INTRODUZIONE AL CORSO DI STATICA

23/02/23

RIPASSO GRANDEZZE FISICHE

DIMENSIONI

- scalari : massa [M], lunghezza [L], pressione $[FL^{-2}]$
- vettoriali : forza [F] ; spostamento [L]

UNITÀ DI MISURA

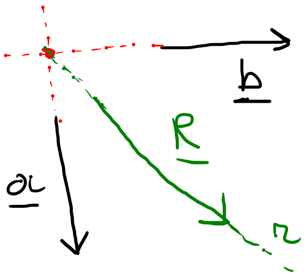
massa : Kg

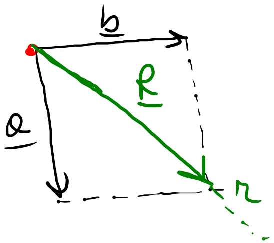
lunghezza : m

forza : N

pressione : $Pa = \frac{N}{m^2} \rightsquigarrow \underset{\underset{N/mm^2}{||}}{MPa} = 10^6 \; Pa \,, \; GPa = 10^9 \; Pa$

QUALCHE CONSIDERAZ. SUI VETTORI





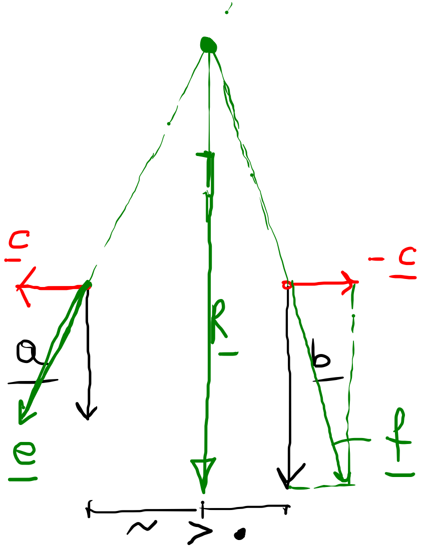
regola del parallelogramma

(5) (2) (3)

$\underline{R} = \underline{a} + \underline{b}$

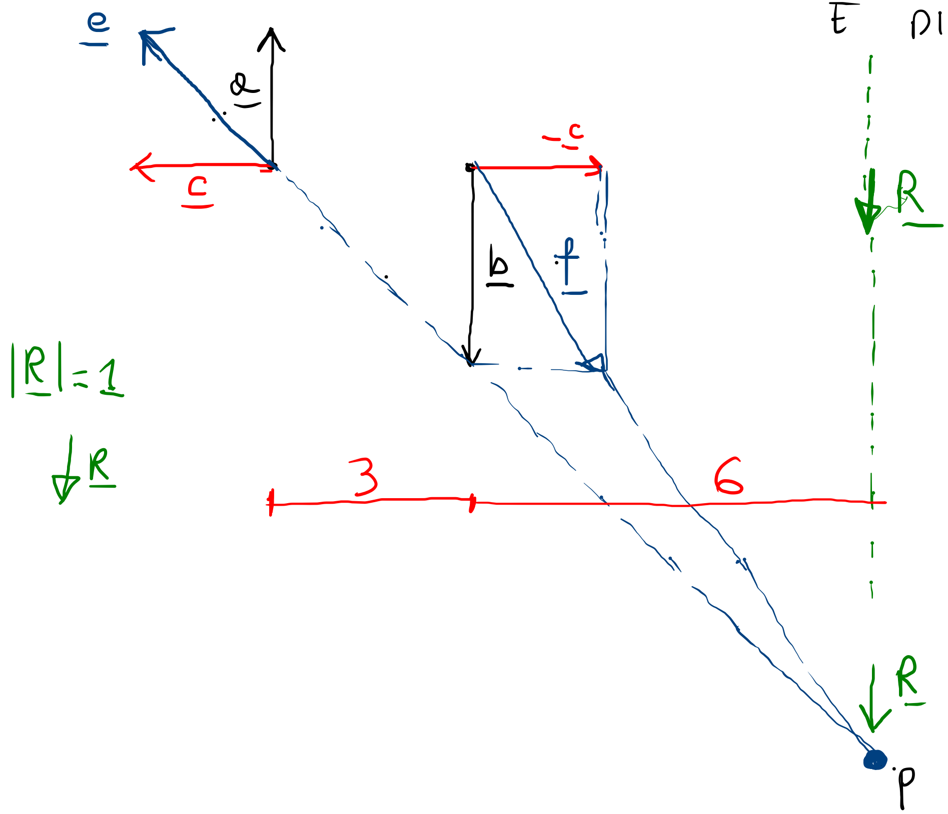
$\underline{R} = \underline{e} + \underline{f}$

