Seismometry

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1 Scope of this article. Historical notes.

1.1 Scope

Seismometry is the technical discipline concerned with the detection and measurement of seismic ground motion. It comprises the design of seismographs, their calibration, their installation, and the quantitative interpretation of seismograms in terms of ground motion. A seismograph basically consists of one or more seismometers, a clock or time-signal receiver, and a recorder. The present article concentrates on the theory and usage of seismometers. It does not describe their technical design in detail, but mentions some general design principles. Emphasis is put on precise methods for calibrating and testing. Although most commercial seismometers are well calibrated and normally need not be recalibrated by the user after installation, such methods are essential to ensure that an instrument is in good working condition.

1.2 Inertial and strain seismometers

There are two basic types of seismic sensors: *inertial* seismometers which measure ground motion relative to an inertial reference (a suspended mass), and *strainmeters* or *extensometers* which measure the motion of one point of the ground relative to another. The wavelength of seismic waves is so large that the differential motion of the ground within a vault is normally much smaller than the motion relative to an inertial reference; strainmeters are therefore generally less sensitive to earthquake signals. However, at very low frequencies it becomes increasingly difficult to maintain an inertial reference, and for the observation of low-order free oscillations of the earth and tidal signals, strainmeters may outperform inertial seismometers. In the presence of gravity, inertial seismometers with a horizontal sensitive axis also respond to tilt, and the better ones are more sensitive to short-term tilt than the majority of dedicated tiltmeters (although inferior in their long-term stability). The principles of operation of horizontal seismometers and tiltmeters are identical. Instruments measuring the angular acceleration, although theoretically required for a complete description of ground motion, have not attained any significance in seismology because the rotational component of seismic signals is in general too small to be directly observed. (Even if the signals were strong enough, existing mechanical sensors would not be able to separate them from the associated large displacements.) We will treat only inertial seismometers in this article. Their theory will be presented as far as it is required for an understanding of specifications, calibration procedures, and operational requirements.

In contrast to most other sensors, inertial seismometers have an inherently frequency-dependent response that must be taken into account when the ground motion is restored from the recorded signal. This is because a suspended mass does not represent a perfect inertial reference. When the ground motion is slow, the mass will begin to follow it, and the output signal for a given ground displacement will therefore diminish. The mechanical system forms a high-pass filter for the ground displacement. Recorders, on the other hand, normally have a constant gain up to some upper cutoff frequency, and contribute only a scale factor to the overall response. We will therefore not discuss their frequency response in detail.

1.3 Historical seismographs

The term *seismograph* is today reserved for instruments recording the waveform of the ground motion versus time. In that sense, the first seismograph was built in Italy by Cecchi in 1875; it was however so unsensitive that its first known seismogram dates from 1887. In meantime, Ewing and colleagues in Japan had built several seismographs (Ewing 1884) and recorded the first earthquake in 1880. One of the instruments was in the same year tested on a shake table! Von Rebeur-Paschwitz (1889) recognized seismic waves from an earthquake in Japan in the records of his tiltmeters at Potsdam and Wilhelmshaven, giving seismology a global dimension. The early history of seismometry to 1900 is described by Dewey and Byerly (1969) in an excellent article with many figures and references.

Going back in time, the "electromagnetic seismograph" built by Palmieri in 1856 was little short of being the first seismograph in a modern sense. It had motion-sensitive electric contacts whose closures were recorded on a strip of paper like Morse code. Earlier constructions, which were only designed to indicate the occurrence and direction of a seismic shock, would today be termed *seismoscopes*. The Chinese Chang Heng is reported to have built one in 132; models of his jar-shaped instrument are exhibited in many seismological institutes but its inner mechanism is unknown.

In the beginning of the 20th century, the technical development concentrated on mechanical seismographs with smoke-paper recording. Viscous damping was introduced by Wiechert around 1900. To overcome the remaining solid friction, the mass had to be increased with the square of the magnification. The largest seismographs had masses from 10 to 20 tons, magnifications around 1000, and stable free periods up to 12 s. Mainka and Wiechert seismographs served in many obervatories until after the second world war, and a few of them are still (or again) operational. De Quervain and Piccard in Zürich (1924, 1927) built a mechanical three-component seismograph with a single mass of 21 tons whose position was stabilized with a water ballast – probably the first feedback-stabilized seismograph. Many of the seismographs of the early 20th century are described in Galitzin's lectures on seismometry (1914) and in a comprehensive handbook article by Berlage (1932).

Photographic recording was occasionally used from the beginning but the higher cost and lower quality of the record put the method at a disadvantage, at least until electric light was available. Later it became a practical alternative, for example with the Wood-Anderson torsion seismograph on which the Richter magnitude scale is based (Anderson & Wood 1925). The electromagnetic seismograph with galvanometer-photopaper recording, invented by Galitzin already in 1904, remained for more than half a century the most sensitive long-period seismograph but had to wait for gradual improvements by LaCoste, Benioff, Press, Ewing, and Lehner (LaCoste 1934; Benioff & Press 1958; Press *et al.* 1958; Lehner 1959) before it was stable enough for wide deployment in the WWSSN (Worldwide Standardized Seismograph Network; Oliver and Murphy 1971).

The next generation of electromagnetic seismographs in the HGLP (High-Gain Long-Period) project (Savino *et al.* 1972) was partially electronic, using galvanometer-phototube amplifiers. The SRO system (Seismic Research Observatory; Peterson et al. 1976) had a fully electronic, broadband, force-balance sensor but did not record the broadband signal. The sensor of the original IDA network (International Deployment of Accelerometers; Agnew et al. 1976, 1986) was a LaCoste-Romberg gravimeter with a slow electrostatic force-balance feedback; although this instrument was not useful as a general-purpose seismometer, its sensitivity in the free-mode band is unsurpassed. An eyewitness account of the emerging electronic era of seismometry from 1947 on is given by Melton (1981a,b).

In the time of transition from electromagnetic to electronic seismographs between 1960 and 1975, two opposite trends can be observed. As long as visible recording was the standard and magnetic tape recording was not much better, the gain could only be increased when the marine microseisms, at periods around 6 s, were suppressed. This resulted in the development of high-gain, narrow-band seismographs which were excellent for studying ground noise and monitoring nuclear explosions but easily saturated by earthquakes. On the other hand, several broadband seismographs with analog or digital magnetic tape recording were developed. They remained experimental because continuous broadband recording and digital or analog post-processing were too inconvenient for routine work. The first digital broad-band seismograph was operated at CALTECH as early as 1962 (Miller 1963) with the intention "to preserve the greatest spectrum, dynamic range, and sensitivity". The installation was discontinued because the digital technology was too inefficient at the time. Block and Moore (1970) built a small broadband quartz accelerometer which was the most sensitive broadband sensor of its time but not a very practical instrument; it required vacuum and a thermostat. An analog very-broad-band seismograph was operated in Czechoslovakia from 1972 on (Plešinger & Horalek 1976); its data archive was later converted to a digital standard format. The first practically successful digital broadband installation is the German GRF array (Harjes & Seidl 1978; Buttkus 1986) which has been operational since 1976. The present generation of digital very-broad-band seismographs covering the full teleseismic bandwidth including the free-mode band was developed from 1984 on (Wielandt & Steim 1986).

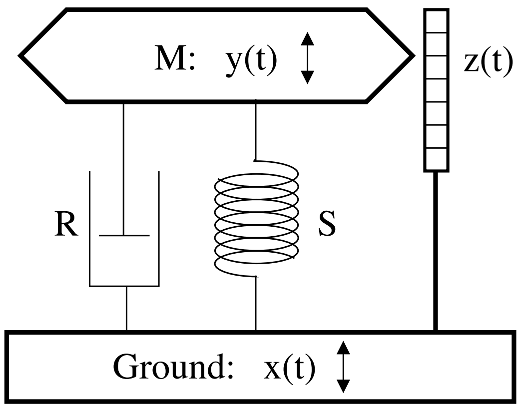

Figure 1: Damped harmonic oscillator

2 Mechanical receivers. Transducers.

2.1 The linear pendulum

The simplest physical model for an inertial seismometer is a mass- and-spring system with viscous damping (fig. 1). The mechanical elements are a mass of M kilograms, a spring with a stiffness S (measured in Newtons per meter), and a damping element with a constant of viscous friction R (in Newtons per meter per second). Let the time-dependent ground motion be $x(t)$, the absolute motion of the mass $y(t)$, and its motion relative to the ground $z(t) = y(t) - x(t)$. An acceleration $\ddot{y}$ of the mass results from any external force f acting on the mass, and from the forces transmitted by the spring and the damper:

$$M\ddot{y} = f - Sz - R\dot{z} \tag{1}$$

Since we are interested in the relationship between z and x, we rearrange this into

$$M\ddot{z} + R\dot{z} + Sz = f - M\ddot{x} \tag{2}$$

Before we solve this equation in the frequency domain, we observe that an acceleration $\ddot{x}$ of the ground has the same effect as an external force of magnitude $f = -M\ddot{x}$ acting on the mass in the absence of ground acceleration. We may thus simulate a ground motion x by applying a force $-M\ddot{x}$ to the mass while the ground is at rest. The force is normally generated by sending a current through an electromagnetic transducer, but it may also be applied mechanically.

2.2 Constraining the motion

Although the mass-and-spring system of fig. 1 is a useful mathematical model for a seismometer, it is incomplete as a practical design. The suspension must suppress five out of the six degrees of freedom of the seismic mass (translational and rotational) but the mass must still move as freely as possible in the remaining direction. Some instruments achieve this with five tensioned threads or five tangential spokes (Geotech S13, Sensonics Mk III). In geophones, the seismic mass is normally suspended between labyrinth springs (which are stamped out of a circular elastic membrane). Most long-period suspensions are of the pendulum type where the seismic mass rotates around a virtual axis defined by flexural hinges (figures 2 to 4). The point bearings shown in our figures 2 and 3 are for illustration only; crossed flexural hinges are normally used.

In principle it is also possible to let the mass move in all directions and observe its motion with three orthogonally arranged transducers, thus creating a three-component sensor with only one suspended mass. Indeed some historical instruments have made use of this concept. It is however difficult to reduce the restoring force and to suppress parasitic rotations of the mass when its translational motion is mechanically unconstrained. Modern three-component seismometers therefore have separate mechanical sensors for the three axes of motion.

2.3 Pendulum type seismometers

These are not only sensitive to translational but also to angular acceleration. The rotational component of seismic shear waves is however too small to have a noticeable effect; its contribution to the output signal is of the order kl where k is the horizontal wavenumber and l the length of the pendulum (Rodgers 1969). In technical applications or on a shake table, effects of rotation may be noticeable.

For small translational ground motions, the equation of motion of a rotational pendulum is formally identical to eq. (2) but z must then be interpreted as the angle of rotation. Since the rotational equivalents to the constants M, R, and S in eq. (2) are of little interest in modern force-balance seismometers, we will not discuss them further and refer the reader instead to the older literature (Berlage Jr. 1932).

2.4 Sensitivity of horizontal seismometers to tilt

We have already seen (eq. 2) that a seismic acceleration of the ground has the same effect on the seismic mass as an external force. The largest such force is gravity. It is normally cancelled by the suspension, but when the seismometer is tilted, the projection of the vector of gravity onto the axis of sensitivity changes, producing a force that is in most cases undistinguishable from a seismic signal. The effect is of second order (proportional to the square of the tilt) and therefore small in well-adjusted vertical seismometers but otherwise of first order. It does not only modify the amplitude with which the horizontal components of long-period SV and Rayleigh waves are recorded (Rodgers 1968) but also introduces noise when the ground is tilted by moving or variable surface loads (traffic, people, wind, barometric pressure). Sensitivity to tilt is the reason why horizontal long-period seismic traces are generally noisier than vertical ones.

