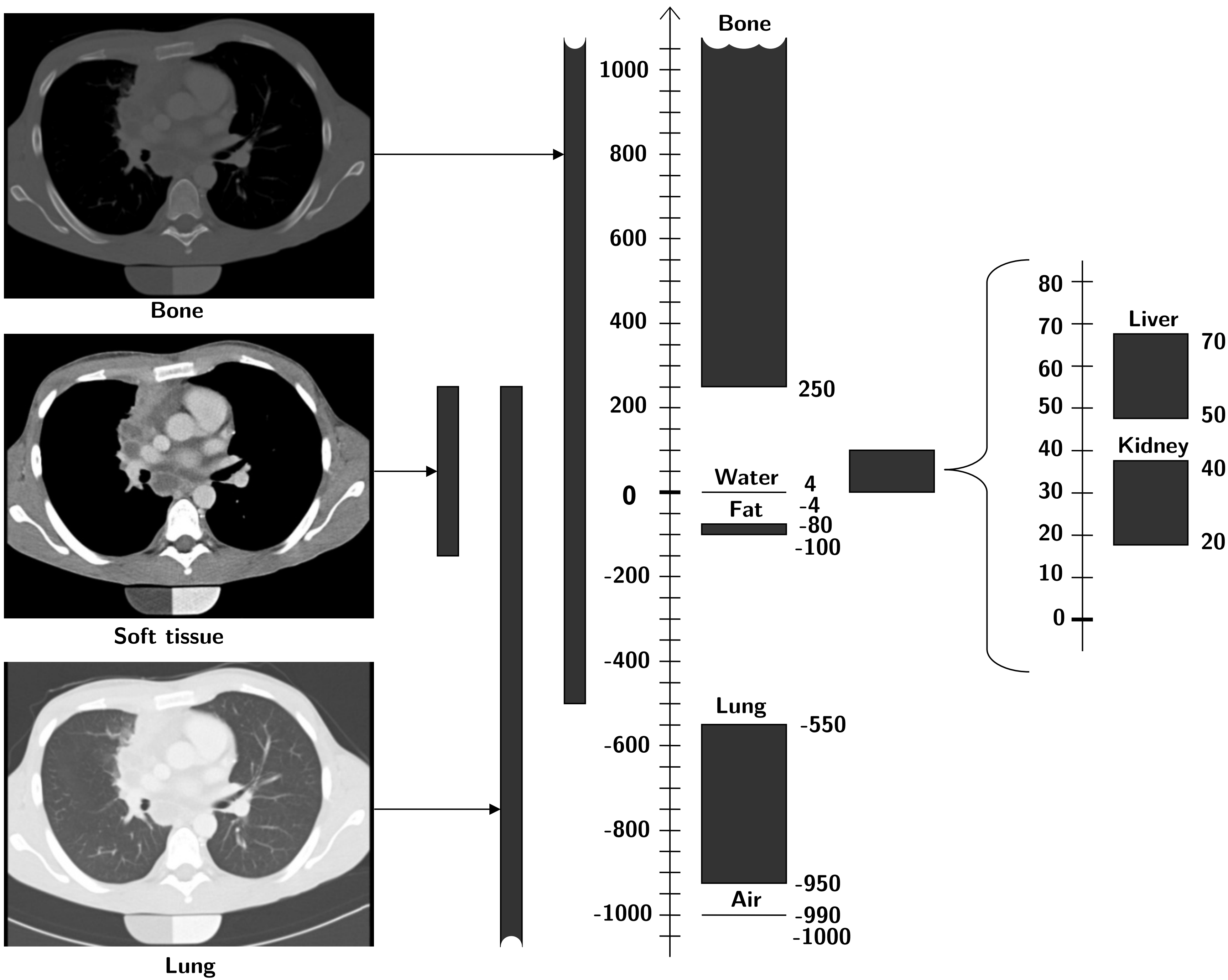
Hounsfield units

Linear attenuation renormalized with water

gray value in an image =

$$\frac{\mu - \mu_{water}}{\mu_{water}} \cdot 1000 \quad HU$$

Hounsfield units



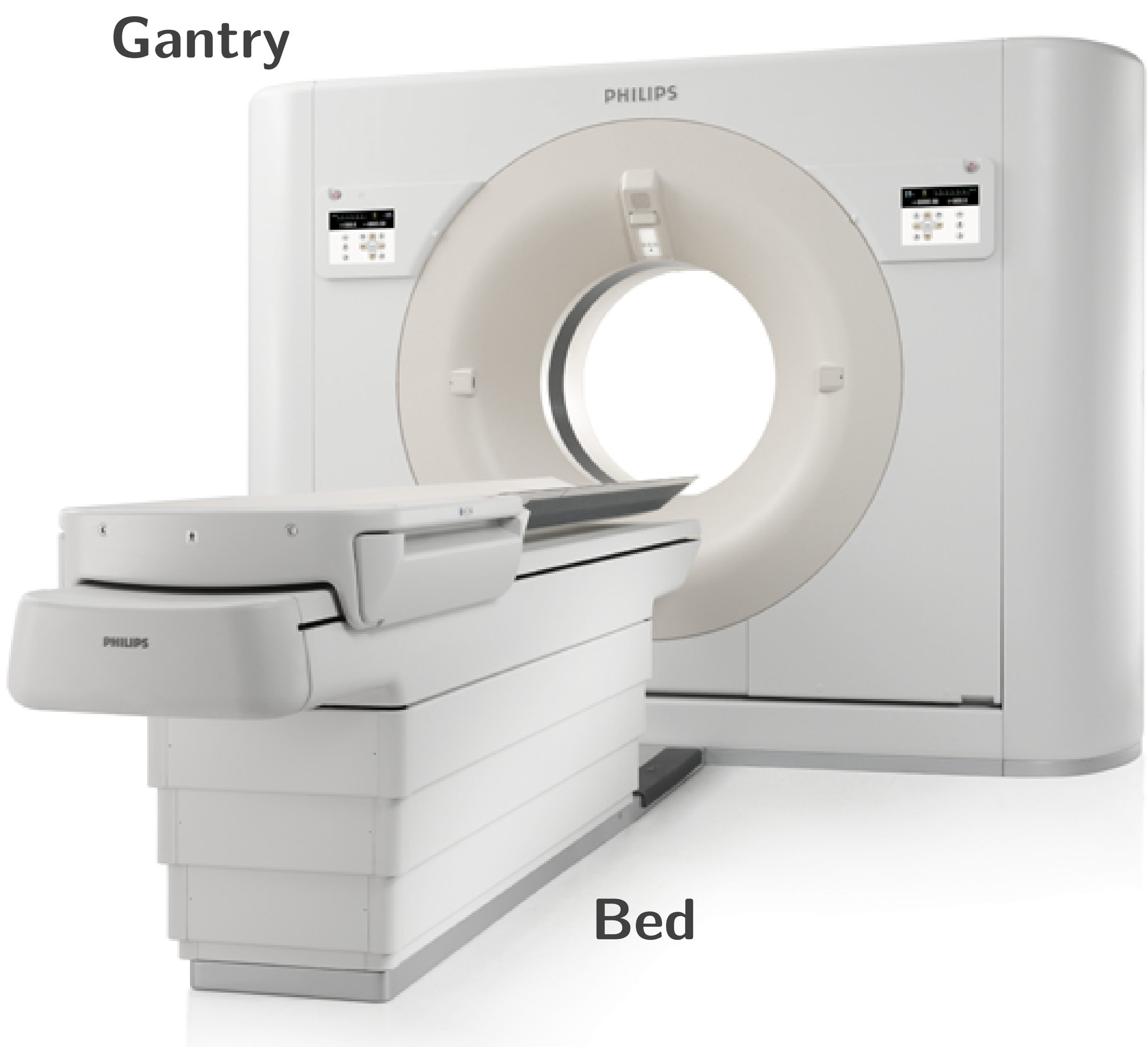
Medical CT

- Mathematical methods developed by Allan M. Cormack in the early 60s.
- First clinically useful CT instrument developed by Godfrey Hounsfield in the early 70s.
- Cormack & Hounsfield were awarded the Nobel prize in 1979 “for the development of computer assisted tomography”.