$|\underline{R}| = 5$



SIST. 2 VETTORI PARALLELI E <u>CONCORDI</u> : LA RISULTANTE PASSA TRA I DUE VETTORI, PIÙ VICINO A QUELLO DI MODULO MAGGIORE

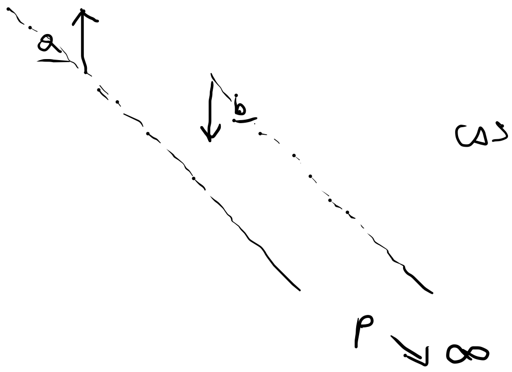
$|\underline{a}| \neq |\underline{b}|$



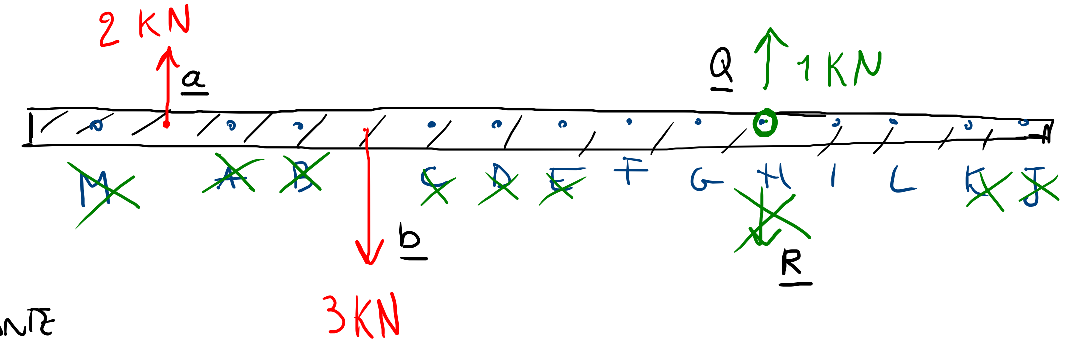
SIST. 2 VETTORI PARALLELI E DISCORDI DI DIVERSA INTENSITÀ:

LA RISULTANTE È ESTERNA AL SISTEMA E PASSA DALLA PARTE DEL VETTORE DI MAGGIORE INTENSITÀ.

$|\underline{a}| = |\underline{b}|$ $\quad$ $\underline{R} = \underline{0}$



CASO DEGENERE



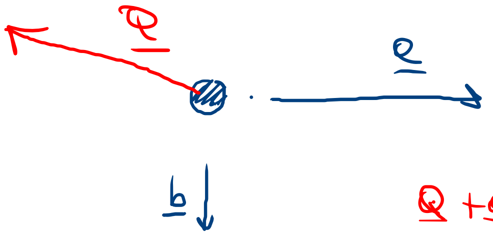
RISULTANTE

$\underline{R} = \underline{a} + \underline{b}$

$\boxed{\underline{a} + \underline{b} + \underline{Q} = \underline{0}}$ EQUILIBRIO

$\underline{Q} = -\underline{R}$

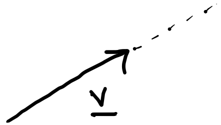
EQUILIBRANTE



$\underline{Q}$ EQUILIBRANTE ?