A short, impulsive tilt excursion is equivalent to a step-like change of the ground velocity and to a ramp-like displacement. It will therefore cause a long-lasting transient in horizontal broadband seismograms. In the near-field of a seismic source, the tilt has the same waveform as the displacement. The tilt signal can then be predicted from the vertical trace and removed from the horizontal traces (Wielandt & Forbriger 1999).

2.5 Decreasing the restoring force

At low frequencies and in the absence of an external force, equation (2) can be simplified to $Sz = -M\ddot{x}$ and read as follows: a relative displacement of the seismic mass by $-\Delta z$ indicates a ground acceleration of magnitude

$$\ddot{x} = (S/M)\Delta z = {\omega_0}^2 \Delta z = (2\pi/T_0)^2 \Delta z \tag{3}$$

where ω_0 is the angular eigenfrequency of the pendulum, and T_0 its eigenperiod. If Δz is the smallest displacement that can be measured electronically, then the formula determines the smallest low-frequency ground acceleration that can be observed. For a given transducer, it is inversely proportional to the square of the free period of the suspension. A sensitive long-period seismometer therefore requires either a pendulum with a low eigenfrequency or a very sensitive transducer (for quantitative examples see 5.2). Since the eigenfrequency of an ordinary pendulum is essentially determined by its size, and seismometers must be reasonably small, astatic suspensions have been invented that combine small overall size with a long free period.

The simplest astatic suspension is the "garden-gate" pendulum used for horizontal seismometers (fig. 2a). The mass moves in a nearly horizontal plane around a nearly vertical axis. Its free period is the same as that of a mass suspended from the point where the plumb line through the mass intersects the axis of rotation. The period is infinite when the axis is vertical,

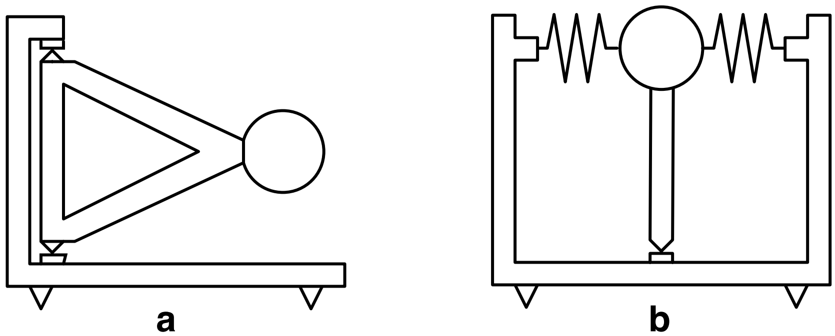

Figure 2: a: Garden-gate suspension; b: Inverted pendulum

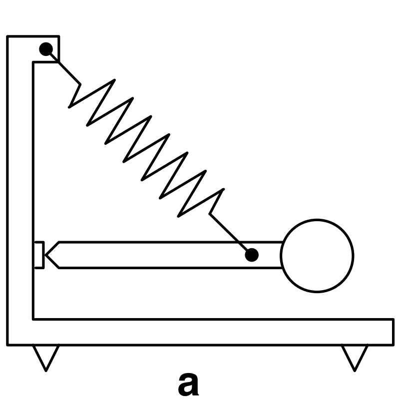

Figure 3: Lacoste suspensions

and is usually adjusted by tilting the whole instrument. This is one of the earliest designs for long-period horizontal seismometers.

Another early design is the inverted pendulum held in stable equilibrium by springs or by a stiff hinge (fig. 2b); a famous example is Wiechert's horizontal pendulum built around 1905 (Berlage Jr. 1932).

An astatic spring geometry for vertical seismometers was invented by LaCoste (1934; fig. 3a). The mass is in neutral equilibrium when three conditions are met: the spring is prestressed to zero length (i.e. the spring force is proportional to the total length of the spring), its end points are seen under a right angle from the hinge, and the mass is balanced in the horizontal position of the boom. A finite free period is obtained by making the angle slightly smaller, or by tilting the frame accordingly. By simply rotating the pendulum, astatic suspensions with a vertical or oblique axis of sensitivity can as well be constructed (fig. 3b).

The astatic leaf-spring suspension (fig. 4a) used in the STS1 seismometer

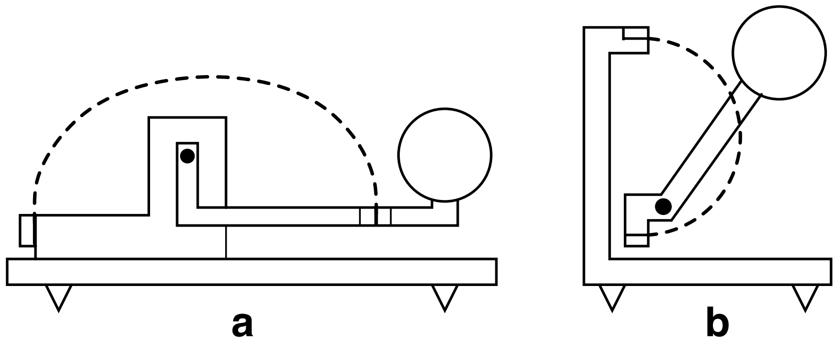

Figure 4: Leaf-spring astatic suspensions

(Wielandt 1975; Wielandt & Streckeisen 1982) is in a limited range around its equilibrium position comparable to a LaCoste suspension but is much simpler to manufacture. A similar spring geometry is also used in the triaxial seismometer STS2 (fig. 4b). The delicate equilibrium of forces in astatic suspensions makes them susceptible to external disturbances such as changes in temperature; they are difficult to operate without a stabilizing feedback system.

Apart from genuinely astatic designs, almost any seismic suspension can be made astatic with an auxiliary spring acting normal to the line of motion of the mass and pushing the mass away from its equilibrium; the Sensonics Mk III seismometer is an example. The long-period performance of such suspensions is however quite limited. Neither the restoring force of the original suspension nor the destabilizing force of the auxiliary spring can be made perfectly linear (i.e. proportional to the displacement). While the linear components of the force may cancel, the nonlinear terms remain and cause the oscillation to become anharmonic and even unstable at large amplitudes. Viscous and hysteretic behaviour of the springs may also cause problems. The additional spring (which has to be soft) may introduce parasitic resonances. Modern seismometers do not use this concept and rely for their sensitivity either on a genuinely astatic spring geometry or on the sensitivity of electronic transducers.

2.6 Effects of temperature and pressure

The equilibrium between gravity and the spring force in a vertical seismometer is disturbed when the temperature changes. Although thermally compensated alloys such as Elinvar are available for springs, a self–compensated spring does not make a compensated seismometer. The geometry of the whole suspension changes with temperature; the seismometer must therefore be compensated as a whole. However, the different time constants involved prevent an efficient compensation at seismic frequencies. Short-term changes of temperature must therefore be suppressed by thermal insulation. Special caution is required with active seismometers: they heat themselves up when insulated and are then very sensitive to air drafts, so the insulation must suppress any possible convection. Long-term (seasonal) changes of temperature do not interfere with the seismic signal but may drive the seismometer out of its operating range. Equation 3 can be used to calculate the thermal drift of a passive vertical seismometer when the temperature coefficient of the spring force is formally assigned to gravity.

Fluctuations of the barometric pressure (resulting from turbulent convection in the atmosphere) have multiple effects on seismic sensors. When the sensor is not sealed, its mass experiences a variable buoyancy. This is a large effect that is intolerable in a vertical broadband seismometer. Changes of pressure also disturb the temperature in the sensor even if it is thermally isolated. On the other hand, a sealed enclosure may be deformed by the barometric pressure and transmit stresses or tilt. The enclosure must therefore either be very rigid or mechanically decoupled from the sensor (see 6.2). Even when the sensor is perfectly shielded, there remain some effects of the barometric pressure that are independent of the sensor: ground tilt due to atmospheric loading, often enhanced by cavity effects (Beauduin *et al.* 1996); the gravitational attraction of the atmosphere (Zürn & Widmer 1995); and the continuous excitation of free modes by global atmospheric turbulence (Tanimoto 1999). Only the first two effects are of local origin and can partially be removed by a correction for the local barometric pressure.

2.7 The homogeneous triaxial arrangement

In order to observe ground motion in all directions, a triple set of seismometers oriented towards East, North, and upward (Z) has been the standard for a century. However, horizontal and vertical seismometers differ in their construction, and it costs some effort to make their responses equal. An alternative way of manufacturing a three–component set is to use three sensors of identical construction whose sensitive axes are inclined against the vertical like the edges of a cube standing on its corner (fig. 5), by an angle of arctan $\sqrt{2}$, or 54.7 degrees.

Presently only one commercial seismometer, the STS2, makes use of this concept, although it was not the first one to do so (Melton & Kirkpatrick 1970). Since most seismologists want finally to see the conventional E, N and Z components , the oblique components U, V, W of the STS2 are electrically recombined according to

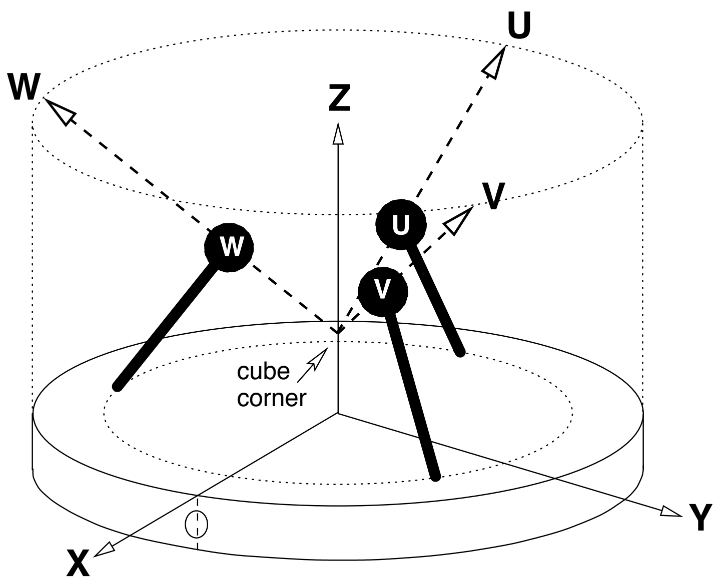

Figure 5: Geometry of the homogeneous triaxial seismometer STS2

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} -2 & 1 & 1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix} \begin{pmatrix} U \\ V \\ W \end{pmatrix} \tag{4}$$

The X and Y axes are normally oriented towards E and N. Noise originating in one of the sensors of a triaxial seismometer will appear on all three outputs (except for Y being independent of U). Its origin can be traced by transforming the X, Y and Z signals back to U, V and W with the inverse (transposed) matrix. Disturbances affecting only the horizontal outputs are unlikely to originate in the seismometer, and are in general due to tilt.

2.8 Electromagnetic velocity sensing and damping

The simplest transducer both for sensing motions and for exerting forces is an electromagnetic (electrodynamic) device where a coil moves in the field of a permanent magnet, like in a loudspeaker. Motion induces a voltage in the coil; a current flowing in the coil produces a force. From the conservation of energy it follows that the responsivity of the coil-magnet system as a force transducer, in Newtons per Ampere, and its responsivity as a velocity transducer, in Volts per meter per second, are identical. The units are in fact the same (remember that $1Nm = 1Joule = 1VAs$). When such a transducer is loaded with a resistor and thus a current is permitted to flow, it generates a force opposing the motion according to Lenz's law. This effect is used to damp the mechanical free oscillation of passive seismic sensors (geophones).

In comparison to technical vibrations, the seismic ground motion is slow and small most of the time. The signals delivered by electromagnetic velocity transducers are therefore normally quite small. For maximum sensitivity, the input stage of the electronic amplifier must be matched to the impedance of the coil (or vice versa). The matter is treated in detail by Riedesel et al. (1990), Rodgers (1992a) and Rodgers (1993).