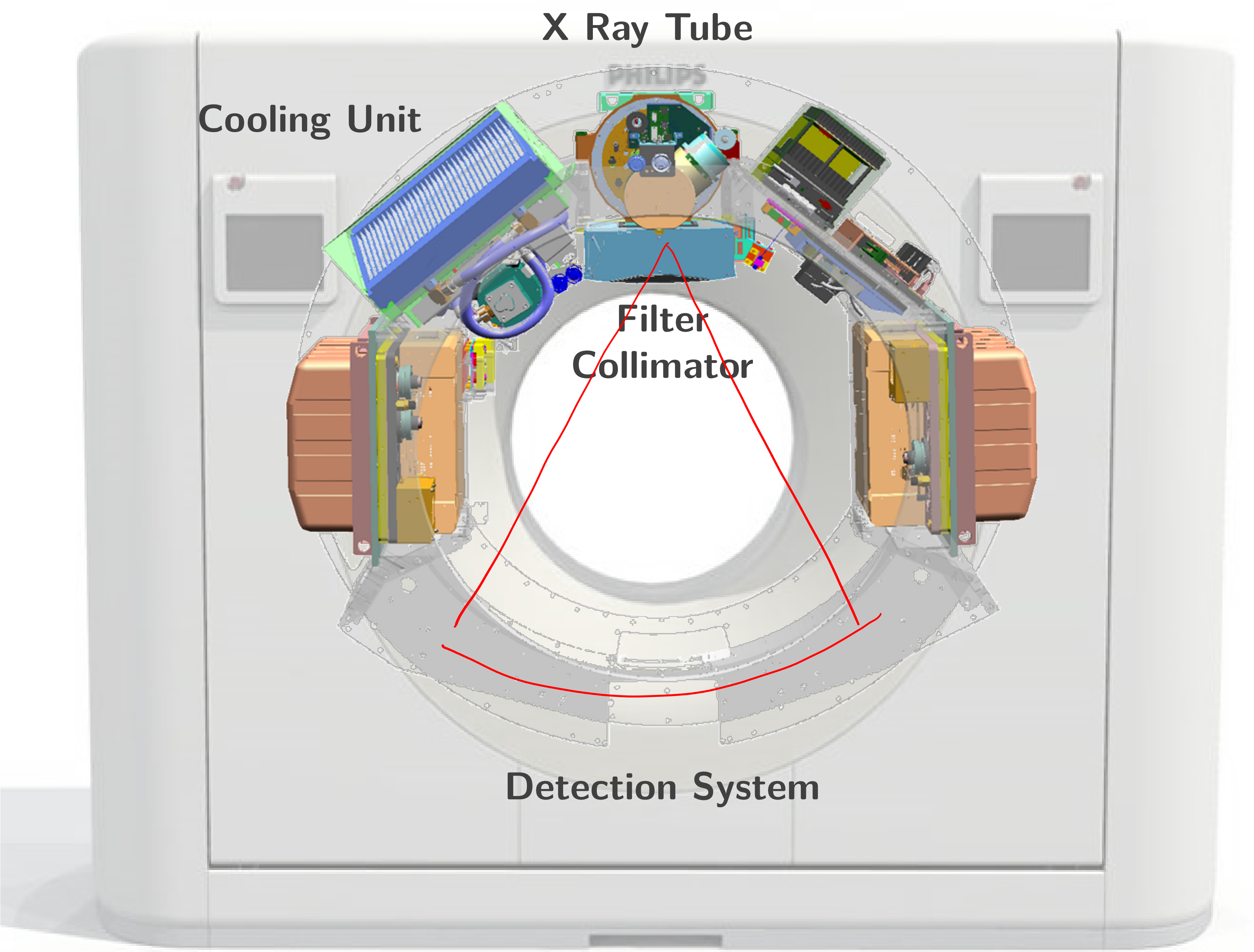
Photo from the Nobel Foundation archive.
Allan M. Cormack

