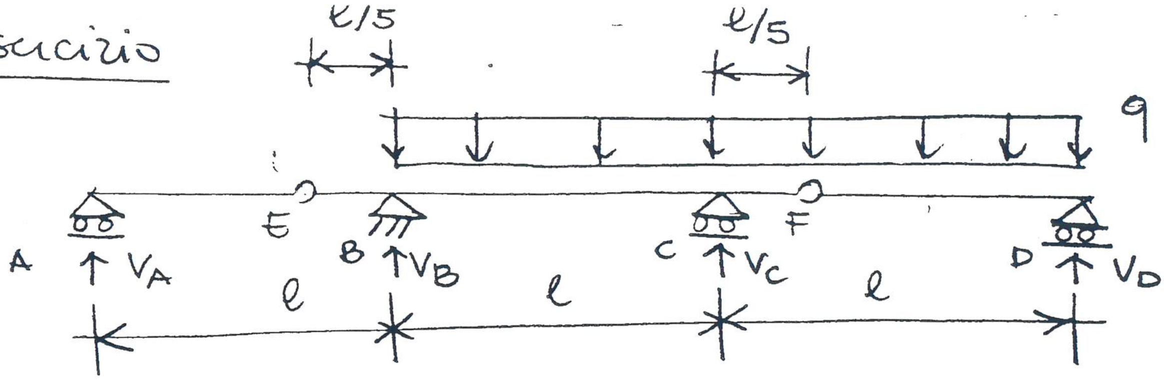
Esercizio



e.e.g:

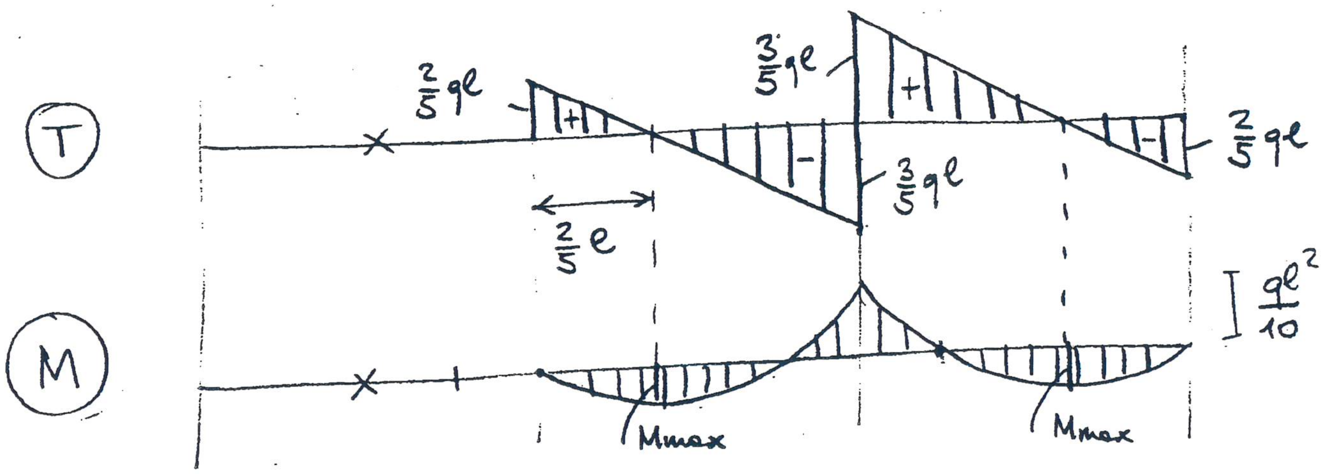
$$
Ⓒ \begin{cases} H_B = 0 \\ \cancel{V_A} + V_B + V_C + V_D = 2q\ell \\ V_D \ell - V_B \ell - \cancel{V_A 2\ell} = 0 \end{cases}
$$

e e a

$$
\begin{cases} M_{EA} = -V_A \frac{4}{5}\ell = 0 & \Rightarrow V_A = 0 \\ M_{FD} = V_D \frac{4}{5}\ell - \frac{q}{2}\left(\frac{4}{5}\ell\right)^2 = 0 & \Rightarrow V_D = \frac{2}{5}q\ell = V_B \end{cases}
$$

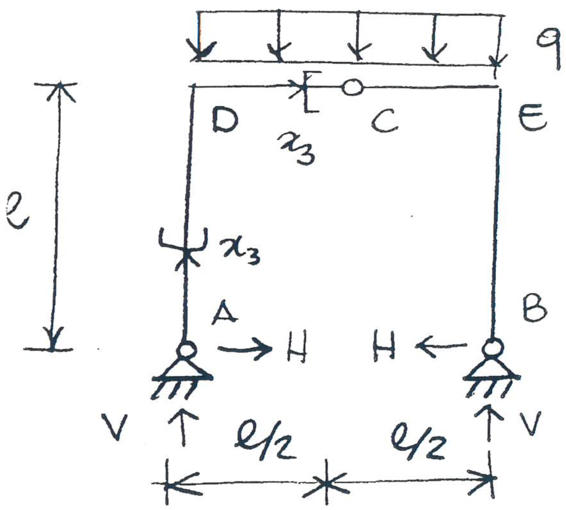
per cui:

$$
V_C = 2q\ell - \frac{4}{5}q\ell = \frac{6}{5}q\ell
$$



$$
M_{max} = \frac{4}{25}q\ell^2 - \frac{q}{2}\frac{4}{25}\ell^2 = \frac{2}{25}q\ell^2
$$

