

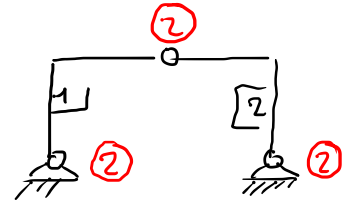
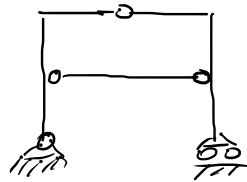
# METODI ALTERNATIVI A QUELLO GENERALE PER LA RISOLUZIONE DI STR. ISOSTATICHE ARTICOLATE

11/05/23

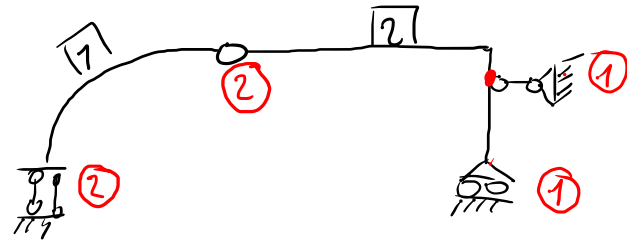
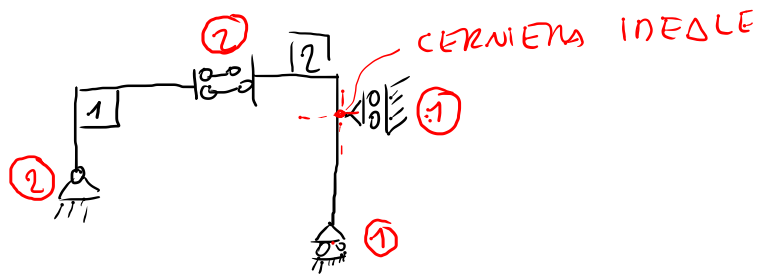
1) STRUTTURE CON GERARCHIA INTERNA (ES. LEZ. 20/04)

2) STRUTTURE ASSIMILABILI ALL'ARCO A 3 CERNIERE (ARCO A 3 CERNIERE "GENERALIZZATO")

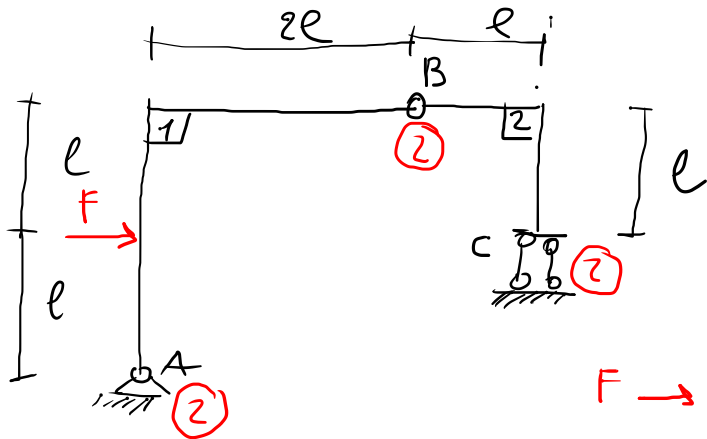
3) STRUTTURE "CHIUSE"



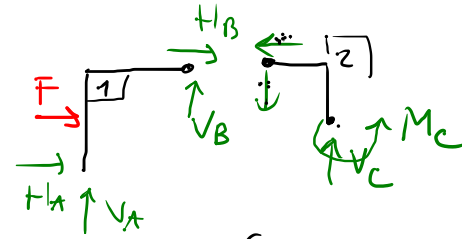
CI SOFFERMAMO SU 2), RICORDANDO CHE FANNO DI QUESTA COT. STRUTTURE DI QUESTO TIPO:



(ES : ARCO A 3 CERNIERE "GENERALIZZATO")



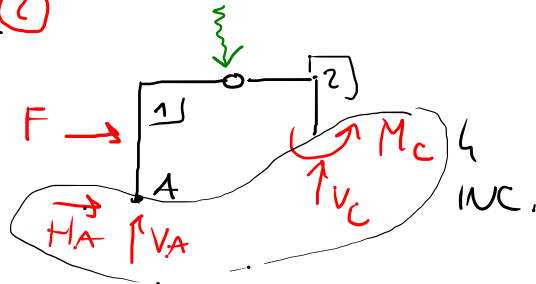
$$\left. \begin{aligned} g &= 6 \\ v &= 6 \\ s &= 6 \end{aligned} \right\} \text{ISOST.}$$



6 INCOGNITE

$H_A = -F$	$V_A = -\frac{F}{2}$
$V_C = +\frac{F}{2}$	$M_C = -\frac{F}{2}l$

Invece di risolvere il sist. di 6 EQ. in 6 INCOGNITE determino PRIMA LE REAZ. ESTERNE (4): SCRIVO SIST. 4 EQUAZ.



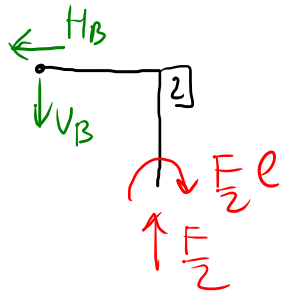
$$\left. \begin{aligned} \rightarrow \Sigma \text{[1] + [2]} \quad & +H_A + F = 0 \\ \uparrow \Sigma \text{[1] + [2]} \quad & +V_A + V_C = 0 \\ \curvearrowright \Sigma \text{[1] + [2]} \quad & -Fl + V_C 3l + M_C = 0 \\ \leftarrow \Sigma \text{[2]} \quad & V_C l + M_C = 0 \end{aligned} \right\}$$

E.C.S.

