Copertina

Corso di Fisica dell'Atmosfera

Coordinate naturali e moti ciclostrofico ed inerziale

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Sommario della lezione

- Il sistema di coordinate naturali
- Modelli semplici nel sistema di coordinate naturali

Sistema di coordinate naturali

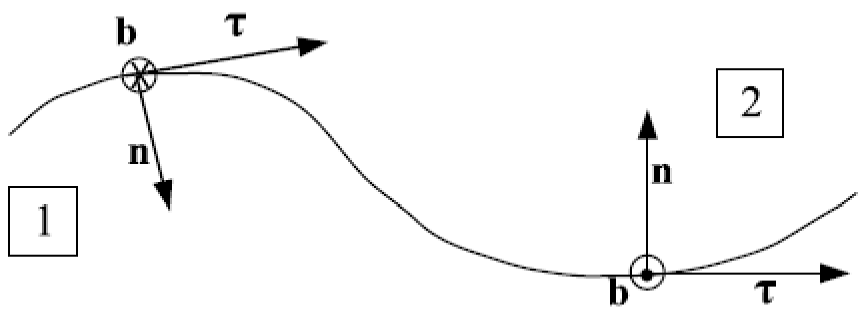


Figure 2. The trajectory of the fluid parcel and the disadvantages of the moving trihedral. The normal unit vector $\boldsymbol{n}$ is always pointing towards the concavity of the curve, while the tangent unit vector $\boldsymbol{\tau}$ is along the motion, so when the curvature changes the resulting unit vector $\boldsymbol{b} = \boldsymbol{\tau} \times \boldsymbol{n}$ experiences abrupt inversions. From position 1 to position 2 $\boldsymbol{b}$ experiences an inversion.

Corollary 3 (The sign of the rarius of curvature).
In the natural coordinate system, given a trajectory $\boldsymbol{r}(l)$, l is the natural parameter of the curve, the curvature is the real number:

$$k := \dot{\boldsymbol{\tau}} \cdot \boldsymbol{n} \qquad \textit{where} \qquad \dot{\boldsymbol{\tau}} = \frac{d\boldsymbol{\tau}}{dl} \tag{3.22}$$

So the curvature k is positive when the concavity of the trajectory is towards the left with respect $\boldsymbol{\tau}$, while it is negative when the concavity is on the opposite side. Consequently the sign of k is hold by the radius of curvature according its definition.

Sistema di coordinate naturali: equazioni per la conservazione della quantità di moto

4. The momentum equations in natural coordinates

Let's consider a fluid parcel moving along its trajectory $\boldsymbol{r}(t)$, t is the time and it is the parameter of the curve. Consider the momentum equation (2.1) in the natural coordinate system $(\boldsymbol{\tau}, \boldsymbol{n}, \boldsymbol{b})$. The right hand side of the equation (2.1) is the parcel acceleration that, according with the (3.2), it is:

