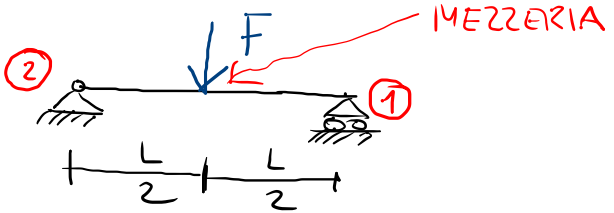
ANALISI STATICA DELLE STRUTTURE PIANE (1 C.R.)

15/03/23



$g=3, v=3, s=3$

ISOSTATICA

A COSA RISPONDE L'"ANALISI STATICA"?

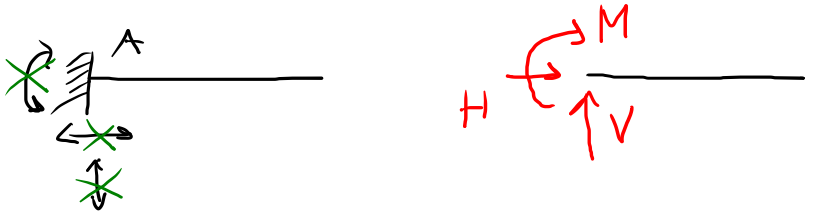
PERMETTE DI CALCOLARE, SE POSSIBILE, LE "REAZ. VINCOLARI", OVVERO, DI CAPIRE COME IL CARICO VIENE TRASFERITO DAI VINCOLI AL "TERRENO".

REAZIONI VINCOLARI: RISPOSTA STATICA DEI VINCOLI (INCOGNITE DEL PROBLEMA)

IL PROBLEMA MATEMATICO DEL CALCOLO DELLE REAZ. VINCOLARI SI CHIAMA "PROBLEMA STATICO"

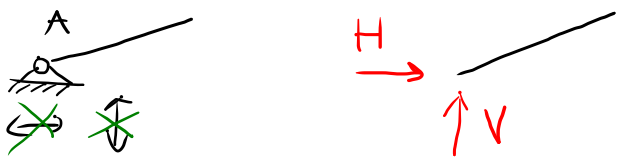
PRESTAZIONI STATICHE DEI VINCOLI

- $v=3$: INCASTRO



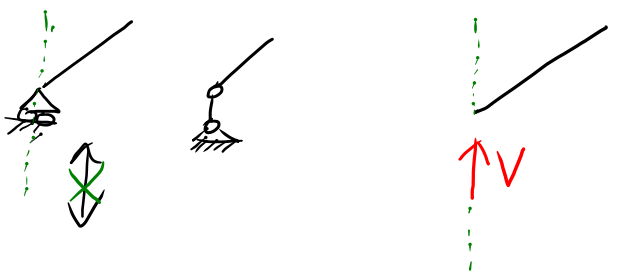
LE REAZ. VINCOLARI NASCONO DUALMENTE ALLE COMPONENTI DI MOVIMENTO IMPEDITE DAL VINCOLO.

- $v=2$: CERNIERA



- $v=2$: DOPPIO PENDOLO

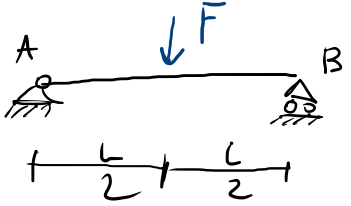
- $v=1$: CARRELLO / PENDOLO



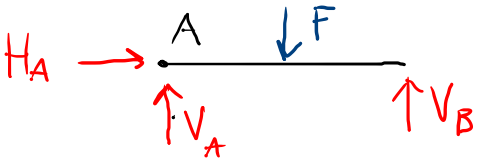
- $v=1$: DOPPIO-DOPPIO PENDOLO

COME IMPOSTARE IL PROBLEMA STATICO

SCHEMA STATICO



1) SOSTITUISCO AI VINCOLI LE REAZ. VINCOLARI SECONDO LO SCHEMA VISTO POCO FA; IL VERSO DI FORZE/MOMENTI E' ARBITRARIO.