(EQUIL. GLOBALE DI [1] + [2])

EQ. AUSILIARIA (CERNIERA)

Adesso calcolo le 2 (NC)-interne

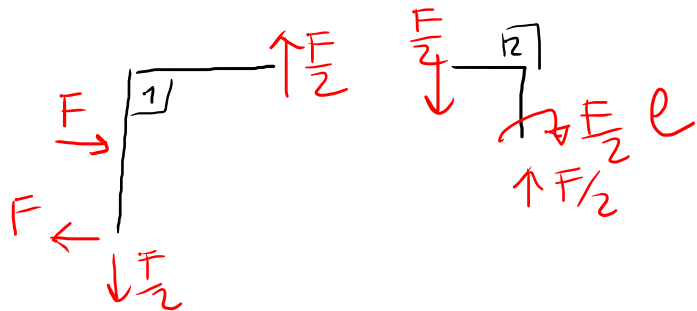


$$\begin{aligned} \rightarrow \square & \quad \left\{ \begin{array}{l} -H_B = 0 \\ -V_B + \frac{F}{2} = 0 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} H_B = 0 \\ V_B = \frac{F}{2} \end{array} \right. \\ \uparrow \square \end{aligned}$$

Per il caso [2]  
queste eq. sono  
le complementari  
all'eq. AUSILIARIA utilizzata  
nella pag. precedente.

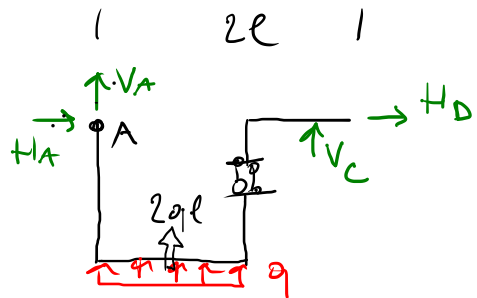
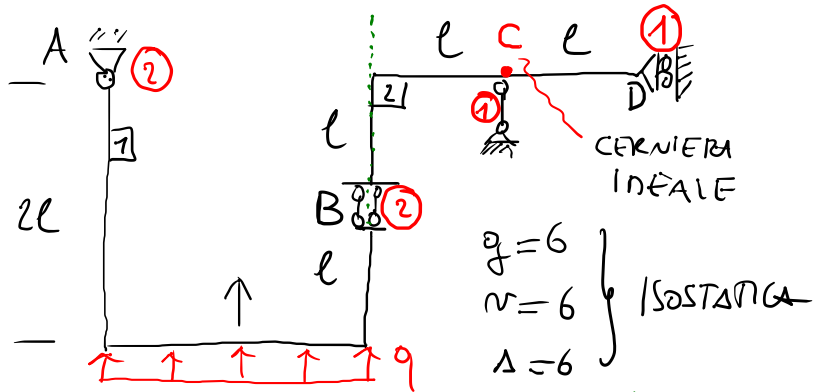
(In questa fase potremo determinare  $H_B, V_B$ )  
imponendo  $\rightarrow \square \quad \uparrow \square$

SCL EQUIL.

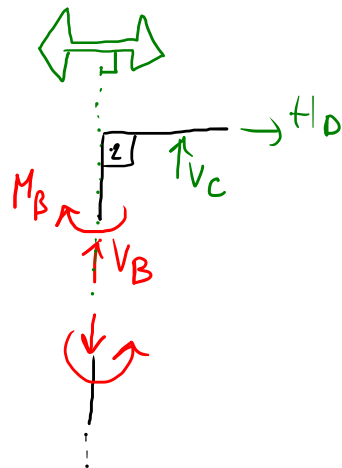


verifichiamo visivamente  
che TUTTI I C.RIGIDI siano  
in EQUILIBRIO !!

ES. (ARCO A 3 CERNIERE GEN.)



SCH. CALCOLO REAZ. ESTERNE



$$\begin{aligned}
 \rightarrow \text{[1+2]} &: H_A + H_D = 0 \\
 +\uparrow \text{ " } &: +V_A + 2ql + V_C = 0 \\
 \curvearrowleft \text{ A }^+ \text{ " } &: +2ql \cdot l + V_C 3l = 0
 \end{aligned}
 \left. \begin{array}{l} \text{E.C.S.} \\ \text{[1+2]} \end{array} \right\}$$

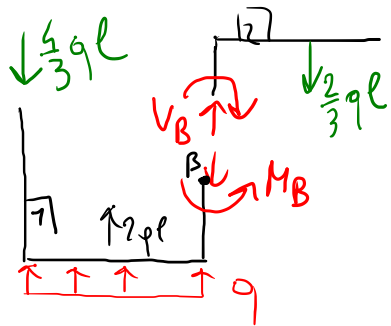

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$$\rightarrow \text{[2]} \quad H_D = 0$$

EQ. AUSILIARIA

Per il doppio pendolo interno l'EQ. AUSILIARIA è l'EQUILIBRIO ALLA TRASL. NELLA DIREZIONE  $\perp$  ALL'ASSE DEL VINCOLO.

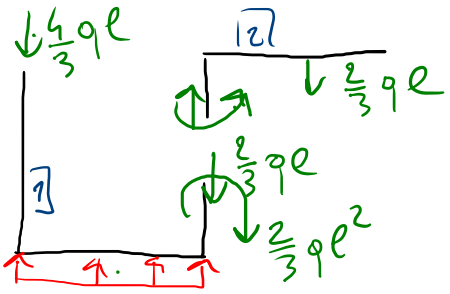
$$\begin{aligned}
 H_D = 0, \quad V_C &= -\frac{2}{3}ql, \quad V_A = -2ql - V_C \\
 &= -\left(2 + \frac{2}{3}\right)ql \\
 &= -\frac{4}{3}ql \\
 H_A &= 0
 \end{aligned}$$



CALCOLO  $V_B, M_B$  CON L'EQUIL. DEL CORPO 1

$$\begin{aligned}
 +\uparrow \text{ 1} : & -\frac{4}{3}q l + q l - V_B = 0 \\
 +\curvearrowright \text{ 1} : & \frac{4}{3}q l \cdot 2l - q l \cdot l + M_B = 0
 \end{aligned}
 \left. \vphantom{\begin{aligned} +\uparrow \text{ 1} : \\ +\curvearrowright \text{ 1} : \end{aligned}} \right\}
 \begin{aligned}
 V_B &= +\frac{2}{3}q l \\
 M_B &= \left(2 - \frac{8}{3}\right) q l^2 \\
 &= -\frac{2}{3}q l^2
 \end{aligned}$$

