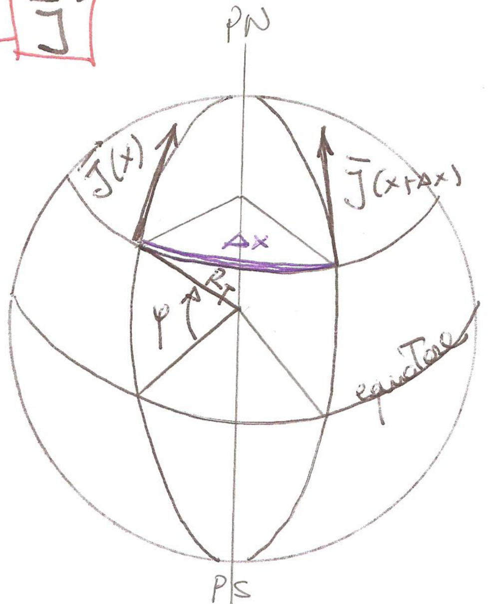
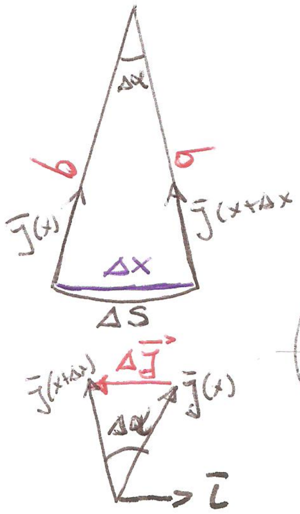
Analisi della variazione del versore $\boxed{\vec{j}}$

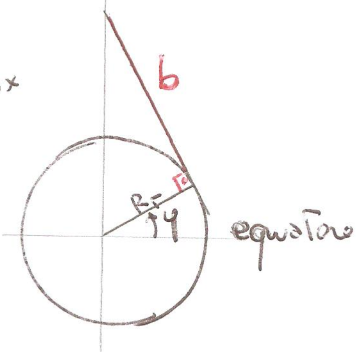


$$\frac{d\vec{j}}{dt} = \frac{\partial \vec{j}}{\partial t} + u\frac{\partial \vec{j}}{\partial x} + v\frac{\partial \vec{j}}{\partial y} + w\frac{\partial \vec{j}}{\partial z}$$

nulla in quanto $\vec{j}(x,y,t)$ ma non dipende esplicitamente da t

$$\frac{\partial \vec{j}}{\partial x} = \lim_{\Delta x \to 0} \frac{\vec{j}(x+\Delta x) - \vec{j}(x)}{\Delta x}$$





Ricordiamo che $|\vec{j}| = 1$ (versore)

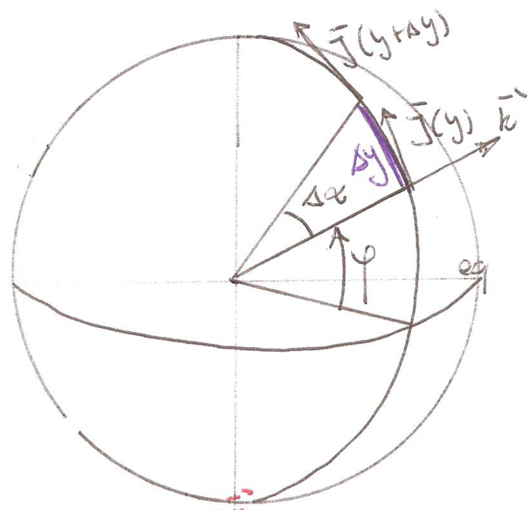
$$|\Delta\vec{j}| = |\vec{j}(x+\Delta x) - \vec{j}(x)| = 1 \cdot \Delta\alpha$$

$$\Delta\alpha = \frac{\Delta S}{b}$$

Osservazione $\lim_{\Delta x \to 0} \Delta S \to \Delta x$ e $b = \dfrac{R_T + z}{tg\varphi} \simeq \dfrac{R_T}{tg\varphi}$

$$\boxed{\frac{\partial \vec{j}}{\partial x}} = \lim_{\Delta x \to 0} \frac{\Delta S}{b}\frac{1}{\Delta x}(-\vec{i}) = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x}\frac{tg\varphi}{R_T}(-\vec{i}) = \boxed{-\frac{tg\varphi}{R_T}\vec{i}}$$