S.C.L.

2) IMPOSTO IL PROBLEMA STATICO SCRIVENDO LE E.C.S. ⇒ SISTEMA LINEARE (1 C.R. ⇒ 3 EQUAZIONI)

$$\begin{cases} \overset{+}{\rightarrow}: \; +H_A = 0 \\ +\uparrow: \; +V_A + V_B - F = 0 \\ \overset{+}{\circlearrowleft_A}: \; -F\frac{L}{2} + V_B L = 0 \end{cases}$$

SIST. LINEARE
3 EQ. IN 3 INCOGNITE

3) CERCO, SE POSSIBILE, LA SOLUZ. DEL SISTEMA

$$\begin{cases} H_A = 0 \\ V_A + V_B - F = 0 \\ -F\frac{L}{2} + V_B L = 0 \end{cases} \qquad \begin{cases} H_A = 0 \\ V_A = F - V_B = \frac{F}{2} \\ V_B = \frac{F}{2} \end{cases}$$

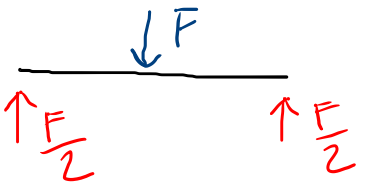
IN QUESTO PROBL. LA SOLUZ. È UNICA.

$H_A = 0$

$V_A = +F/2$

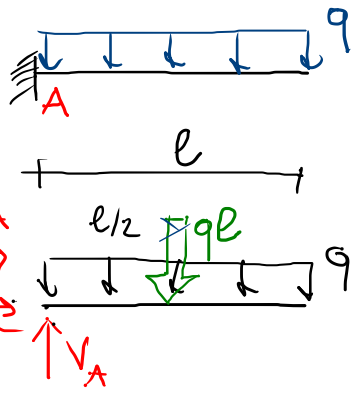
$V_B = +F/2$

4) DISEGNO LO SCHEMA DI CORPO LIBERO (S.C.L.) EQUILIBRATO, CON LE REAZ. VINCOLARI OTTENUTE NEL PUNTO 3)



S.C.L. EQUILIBRATO

ES.



$g = 3$
$v = 3$
$\delta = 3$

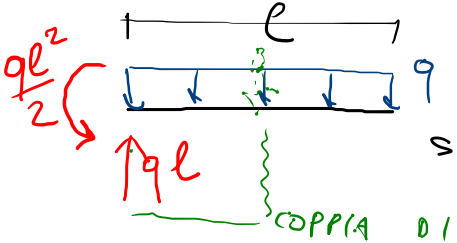
ISOSTATICA

S.C.L.

$$\begin{cases} \rightarrow : +H_A = 0 \\ +\uparrow : +V_A - q\ell = 0 \\ \circlearrowleft_A^{+} : -M_A - q\ell \cdot \frac{\ell}{2} = 0 \end{cases}$$

$$\begin{cases} H_A = 0 \\ V_A = +q\ell \\ M_A = -q\frac{\ell^2}{2} \end{cases}$$

SOLUZ. UNICA

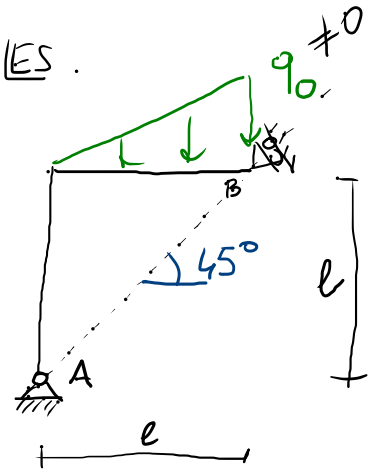


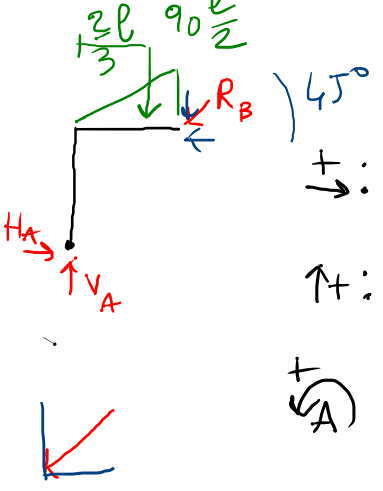
S.C.L. EQUILIBRATO

IL SIST. HA SOLUZIONE?