$$\boldsymbol{v} = v\boldsymbol{\tau} \tag{4.1}$$

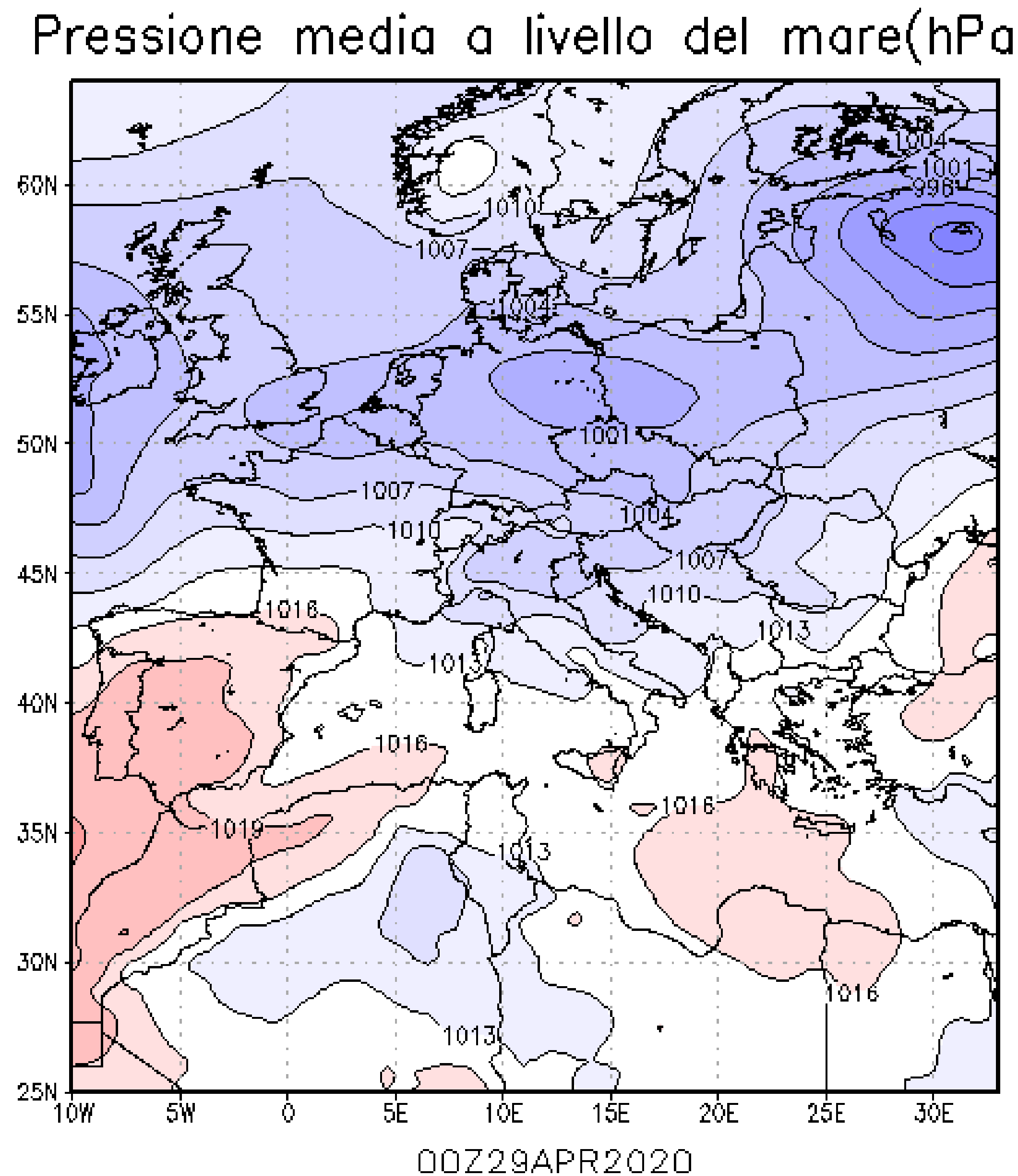
and computing of the derivative of $\boldsymbol{v}$ with respect the time, keeping in mind that $\dot{\boldsymbol{v}}$ can be expressed as a function of the natural parameter of the curve l, we get an expression for the acceleration that allows the use of the Frenet-Serret formulas for the $\dot{\boldsymbol{\tau}}$. In fact.

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial l} \qquad \text{equation for } \boldsymbol{\tau} \tag{4.12}$$