Portale a 3 cerniere



Struttura simmetrica

carico simmetrico

$$V_A = V_B = V$$

$$H_A = H_B = H$$

equilibrio globale:

$$2V - q\ell = 0 \quad \Rightarrow V = \frac{q\ell}{2}$$

le altre due eq. e.g. sono identicamente verificate, ho bisogno di una eq. ausiliaria per determinare H

$$M_{CA} = H\ell - V\frac{\ell}{2} + q\frac{\ell}{2}\cdot\frac{\ell}{4} = 0$$

da cui

$$H = \frac{V}{2} - \frac{q\ell}{8} = \frac{q\ell}{8}$$

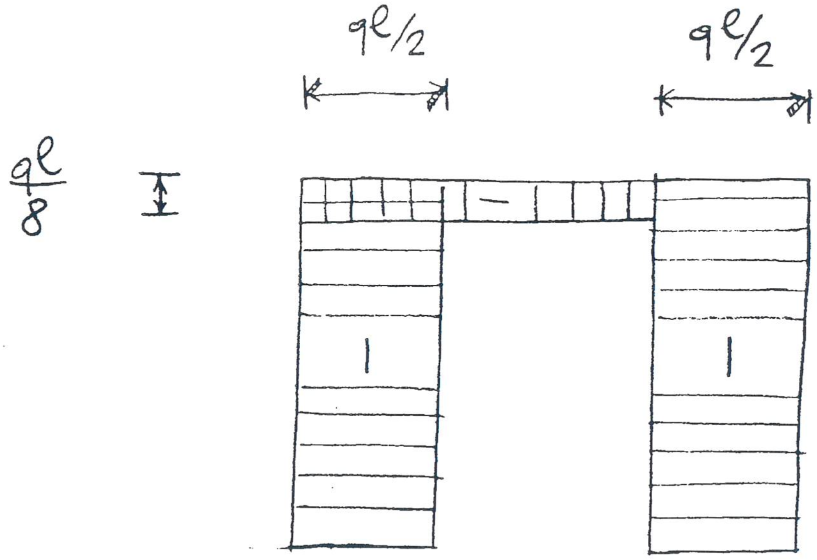
AD : $0 \leq x_3 \leq \ell$

$$\begin{cases} N = -\frac{q\ell}{2} \\ T = -\frac{q\ell}{8} \\ M = -\frac{q\ell}{8}x_3 \end{cases} \qquad \begin{cases} M_A = 0 \\ M_D = -\frac{q\ell^2}{8} \end{cases}$$

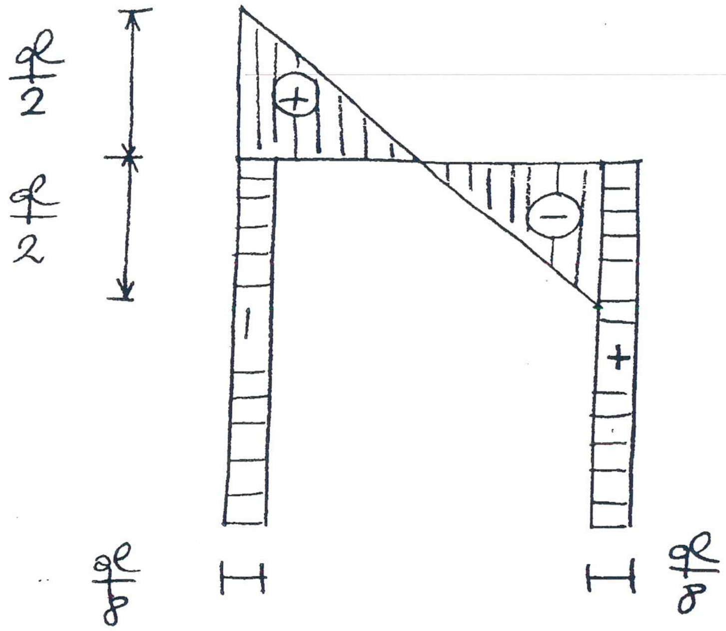
DE : $0 \leq x_3 \leq \ell$

$$\begin{cases} N = -\frac{q\ell}{8} \\ T = \frac{q\ell}{2} - qx_3 \\ M = \frac{q\ell}{2}x_3 - \frac{qx_3^2}{2} - \frac{q\ell^2}{8} \end{cases} \qquad \begin{cases} T_D = \frac{q\ell}{2} \\ T_E = -\frac{q\ell}{2} \end{cases} \qquad \begin{cases} M_A = \frac{q\ell^2}{8} \\ M_E = - '' \end{cases}$$