SCL EQUILIBRATO

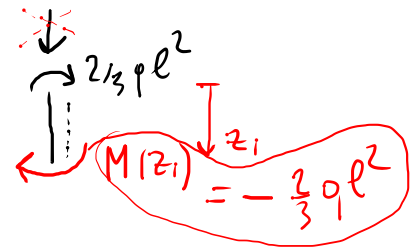
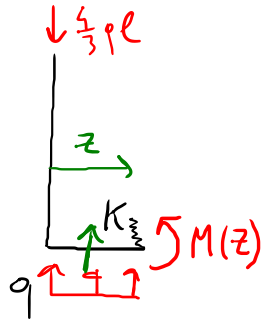
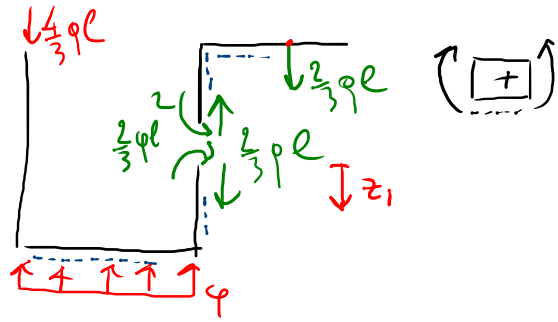


IN ALCUNI CASI LE EQUAZ. AUSILIARIE  
 VENGONO ANCHE INDICATE CON IL  
 NOME EQ. DI EQUILIBRIO PARZIALE

Per il corpo 1 non è immediato verificare l'EQUILIBRIO. Scrivere le equazioni di equilibrio per verificarlo.

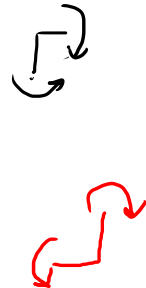
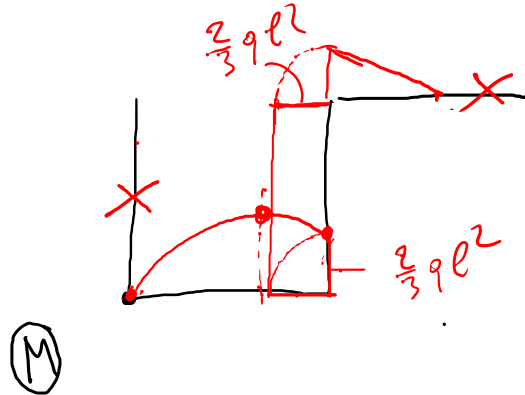
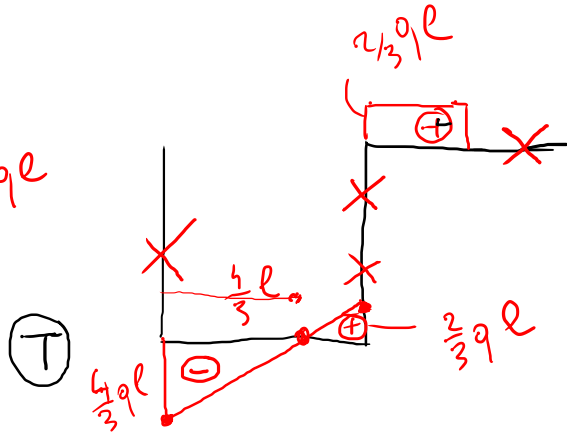
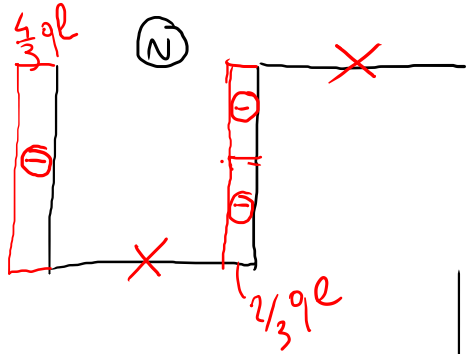
Le strutture chiuse saranno oggetto di attenzione più avanti.

DISEGNO DEI DIAGR. CDS delle strutture appena risolte.

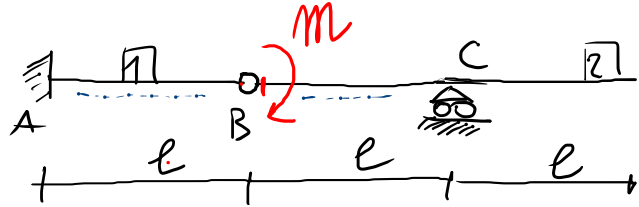


$$\sum K_i: \frac{4}{3} ql \cdot z - qz \cdot \frac{z}{2} + M(z) = 0$$

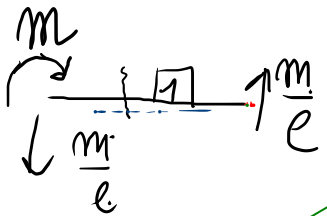
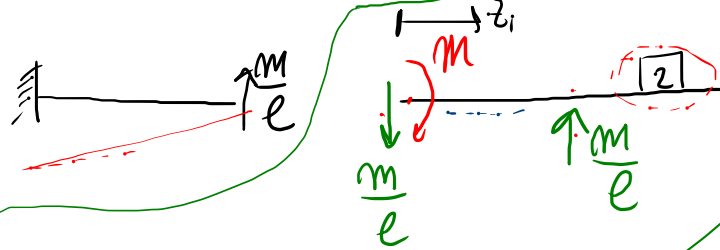
$$M(z) = q \frac{z^2}{2} - \frac{4}{3} qlz \quad \left\{ \begin{array}{l} M(0) = 0 \\ M(2l) = -\frac{2}{3} ql^2 \end{array} \right.$$



ES. RISOLVERE LA STR. ISOST. E DISEGNARE I DIAGRAMMI DELLE CDS (N, T, M)