NO

ES.





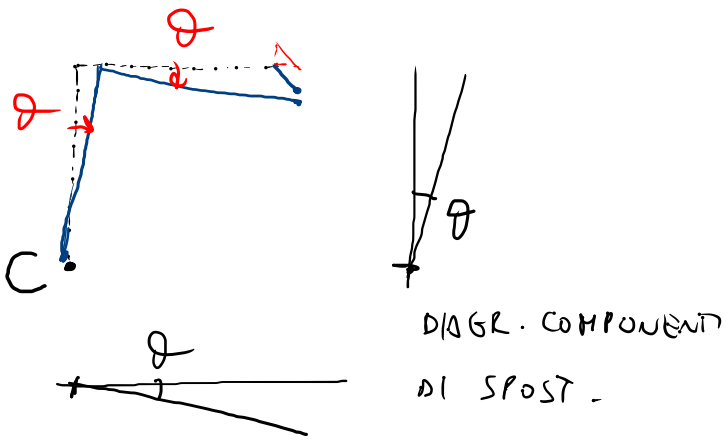
$$\rightarrow : +H_A - R_B \frac{1}{\sqrt{2}} = 0$$

$$\uparrow + : +V_A - \frac{q_0\ell}{2} - R_B \frac{1}{\sqrt{2}} = 0$$

$$\circlearrowleft_A^{+} : -q_0\frac{\ell}{2} \cdot \frac{2}{3}\ell = 0$$

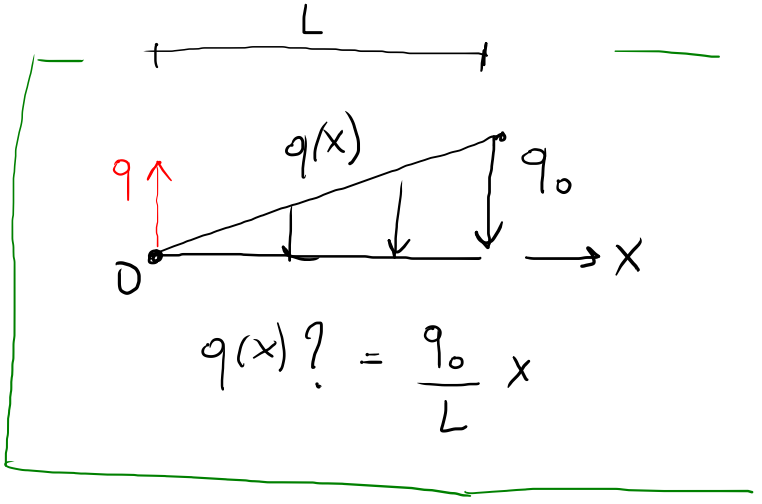
$\rightarrow$ EQ. NON SODDISFATTA

NELL'ES. I VINCOLI NON SONO IN GRADO DI BLOCCARE QUESTA ROTAZIONE

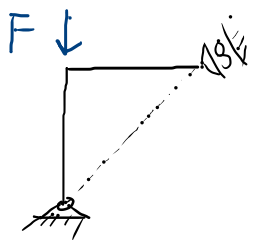


DIAGR. COMPONENTI DI SPOST.

STR. LABILE



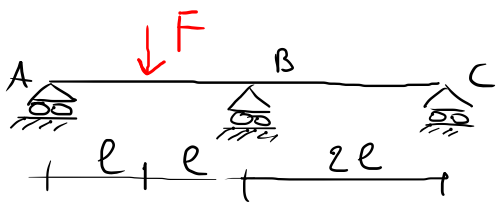
ES DA SVOLGERE (STR. LABILI)



ESISTE LA SOLUZIONE

NON ESISTE LA SOLUZ.