2.9 Electronic displacement sensing

At very low frequencies, the output signal of electromagnetic transducers becomes too small to be useful for seismic sensing. One then uses active electronic transducers where a carrier signal, usually in the audio frequency range, is modulated by the motion of the seismic mass. The basic modulating device is an inductive or capacitive half-bridge. Inductive half-bridges are detuned by a movable magnetic core. They require no electric connections to the moving part and are environmentally robust; however their sensitivity appears to be limited by the granular nature of magnetism, and they may push back on the seismometer mass. Capacitive half-bridges are realized as three-plate capacitors where either the central plate or the outer plates move with the seismic mass. Their sensitivity is limited by the ratio between the electrical field strength and the electronic noise of the demodulator; it is typically a hundred times better than that of the inductive type. The comprehensive paper by Jones and Richards (1973) on the design of capacitive transducers still represents the state of the art in all essential aspects.

3 Mathematical representation of the response

3.1 The transfer function

We can give here only a very brief outline of the theory of linear systems. The reader who is not familiar with it should consult a textbook such as Oppenheim and Willsky (1983); for digital signal processing, Oppenheim and Schafer (1975) is a standard text. An instructive book on the Fourier transformation and its applications is Bracewell (1978). Seismological applications of the theory are treated in Plešinger et al. (1996) and in Scherbaum (1996).

Seismometers, amplifiers and filters are designed as linear and time-invariant systems, i.e. the mathematical relationship between the time-dependent input and output signals is supposed to be a linear differential equation with constant coefficients. This has the mathematical consequence that sinusoidal input signals produce sinusoidal output signals. The response of such a system can be described by a complex gain factor T at each frequency: when the input signal is $x(t) = Xe^{j\omega t}$ and the output signal is $z(t) = Ze^{j\omega t}$, then the gain is $T = Z/X$. It may have a physical dimension (when the system is a transducer) and will in general depend on frequency. The function $T(\omega)$ is called the *complex frequency response*. Its absolute value $|T(\omega)|$ is the *amplitude response* and its phase is the *phase response* of the system. Signals of arbitrary time dependence can be represented as an integral or sum over sinusoidal signals of different frequency; knowledge of the response function $T(\omega)$ at all relevant frequencies is therefore sufficient to calculate the response of the system to any input signal. The decomposition of arbitrary signals into sinusoidal (time-harmonic) components is a *Fourier transformation*, and their synthesis from such components is an *inverse Fourier transformation*. For transient signals such as analog seismograms, the Fourier transformation is formulated as a pair of integral transformations:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} a(\omega)\ e^{j\omega t}\ d\omega, \qquad a(\omega) = \int_{-\infty}^{\infty} f(t)\ e^{-j\omega t}\ dt \tag{5}$$

The most general class of signals for which the response of a linear and time-invariant system can be described by a gain factor are sinusoidal oscillations with exponentially growing or decaying amplitudes: $a(t) = Ae^{st}$ with complex $s = \sigma + j\omega$. (The complex exponential function is defined as $e^{(\sigma+j\omega)t} = e^{\sigma t}(\cos\omega t + j\sin\omega t)$). Again the system can be characterized by a complex gain factor H(s): the input signal $x(t) = Xe^{st}$ produces the output signal $z(t) = Ze^{st} = H(s)Xe^{st}$, thus $Z = H(s)X$. Other signals are decomposed and synthesized with the *Laplace transformation*:

$$f(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} b(s)\ e^{st}\ ds \quad \text{for} \quad t > 0, \qquad b(s) = \int_{0}^{\infty} f(t)\ e^{-st}\ dt \tag{6}$$

The Fourier transformation may be considered as a special case of the Laplace transformation with a purely imaginary $s = j\omega$, although the mathematical concepts behind both transformations are somewhat different. $H(s)$ is the *transfer function* of the system. It follows that $T(\omega) = H(j\omega)$; the functions $T(\omega)$ and $H(s)$ are related by a simple substitution of the frequency variable but are not identical. When their poles and zeroes or other mathematical properties are discussed, it is important to state which one of the functions is meant. The terminology is however not uniform in the literature; sometimes $T(\omega)$ is also referred to as a transfer function.

Transfer functions of seismometers and also of a wide class of other mechanical and analog electronic systems are rational functions of frequency with real coefficients: $H(s) = P(s)/Q(s)$ where $P(s)$ and $Q(s)$ are polynomials in s. This is a consequence of the fact that the differential equation of the system is transformed into an algebraic equation for the complex signal amplitudes when only signals with a time dependence of the form e^{st} or $e^{j\omega t}$ are admitted; eq. (8) in the next paragraph is an example. A polynomial $P(s)$ is up to a factor determined by its zeros, i.e. by those complex values of s for which $P(s) = 0$. The zeros of $P(s)$ are at the same time those of $H(s)$; the zeros of $Q(s)$ are poles of $H(s)$. Rational transfer functions can therefore be specified with a limited number of numerical coefficients, by listing either the the complex poles and zeros or the real polynomial coefficients. Different factorizations of the polynomials are possible in the latter case. The essential options are:

1. The poles and zeroes in the complex s plane are listed, together with a constant gain factor.
2. The polynomial coefficients of the nominator and denominator are listed (they are real when $s = j\omega$ is chosen as the frequency variable).
3. The polynomials are decomposed into normalized first- and second-order factors, each of which is defined by its corner frequency and, in case of second order, numerical damping. The individual factors can normally be attributed to physical subunits of the system. An overall gain factor is also required.

3.2 The frequency response of geophones

Using time-harmonic signals, the solution of the differential equation (2) becomes very simple. Let the input signal be $x(t) = Xe^{j\omega t}$, the output signal $z(t) = Ze^{j\omega t}$ and the external force $f(t) = Fe^{j\omega t}$. Eq. (2) then reduces to

$$(-\omega^2 M + j\omega R + S)Z = F + \omega^2 MX \tag{7}$$

$$Z = \frac{F/M + \omega^2 X}{-\omega^2 + j\omega R/M + S/M} \tag{8}$$

The mechanical pendulum is thus a second-order high-pass filter for displacements, and a second-order low-pass filter for accelerations and external forces. Its angular corner frequency is $\omega_0 = \sqrt{S/M}$. At this frequency, the ground motion X is amplified by a factor $\omega_0 M/R$ and phase shifted by $\pi/2$. The imaginary term in the denominator is usually written as $2j\omega\omega_0 h$ where $h = R/(2\omega_0 M)$ is the numerical damping, i.e. the ratio of the actual to the critical damping.

In order to convert the motion of the mass into an electric signal, the mechanical pendulum is in the simplest case coupled with an electromagnetic velocity transducer (see 2.8) whose output voltage we denote with E. We then have an electromagnetic seismometer, also called a geophone when designed for seismic exploration. When the responsivity of the transducer is σ (volts per meter per second; $E = \sigma j\omega Z$) we get

$$E = j\omega\sigma(F/M + \omega^2 X)/(-\omega^2 + 2j\omega\omega_0 h + \omega_0^2) \tag{9}$$

from which, in the absence of an external force, we obtain the complex response functions

$$T_d = E/X = j\omega^3\sigma/(-\omega^2 + 2j\omega\omega_0 h + \omega_0^2) \tag{10}$$

for the displacement,

$$T_v = E/(j\omega X) = \omega^2\sigma/(-\omega^2 + 2j\omega\omega_0 h + \omega_0^2) \tag{11}$$

for the velocity, and

$$T_a = E/(-\omega^2 X) = -j\omega\sigma/(-\omega^2 + 2j\omega\omega_0 h + \omega_0^2) \tag{12}$$

for the acceleration. The geophone is a second-order high-pass filter for the velocity, and a band-pass filter for the acceleration.

We have so far treated the damping as if it were a viscous effect in the mechanical receiver. Actually, only a small part h_m of the damping is due to mechanical causes. The main contribution normally comes from the electromagnetic transducer which is suitably shunted for this purpose (see 2.8). Its contribution is

$$h_{el} = \sigma^2/2M\omega_0 R_d \tag{13}$$

where R_d is the total damping resistance (the sum of the resistances of the coil and of the external shunt). The total damping $h_m + h_{el}$ is preferably chosen as $1/\sqrt{2}$, a value that defines a second-order Butterworth filter characteristic, and gives a maximally flat response in the passband.

3.3 The impulse response

Alternatively, the transfer properties of a seismometer can be described in the time domain by its impulse response, which is the response of the system to an impulsive input signal. (An impulse in this sense is any signal whose time integral is undistinguishable from a unit step). The impulse response and the transfer function are Laplace transforms of each other, so they offer mathematically equivalent descriptions of the system. In the same way, the complex frequency response is the Fourier transform of the impulse response. The impulse response can directly be calculated from the poles and zeros of the transfer function. For a practical specification, the impulse response is less suitable because it is a transcendental function of infinite duration that is inconvenient to formulate.

The response of a seismograph to an arbitrary input signal can in principle be computed as the convolution of that signal with the impulse response. However, due to the infinite length of the latter, this may not be an efficient procedure. Also, a sampled version of the impulse response may not represent the analytical form correctly when the system is not strictly bandlimited. So computing the response of a system by convolution requires some precautions, and one would in most cases prefer to either do the computation in the frequency domain with the Fourier transformation, or to filter the input signal with a recursive filter that represents the seismograph, as explained in the next paragraph.

3.4 Representing a seismograph by a recursive filter

For a general theory of recursive (or IIR) filters, we must refer the reader to the pertinent literature (Oppenheim & Schafer 1975; Robinson & Treitel 1980; Plešinger *et al.* 1996). For mathematical reasons, recursive filters can only approximate, but not have, the rational transfer functions of seismographs. In a restricted sense, however, an exact equivalent exists. When we filter all signals with an anti-alias filter that has a rational transfer function and is sharp enough to practically prevent aliassing, then we can precisely model the overall transfer function with an "impulse-invariant" recursive algorithm (Schuessler 1981). Remarkably, the rational anti-alias filter need not exist in hardware but only as part of the computer algoritm. So we can, in effect, precisely model any seismograph in a bandwidth that is by a factor of two or so smaller than theoretically permitted by the sampling theorem. Even this restriction can be overcome by sampling the signals twice as fast, or by resampling them numerically. The method is especially useful for the purpose of calibrating seismographs with arbitrary signals (see section 9).

4 Force-balance accelerometers and seismometers

4.1 The force-balance principle

The precision of a conventional, passive seismometer depends on its two functional subunits: the mechanical suspension and the displacement or velocity transducer. An inertial seismometer basically measures the inertial force acting on the seismic mass in an accelerated local frame of reference (eq.2). The suspension converts the inertial force into a displacements of the mass, and the transducer converts this into an electric signal. Neither one of these conversions is inherently precise. As discussed under 2.5, a sensitive seismometer must have a suspension with a small restoring force so that small accelerations produce noticeable displacements of the seismic mass. Then, of course, larger seismic signals or environmental disturbances produce large displacements that change the geometry of the spring and destroy the linear relationship between displacement and force. When the restoring force is diminished, undesired effects such as hysteresis and viscous behaviour retain their absolute magnitudes and thus become relatively larger. Finally, it is difficult to build linear transducers with a large range. A passive seismic sensor therefore cannot be optimized for sensitivity and precision at the same time.

These problems are well known from the design of precision instruments, especially of laboratory balances. They are solved by compensating the unknown force with a known force, rather than determining it indirectly from the elongation of a spring. The compensating force is generated in an electromagnetic transducer and is controlled by a servo circuit (fig. 6) that senses the position of the seismic mass and adjusts the force so that the mass returns to its center position. Such a system is most effective when it contains an integrator, in which case the offset of the mass is exactly nulled in the time average. Due to unavoidable delays in the feedback loop, servo systems have a limited bandwidth; however at frequencies where they are effective, they force the mass to move with the ground by generating a feedback force strictly proportional to ground acceleration. When the force is proportional to the current in the transducer, then the current, the voltage across the feedback resistor R_1, and the output voltage are all proportional to ground acceleration. We have thus converted the acceleration into an electric signal without relying on the mechanical precision of the spring. The suspension still serves as a detector but not as a converter, and may now be optimized for sensitivity without giving up precision.