Photo from the Nobel Foundation archive.
Godfrey N. Hounsfield

CT systems

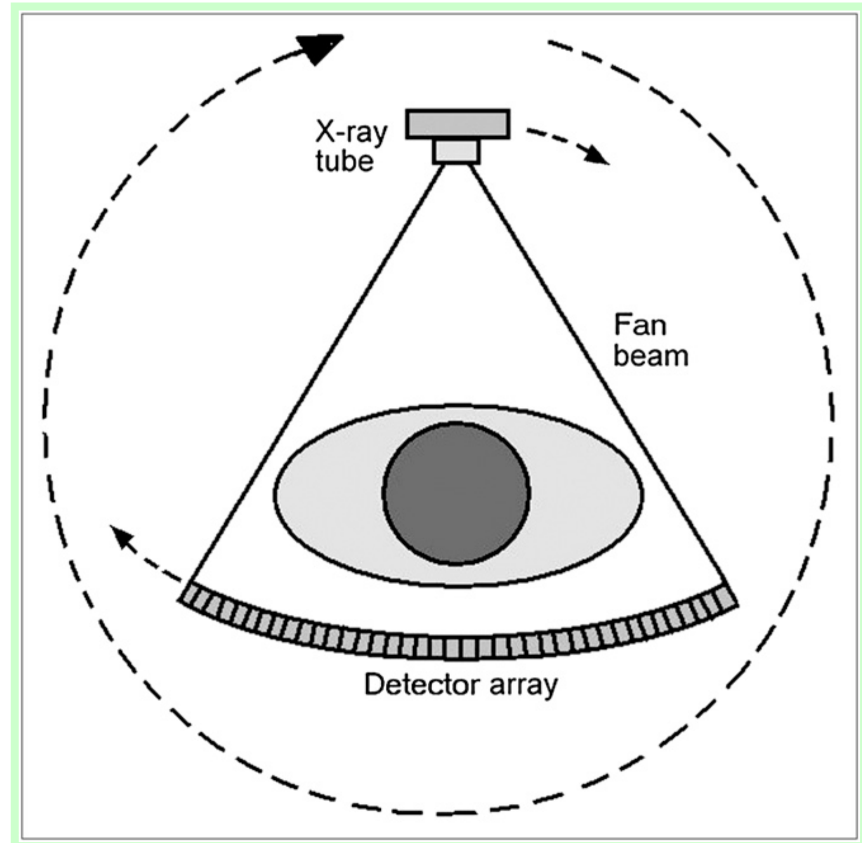


CT systems


X Ray Tube
Cooling Unit
Filter
Collimator
Detection System

CT systems

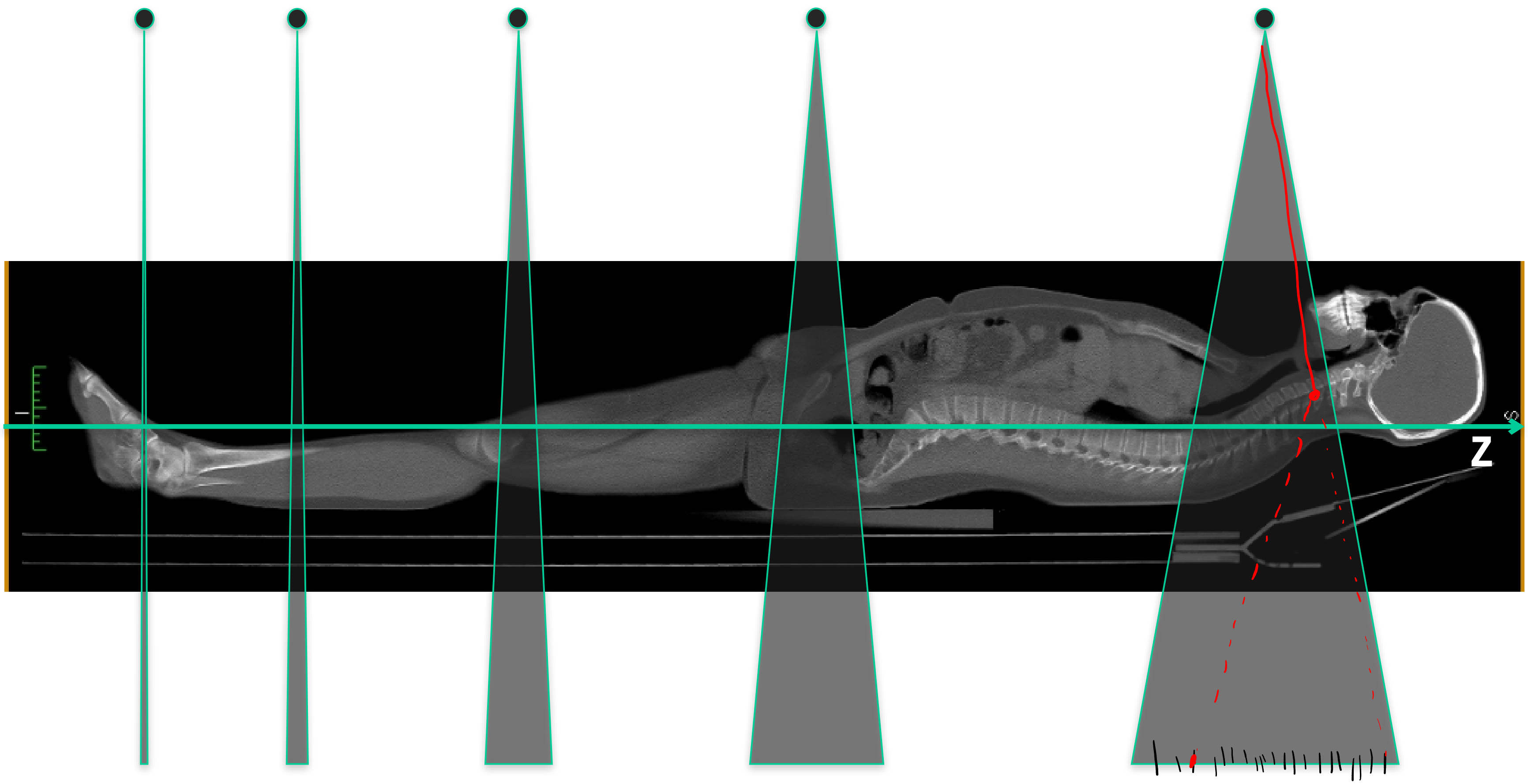
- 3^{rd} generation scanners:
  - 1 X-ray source
  - 1D detector
  - *Fan beam* geometry
  - Total scan time less than 5 second.



first scanners
3rd generation

CT systems

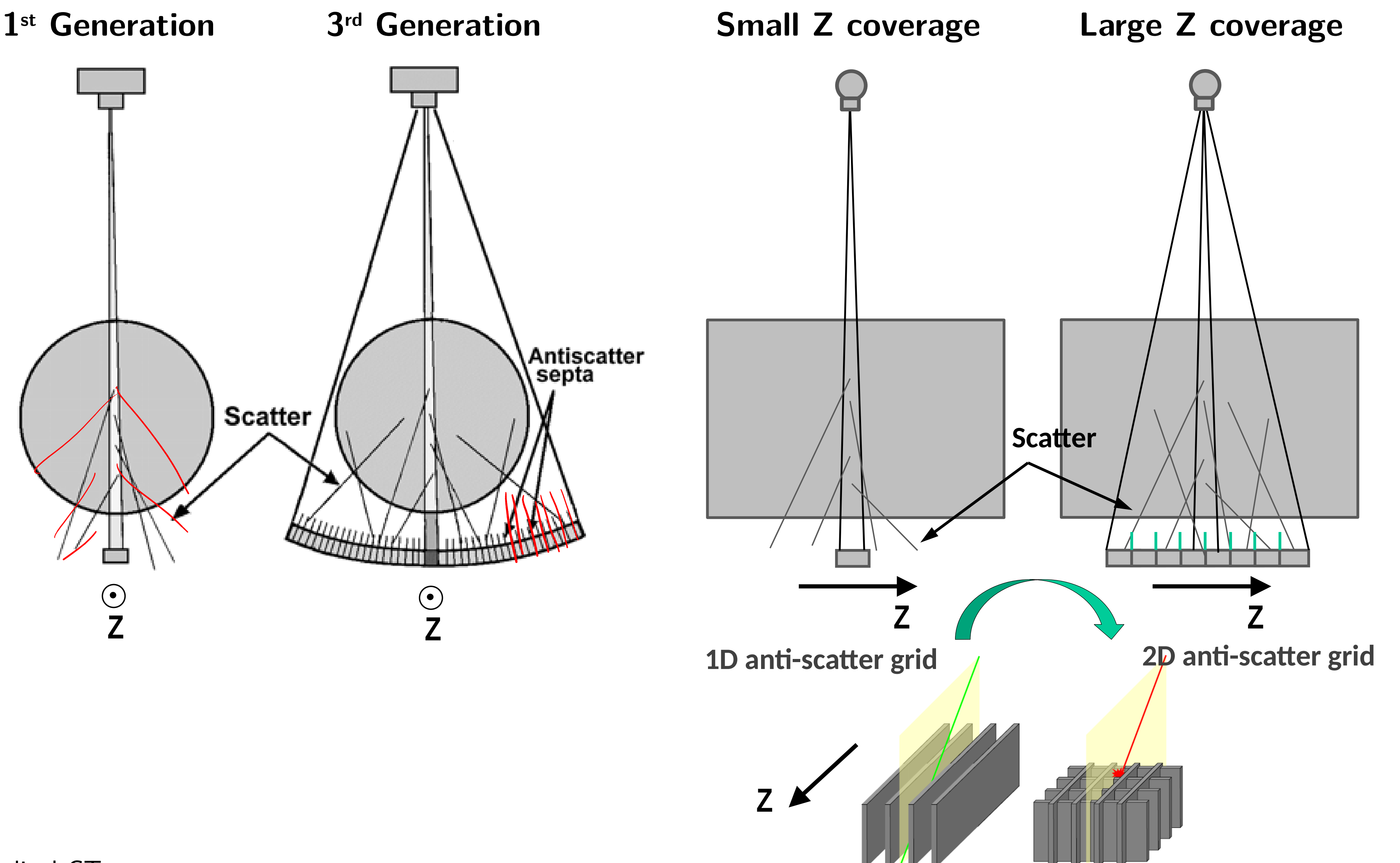
- Recent scanners: 2D detectors (cone beam)



NxT	1x5mm	4x1mm	16x0.75mm	64x0.6mm	320x0.5mm
t_{rot}	0.75s	0.5s	0.42s	0.33s	0.27s
year	1995	1998	2001	2004	2008