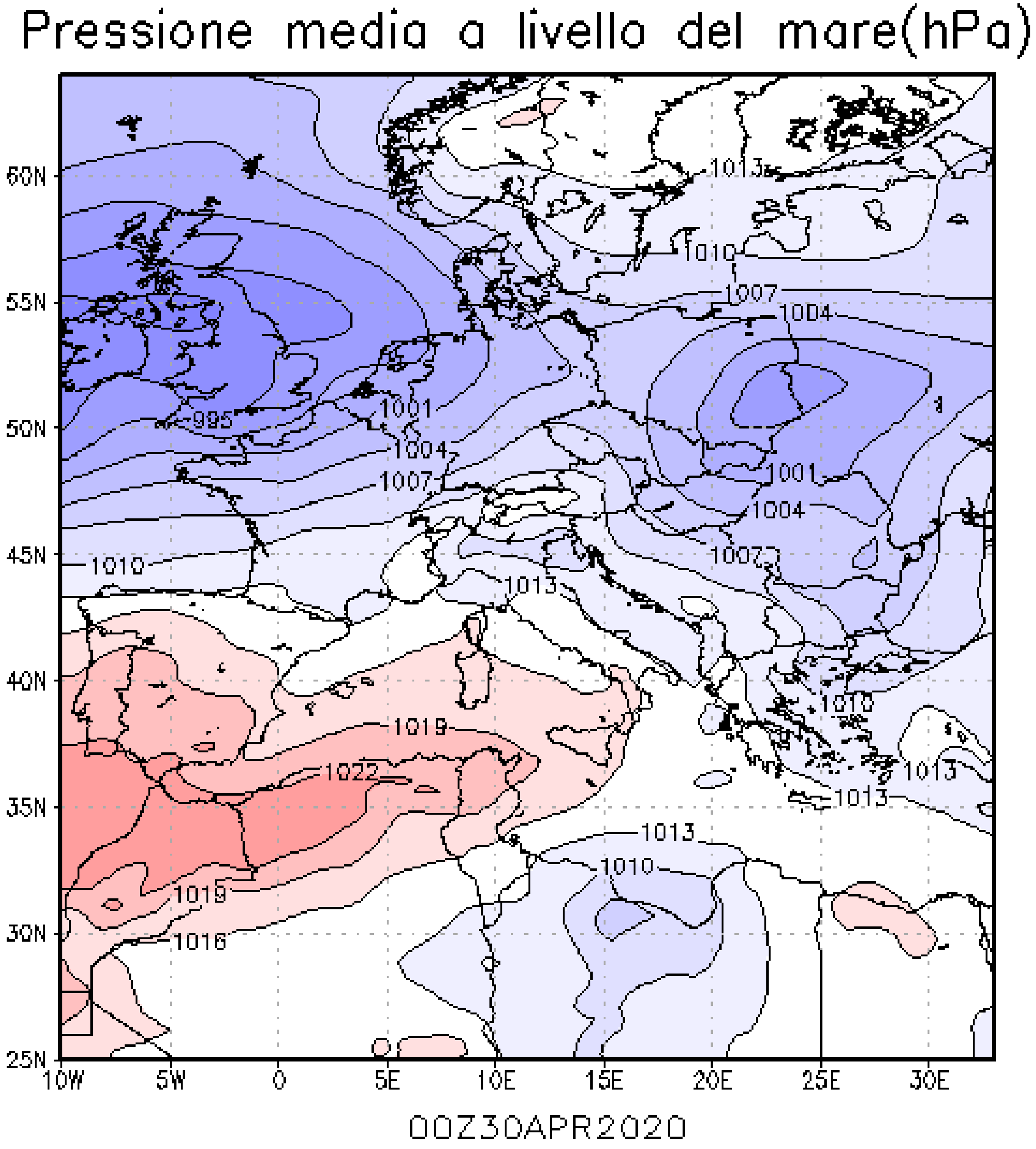
$$\frac{v^2}{R} = v^2 k = -fv - \frac{1}{\rho}\frac{\partial p}{\partial n} \qquad \text{equation for } \boldsymbol{n} \tag{4.13}$$

$$0 = 2\Omega_n v - \frac{1}{\rho}\frac{\partial p}{\partial z} - g \qquad \text{equation for } \boldsymbol{b} \tag{4.14}$$

Sistema di coordinate naturali: le oscillazioni inerziali



Sistema di coordinate naturali: le oscillazioni inerziali



Sistema di coordinate naturali: le oscillazioni inerziali

L — Situazione ore 00:00 — H

Venezia — Otranto

H — Situazione ore 24:00 — H

Venezia — Otranto

Durata del giorno

$$\frac{\partial p}{\partial l} = 0 \quad \text{and} \quad \frac{\partial p}{\partial n} = 0 \qquad T = \frac{D}{2\,|\sin\phi\,|}$$