The effectiveness of a servo system is measured by its loop gain L which

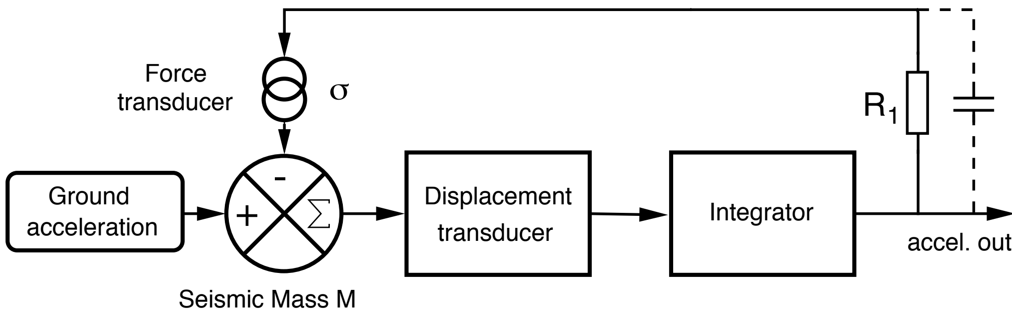

Figure 6: FBA feedback circuit

is the amplitude ratio of the feedback force to the uncompensated residual of the inertial force. If f is the inertial force and r the residual, then $r = f/(1+L)$. A large loop gain implies a small residual and thus an output signal that represents the acceleration with a small error. The integrator provides a large loop gain and thus a high precision at low frequencies. At high frequencies, the loop gain is limited by stability conditions and cannot be increased arbitrarily; this topic is treated in textbooks on network analysis or system theory (Oppenheim & Willsky 1983). Some minor modifications of the basic circuit, such as the addition of a small capacitor C parallel to the feedback resistor R_1, may be required to ensure stability.

The response of a servo system is approximately inverse to the gain of the feedback path. It can easily be modified by giving the feedback path a frequency-dependent gain. For example, if we make the capacitor C large so that it determines the feedback current, then the gain of the feedback path increases linearly with frequency, and we have a system whose responsivity to acceleration is inverse to frequency and thus flat to velocity over a certain passband. We will look more closely at this option in 4.3.

4.2 Force-balance accelerometers

By equating the inertial and the electromagnetic force, it is easily seen that in the circuit of fig. 6 the factor of proportionality between the output voltage and the acceleration is

$$A = MR_1/\sigma \tag{14}$$

where M is the seismic mass, R_1 the resistance of the feedback path (including the coil), and σ the responsivity of the forcer (in N/A). The conversion is determined by only three passive components of which the mass is error-free by definition (it defines the inertial reference), the resistor is a nearly ideal component, and the force transducer can be very precise because the motion is small.

Figure 6 is the circuit of a force-balance accelerometer (FBA), a device that is widely used for earthquake strong-motion recording, measuring tilt, and inertial navigation. Since the dynamic range (see 5.1) of a feedback system is mainly determined by its feedback path, which is in this case composed of passive components whose range is not limited by semiconductor noise or clipping, FBAs can have a very large dynamic range (up to 160 decibels). The operating range can conveniently be adjusted by changing the feedback resistor, which is external in some types. FBAs work down to zero frequency but the servo loop becomes ineffective at some high frequency (typically between 100 and 1000 Hz), above which the arrangement acts like an ordinary inertial displacement sensor. FBAs are therefore low-pass filters for ground acceleration and high-pass filters for ground displacement.

4.3 Velocity broad-band seismometers

For broadband seismic recording with high sensitivity, an output signal proportional to ground acceleration is unfavourable. At high frequencies, sensitive accelerometers are easily saturated by traffic noise or impulsive disturbances. At low frequencies, a system with a response flat to acceleration generates a voltage at the output as soon as the suspension is not completely balanced. Such a system would easily be saturated by the offset voltage resulting from thermal drift or tilt. What we need is a band-pass response in terms of acceleration, or equivalently a high-pass response in terms of ground velocity, like that of a normal electromagnetic seismometer but with a lower corner frequency. Essentially the same considerations are expressed in paragraph 5.5 as the rule that the response of a broadband seismometer should be approximately inverse to the spectral distribution of the noise.

The desired velocity broad-band (VBB) response is obtained from the FBA circuit by adding paths for differential feedback and integral feedback (fig. 7). The capacitor C is chosen so large that the differential feedback dominates throughout the desired passband. While the feedback current is still proportional to ground acceleration as before, the voltage across the capacitor C is a time integral of the current, and thus proportional to ground velocity. This voltage serves as the output signal. The factor of proportionality – the apparent generator constant of the feedback seismometer – is

$$V = M/\sigma C \tag{15}$$

Again the response is essentially determined by three passive components. Although a capacitor with a solid dielectric is not quite as ideal a component as a good resistor, the response is still linear and very stable.

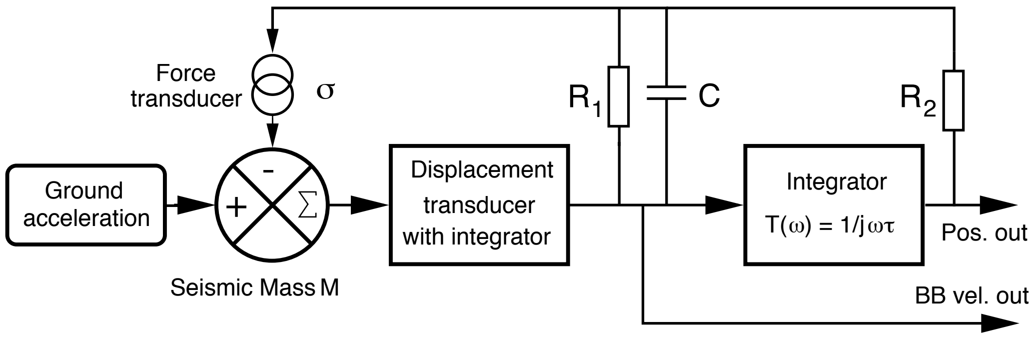

Figure 7: VBB feedback circuit

The output signal of the integrator is normally accessible at the "mass position" output. It does not indicate the actual position of the mass but indicates where the mass would go if the feedback were switched off. "Centering" the mass of a feedback seismometer (by adjusting its weight or the spring force) has the effect of discharging the integrator so that its full operating range is available for the seismic signal. The mass-position output is not normally used for seismic recording but is useful as a state-of-health diagnostic, and is used in some calibration procedures.

The relative strength of the integral feedback increases at lower frequencies while that of the differential feedback decreases. At some frequency, the two contributions are of equal strength but opposite phase ($-\pi/2$ and $\pi/2$, respectively). This is the lower corner frequency of the feedback system below which the response rolls off with the second derivative of ground velocity. The differential and the integral parts of the feedback cancel at the corner frequency but the proportional feedback remains and damps the resonance that would otherwise occur. As a result, the feedback system behaves like a conventional electromagnetic seismometer and can be described by the usual three parameters: free period, damping, and generator constant. In fact most of the presently used electronic broadband seismometers follow the simple theoretical response of electromagnetic seismometers more closely than these ever did.

At some high frequency, the loop gain falls below unity. This is the upper corner frequency of the feedback system which marks the transition between a response flat to velocity and one flat to displacement. A well-defined and nearly ideal behaviour of the seismometer like at the lower corner frequency should not be expected there, both because the feedback becomes ineffective and because most suspensions have parasitic resonances slightly above the electrical corner frequency (otherwise they could have been designed for a larger bandwidth). The detailed response at the high-frequency corner does however rarely matter since the upper corner frequency is usually outside the passband of the recorder. Its effect on the transfer function can in most cases be modelled as a small, constant delay (a few milliseconds) over the whole VBB passband.

4.4 The response of force-balance seismometers

For completeness we give here a slightly simplified formula for the complex frequency response of a broadband force-balance seismometer. The simplification consists in ignoring the mechanical restoring force and damping which can be absorbed into the electric parameters, and the impedance of the feedback coil which can be made small. When X is the Fourier amplitude of the ground displacement and E that of the output voltage, then the response to ground velocity is

$$E/j\omega X = \frac{\omega^2 M/\sigma C}{-j\omega^3 M/\alpha\sigma C - \omega^2 + j\omega/R_1 C + 1/\tau R_2 C} \tag{16}$$

The leading term in the denominator disappears when the responsivity α of the displacement transducer is large. What remains is an ordinary high-pass response with a corner frequency $1/\sqrt{\tau R_2 C}$, damping $\sqrt{\tau R_2/4CR_1^2}$, and generator constant $M/\sigma C$. The response is thus identical to that of a conventional electrodynamic seismometer (eq. 11). The realization of a "very-broad-band" type of seismometer, with a free period of 100 s or more, depends on the ability to build an integrator with a long time-constant τ and low electronic noise (Wielandt & Steim 1986). At high frequencies, the bandwidth is limited by the finite responsivity α of the displacement transducer. The loop gain falls below unity and the velocity response rolls off with $1/j\omega$ at frequencies above $\alpha\sigma C/2\pi M$; the response is then flat to displacement. The resistance and inductance of the feedback coil, phase delays in the electronic circuit, and parasitic resonances in the suspension make the feedback loop unstable for large α and thus set a limit to the upper corner frequency of the response.

The reader is referred to publications by Usher et al. (1978), Wielandt (1983), and Wielandt et al. (1982,1986) for a deeper discussion.

5 Design criteria for broadband seismographs

5.1 The concept of very-broad-band seismometry

Earthquakes and other seismic sources radiate signals in a large range of frequencies and amplitudes. In a logarithmic scale, the seismic frequency band is much broader than that of audible tones. Only a limited range of these signals can be visibly displayed at a time. So any seismograph recording on paper or film has to act as a filter and suppress most of the available information. Quite a number of analog seismographs with different characteristics had to be operated in parallel in order to preserve a reasonable choice of signals. Digital technology now permits the recording of all useful seismic signals on the same medium in a single data stream. Such a system is called a very-broad-band (VBB) seismograph. It must meet the following requirements (IRIS 1985; Wielandt & Steim 1986):

1. The system must have a sensitivity sufficient to resolve signals at the level of minimum ground noise at all frequencies of interest.

2. Its operating range must be large enough to record the largest earthquakes at regional to teleseismic distances.

3. The largest ground noise, natural or artificial, that is likely to occur in any part of the spectrum must not interfere with the resolution of small signals at other frequencies.

A few explanations: a signal is said to be *resolved* when it is present in the seismic record and not masked by instrumental noise. The minimum ground noise is different at each station but for a uniform system intended for global deployment, the instrumental sensitivity must be designed for the global minimum. The present standard is the New Low Noise Model (NLNM) compiled by Peterson (1993) . A simple computer program converting noise data into different units and comparing them to the NLNM is available from the author's FTP site (see section 11). The *operating range* is the maximum signal amplitude that can pass through the system without serious distortion.

The above requirements can be visualized in a doubly-logarithmic diagram like fig. 8 where the levels of ground noise and earthquake signals are expressed in common units, here as average peak values in 1/3 octave or rms values in 1/6 decade (these two measures happen to coincide within a few percent). In other contexts, noise levels are normally expressed as power spectral densities which cannot directly be compared with the amplitudes of transient signals. The amplitude ratio between the instrumental noise and the clipping level of a sensor or a recorder is called its *dynamic range*. It is usually expressed in decibels and depends on frequency and on the bandwidth in which it is measured; without this information its specification is meaningless. The reference bandwidth should be chosen with a view to the narrowest bandwidth useful for a waveform plot at each frequency. *Absolute dynamic range* is short for a specification of the dynamic range in absolute signal levels.