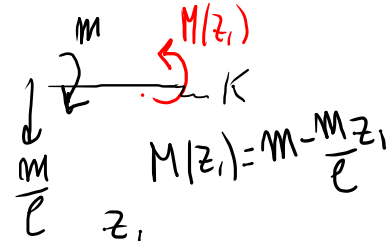
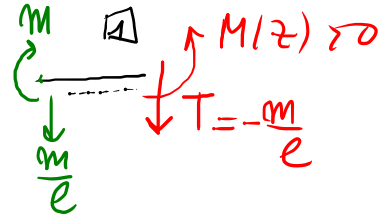
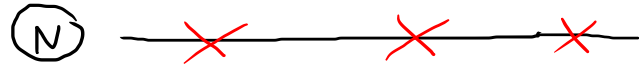
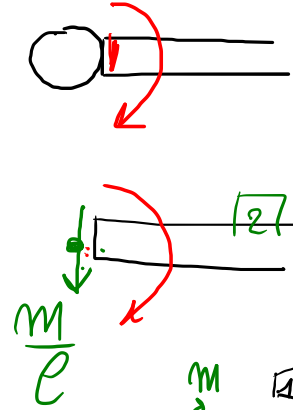
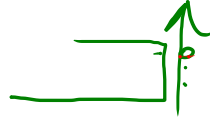


Risolvere [2] e poi [1]:



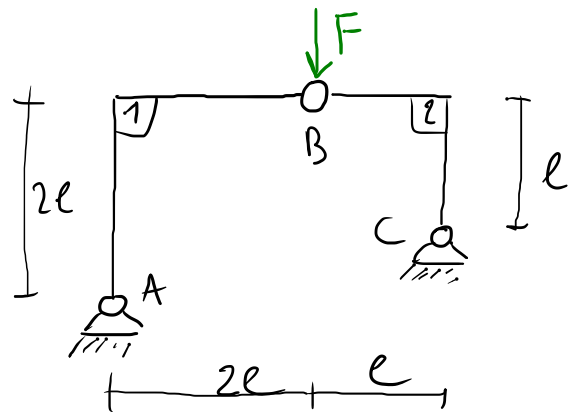
SCL EQUILIBRATO

STR. A SCHEMA GERARCHICO

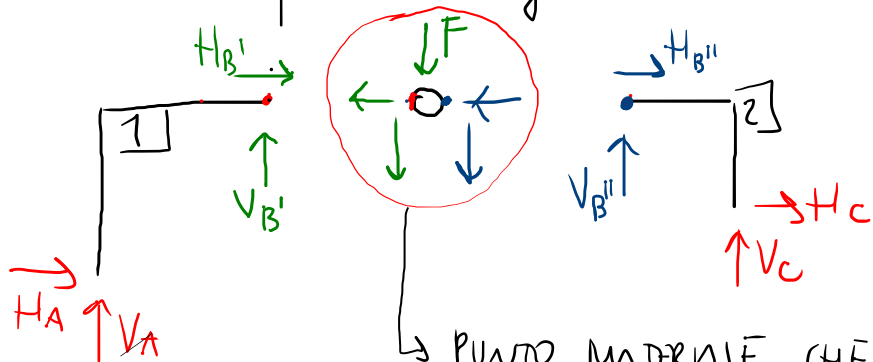


$$K) : -m + \frac{m}{e} z_1 + M(z_1) = 0$$

ES. ARCO A 3 CERNIERE CON CERNIERA INTERNA CARICATA.



Domande: quante incognite ho adesso? E quante equazioni?



Le incognite sono:

$$V_A, H_A, V_C, H_C, V_{B'}, H_{B'}, V_{B''}, H_{B''} \Rightarrow 8$$

NEL CASO IN ESAME  $\otimes$  SI SCRIVONO:

$$\left. \begin{aligned} \rightarrow: -H_{B'} - H_{B''} &= 0 \\ \uparrow: -V_{B'} - F - V_{B''} &= 0 \end{aligned} \right\}$$

PUNTO MATERIALE CHE VA EQUILIBRATO

- EQUL. ORIZZONTALE  $\otimes$
- EQUL. VERTICALE

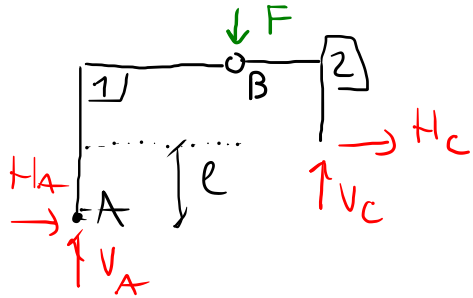
2 EQUAZ. CHE NON SONO DISPONIBILI QUANDO LA CERNIERA E' SCARICA  $\Rightarrow$  IN TOTALE ORA

ABBIAMO 8 EQUAZ. DI EQUILIBRIO

3	1
3	2
2	cerniere



Risolvo la struttura determinando PRIMA le reat. esterne (4)