ES



$g = 3$
$v = 3$
$s = 2$

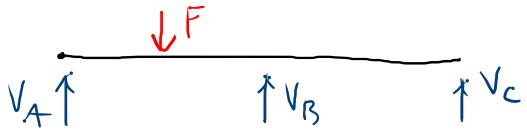
STR LABILE

$$\begin{cases} \rightarrow : \quad 0 = 0 \qquad (\text{IDENTITÀ}) \\ +\uparrow : \quad V_A + V_B + V_C - F = 0 \\ +\circlearrowleft_A : \quad -F\ell + V_B 2\ell + V_C 4\ell = 0 \end{cases}$$

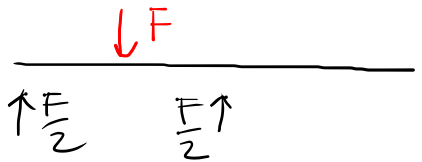
2 EQ. INDIPENDENTI IN 3 INCOGNITE

$\Rightarrow \infty^{3-2}$ SOLUZ.

SCL



MOSTRO DUE POSSIBILI SOLUZ. TRA LE INFINITE POSSIBILI



① $V_A = +\frac{F}{2}, \quad V_B = +\frac{F}{2}, \quad V_C = 0$

$$\begin{cases} \frac{F}{2} + \frac{F}{2} + 0 - F = 0 \quad \text{OK} \\ -F\ell + \frac{F}{2} 2\ell + 0 = 0 \quad \text{OK} \end{cases}$$

② $V_A = \frac{3}{4}F, \quad V_B = 0, \quad V_C = \frac{1}{4}F$

$$\begin{cases} \frac{3}{4}F + 0 + \frac{F}{4} - F = 0 \quad \text{OK} \\ -F\ell + \frac{1}{4}F \cdot 4\ell = 0 \quad \text{OK} \end{cases}$$

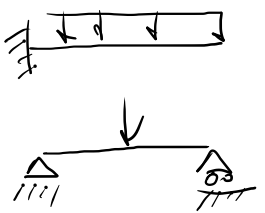
CLASSIFICAZIONE DEI PROBLEMI STATICI (P.S.)

- P.S. STATICAMENTE DETERMINATO: ∃ UNA SOLUZ. UNICA DEL SISTEMA DELLE EQUAZIONI DI EQUILIBRIO

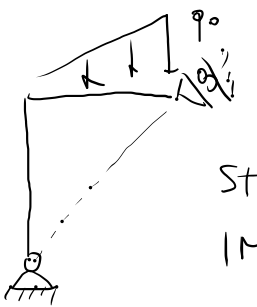
- P.S. " IMPOSSIBILE: NON ESISTE NESSUNA SOLUZIONE DEL SIST. DELLE EQ. DI EQUILIBRIO

- P.S. " INDETERMINATO: ∃ INFINITE SOLUZIONI .....

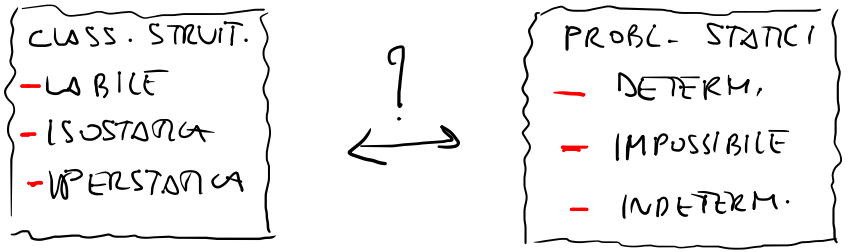
STAT. INDET.



STAT. DET.



STAT. IMPOSS.