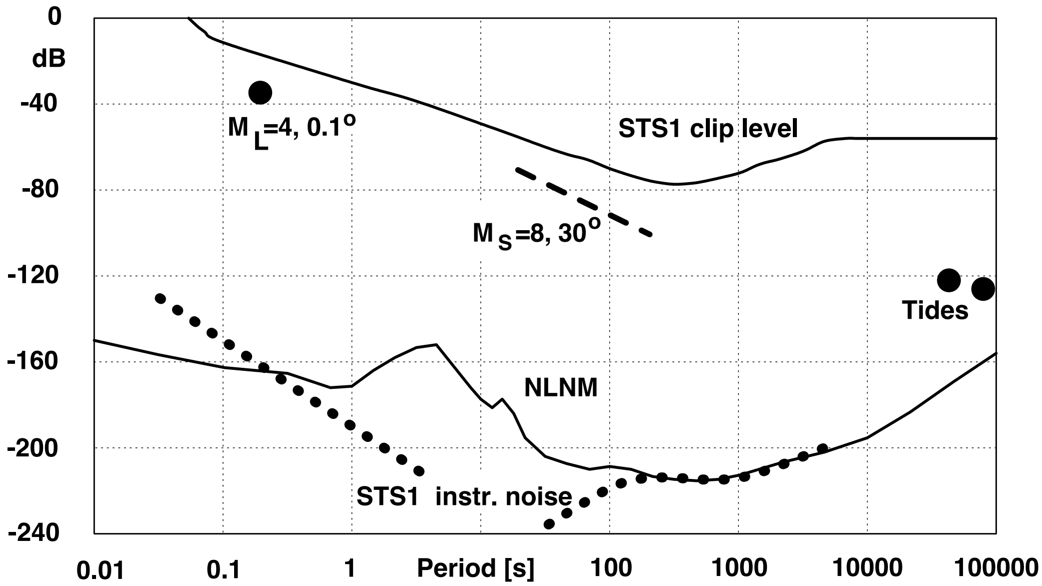

Figure 8: A representation of the USGS New Low Noise Model in comparison with the self-noise and operating range of the STS1 very-broad-band seismometer. Signal levels are expressed as decibels relative to $1m/s^2$ and may be understood as rms values in a bandwidth of 1/6 decade or average peak values in a bandwidth of 1/3 octave. At periods longer than 100 s, the NLNM is mainly based on STS1 data. Its coincidence with the instrumental noise beyond 200 s suggests that the STS1 seismometer may not resolve ground noise at the quietest sites (Peterson 1993).

5.2 The sensitivity of force-balance seismometers

The spectral distribution of minimum ground noise is such that noise is difficult to resolve both at the short-period and long-period ends of the seismic band. At short periods, the ground displacements in the NLNM (fig. 8) are very small: 6 pm average peak in a bandwidth of one-third octave at 5 Hz, 0.4 pm at 30 Hz. Capacitive displacement transducers (Blumlein bridges) used in VBB seismometers typically resolve between 0.1 and 1 pm at these frequencies. The resolution cannot easily be improved without increasing the undesired effects of power consumption, heat production, electrostatic forces, and viscous (air) damping in the displacement transducer.

At long periods, the seismic noise is relatively large in terms of displacement, but the acceleration is small. Around a period of 300 s the acceleration associated with the minimum noise is about $15pm/s^2$ $(1.5 * 10^{-12}g)$ in one-third octave. The relationship $\Delta g = \omega_0^2 \Delta z$ (eq. 3 with Δg written for the ground acceleration $\ddot{x}$) tells us then that we need a displacement resolution of

0.1 pm when the pendulum has a free period of 0.5 s, 10 pm at a free period of 5s, and 350 pm at 30 s (the numbers are estimates for the CMG3, STS2 and STS1 seismometers, respectively). The difficulty of building a sensitive VBB seismometer with a mechanical free period substantially shorter than 1 s is obvious.

A final limit to the sensitivity at long periods is set by the Brownian (thermal) motion of the seismic mass. Air molecules hitting the pendulum not only damp its motion but also exert a force whose spectral noise power density N is given by the same well-known formula that quantifies the voltage noise of a resistor:

$$N = 4kTR \tag{17}$$

where R is the viscous friction experienced by the seismic mass, k the Boltzmann constant, and T the absolute temperature. In a typical force-balance seismometer, damping is mainly caused by the air gap of the capacitive displacement transducer. Even with a favourable capacitor geometry and with ventilation holes in the plates, thermal noise is likely to mask the minimum long-period ground noise when the seismic mass is reduced to some 10 grams or less. The problem can in principle be solved by evacuating the sensor but it would be very difficult to maintain a sufficiently high vacuum in a sensor of conventional design.

While failure to meet the basic criteria must spoil the ability to resolve minimum noise, their observation is no guarantee for a sensor to achieve the desired resolution. Additional noise may result from various mechanical, electric, magnetic, thermal and even chemical effects, and it is difficult to predict which ones will infest a specific design.

5.3 The operating range of force-balance seismometers

The electric operating range of a well-designed seismometer is, at least in its passband, limited by clipping at the output, at a signal level slightly below the internal supply voltage. The seismic operating range is the electric range divided by the amplitude response, and thus in general frequency-dependent. Clipped waveforms can normally be recognized when the signal has been recorded with adequate bandwidth. However, when clipping occurs in earlier stages of the circuit or in the feedback loop, or when the signal is low-pass filtered in the recorder, clipping may go unnoticed and the record may look quite normal even if it is severely distorted; an example is presented in Wielandt (1983) .

The operating range at very low frequencies requires a separate consideration. Due to the presence of an integrator in the feedback loop, the condition that clipping should occur only at the output cannot be maintained at very low frequencies. The operating range is then limited by saturation of the integrator. Since the integrator also generates noise, it determines the dynamic range of the whole system at long periods. This has direct consequences for the ranges of drift and tilt in which force-balance seismometers can be operated. We give an order-of-magnitude estimate: the integrator may have a noise level of $0.1\mu V$ rms (in an appropriate bandwidth) and a saturation level of $\pm 10V$; the sensor may have been designed to resolve 10^{-12} g rms. Then the integrator will be saturated by static accelerations of $\pm 10^{-4}g$. A vertical sensor whose suspension has a temperature coefficient of 10^{-5} per Kelvin could be operated with this feedback system in a temperature range of ± 10 Kelvin; a horizontal sensor would have to be levelled to within 0.1 mm per meter.

The designer of a force-balance seismometer has a considerable freedom in the choice of the responsivity and thus of the seismic operating range. The dynamic range of sensors and recorders is however limited, so a decision must be made whether the operating range or the self-noise of a system should be specified in the first place. Recording systems for strong motion usually have a certain level of ground acceleration specified as the operating range. General-purpose seismographs are normally designed to resolve ground noise; their operating range is then made as large, and their gain as small, as this requirement permits.

5.4 Digitizer noise

An ideal digitizer rounds the samples of the input signal to integer multiples of a quantum q (one count) which can be expressed as an equivalent step of the input voltage. The sequence of rounding errors constitutes the quantization noise. Under certain conditions it is white and has a constant total power of $q^2/12$ in the band from zero to the Nyquist frequency (half the sampling frequency). Its spectral density therefore depends on the sampling rate, and can be reduced by oversampling and subsequent bandwidth reduction. However, digitizers also exhibit ordinary electronic noise both of the white and the 1/f - variety, which cannot be reduced by oversampling. Filters for bandwidth reduction must have some gain or use floating-point arithmetic in order to avoid additional quantization errors. Nonlinear distortions in modern 24-bit digitizers are normally so small that they can safely be ignored.

5.5 Matching sensors and recorders

The clipping level of a seismic recording system - sensor plus recorder - is the minimum of the clipping levels of the components. On the other hand, the supposedly uncorrelated noise levels of the components add up like orthogonal vectors. The overall dynamic range of the system is therefore always smaller than that of each subsystem. (To be comparable, all signal levels must be referred to the same point in the circuit, or expressed as ground motion.) To make best use of the absolute dynamic ranges of the components, the smallest one must be contained in all others. When the dynamic ranges of the sensor and the recorder are of similar magnitude, as is the case with modern broadband sensors and 24-bit digitizers, this condition determines the optimum gain of the sensor within narrow limits.

Similar considerations apply to the seismic input signals. The earth may in this context be considered as another signal-transmitting system with its own noise and operating range. For a system to resolve ground noise, the sensor must not only have a self-noise below that noise but must also put out the combined ground and sensor noise as an electric signal above the digitizer noise. The necessary gain margin has occasionally been a matter of controversial discussion. A factor of 2 should normally be sufficient; the digitizer noise then increases the noise amplitude by only 12%. Using a larger gain margin would unnecessarily reduce the seismic operating range. A small gain margin does not preclude the extraction of coherent signals from incoherent noise by bandwidth reduction, stacking, or beamforming. When the system resolves incoherent ground noise, it also resolves all coherent signals that can theoretically be extracted from it.

If the digitizer noise were white, the optimum response of the sensor would be inverse to the spectral distribution of the minimum noise. The latter is however too complicated to be modelled in a practical sensor. Experience has shown that a response flat to velocity, from short periods to a long-period corner of 100 s or more, is sufficiently close to the optimum. A more detailed discussion of this topic is found in Wielandt and Steim (1986) .

5.6 Scaling down seismometers

Seismometers should be as small as possible for easier transportation, installation, and shielding, and because internal air convection (which causes noise) is more easily suppressed in a smaller volume. It is therefore interesting to see what happens if a given design is scaled down by a factor of γ. A simple calculation shows that strict scaling is not possible for vertical sensors which are in equilibrium with terrestrial gravity. The mass, the spring force, and the spring stiffness scale with different powers of γ (3, 2, and 1, respectively). The spring would be too strong in a downscaled version and must be made slightly thinner than to scale; nevertheless it remains too stiff. The mechanical sensitivity to long-period accelerations is thereby reduced by a factor of γ, and the period becomes shorter by a factor $\sqrt{\gamma}$ like for a simple pendulum. On the other hand, the resolution of the displacement transducer will suffer from the smaller size of the capacitor plates. Air damping in the displacement transducer is also undesirably increased (see 5.2 and Jones and Richards 1973). It is, by the way, unlikely that one would want to reduce the gap between the plates because if this were practical, one would have done it in the first place.) The signal-to-noise ratio at long periods will therefore go down at least with the square of the scaling factor. Undesired forces causing nonlinearity or noise will in general scale with a smaller power of γ than the mass, and thus become relatively larger. Finally, Brownian noise will come up when the mass is too small, as mentioned above. So it appears that miniature seismometers require different designs and are unlikely to reach the sensitivity of presently available VBB seismometers in the near future. There is no doubt, however, that existing designs could be scaled down by a factor of two or so, which would make the instruments very handy.

6 Site selection. Installation. Shielding.

6.1 Site selection

Site selection for a permanent seismic station is always a compromise between two conflicting requirements: infrastructure and low seismic noise. The noise level depends on the geological situation and on the proximity of sources, some of which are usually associated with the infrastrucure. A seismograph installed on solid basement rock can be expected to be fairly insensitive to local disturbances while one sitting on a thick layer of soft sediments will be noisy even in the absence of identifiable sources. As a rule, the distance from potential sources such as houses and roads should be large compared to the thickness of the sediment layer. Broadband seismographs can be successfully operated in major cities when the geology is favourable; in unfavourable situations such as in sedimentary basins, only deep mines and boreholes may offer acceptable noise levels.