CT systems

Anti-scatter



CT systems

~~Spiral~~ CT

Helical CT

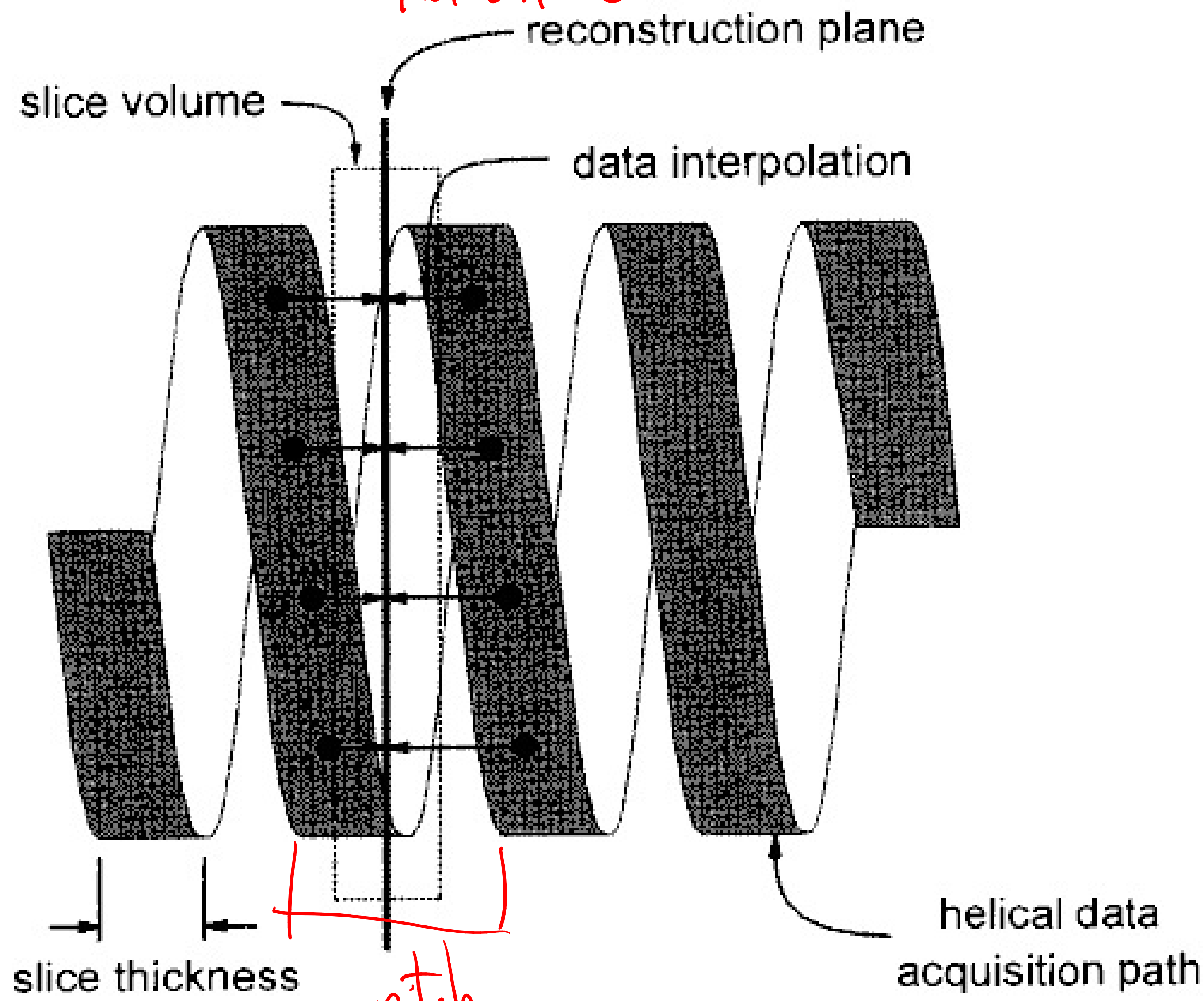
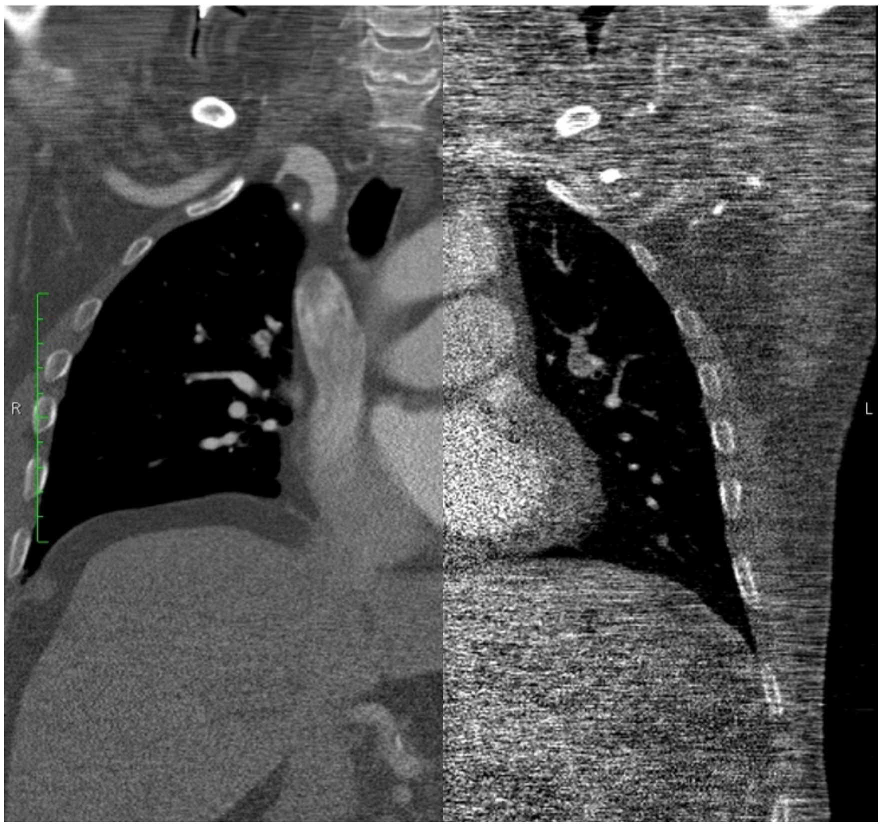


Image quality

stochastic

systematic

Signal to noise



Artifacts

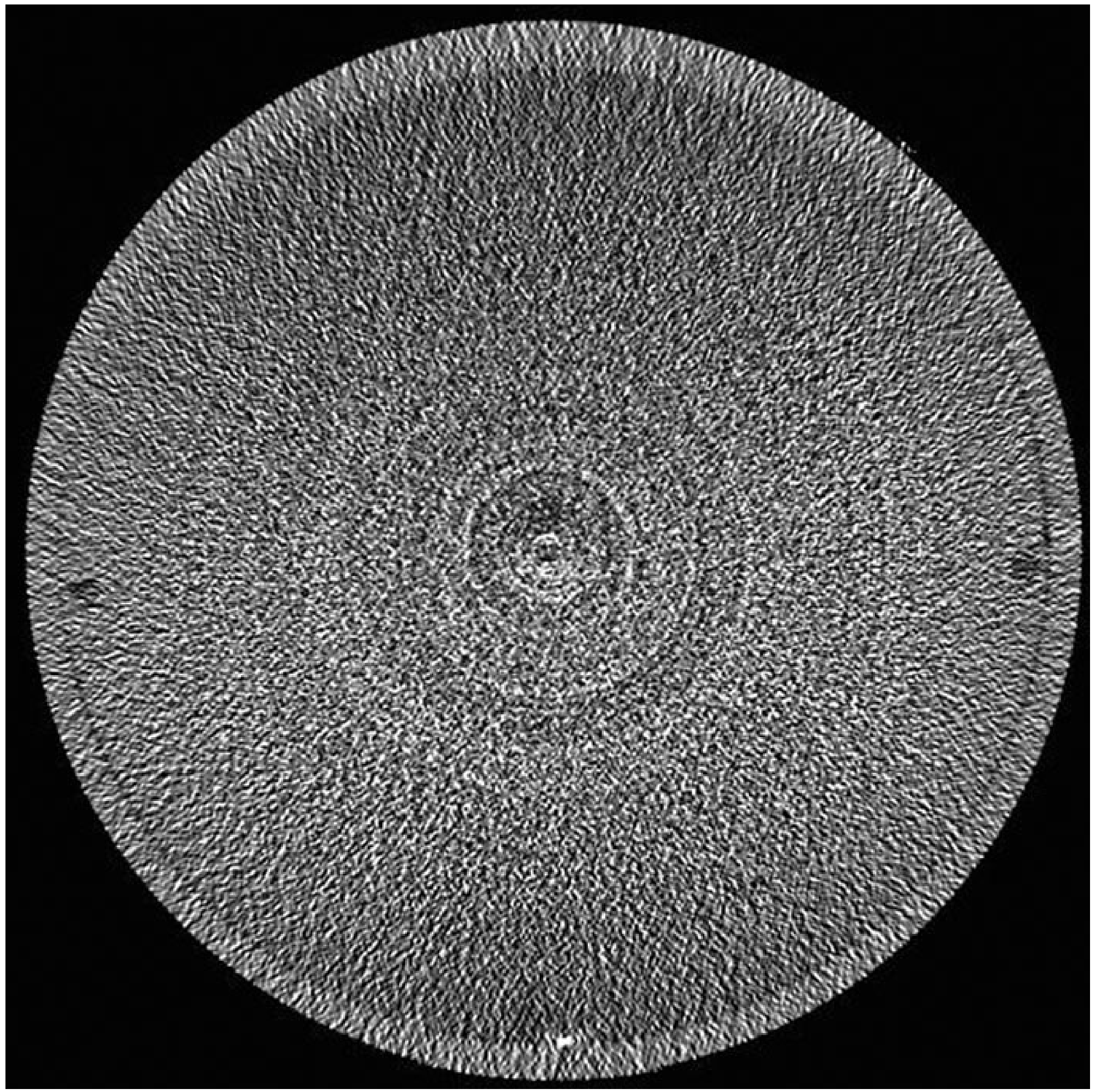
Artifacts

Ring artifacts: caused by damaged or miscalibrated detector pixels.