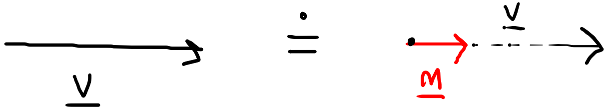
$\underline{Q} + \underline{a} + \underline{b} = \underline{0}$ EQUILIBRIO SODDISFATTO

VETTORI, LORO RAPPRESENTAZIONE E OPERAZIONI



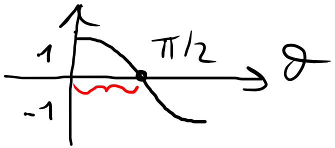
$|\underline{v}|, v$: INTENSITÀ o MODULO (LUNGHEZZA DEL SEGMENTO)

• VERSORE: VETTORE DI MODULO UNITARIO ($\underline{m}$ versore $\rightarrow |\underline{m}| = 1$)



; $\underline{v} = 3\,\underline{m}$

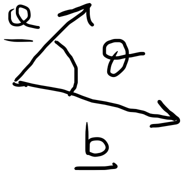
$v = 3$



• DUE OPERAZ. SUI VETTORI

1) PRODOTTO SCALARE

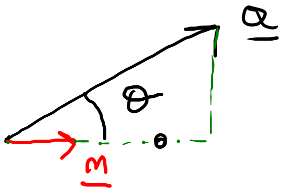
$\underline{a} \cdot \underline{b} \Rightarrow$ SCALARE



$\theta < \pi/2$

$\underline{a} \cdot \underline{b} = a\,b\cos\theta \in \mathbb{R} \gtreqless 0 \Rightarrow \theta = \frac{\pi}{2} \quad \underline{a} \perp \underline{b}$

$\frac{\pi}{2} < \theta < \frac{3}{2}\pi$



$\underline{a} \cdot \underline{n} = a \cdot 1 \cdot \cos\theta = a\cos\theta$; PROIEZIONE DI $\underline{a}$ NELLA DIREZ. DI $\underline{n}$

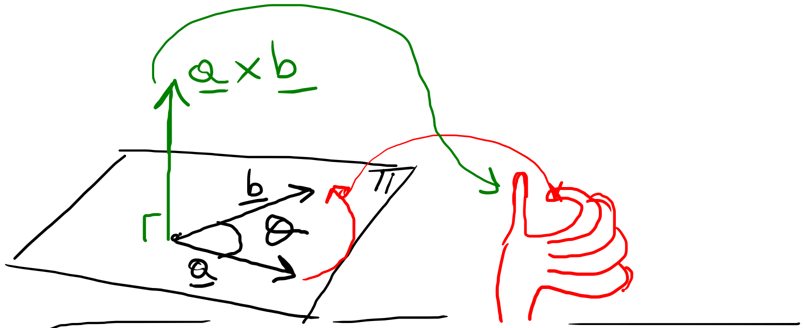
versore

$a\cos\theta$

2) PRODOTTO VETTORIALE

$\underline{a}, \underline{b} \Rightarrow \underline{a} \times \underline{b}$ è un vettore

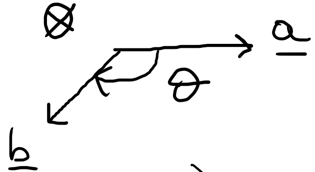
$|\underline{a} \times \underline{b}| = ab\sin\theta$ — $= 0$ se $\theta = 0$

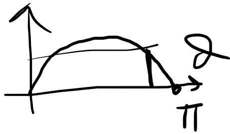


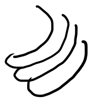
ES.

$\underline{a} \times \underline{b}$? ENTRA NEL FOGLIO

$ab\sin\theta$

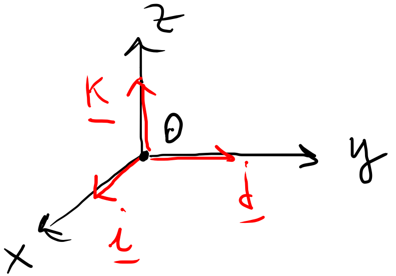


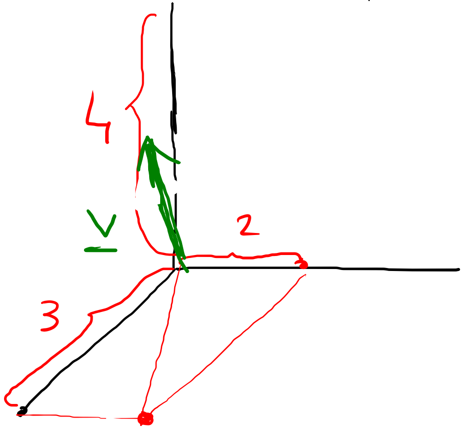




• RAPPRESENTAZ. CARTESIANE ($\underline{i}, \underline{j}, \underline{k}$: versori) $\{\underline{i}, \underline{j}, \underline{k}\}$: BASE ORTONORMALE DELLO SPAZIO TRIDIMENSIONALE.