$$\rightarrow : H_A + H_C = 0$$

$$\uparrow : V_A - F + V_C = 0$$

$$\curvearrow^+ : -F2l - H_C l + V_C 3l = 0$$

$$\curvearrow^- : V_C l + H_C l = 0$$

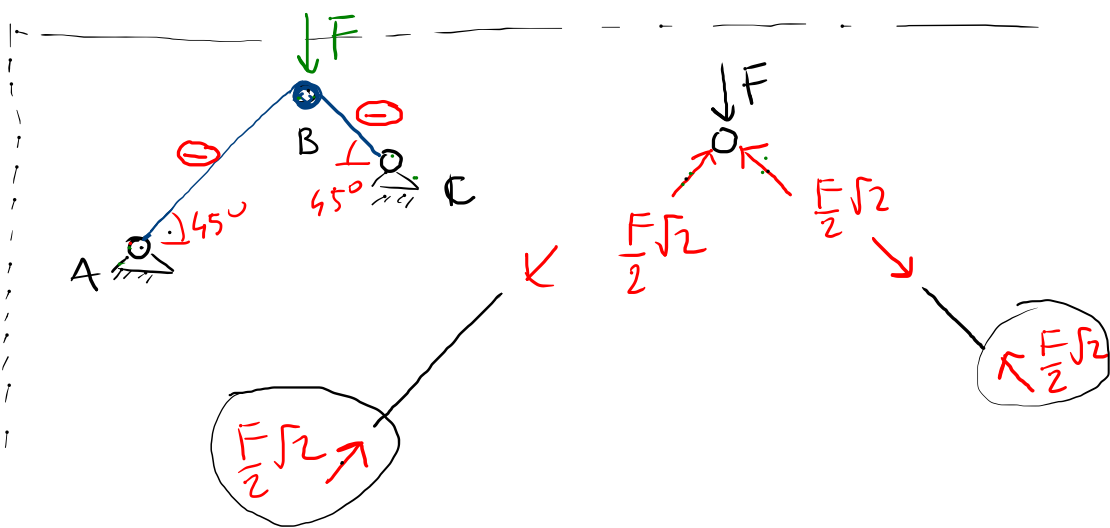
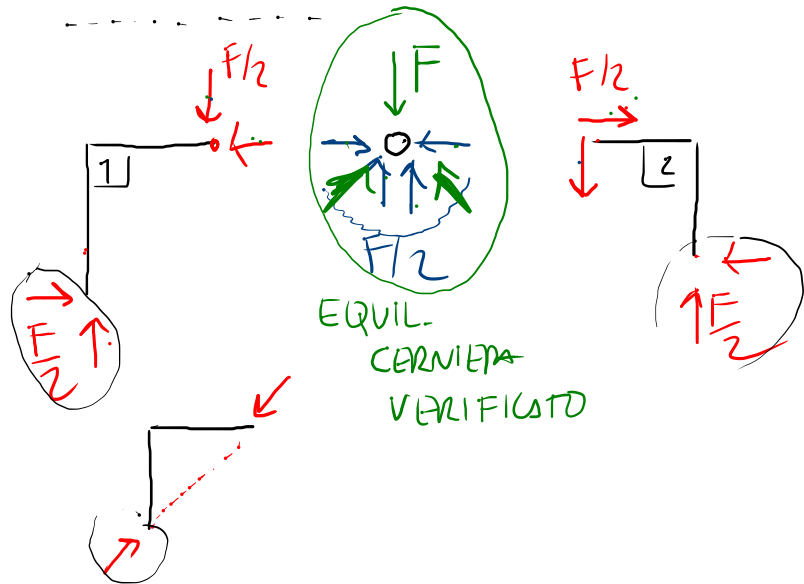
E.C.S. (GLOBALE)

EQ. AUSILIARIA (PARZIALE)

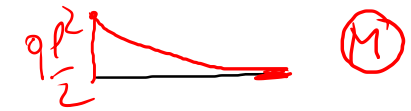
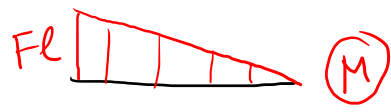
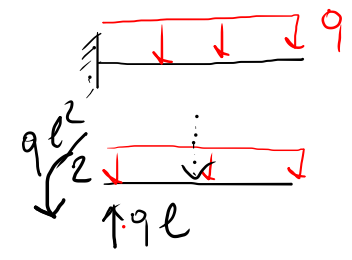
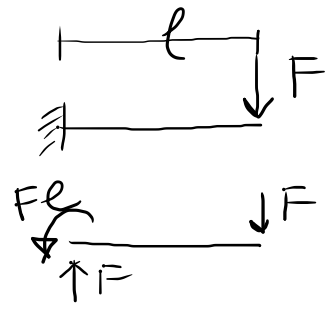
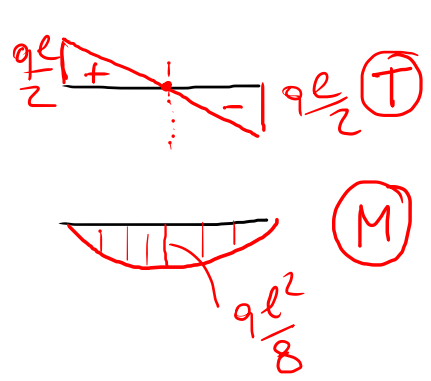
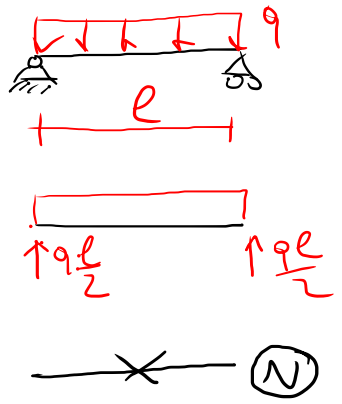
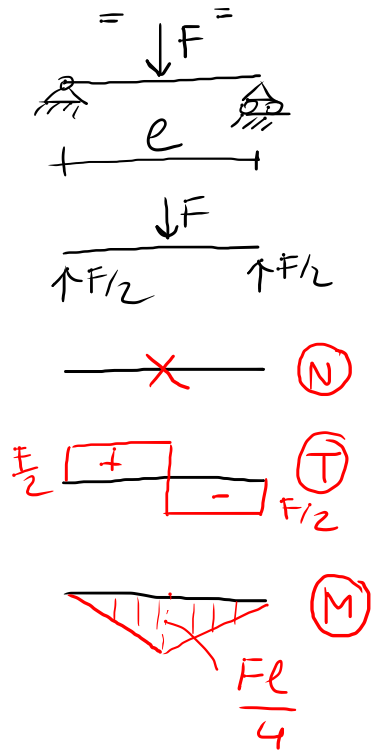
$$H_A = V_A = +\frac{F}{2}$$