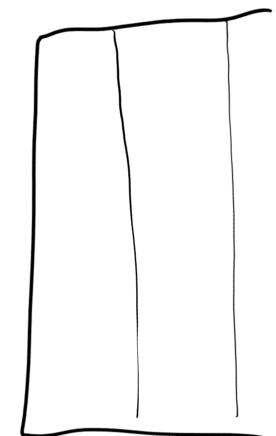
ring correction

$\Updownarrow$

filters in sinogram space



sinogram



bad pixels

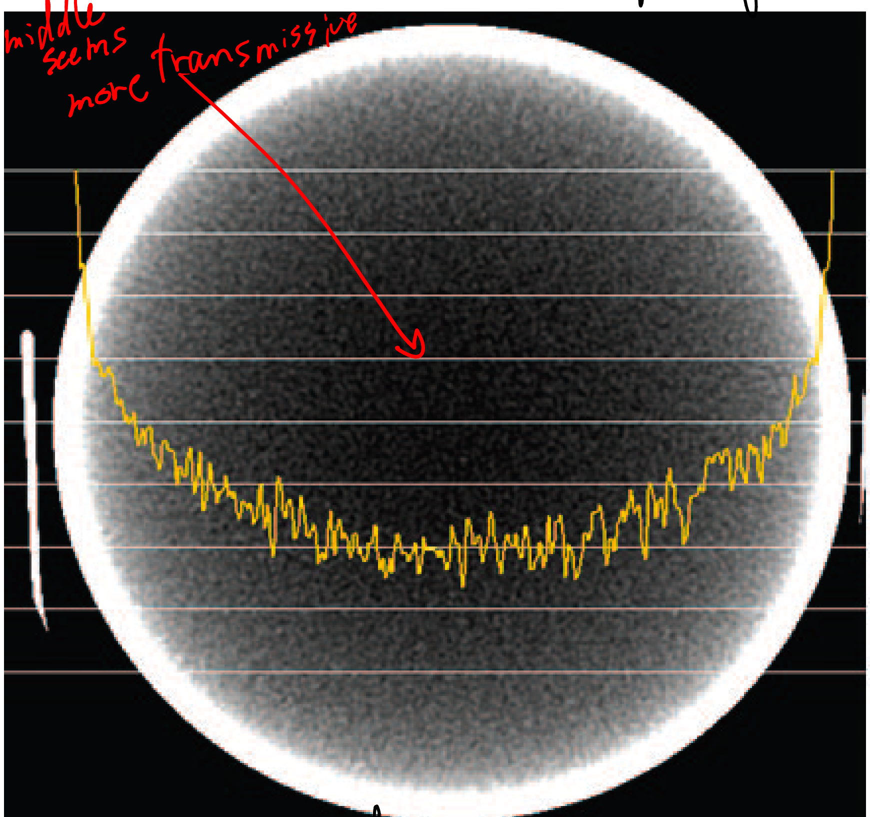
line → ring

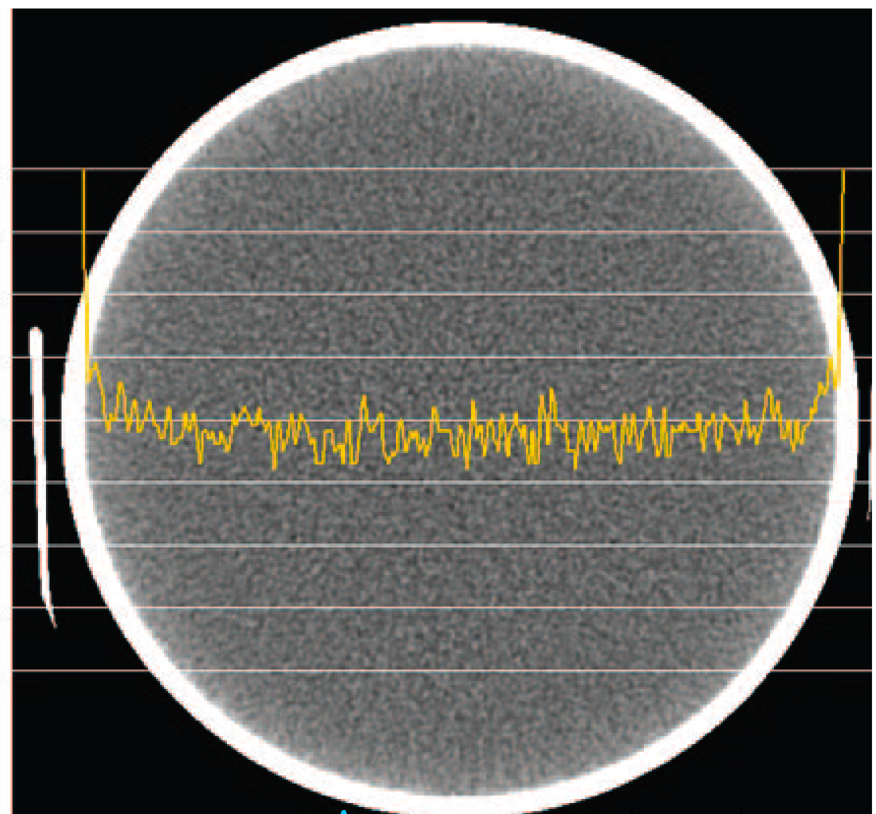
common way to reduce ring artifacts:
translation of the detector ("typewriter")

Artifacts

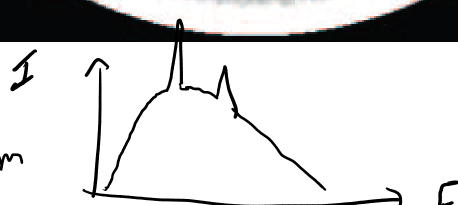
Beam hardening: deviation from the exponential law caused by the attenuation of a broad spectrum (thicker objects seem more transmissive than they should)