$\perp$ tra di loro





$\underline{v} = 3\underline{i} + 2\underline{j} + 4\underline{k}$; RAPPRESENT. CARTES. DI $\underline{v}$

$\underline{b} = -2\underline{i} + \underline{j}$

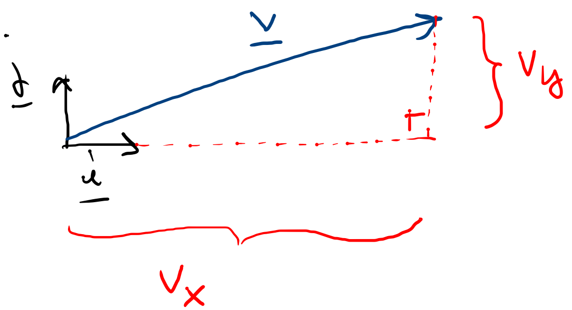
$$\underline{v} \cdot \underline{b} = (3 \cdot \underline{i} + 2\underline{j} + 4\underline{k}) \cdot (-2\underline{i} + \underline{j}) = 3\underline{i} \cdot (-2\underline{i}) + 3\underline{i} \cdot \underline{j} + 2\underline{j} \cdot (-2\underline{i}) +$$

$$+ 2\underline{j} \cdot \underline{j} + 4\underline{k} \cdot (-2\underline{i}) + 4\underline{k} \cdot \underline{j} = -6 \underbrace{\underline{i} \cdot \underline{i}}_{1} + 3 \underbrace{\cancel{\underline{i} \cdot \underline{j}}}_{\perp \Rightarrow 0} + (-4) \underbrace{\cancel{\underline{j} \cdot \underline{i}}}_{\perp \Rightarrow 0}$$

$$+ 2 \underbrace{\underline{j} \cdot \underline{j}}_{1} - 8 \underbrace{\cancel{\underline{k} \cdot \underline{i}}}_{\perp \Rightarrow 0} + 4 \underbrace{\cancel{\underline{k} \cdot \underline{j}}}_{\perp \Rightarrow 0} = -6 + 2 = -4$$

$$\underline{v} \cdot \underline{b} = 3(-2) + 2 \cdot 1 = -4$$

MODULO O INTENSITÀ DI UN VETTORE IN RAPPRESENT. CARTESIANA



$\underline{v} = v_x \underline{i} + v_y \underline{j}$; $|\underline{v}|$? $v = \sqrt{v_x^2 + v_y^2}$

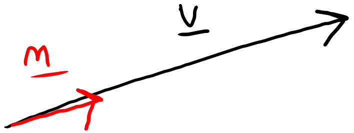
$|\underline{v}| \Leftrightarrow \underline{v} \cdot \underline{v}$

$\underline{v} \cdot \underline{v} = v_x^2 + v_y^2 \Rightarrow |\underline{v}| = \sqrt{\underline{v} \cdot \underline{v}}$

$\underline{a} = a_x \underline{i} + a_y \underline{j} + a_z \underline{k}$

$|\underline{a}| = \sqrt{\underline{a} \cdot \underline{a}}$

PROBLEMA: HO UN VETTORE $\underline{v}$ IN RAPPR. CARTESIANA $\overset{?}{\Rightarrow}$ IL VERSORE $\underline{m}$ ASSOCIATO



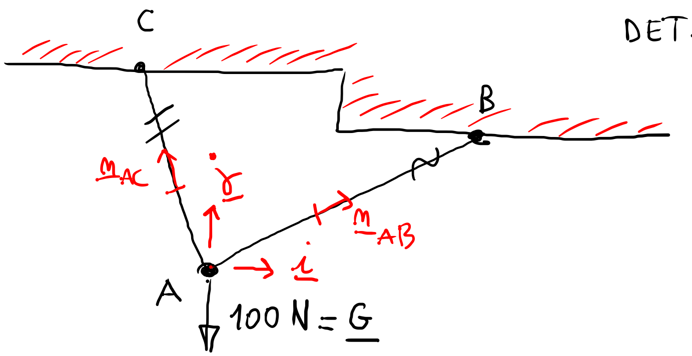
$\underline{v} = v \, \underline{m}$

ES. $\underline{v} = 3 \underline{i} + 4 \underline{j} \Rightarrow |\underline{v}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$; $\underline{m} = \frac{\underline{v}}{5} = \frac{3}{5} \underline{i} + \frac{4}{5} \underline{j}$

$\underline{m}$?