$$V_C = +\frac{F}{2}$$

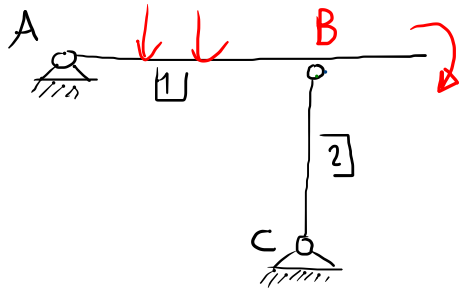
$$H_C = -\frac{F}{2}$$



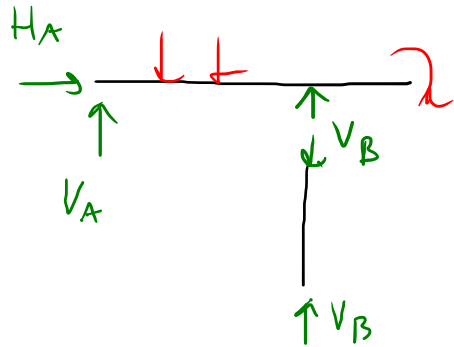
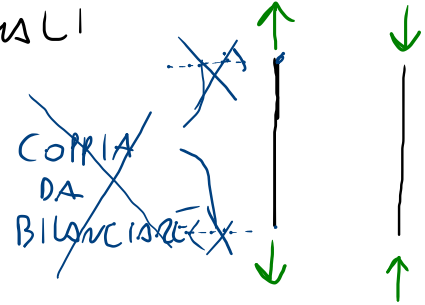
# SCHEMI STATICI DA SAPERE A MEMORIA



# CONCETTO DI "BIELLA ESTESA"



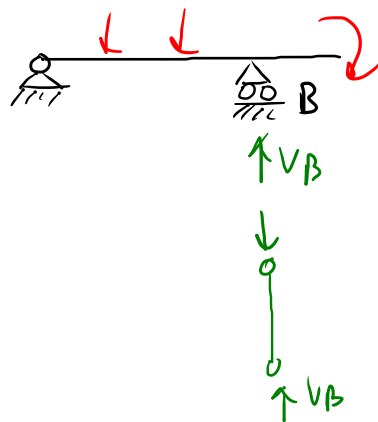
CORPO [2] è SOTTO CARICO e può essere sollecitato solo da FORZE NORMALI



3 INCOGNITE ESTERNE

$V_A, H_A, V_B$

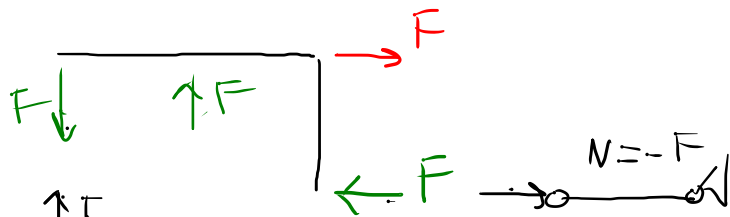
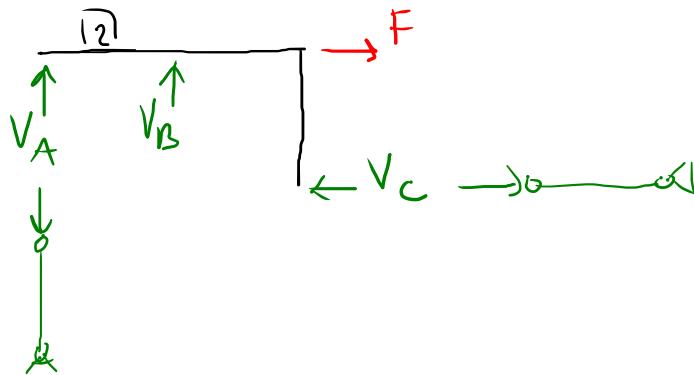
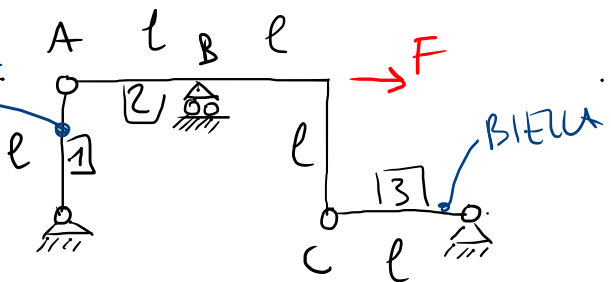
$\Rightarrow$  EQUILIBRIO CORPO [1]



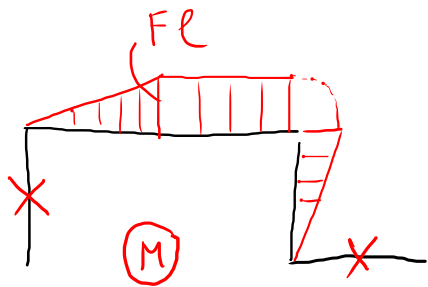
BIELLA ESTESA  $\left\{ \begin{array}{l} N \neq 0 \\ T, M = 0 \end{array} \right.$

LES

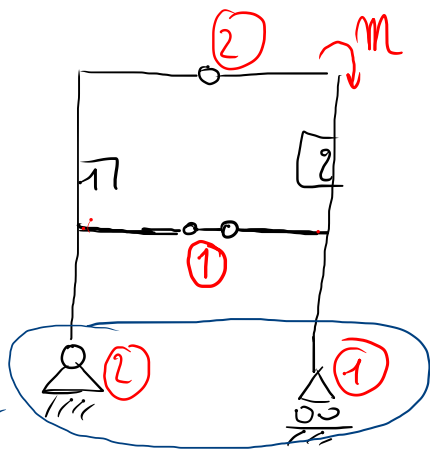
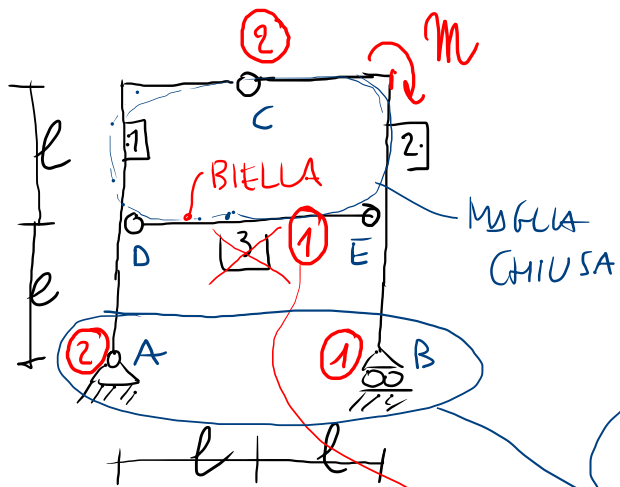
BIELWA