"cupping" artifact

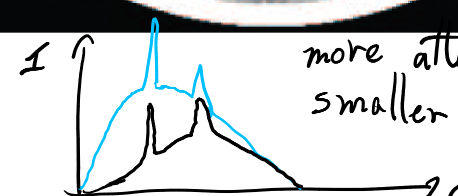




incident spectrum



trough object

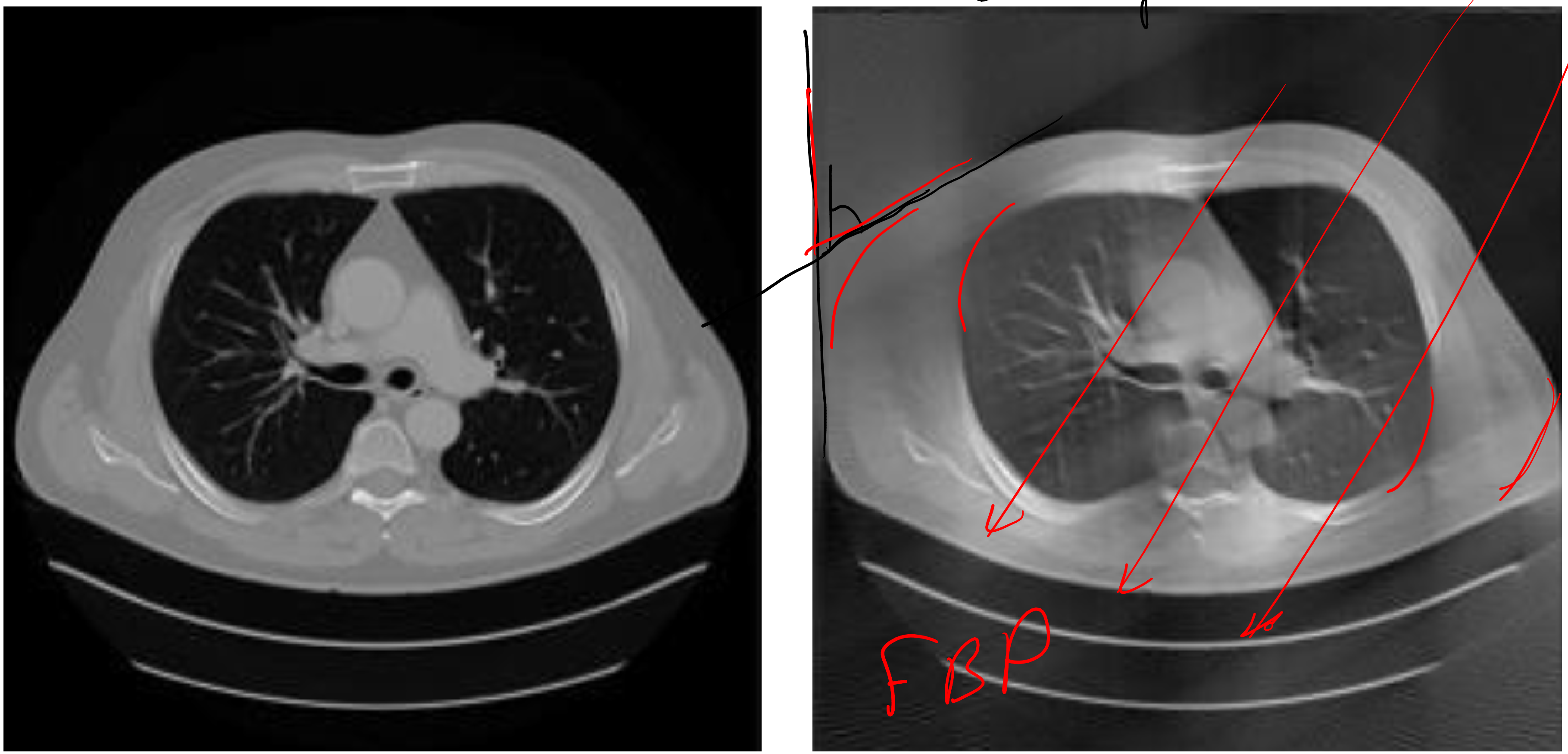


more attenuation for smaller photon energies

Artifacts

"Missing wedge": caused by an incomplete sinogram (also called limited angle tomography)

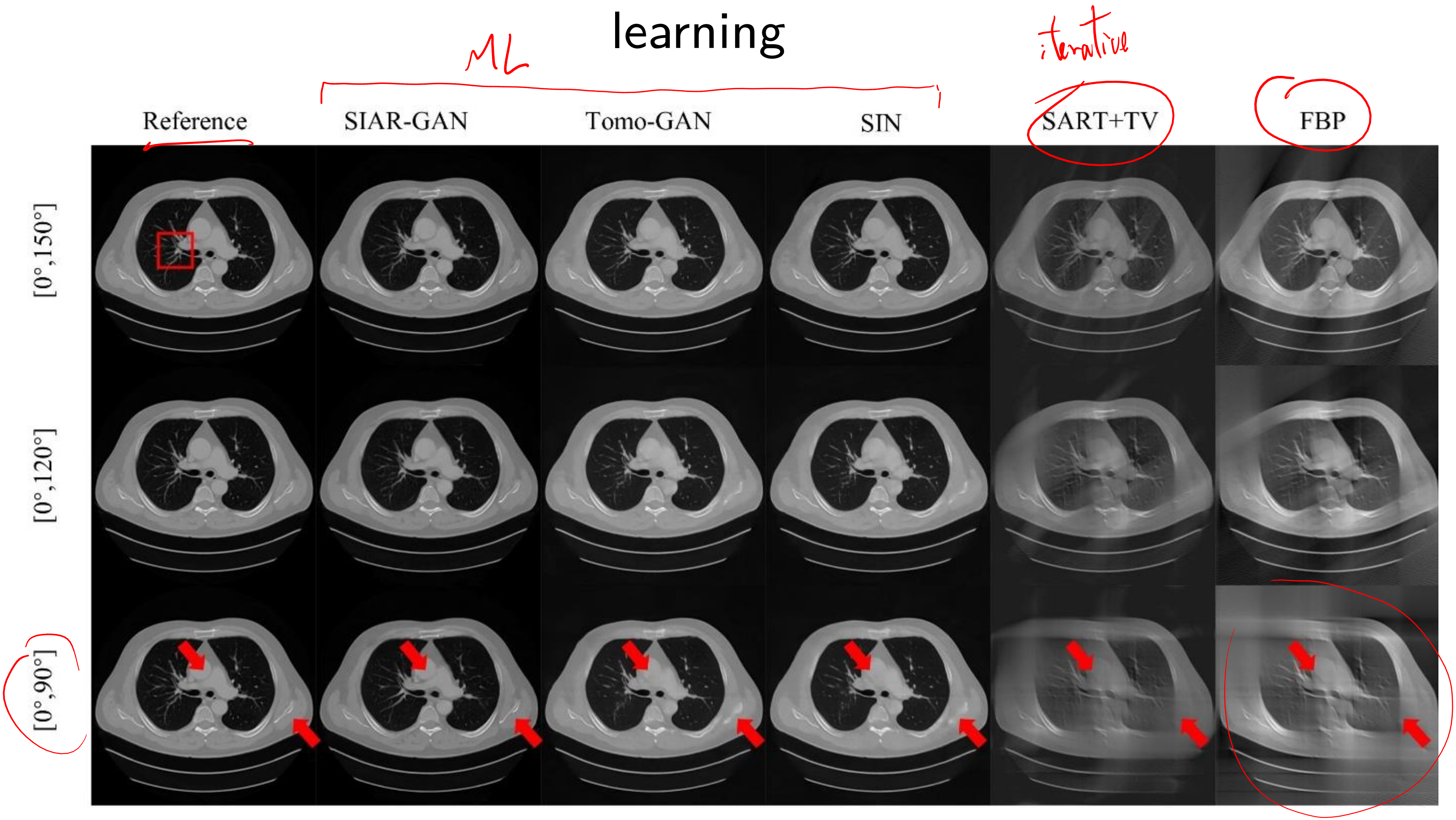
60° missing out of 180°



iterative techniques much better suited for these measurements

Artifacts

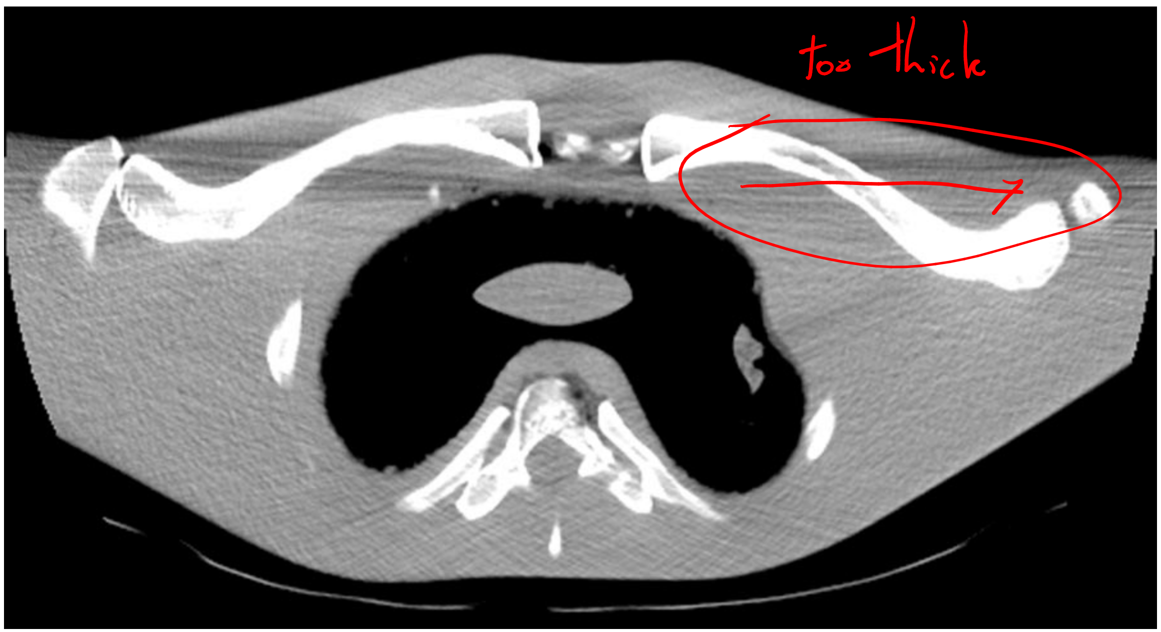
“Missing wedge”: promising results using machine learning