Seismic noise has many different causes. Short-period noise is at most sites predominantly manmade and somewhat larger in the horizontal components than in the vertical. At intermediate periods (2 to 20 s), marine microseisms dominate with similar amplitudes in the horizontal and vertical directions. At long periods, horizontal noise may be larger than vertical noise by a factor up to 300, the factor increasing with period. This is mainly due to tilt which couples gravity into the horizontal components but not into the vertical. Tilt may be caused by traffic, wind, or the barometric pressure. Large tilt noise is sometimes observed on concrete floors when an unventilated cavity exists underneath; the floor then acts like a membrane. Such noise can be identified by its linear polarization and its correlation with the barometric pressure. The sound of a hammer stroke tells a lot about the solidity of a floor. Even on an apparently solid foundation, the long-period noise often correlates with the barometric pressure (Beauduin *et al.* 1996). If the situation cannot be remedied otherwise, the barometric pressure should be recorded with the seismic signal and used for a correction. For very-broad-band seismographic stations, barometric recording is generally recommended.

Besides ground noise, environmental conditions must be considered. Although seismometers can and must be shielded agains these, an aggressive atmosphere may cause corrosion, short-term variations of temperature may induce noise and seasonal variations of temperature may exceed the drift specifications. As a precaution, cellars and vaults should be checked for signs of occasional flooding.

6.2 Seismometer installation

We shortly describe the installation of a portable broad-band seismometer inside a building, vault, or cave. The first act is to mark the orientation of the sensor on the floor. This is best done with a geodetic gyroscope but a magnetic compass will do in most cases. The magnetic declination must be taken into account. Since a compass may be deflected inside a building, the direction should be taken outside and transferred to the site of installation. A laser pointer may be useful for this purpose. When the declination is unknown or unpredictable (such as in high latitudes or volcanic areas), the orientation should be determined with a sun compass.

To isolate the seismometer from stray currents, small glass or perspex plates are cemented to the ground under its feet. Then the seismometer is installed, tested, and wrapped with a thick layer of thermally insulating material. The type of material seems not to matter very much; alternate layers of fibrous material and heat-reflecting blankets are probably most effective. The edges of the blankets should be taped to the floor around the seismometer. Electronic seismometers produce heat and may induce convection in any open space inside the insulation; it is therefore important that the insulation has no gap and fits the seismometer tightly. Another method of insulation is to surround the seismometer with a large box which is then filled with fine styrofoam seeds. For a permanent installation under unfavourable environmental conditions, the seismometer must be enclosed in a hermetic container. A problem with such containers (as with all seismometer housings) is however that they cause tilt noise when they are deformed by the barometric pressure. Essentially three precautions are possible: either the baseplate is carefully cemented to the floor, or it is made so massive that its deformation is negligible, or a "warp-free" design is used as desribed by Holcomb and Hutt (1992) for the STS1 seismometers. Some desiccant should be placed inside any hermetic container, even into the vacuum bell of STS1 seismometers.

Guidelines for installing broadband seismic instrumentation are offered at the web site of the Seismological Lab at Berkeley (Uhrhammer & Karavas 1997). Detailed instructions for the design of seismic vaults are given by Trnkoczy (1998).

6.3 Installation of strong-motion instruments

Although strong-motion instruments are sometimes installed side by side with broadband instruments, their site is normally selected after different criteria. For purposes of seismic engineering, strong-motion records may be desired where one would not normally install a seismic station - for example in power stations or high-rise buildings. The instruments must be cemented or bolted to the structure; in regions of high epicentral intensity, they should also be protected from falling debris. Other considerations apply when free-field signals are to be recorded. At high frequencies and when the soil is soft, nearby buildings and even the instrument vault itself may modify the signal. The vault should in this case be a small, rigid structure that is firmly coupled to the ground on all sides. Since strong-motion recorders are normally operated in an event-triggered mode and may not record any ground motion for long intervals of time, their state of health must be regularly checked with test signals.

6.4 Magnetic shielding

Broadband seismometers are to some degree sensitive to magnetic fields because all thermally compensated spring materials are slightly magnetic. This may become noticeable when seismometers are operated in industrial areas or in the vicinity of dc-powered railway lines. When long-period noise is found to follow a regular time table, magnetic interferences should be suspected. Shields can be manufactured from permalloy metal but are expensive and of limited efficiency. An active compensation is often preferable. It may consist of a three-component fluxgate magnetometer that senses the field near the seismometer, an electronic driver circuit in which the signal is integrated with a short time constant (a few milliseconds), and a three-component set of Helmholtz coils which compensate changes of the magnetic field. The permanent geomagnetic field should not be compensated; the resulting offsets of the fluxgate outputs can be be compensated with a permanent magnet or electrically.

7 Deconvolution

7.1 General deconvolution

It is often necessary to restore the original ground motion from a seismogram. For this purpose, the seismogram must be filtered with the inverse response of the seismograph, a process known as deconvolution. It is rarely carried out in its exact mathematical form because the signal-to-noise ratio deteriorates outside the passband of the seismograph, and the deconvolution must be limited to a passband where the signal-to-noise ratio is still acceptable. Generally speaking, the ground motion is not entirely determined by the seismic record, and its reconstruction is a geophysical inverse problem whose solution must be constrained by a-priori information. Similar considerations apply to the case that records from seismographs with different responses must be homogenized. The PREPROC software package (Plešinger *et al.* 1996) contains different routines for a band-limited deconvolution and its manual offers a concise introduction into the problem.

A deconvolution can be realized in frequency or time domain. The frequency domain is convenient for the construction of approximate inverses with a view to the frequency-dependent signal-to-noise ratio of the data. Causal and acausal solutions are available; waveforms may be better preserved with an acausal inverse but the resulting precursory signals may give rise to misinterpretation. A division-by-zero problem exists in the low-frequency limit and prevents the construction of an exact inverse. In the time domain, both approximate and quasi-exact inverses can conveniently be realized with recursive (IIR) filters. They are always causal. Short time series can often be deconvolved with a quasi-exact inverse; no division-by-zero problem is encountered but a quadratic or cubic trend may be generated, which can be removed afterwards by polynomial fitting.

7.2 Removing the noncausal response of digital recorders

The response of digital recorders is normally determined by digital, finite impulse response (FIR) filters which decimate the oversampled data stream. The transfer functions of such filters are transcendental functions, and it is neither correct nor practical to specify them by poles and zeroes in frequency domain as if they were rational functions.

Essentially two types of FIR filters are used. Most recorders have zero-phase filters that do not introduce phase shifts; their response can simply be ignored in the overall response of a digital seismograph, except for their limiting the bandwidth. Zero-phase filters are however noncausal and can produce spurious precursors of seismic arrivals. This is a problem when first arrivals must be picked, in which case it is preferable to have minimum-phase decimation filters. These, on the other hand, introduce frequency-dependent delays that must be removed from the data in other applications. It is possible to convert a zero-phase filtration into a minimum-phase filtration, and vice versa, by applying a suitable all-pass filter to the data (Scherbaum 1996). The problem is treated elsewhere in this volume.

Problems with acausal zero-phase filters can also be avoided by filtering the data with a causal filter of smaller bandwidth such as a Butterworth low-pass filter, or with the response of a classical electromagnetic seismograph. The influence of the original decimation filter then disappears and only the response of the user-specified filter must be considered.

8 Testing for linearity and noise

8.1 Testing for linearity

Seismographs need not record the ground motion with extreme precision; most methods of interpretation do not depend on minor amplitude errors or waveform distortions. In one respect, however, a broadband seismograph must be precise: it must be a linear system, that is, it must conform to the principle of superposition. The sum of two input signals must produce the sum of the corresponding output signals and nothing else. Then, the presence of any number of input signals in a given frequency band will not produce an output signal in another band. This is a prerequisite for being able to record small long-period signals in the presence of strong short-period signals, or vice versa.

The electric linearity of a seismometer is usually checked with a two-tone test (Peterson *et al.* 1980). Two sinusoidal signals of equal amplitude and nearly equal frequency (for example, 1.00 and 1.02 Hz) are applied to the calibration input and their low-frequency (0.02 Hz) intermodulation product is observed at the output. The measurement is facilitated by the fact that the gain of a VBB seismometer at 0.02 Hz is 50 times larger than at 1 Hz, so even a small amount of distortion will be noticeable. Another way of testing linearity is to calibrate the system with the CALEX routine (see 9.5) using a sweep or another nearly sinusoidal signal as an input. Nonlinear distortions, if present, are visible in the residual signal provided that the transfer function has been modelled correctly so that the linear distortions (errors in amplitude and phase) are small. Expressing the result in meaningful numbers is another problem. Linearity is not a simple concept in a system with a frequency-dependent response; the test conditions must be precisely defined. Referring the result to the input (acceleration) in place of the output (velocity) may have a large cosmetic effect.

Good electrical linearity unfortunately does not ensure that the system is equally linear for seismic signals. Mechanical forces in the sensor and non-linear behaviour of the force transducer may differ between an electrical and a mechanical experiment. Moreover, a sensor might respond in a nonlinear way to transverse accelerations that cannot be excited electrically. Unfortunately, it is nearly impossible to observe the mechanical nonlinearity of a force-balance seismometer on a shake table. Such experiments invariably end up as tests of the table with the seismometer as the reference. Even a small tilt component in the motion of the table can produce stronger spurious signals than a suspected nonlinearity of the sensor. A seismometer may however be considered as sufficiently linear when it produces undisturbed long-period records during local earthquakes (Wielandt & Streckeisen 1982).

8.2 Measuring instrumental noise

Force-balance sensors cannot be tested for noise with the mass locked, so the instrumental noise can only be observed in the presence of seismic noise. This may nevertheless be useful as a first check; a broadband seismograph that resolves tides or free oscillations of the earth cannot be too bad at long periods. For a quantitative assessment of instrumental noise, however, the two contributions must be separated from each other. The usual procedure is to perform a frequency-domain analysis of the coherency between the output signals of two seismometers, and to assume that coherent noise is seismic and incoherent noise is instrumental. This is however questionable because the two instruments may respond coherently to environmental disturbances such as caused by barometric pressure, temperature, the supply voltage, magnetic fields, vibrations, or electromagnetic waves. Nonlinear behaviour (intermodulation) may produce coherent but spurious long-period signals. The reference instrument should therefore be one whose (relative) immunity to intermodulation and environmental conditions is known; otherwise two instruments of a different type or at least in a different setup and with different power supplies should be used, hoping that they will not respond to disturbances in a coherent manner.

The analysis for coherency is somewhat tricky in detail. When the transfer functions of both instruments are known, it is in fact theoretically possible to measure the seismic signal and the instrumental noise of each instrument separately as a function of frequency. However, the transfer functions must be known with great precision for this purpose. Alternatively, one may assume that the reference instrument is noise-free; in this case the noise and the transfer function of the other instrument can be determined. Long time series are required for reliable results: the analysis is made in a set of narrow frequency bands, and the length of the record multiplied with the smallest bandwidth must be a large number. We offer a computer program UNICROSP for the analysis.

8.3 Transient disturbances

Most new seismometers produce spontaneous transient disturbances, quasi miniature earthquakes caused by stresses in the mechanical components. Although they do not necessarily originate in the spring, their waveform at the output seems to indicate a sudden and permanent (step-like) change in the spring force. Long-period seismic records are sometimes severely degraded by such disturbances. The transients often die out within some months or years; if not, and especially when their frequency increases, corrosion must be suspected. Manufacturers try to mitigate the problem with a low-stress design and by aging the components or the finished seismometer (by extended storage, vibrations, or alternate heating and cooling cycles). It is sometimes possible to virtually eliminate transient disturbances by hitting the pier around the seismometer with a hammer, a procedure that is recommended in each new installation.

9 Calibration

9.1 Electrical and mechanical calibration

The calibration of a seismograph establishes knowledge of the relationship between its input (the ground motion) and its output (an electric signal), and is a prerequisite for a reconstruction of the ground motion. Since precisely known ground motions are difficult to generate, one makes use of the equivalence between ground accelerations and external forces on the seismic mass (eq.2), and calibrates seismometers with an electromagnetic force generated in a calibration coil. If the factor of proportionality between the current in the coil and the equivalent ground acceleration is known, then the calibration is a purely electrical measurement. Otherwise, the missing parameter -

either the transducer constant of the calibration coil, or the responsivity of the sensor itself - must be determined from a mechanical experiment in which the seismometer is subject to a known mechanical motion on or a known tilt. This is called an absolute calibration. Since it is difficult to generate precise mechanical calibration signals over a large bandwidth, one does normally not attempt to determine the complete transfer function in this way.