$N = +F$



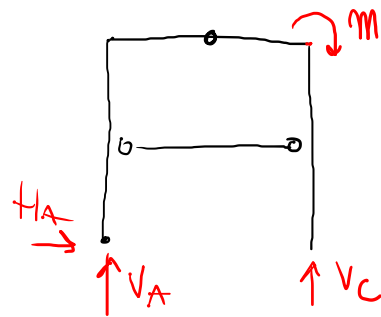
### 3) STRUTTURE ISOSTATICHE "CHIUSE"



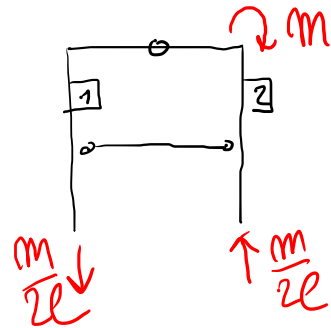
L'INSIEME DELLE  
REAZIONI ESTERNE  
È FORMATO DA  
3 INCOGNITE

$$V_A, H_A, V_B$$

⇒ LE DETERMINO CON  
LE E.C.S. PER TUTTA LA STRUTTURA



$$\begin{cases} \rightarrow : H_A = 0 \\ \uparrow : V_A + V_C = 0 \\ \curvearrowright^+ : V_C 2l - M = 0 \end{cases} \left. \begin{array}{l} H_A = 0 \\ V_A = -\frac{M}{2l} \\ V_C = \frac{M}{2l} \end{array} \right\}$$

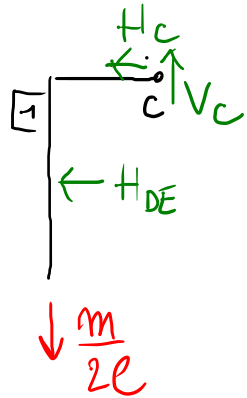


È ISOSTATICA?

$$g = 6 \text{ (2 C. RIGIDI)}$$

$$v = 2 + 1 + 2 + \textcircled{1} \text{ BIELLA ESTESA INTERNA} \\ = 6$$

Adesso passo alle seconde fase: calcolo reazioni interne (tramite corpo  $\boxed{1}$ )



Con 3 EQ. DI EQUILIBRIO calcolo le tre incognite interne.

$$\rightarrow : -H_{DE} - H_c = 0$$

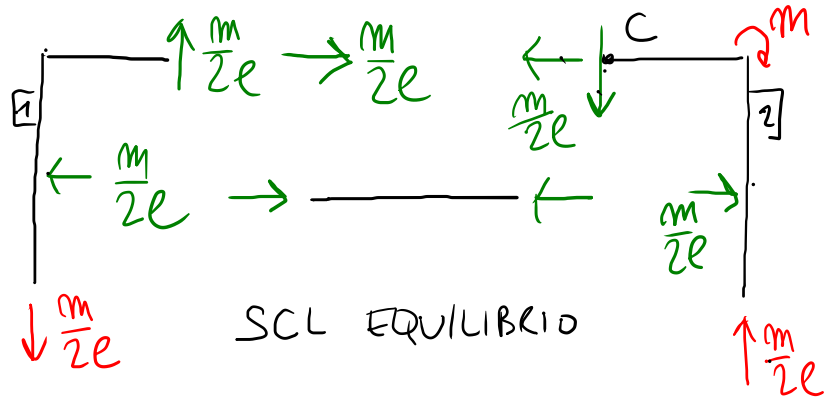
$$\uparrow : -\frac{m}{2l} + V_c = 0$$

$$\curvearrow^+ : \frac{m}{2l} l - H_{DE} l = 0$$

$$V_c = \frac{m}{2l}$$

$$H_c = -\frac{m}{2l}$$

$$H_{DE} = \frac{m}{2l}$$



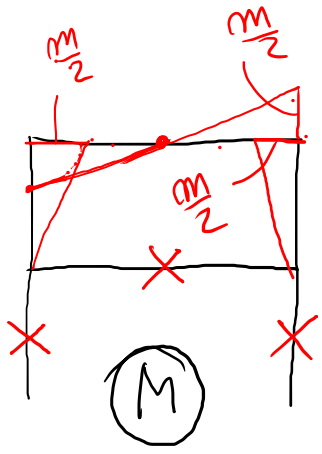
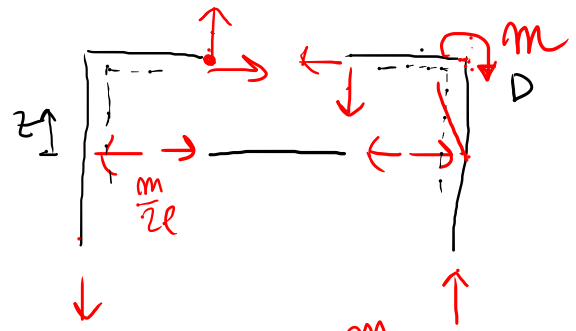
Terza fase: verificare che il corpo  $\boxed{2}$  sia in EQUILIBRIO.