Sistema di coordinate naturali: le oscillazioni inerziali

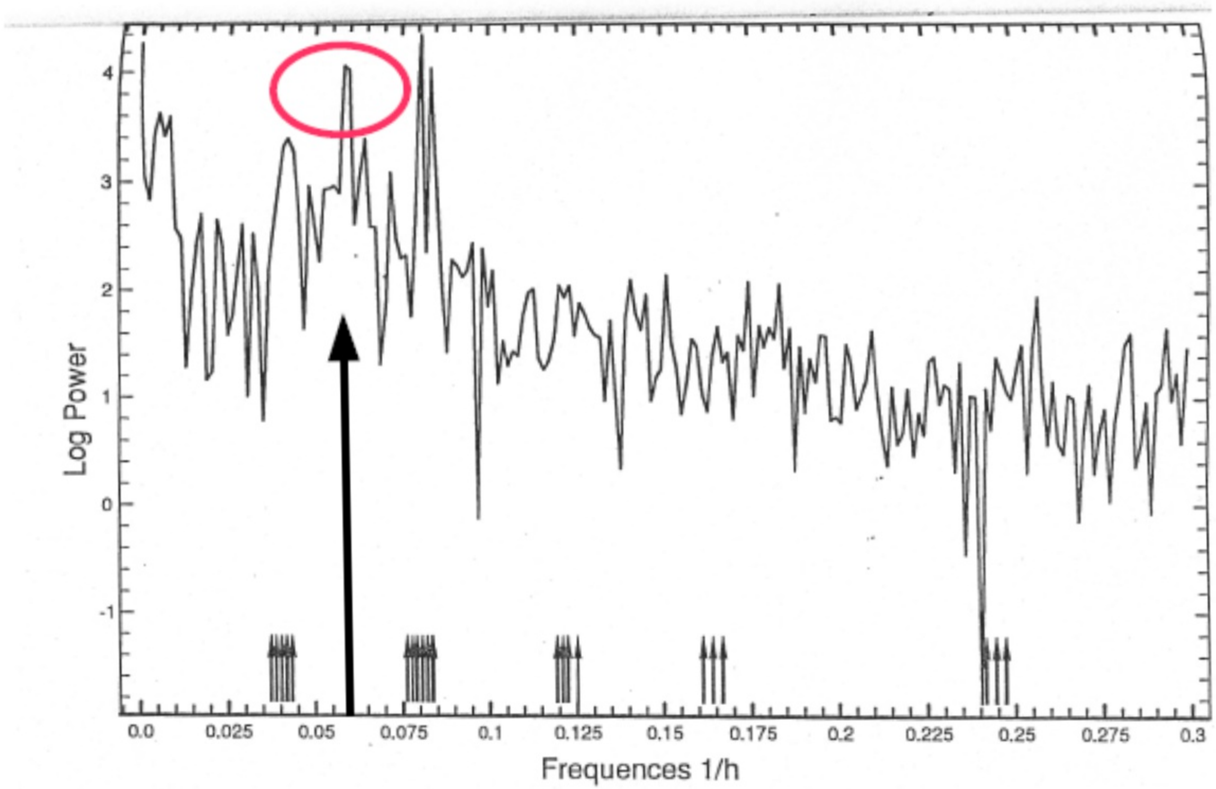
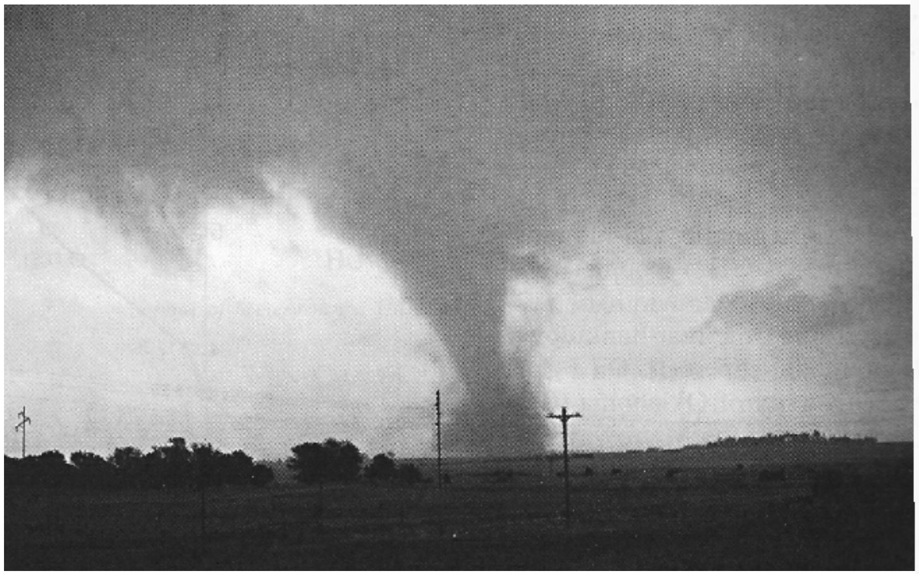


FIGURE 4. Power spectra of the North-South component of the horizontal vector for Adriatic sea currents of the time series measurements made in the frame of the PRISMA project by Osservatorio Geofisico Sperimentale (Trieste ? ITALY), during the period [1995-07-12; 1995-08-18], by a NBA 6 current meter moored 23 m below the sea surface, at the geographical position 44.382° N, 13.189° E, where the bottom depth is 50 m, (Giaiotti & Ursella 1996). Tide contributions to the currents are marked with small arrows and the peak of the power corresponding to the typical frequency of the inertial motion ($1/17\ h^{-1}$) is marked with a big arrow and a circle.