The present section is mainly concerned with the electrical calibration although the same methods may also be used for the mechanical calibration on a shake table. Specific procedures for the mechanical calibration without a shake table are presented in section 10.

9.2 General conditions

Calibration experiments are disturbed by seismic noise and tilt, and should therefore be carried out in a basement room. However, the large operating range of modern seismometers permits a calibration with relatively large signal amplitudes, making background noise less of a problem than one might expect. Thermal drift is more serious because it interferes with the long-period response of broadband seismometers. These should be protected from draft and allowed sufficient time to reach thermal equilibrium. Visible and digital recording in parallel is recommended. Recorders must themselves be absolutely calibrated before they can serve to calibrate seismometers. The input impedance of recorders as well as the source impedance of sensors should be measured so that a correction can be applied for the loss of signal in the source impedance.

9.3 Calibration of geophones

Simple electrodynamic seismometers (geophones) have no calibration coil. The calibration current must then be sent through the signal coil. There it produces an ohmic voltage in addition to the output signal generated by the motion of the mass. The simplest way of circumventing this difficulty is to excite the geophone by interrupting a constant current through the coil, and to evaluate the resulting transient response either graphically or with any suitable program for system identification (Rodgers *et al.* 1995). The method will also supply the generator constant when the size of the mass is known and its motion is linear. When other calibration signals are used, the undesired voltage can be compensated in a bridge circuit (Willmore 1959); the bridge is zeroed with the geophone clamped or inverted. However, when both the calibration current and the output voltage are digitally recorded, it is more convenient to use only a half-bridge (fig. 9) and to compensate the ohmic voltage numerically. The program CALEX decribed below has provisions to do this automatically.

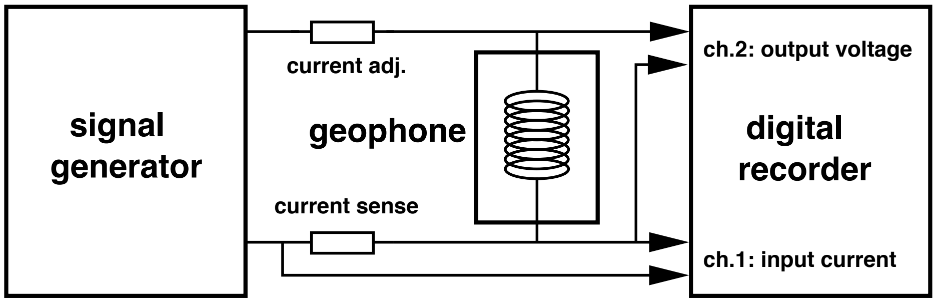

Figure 9: Half-bridge circuit for calibrating geophones

Eigenfrequency and damping of geophones (and of most other seismometers) can be determined graphically with a set of standard resonance curves on doubly-logarithmic paper. The measured amplitude ratios are plotted on the same type of paper and overlain with the standard curves (fig. 10). The desired quantities can directly be read. The method is simple but not very precise.

Geophones whose seismic mass moves along a straight line require no mechanical calibration when the size of the mass is known. The electromagnetic part of the numerical damping is inversely proportional to the total damping resistance (eq. 13); the factor of proportionality is $\sigma^2/2M\omega_0$, so the generator constant σ can be calculated from electrical calibrations with different resistive loads.

9.4 Calibration with sinewaves

With a sinusoidal input, the output of a linear system is also sinusoidal, and the ratio of the two signal amplitudes is the amplitude response (the modulus of the complex frequency response). An experiment with sinewaves therefore permits an immediate check of the response, without any a-priori knowledge of its mathematical form and without waveform modelling; this is often the first step in the identification of an unknown system. A computer program would however be required for converting discrete values of the response function into a parametric representation; a calibration with arbitrary signals as described later is more straightforward for this purpose.

When only analog equipment is available, the calibration coil or the shake table may be driven with a sinusoidal test signal and the input and output signals recorded with a chart recorder or an X-Y recorder. On the latter,

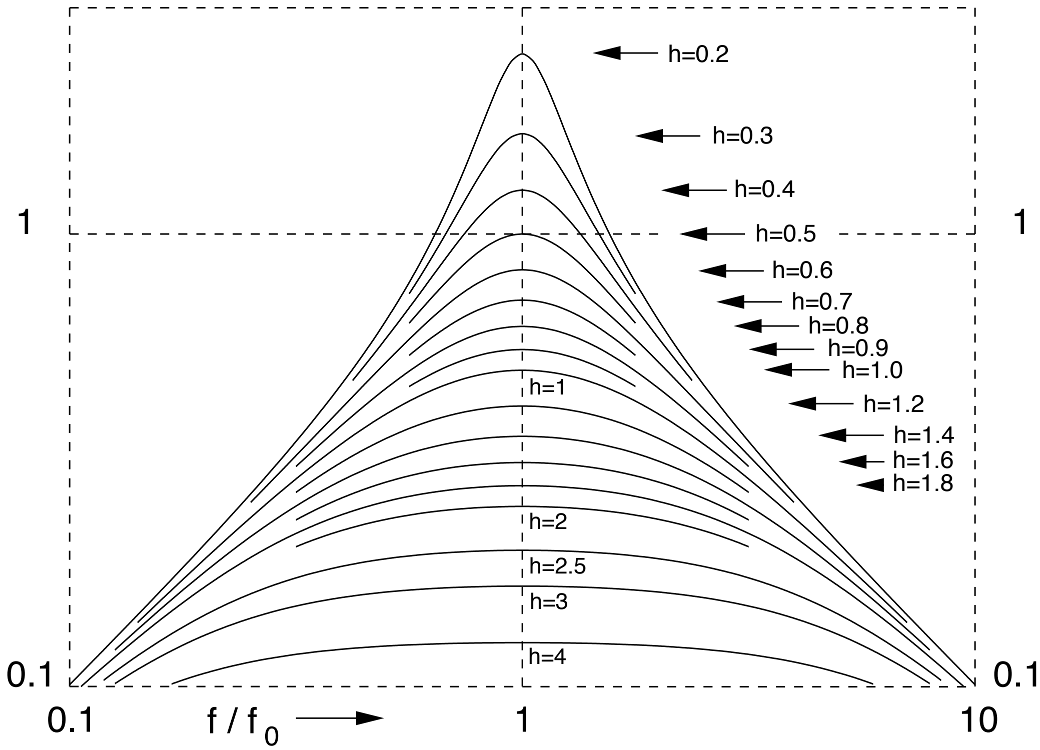

Figure 10: Normalized resonance curves

the signals should be be plotted as a Lissajous ellipse from which both the amplitude ratio and the phase can be read with good accuracy (Mitronovas & Wielandt 1975). The signal period should be measured with a counter or a stop watch because the frequency scale of sinewave generators is often inaccurate.

The accuracy of the graphic evaluation depends on the purity of the sinewave. A better accuracy can of course be obtained with a numerical analysis of digitally recorded data. By fitting sinewaves to the signals, amplitudes and phases can be extracted for just one precisely known frequency at a time; distortions of the input signal then don't matter. For best results, the frequency should be fitted as well, the fit should be computed for an integer number of cycles, and offsets should be removed from the data. A FORTRAN program "SINFIT" is offered for this purpose (section 11).

9.5 Calibration with arbitrary signals

The purpose of calibration is in most cases to obtain the parameters of an analytic representation of the transfer function. Assuming that its mathematical form is known, the task is to determine its parameters from an experiment in which both the input and the output signals are recorded. This is a classical inverse problem that can be solved with standard least-squares methods.

The general concept is as follows. A computer algorithm is implemented that represents the seismometer as a filter, and permits the computation of its response to an arbitrary input. An inversion algorithm is programmed around the filter algorithm in order to find best-fitting parameters for a given pair of input and output signals. A calibration experiment is then made with a test signal for which the response of the system is sensitive to the unknown parameters but which is otherwise arbitrary. When the system is linear, parameters determined from one test signal will also predict the response to any other signal.

The approximation of a rational transfer function with a discrete filtering algorithm is not trivial (see 3.4). For the program CALEX that accompanies this article, we have chosen an impulse-invariant recursive filter after Schuessler (1981). The method formally requires that the seismometer has a negligible response at frequencies outside the Nyquist bandwidth of the recorder, a condition that is severely violated by most digital seismographs; but this problem can be circumvented with an additional sharp low-pass filtration within the program. Figure 11 shows signals from a typical calibration experiment with CALEX. The test signal is a sweep, which permits the residual error to be visualized as a function of frequency even when the whole algorithm works in time domain. When the transfer function has been correctly parametrized and the inversion has converged, then the residual error consists mainly of noise, drift, and nonlinear distortions. At a signal level of about one-third of the operating range, typical residuals are 0.03% to 0.05% rms for force-balance seismometers and $\geq$1% for passive electrodynamic sensors.

With an appropriate choice of the test signal, other methods like the calibration with sinewaves, step functions, random noise or random telegraph signals can be duplicated and compared to each other. An advantage of the CALEX algorithm is that it makes no use of special properties of the test signal, such as being sinusoidal, periodic, step-like, or random. Therefore, test signals can be short (a few times the free period of the seismometer), and they can be generated with the most primitive means, even by hand. A breakout box or a special cable may however be required for feeding the calibration signal into the digital recorder.

Alternative routines for seismograph calibration are contained in the free PREPROC software package (Plešinger et al. 1996; see section 11).

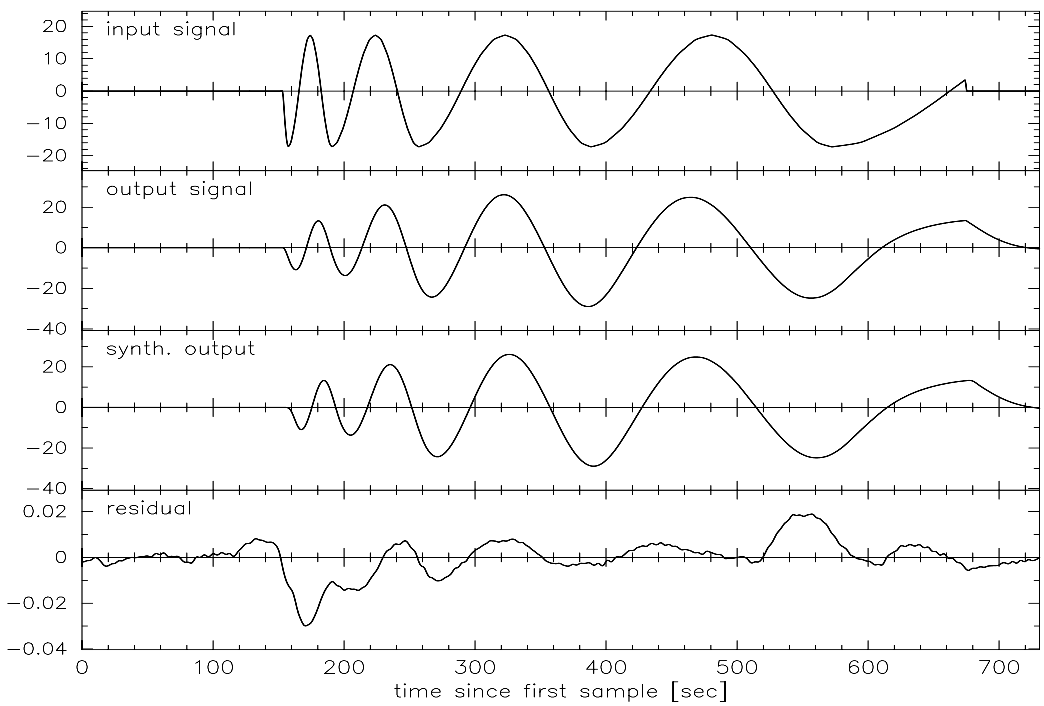

Figure 11: Electrical calibration of a STS2 seismometer with CALEX. Traces from top to bottom: input signal (a sweep with a total duration of 10 min); observed output signal; modelled output signal; residual. The rms residual is 0.05% of the rms output.