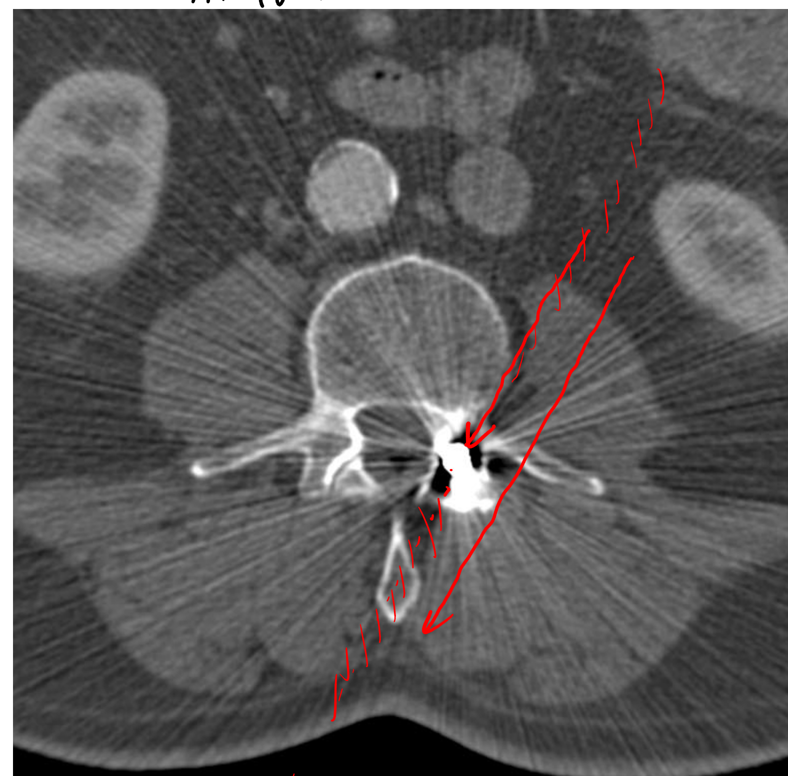
Medical CT

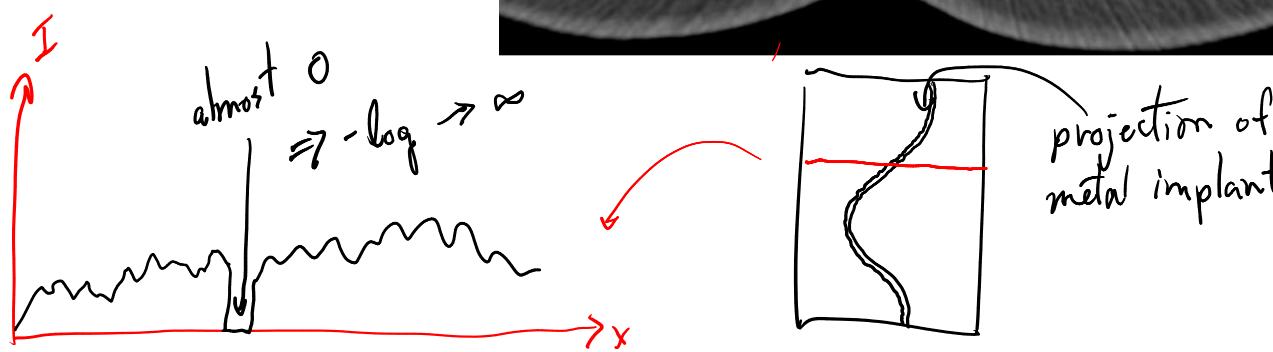
Artifacts

Photon starvation: strongly absorbing features discard useful signal for the reconstruction of nearby areas



metal artifact





Dose and image quality

Image quality $\uparrow$ with dose $\uparrow$

* X-ray generator current (mA)

  current is doubled $\Longrightarrow$ dose is doubled

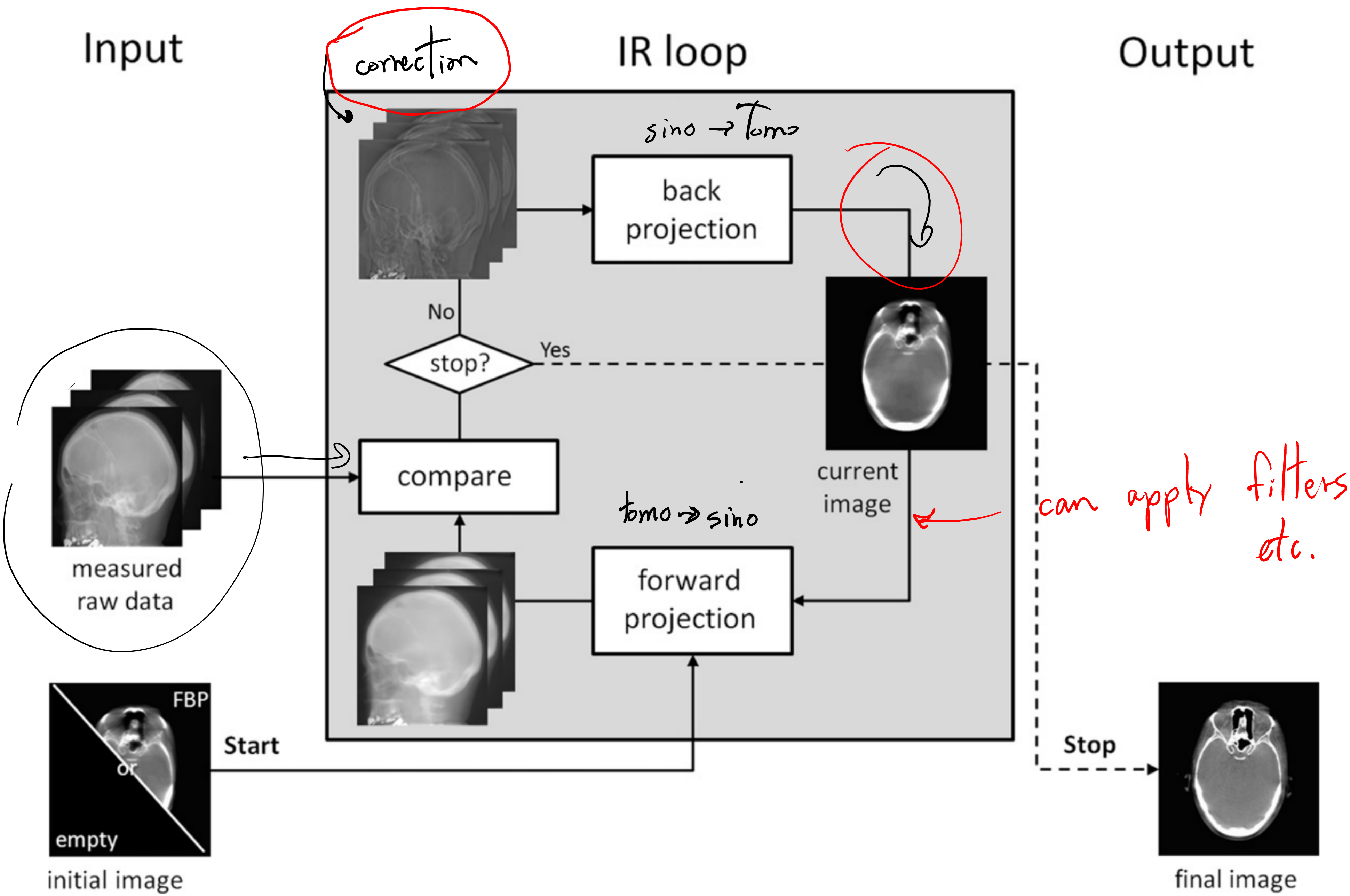
* X-ray generator voltage:

  e.g. 120 kV → 140 kV → double deposited energy = dose

* Slice thickness

  same dose : double thickness → double signal (at the cost of resolution)