Moto ciclostrofico – Tornadoes e dust devils



$$\left|\frac{v^2}{R}\right| \gg |fv|$$

$$\frac{v^2}{R} + \frac{1}{\rho}\frac{\partial p}{\partial n} = 0$$

$$v = \pm\sqrt{-\frac{R}{\rho}\frac{\partial p}{\partial n}}$$

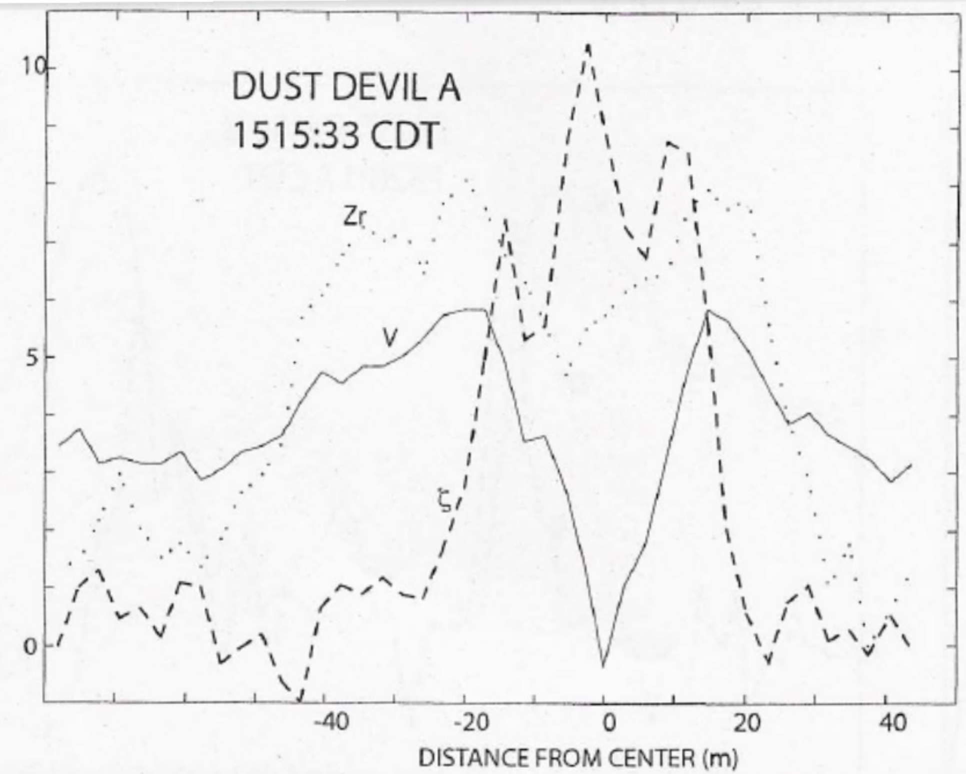


FIG. 9. Approximate vortex-relative azimuthal wind component V (solid line; m s^{-1}), vorticity ζ (dashed line; $\times 10$ s^{-1}), and relative radar reflectivity $Zr = (Z - Z_{noise})/C$, where Z is the reflectivity factor (dBZ), Z_{noise} (dBZ) is a subjectively determined approximate noise floor, and C is a dimensionless compression factor chosen subjectively so that the range of relative reflectivity remains within the scale of the figure, in dust devil A, at 1515:33 CDT, as a function of distance from the center of the vortex (m). Negative (positive) distances are measured to the left (right) of the vortex center, as viewed by the radar. The vorticity at the center is estimated as the average of the vorticities computed just to the right and left of the center.