VERIFICA $|\underline{m}| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25} + \frac{16}{25}} = 1$

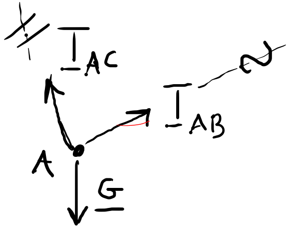
ES. EQUILIBRIO DI UN PUNTO MATERIALE "PESANTE" SOSPESO



DET. LE TENSIONI NEI FILI AB e AC

1) METODO GRAFICO

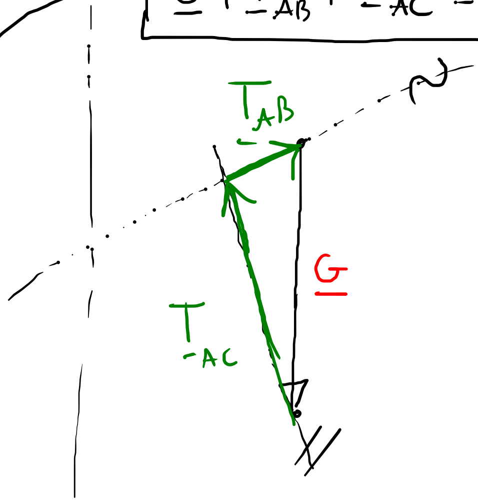
incognite : T_{AC}, T_{AB}
(2 scalari)



$$\underline{G} + \underline{T}_{AB} + \underline{T}_{AC} = \underline{0}$$

3 FORZE IN EQUILIBRIO:

- DEVONO PASSARE PER LO STESSO PUNTO (OVVIO IN QUESTO PROBLEMA)
- DEVONO CHIUDERE IL TRIANGOLO DELLE FORZE ?



BASANDOMI SUL VETTORE $\underline{G}$ NOTO, POSSO CALCOLARE (O MISURARE) LE LUNGHEZZE DI $\underline{T}_{AB}$ e $\underline{T}_{AC}$

2) METODO ANALITICO

$$\underline{G} = -100\,\underline{j}$$

$$\underline{T}_{AB} = T_{AB}\,\underline{m}_{AB} = T_{AB}\,\underbrace{(4\underline{i}+2\underline{j})\frac{1}{\sqrt{20}}}_{\underline{m}_{AB}}$$

$$\underline{T}_{AC} = T_{AC}\,\underline{m}_{AC} = T_{AC}\,\underbrace{(-\underline{i}+3\underline{j})\frac{1}{\sqrt{10}}}_{\underline{m}_{AC}}$$

$$\underline{G}+\underline{T}_{AB}+\underline{T}_{AC} = -100\,\underline{j} + T_{AB}\left(4\,\underline{i}+2\,\underline{j}\right)\frac{1}{\sqrt{20}} + T_{AC}\left(-\underline{i}+3\,\underline{j}\right)\frac{1}{\sqrt{10}} = \underline{0}$$

$$\begin{cases} \underline{i}: & T_{AB}\,4\,\frac{1}{\sqrt{20}} + T_{AC}\left(-\frac{1}{\sqrt{10}}\right) = 0 \\ \underline{j}: & -100 + T_{AB}\,\frac{2}{\sqrt{20}} + T_{AC}\,\frac{3}{\sqrt{10}} = 0 \end{cases}$$

SIST. DI 2 EQZ IN 2 INCOGNITE (T_{AB}, T_{AC})

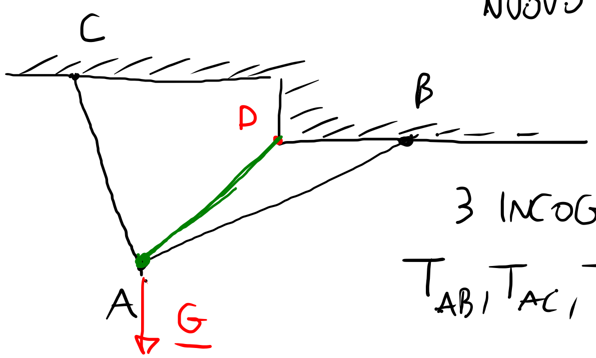
$\Rightarrow$ 1 SOLUZ. UNICA ; $T_{AC}, T_{AB} \neq 0$

$$T_{AB} = \frac{100\sqrt{5}}{7}\,N \;;\quad T_{AC} = \frac{200\sqrt{10}}{7}\,N$$

$\oplus$ $\oplus$

CONFERMA IL VERSO DEI VETTORI $\underline{T}_{AB}$ e $\underline{T}_{AC}$ SCELTO ARBITRARIAMENTE NELLO SCH. DELLA PAGINA PREC.