Iterative methods

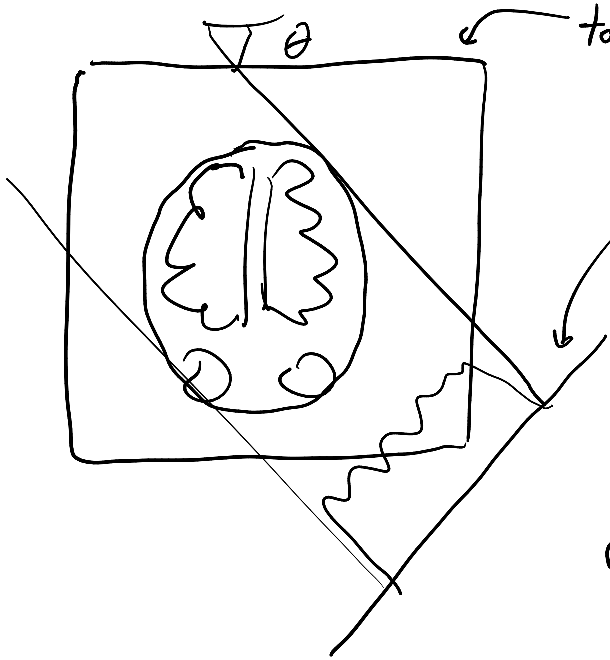


Iterative methods

Abbreviation	Meaning	
ART	Algebraic reconstruction technique	Gordon et al. 1970
SART	Simultaneous ART	Anderson & Kak, 1984
SIRT	Simultaneous iterative reconstruction technique	Gilbert 1972
OS-SIRT	Ordered subset SIRT	Gordon et al. 1970
MART	Multiplicative algebraic reconstruction technique	
ML-EM	Maximum likelihood expectation-maximization	Lange & Carson 1984
OS-EM	Ordered subset expectation-maximization	Manglos et al 1995
OSC	Ordered subset convex algorithm	Kamphuis & Beekman 1998 Erdogan & Fessler 1999
ICD	Iterative coordinate descent	
OS-ICD	Ordered subset ICD	...
MBIR	Model-based iterative reconstruction	

model not only noise, but other potential sources of artifacts

Iterative Reconstruction formulation



tomogram slice "T"

projection at angle θ

=

line θ of the complete sinogram S^θ

Reminder: Radon transform is a linear operation

$$S = M T$$

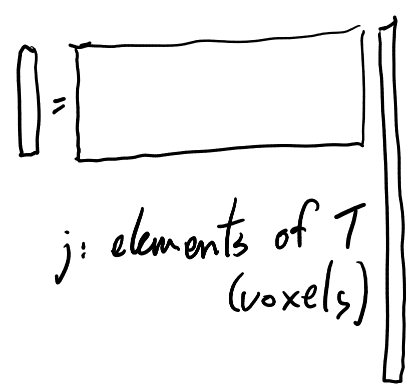
"system matrix"

M encodes the summation process as a ray passes through the sample

M is sparse (full of 0s) but its (pseudo) inverse is not sparse and it's <u>big</u>

Iterative methods break up the linear system in pieces.

for one angle: $S^\theta = A^\theta T$

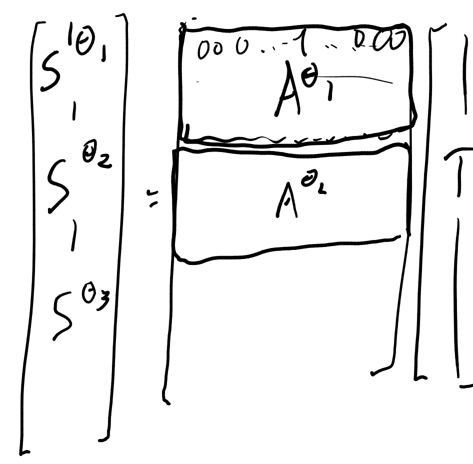


k: pixel index in the sinogram

$$S^\theta_k = \sum_j A^\theta_{kj} T_j$$

j: elements of T (voxels)

$S = MT$



Iterative Reconstruction formulation

Problem formulation: given a current tomogram estimate T_0, what is the new tomogram T, as close as possible to T_0 such that the constraint $S^\theta = A^\theta T$ is satisfied?

$\Rightarrow$ cost function to minimize:

$$D = \sum_j |T_j - T_{0j}|^2 + \sum_k \lambda_k \left(\sum_j A^\theta_{kj} T_j - S^\theta_k \right)$$

Lagrange multipliers

ART: solves this system for a single k (and single θ) at a time
one ray at a time

SART: solves this system for all k but a single θ at a time

SIRT: same as SART except that the corrections found at different θ are applied together after looping through all the angles.

Constrained least square problem

$$\frac{\partial}{\partial T_j} O = 2(T_j - T_{j0}) + \sum_k \lambda_k A^\theta_{kj} = 0$$

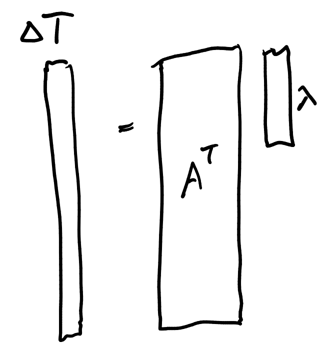
$$T_j = T_{0j} - \frac{1}{2} \sum_k \lambda_k A^\theta_{kj}$$

we still have to find λ!

$$(*) \quad T = T_0 - \frac{1}{2} A^T \lambda$$

vector of all λ_k

(BTW A^T = backprojection operation!)



To find λ: apply constraint on the solution

$$S^\theta_k = \sum_j A^\theta_{kj} \left(T_{0j} - \frac{1}{2} \sum_{k'} \lambda_{k'} A^\theta_{k'j} \right)$$

$$= \sum_j A^\theta_{kj} T_{0j} - \frac{1}{2} \sum_{j k'} \lambda_{k'} A^\theta_{k'j} A^\theta_{jk}$$

$$S^\theta = A^\theta T_0 - \frac{1}{2} A A^T \lambda$$

$$S^\theta = P^\theta - \frac{1}{2} A A^T \lambda \Rightarrow \lambda = 2 (A A^T)^{-1} (P^\theta - S^\theta) \quad (**)$$

$* \, P^\theta$: forward projection of current tomogram estimate T_0

ART

$$T_j = T_{0j} - \frac{1}{2}\lambda_k A_{kj} \qquad \text{(no sum over } k\text{)}$$

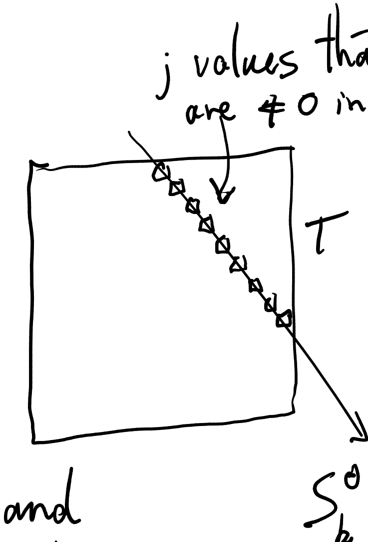
apply constraint:

$$\sum_j A^\theta_{kj} T_j = S^\theta_k = \sum_j A^\theta_{kj}\left(T_{0j} - \frac{1}{2}\lambda_k A_{kj}\right)$$

$$= P^\theta_k - \frac{1}{2}\sum_j (A_{kj})^2 \lambda_k$$

$$\lambda_k = \frac{2}{\sum_j (A_{kj})^2}\left(P^\theta_k - S^\theta_k\right)$$

$$\Rightarrow T_j = T_{0j} + \sum_k \overset{\text{back projection}}{A_{kj}} \underbrace{\frac{1}{\sum_j (A_{kj})^2}\left(S^\theta_k - P^\theta_k\right)}_{\text{diffenence betwen sinogram pixel and modelled projection normalized with ray length!}}$$



ART: apply this equaton sequentially over all k and θ

SART

Substitute (**) into (*):

$$T = T_0 + \overbrace{A^T}^{\text{back projection}} \underbrace{(AA^T)^{-1}}_{\sim \text{diagonal matrix with inverse ray length along the diagonal}} \overbrace{(S - P)}^{\text{difference between measurement and model}}$$

Recipe:

1) Project $T_0 \rightarrow P^\theta$
2) compute $\Delta P^\theta = S^\theta - P^\theta$
3) renormalize: $(AA^T)^{-1} \Delta P^\theta$ (divide each pixel in ΔP^θ by ray length)
4) back-project $\rightarrow \Delta T^\theta$
5) add result to $T_0 \longrightarrow T = T_0 + \Delta T^\theta$ ← SART

5*) accumulate ΔT^θ for all angle, then $T = T_0 + \sum_\theta \Delta T^\theta$ ← SIRT