9.6 Calibration of triaxial seismometers

In a triaxial seismometer such as the STS2 (fig. 5), transfer functions in a strict sense can only be attributed to the individual U,V,W sensors, not to the X,Y,Z outputs. Formally, the response of a triaxial seismometer to arbitrary ground motions is described by a nearly diagonal 3 x 3 matrix of transfer functions relating the X,Y,Z output signals to the X,Y,Z ground motions. (This is also true for conventional three-component sets if they are not perfectly aligned; only the composition of the matrix is slightly different.) If the U,V,W sensors are reasonably well matched, the effective transfer functions of the X,Y,Z channels have the traditional form and their parameters are weighted averages of those of the U,V,W sensors. The X,Y,Z outputs can therefore be calibrated as usual. For the simulation of horizontal and vertical ground accelerations via the calibration coils, each sensor must receive an appropriate portion of the calibration current. For the vertical component this is approximately accomplished by connecting the three calibration coils in parallel. For the horizontal components and also for a more precise excitation of the vertical, the calibration current must be split into three individually adjustable and invertible components. These are then adjusted so that the signal appears only at the desired output of the seismometer.

10 Procedures for the mechanical calibration

10.1 Calibration on a shake table

Using a shake table is the most direct way of obtaining an absolute calibration. In practice, however, precision is usually poor outside a frequency band roughly from 0.5 to 5 Hz. At higher frequencies, a shake table loaded with a broadband seismometer may develop parasitic resonances, and inertial forces may cause undesired motions of the table. At low frequencies, the maximum displacement and thus the signal-to-noise ratio may be insufficient, and the motion may be nonuniform due to friction or roughness in the bearings. Still worse, most shake tables do not produce a purely translational motion but also some tilt. This has two undesired side-effects: the angular acceleration may be sensed by the seismometer, and gravity may be coupled into the seismic signal (see 2.4). The latter effect can be catastrophic for the horizontal components at long periods since the error increases with the square of the signal period. One might think that a tilt of 10 μrad per mm of linear motion should not matter; however such a tilt will, at a period of 30 s, induce seismic signals twice as large as those originating from the linear motion. At a period of 1 s, the same tilt would be negligible. Long-period measurements on a shake table, if possible at all, require extreme care.

Although all calibration methods mentioned in the previous section are applicable on a shake table, the preferred method would be to record both the motion of the table (as measured with a displacement transducer) and the output signal of the seismometer, and to analyze these signals with CALEX. Depending on the definition of active and passive parameters, only the absolute gain (responsivity, generator constant) or any number of additional parameters of the transfer function may be determined.

10.2 Calibration by stepwise motion

The movable tables of machine tools like lathes and milling machines, and of mechanical balances, can replace a shake table for the absolute calibration of seismometers. The idea is to place the seismometer on the table, let it come to equilibrium, then move the table manually by a known amount and let it rest again. The total motion can then be calculated from the seismic signal and compared to the actual mechanical displacement. Since the calculation involves triple integrations, offset and drift must be carefully removed from the seismic trace. The main contribution to drift in the apparent horizontal "ground" velocity comes from tilt associated with the motion of the table. With the method subsequently described, it is possible to separate the contributions of displacement and tilt from each other so that the displacement can be reconstructed with good accuracy. This method of calibration is most convenient because it uses only normal workshop equipment; the inherent precision of machine tools and the use of relatively large displacements eliminate the problem of measuring small displacements. A FORTRAN program named DISPCAL is available for the evaluation.

The precision of the method depends on minimizing errors from two sources:

1. The numerical restoration of ground displacement from the seismic signal (a process of inverse filtration) is uncritical for broadband seismometers but requires a precise knowledge of the transfer function of short-period seismometers. Instruments with unstable parameters must be electrically calibrated while installed on the test table. However, when the response is known, the restitution of absolute ground motion is no problem even for a geophone with a free period of 0.1 s.

2. The effect of tilt can only be removed from the displacement signal when the motion is sudden and short. The tilt is unknown during the motion, and is integrated twice in the calculation of the displacement. So the longer the interval of motion, the more effect the unknown tilt will have on the displacement signal. Practically, the motion may last about one second on a manually operated machine tool, and about a quarter-second on a mechanical balance. It may be repeated at intervals of a few seconds.

Static tilt before and after the motion produces linear trends in the velocity which are easily removed before the integration. The effect of tilt during the motion can however only approximately be removed by interpolating the trends before and after the motion. The computational evaluation consists in the following major steps (fig. 12):

1. The trace is deconvolved with the velocity transfer function of the seismometer.

2. The trace is piecewise detrended so that it is close to zero in the motion-free intervals; interpolated trends are removed from the intervals of motion.

3. The trace is integrated.

4. The displacement steps are measured and compared to the actual motion.

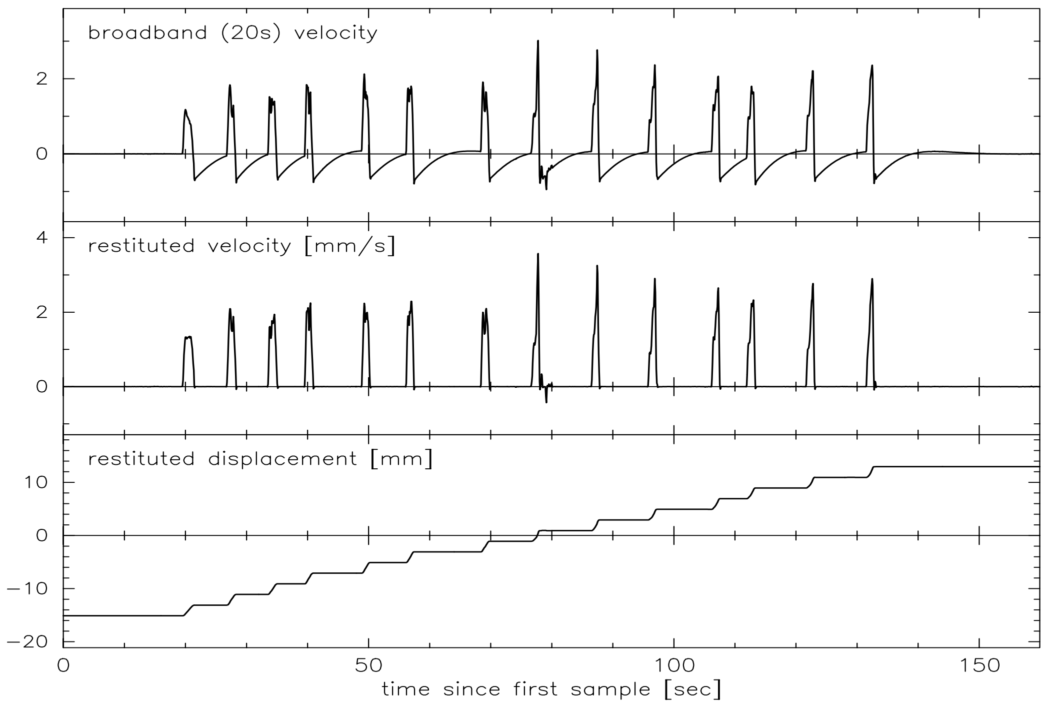

Figure 12: Absolute mechanical calibration of an STS1-BB seismometer on the table of a milling machine, evaluated with DISPCAL. Traces from top to bottom: recorded BB output signal; restored and detrended velocity; restored displacement.

In principle a single steplike displacement is all that is needed. However, the experiment takes so little time that it is convenient to produce a dozen or more equal steps, average the results, and do some error statistics. On a milling machine or lathe, it is recommended to install a mechanical device that stops the motion after each full turn of the spindle. On a balance, the table is repeatedly moved from stop to stop. Its amplitude may be measured with a micrometer dial but may also be determined from the lever ratio, which is in turn is obtained from the ratio of equivalent weights.

From the mutual agreement between a number of different experiments, and from the comparison with a calibration on a precise shake table, the absolute accuracy of the method is estimated to be better than 1%.

10.3 Calibration with tilt

Accelerometers can be statically calibrated on a tilt table. Starting from a horizontal position, the fraction of gravity coupled into the sensitive axis equals the sine of the tilt angle. (A tilt table is not required for accelerometers with an operating range exceeding $\pm 1g$; these are are simply turned over.) Force-balance seismometers normally have a mass-position output which is a slowly responding acceleration output. This output can, with some patience, likewise be calibrated on a tilt table; the small static tilt range of sensitive broadband seismometers may however be inconvenient. The transducer constant of the calibration coil is then obtained by sending a direct current through it and comparing its effect with the tilt calibration. Finally, by exciting the coil with a sinewave whose acceleration equivalent is now known, the absolute calibration of the broadband output is obtained. The method is not explained in more detail here because a simpler method exists. Anyway, triaxial seismometers cannot be calibrated in this way because they do not have X,Y,Z mass-position signals.

The method which we propose (for horizontal components only; program TILTCAL) is similar to what was described under 10.2 but this time we excite the seismometer with a known step of tilt, and evaluate the recorded output signal for acceleration rather than displacement. This is simple because we only have to look at the drift rate of the deconvolved velocity trace before and after the step; no baseline interpolation is involved. In order to produce repeatable steps of tilt, it is useful to prepare a small lever by which the tilt table or the seismometer can quickly be tilted forth and back by a known amount. The tilt may be larger than the static operating range of the seismometer; one then has to watch the output signal and reverse the tilt before the seismometer goes to a stop.

11 Free Software

FORTRAN source code of six computer programs mentioned in the text is included in the CD-ROM and can be downloaded from the author's FTP site (see below). These are stand-alone programs for calibrating and testing seismometers and for standardizing noise data; they do not form a package for general seismic processing such as SAC, SEISMIC UNIX, PITSA, or PREPROC (see below). A README file with explanations, a set of test data, and output files are provided with each program. No graphics are included. The data files have the following format:
- a header line (text only)
- a line in the format (i10, a20, f10.x) that contains the number of samples, their format, and the sampling interval
- samples in ASCII

It is suggested that the user modifies the input and output routines to conform with his own preferred data format. The programs are:

- CALEX: Determines parameters of the transfer function of a seismometer from the response to an arbitrary input signal (which must be recorded together with the output signal). The transfer function is implemented in the time domain as an impulse-invariant recursive filter. The inversion uses the method of conjugate gradients (moderately efficient but quite failsafe).
- DISPCAL: Determines the generator constant of a horizontal or vertical seismometer from an experiment where the seismometer is moved stepwise on the table of a machine tool or a mechanical balance.
- NOISECON: converts noise specifications into all kind of standard and nonstandard units and compares them to the USGS New Low Noise Model (Peterson 1993). Interactive program available in BASIC, FORTRAN, C and as a DOS-Executable.
- SINFIT: fits sinewaves to a pair of sinusoidal signals and determines their frequency and the relative amplitude and phase.
- TILTCAL: Determines the generator constant of a horizontal seismometer from an experiment where the seismometer is tilted stepwise.
- UNICROSP: Estimates seismic and instrumental noise separately from the coherency of the output signals of two seismometers.

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Free seismic software packages:

- SAC: available by FTP from s122.es.llnl.gov/sac2000 (128.115.4.141)
- SEISMIC UNIX: available by FTP from ftp.cwp.mines.edu (138.67.12.4)

- PITSA: available by download from the website http://www.rz.uni-potsdam.de/u/Geowissenschaft/index.htm
- PREPROC: available by FTP from hamy.ig.cas.cz (147.231.73.10) or gldfs.cr.usgs.gov (136.177.20.1)

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