LE POSSIBILI RELAZIONI SONO:

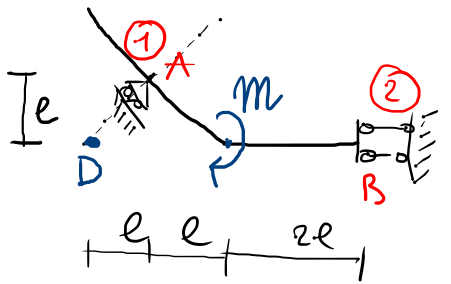
- STR. ISOSTATICA $(g = v = 3) \Longrightarrow$ P.S. STATICAM. DETERMINATO
- STR. IPERSTATICA $(g = 3 < v) \Longrightarrow$ P.S. " INDETERMINATO (∞ SOLUZ.)

questo perché: v: n° di INCOGNITE

g: n° di EQ. DISPONIBILI

Per le Str. LABILI NON SI PUÒ DIRE NULLA (LA NATURA DEL P.S. DIPENDE DAI CARICHI)

ES

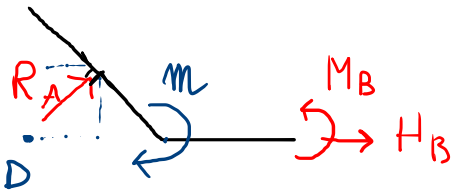


$g = 3$

$v = 3$

$\Delta = 3$

STR ISOST.

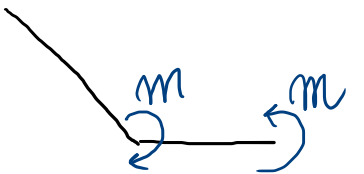


$\rightarrow$: $R_A \frac{1}{\sqrt{2}} + H_B = 0$ $\quad H_B = 0$

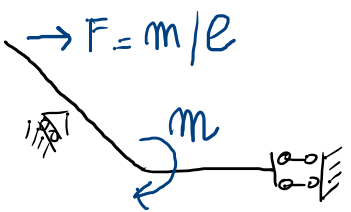
$+\uparrow$: $+R_A \frac{1}{\sqrt{2}} = 0$ $\quad R_A = 0$

$+\circlearrowleft_D$: $-m + M_B = 0$ $\quad M_B = m$

} R.S. STAT. DETERM.



S.C.L. EQUIL.



LO RISOLVO IN DUE MODI

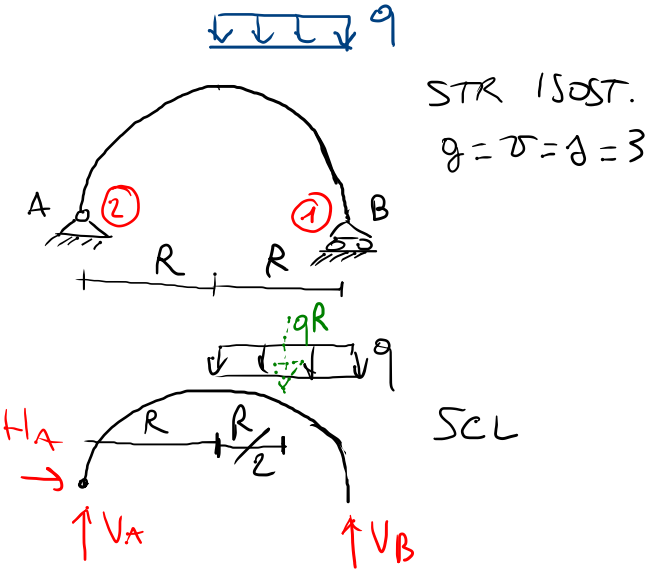
1- STUDIO IL NUOVO P.S. CON F, m

2- STUDIO SEPARATAMENTE LE DUE CONDIZ. DI CARICO

IN ENTRAMBI I CASI LA SOLUZ. È LA STESSA

LA SEPARAZ. IN PIÙ CONDIZ. DI CARICO SI CHIAMA "SOVRAPPOSIZ. DEGLI EFFETTI" ED È POSSIBILE PERCHÉ LE EQ. DI EQUILIBRIO SONO LINEARI NELLE INCOGNITE.

ES.