NUOVO PROBLEMA



3 INCOGNITE

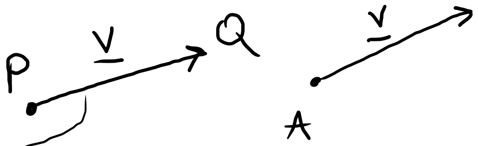
T_{AB}, T_{AC}, T_{AD}

RIUSCIAMO A CALCOLARE UNIVOCAMENTE LE 3 TENSIONI CON I METODI VISTI SOPRA ? HO 2 EQ. IN 3 INCOGNITE

$\Rightarrow$ INDETERMINATI $\infty^{3-2} = \infty^1$

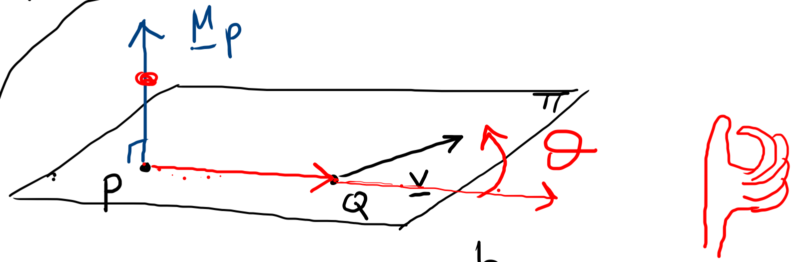
VETTORI APPLICATI

UN VETTORE APPLICATO È UNA COPPIA (P, $\underline{v}$) DOVE P È IL PUNTO DI APPLICAZIONE DEL VETTORE $\underline{v}$ NELLO SPAZIO.



2 VETTORI APPLICATI DIVERSI

PQ ; $|PQ| = v$



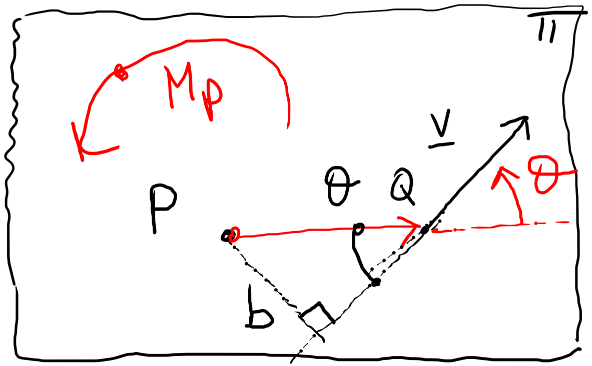
$$|\underline{M}_P| = |PQ|\, v \sin\theta = v \cdot b$$

b : braccio di $\underline{v}$ = DISTANZA DELLA RETTA D'AZIONE DEL VETTORE $\underline{v}$ DAL PUNTO P.

DEFINIZIONE DI MOMENTO DI VETTORE APPLICATO

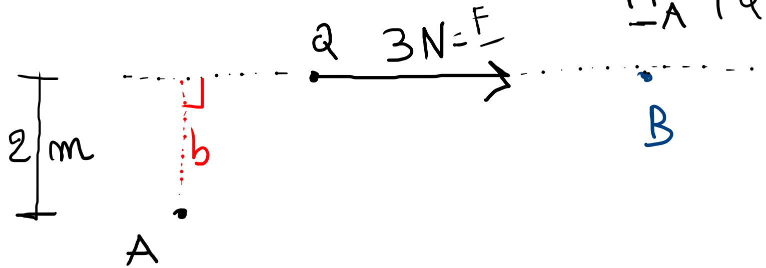
$$\underline{M}_P(Q, \underline{v}) = PQ \times \underline{v}$$

POLO DI RIDUZIONE

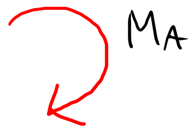


ES

$\underline{M}_A(Q,\underline{F})$?



$M_A = 3 \cdot 2 = 6\ Nm$



$\underline{M}_B(Q,\underline{F}) = \underline{0}$ (braccio è NULLO)