$$\rightarrow : -\frac{m}{2l} + \frac{m}{2l} \stackrel{?}{=} 0 \quad \text{Sì}$$

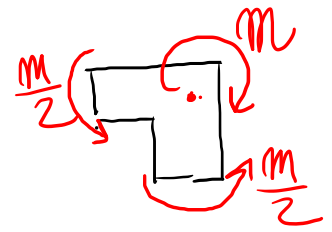
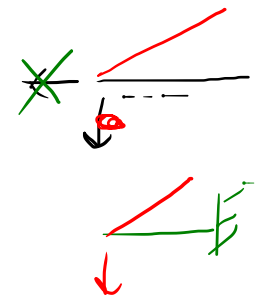
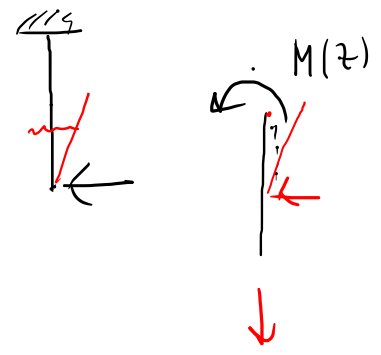
$$\uparrow : -\frac{m}{2l} + \frac{m}{2l} \stackrel{?}{=} 0 \quad \text{Sì}$$

$$\curvearrow^+ : -m + \frac{m}{2l} l + \frac{m}{2l} l \stackrel{?}{=} 0 \quad \text{Sì}$$

# DISEGNAMO IL DIAGR DEL MOMENTO

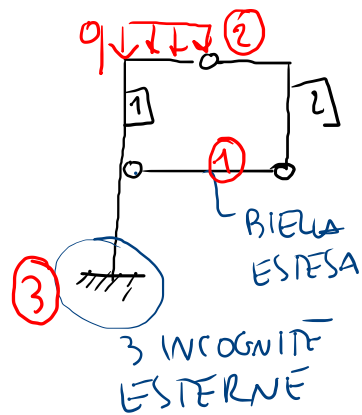


VERIFICA  
EQUILIBRIO  
MOMENTI IN  
D

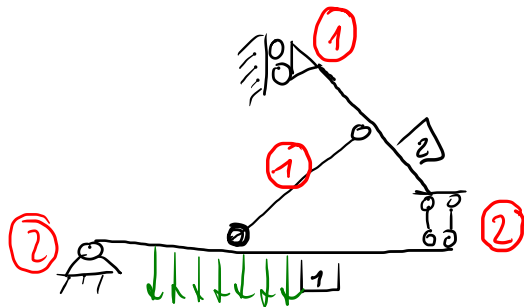
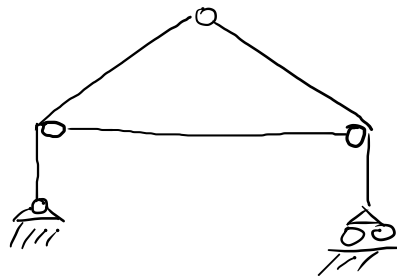
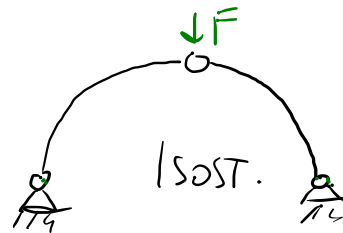
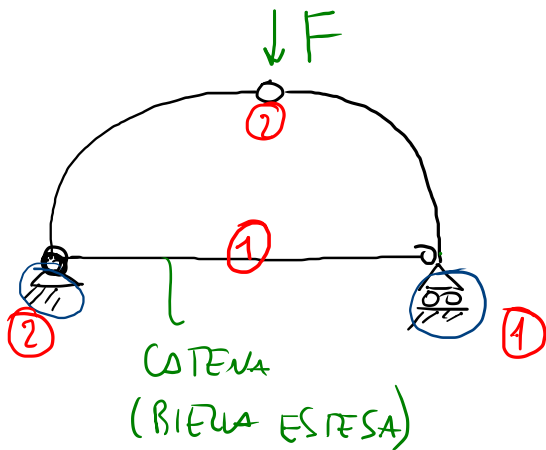


VERIFICA  
SODDISFATTA

STRUTTURE CHIUSE ANALOGHE A QUELLA STUDIATA IN PRECEDENZA

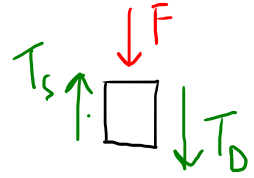
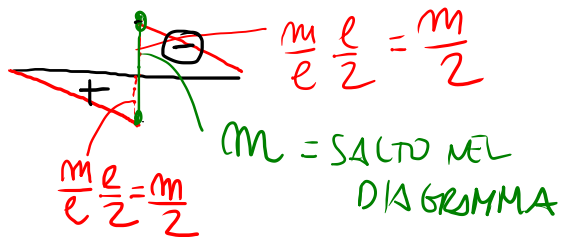
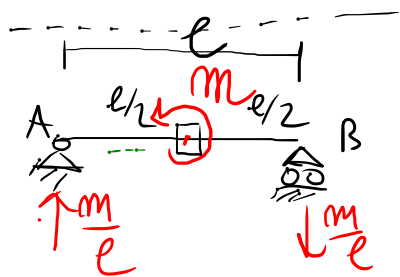
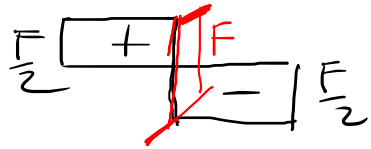


$g = 6$   
 $v = 3 + 2 + 1 = 6$   
 $\delta = 6$



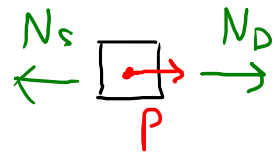


# OSSERVAZ. SULLA DISCONTINUITA' NEI DIAGRAMMI DELLE CDS



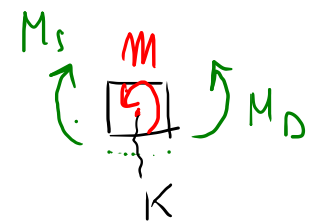
$$\uparrow: T_s - F - T_D = 0 \Rightarrow T_D - T_s = -F$$

SALTO NEL DIAGRAMMA DEL TAGLIO =  $-F$



$$\rightarrow: -N_s + P + N_D = 0 \Rightarrow N_D - N_s = -P$$

SALTO DIAGR.  $(N) = -P$



$$\curvearrowright: -M_s + m + M_D = 0 \Rightarrow M_D - M_s = -m$$

SALTO DIAGR.  $(M) = -m$