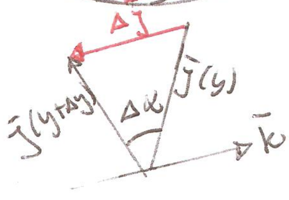
$$\frac{\partial \vec{j}}{\partial y} = \lim_{\Delta y \to 0} \frac{\vec{j}(y+\Delta y) - \vec{j}(y)}{\Delta y}$$



Ricordiamo che $|\vec{j}| = 1$ (versore)

$$|\Delta\vec{j}| = |\vec{j}(y+\Delta y) - \vec{j}(y)| = 1 \cdot \Delta\alpha$$

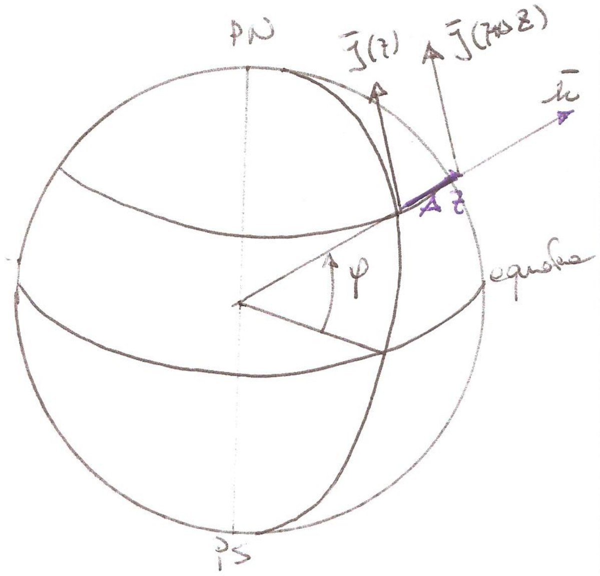
$$\Delta\alpha = \frac{\Delta y}{R_T + z} \simeq \frac{\Delta y}{R_T}$$



$\Delta\vec{j}$ al limite $\Delta y \to 0$ ha la stessa direzione di $\vec{k}$ ma verso opposto

$$\boxed{\frac{\partial \vec{J}}{\partial y}} = \lim_{\Delta y \to 0} \frac{\Delta \vec{J}}{\Delta y} = \lim_{\Delta y \to 0} \frac{\Delta y}{R_T} \frac{1}{\Delta y} (-\vec{k}) = \boxed{-\frac{1}{R_T} \vec{k}}$$

$$\frac{\partial \vec{J}}{\partial z} = \lim_{\Delta z \to 0} \frac{\vec{J}(z+\Delta z) - \vec{J}(z)}{\Delta z}$$



Osservazione $\vec{J}(z)$ e $\vec{J}(z+\Delta z)$ sono due versori paralleli (identici)

da cui $\Delta \vec{J} = \vec{J}(z+\Delta z) - \vec{J}(z) = 0$

Quindi $$\boxed{\frac{\partial \vec{J}}{\partial z} = 0}$$

Sommando tutti gli addendi della $\frac{d\vec{J}}{dt}$ si ottiene

$$\frac{d\vec{J}}{dt} = 0 + u\left(-\frac{tg\varphi}{R_T}\right)\vec{i} + v\left(-\frac{1}{R_T}\right)\vec{k} + w \cdot 0$$

Quindi

$$\boxed{\frac{d\vec{J}}{dt} = -\frac{u}{R_T} tg\varphi\, \vec{i} - \frac{v}{R_T} \vec{k}}$$

Osservazione

La variazione di $\vec{J}$ dà due contributi su assi distinti da $\vec{J}$