Accelerazione di Coriolis – agisce anche lungo la verticale

Conservazione della quantità di moto – equazioni

$$i)\quad \frac{du}{dt} - \frac{vw}{R_T}\operatorname{tg}\varphi + \frac{wu}{R_T} = fv - f^{*}w - \frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (2.1)$$

$$j)\quad \frac{dv}{dt} + \frac{u^2}{R_T}\operatorname{tg}\varphi + \frac{wv}{R_T} = -fu - \frac{1}{\rho}\frac{\partial p}{\partial y} \qquad (2.2)$$

$$k)\quad \frac{dw}{dt} - \frac{u^2 + v^2}{R_T} = \boxed{f^{*}u} - g - \frac{1}{\rho}\frac{\partial p}{\partial z} \qquad (2.3)$$

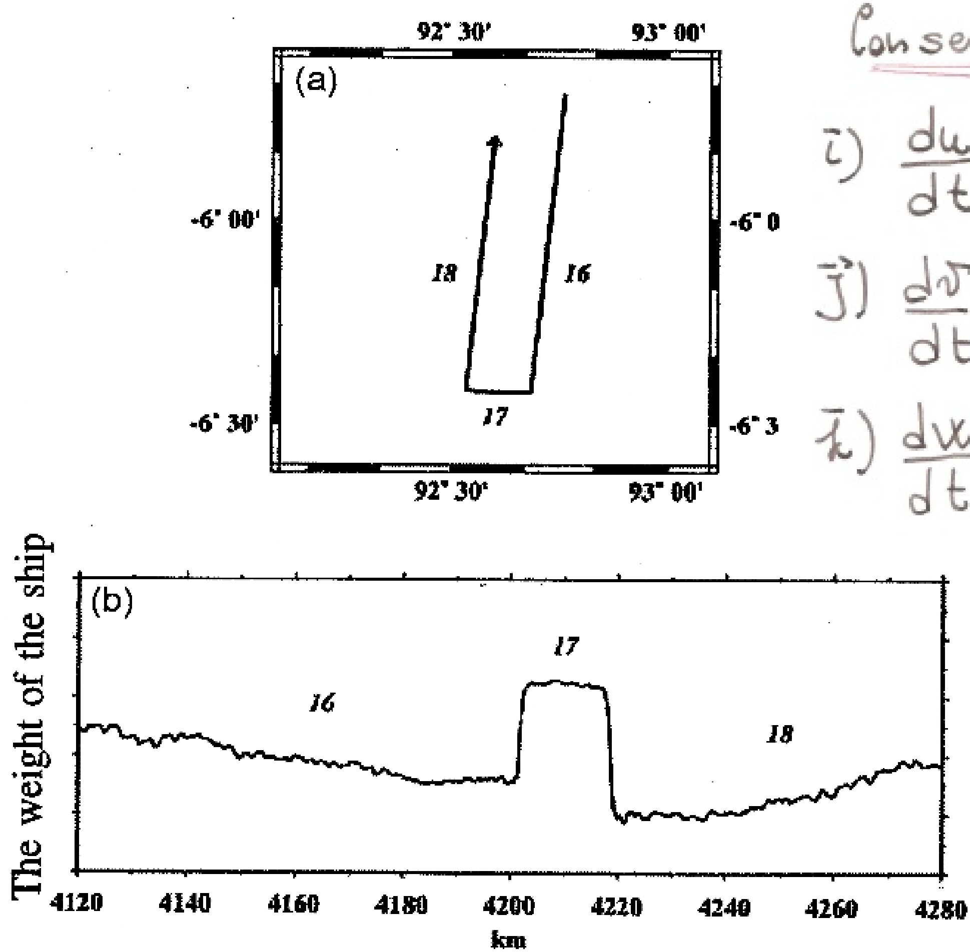


FIGURE 3. Measurements of gravity on a ship sailing in the Pacific ocean. The route is reported (a) on a longitude-latitude box, while the gravity data are plotted as a function of the distance covered by the ship (b) in arbitrary units. The parts of the route parallel to the meridians, 16 and 18 in (a) show a light gravity with respect to the measurements made during the westward part of the route, 17, due to the Coriolis contribution to the acceleration. Figure taken from Persson (2001)

Bibliografia

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[2] Dynamics of the atmosphere, 1995, Dutton J. A., New York Dover publication inc., 617 pp, - disponibile in Biblioteca Tecnico Scientifica (BS/04./M/0022). [pagine 220-243, 289-293, 304-306]

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