STR ISOST.
$g = v = a = 3$

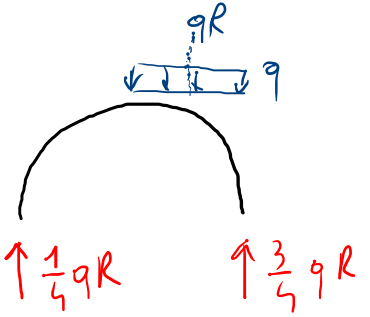
SCL

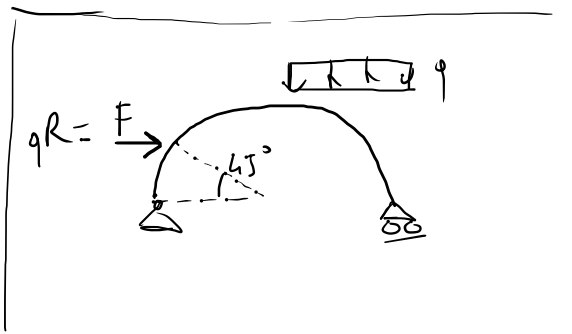
$\rightarrow : H_A = 0$

$+\uparrow : +V_A - qR + V_B = 0$

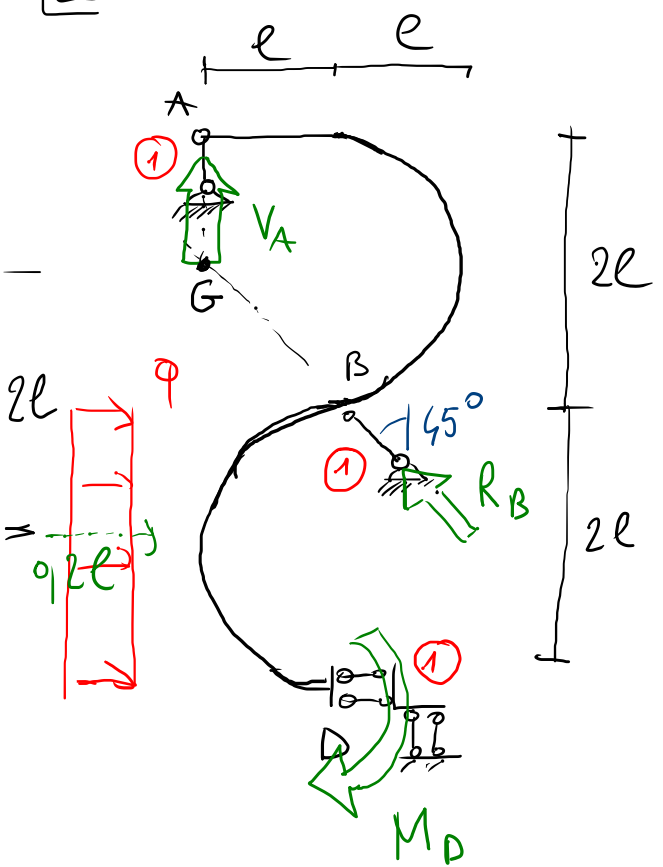
$+\circlearrowleft_A : -qR \cdot \frac{3}{2}R + V_B \, 2R = 0$

$$H_A = 0$$
$$V_A = \frac{1}{4} qR$$
$$V_B = \frac{3}{4} qR$$





ES



$g = 3$

$n = 3$

$s = 3$ (~~#~~ NESSUN C COMPATIBILE CON I 3 VINCOLI)

$$\rightarrow: \; -R_B \frac{1}{\sqrt{2}} + q\,2\ell = 0 \qquad R_B = 2\sqrt{2}\,q\ell$$

$$+\uparrow: \; V_A + R_B \frac{1}{\sqrt{2}} = 0 \qquad V_A = -\frac{R_B}{\sqrt{2}} = -2q\ell$$

$$+\circlearrowleft_G: \; +q2\ell \cdot 2\ell - M_D = 0 \qquad M_D = 4q\ell^2$$