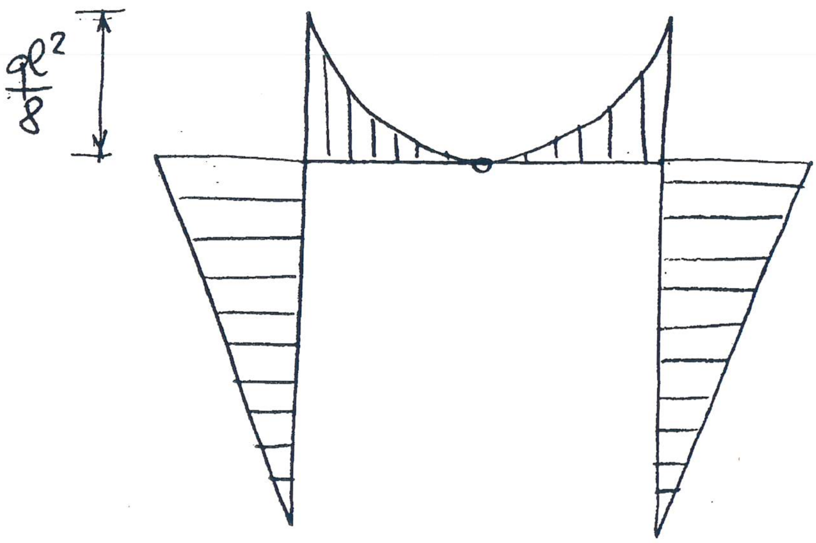
le azioni interne sono continue nel tratto DE poiché non agiscono forze o coppie concentrate.



N

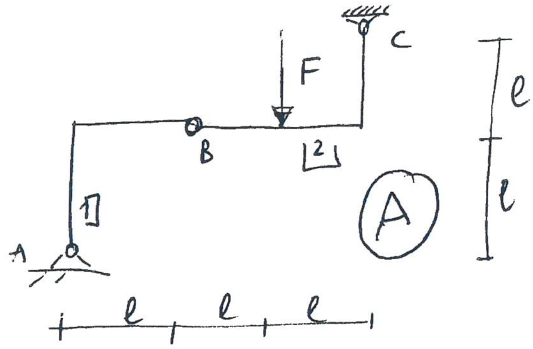


T



M

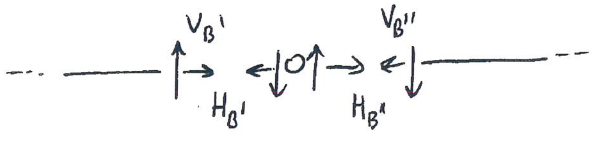
NOTE SULLE CERNIERE CARICATE CON FORZE CONCENTRATE



In questo caso le incognite statiche sono 6: H_A, V_A, H_C, V_C V_B, H_B.

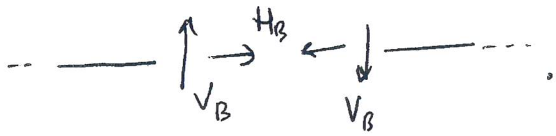
Le equaz. a disposizione sono 6: 3 per il corpo [1], 3 per il corpo [2].

In questo schema si dà per scontato che la cerniera B sia in equilibrio:

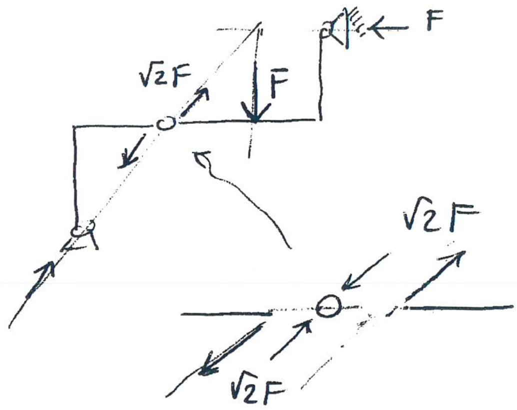


l'eq. della cerniera:

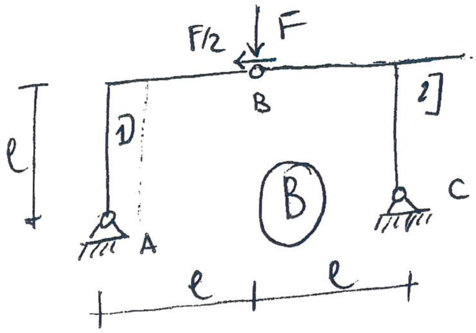
$H_{B''} = H_{B'}$; $V_{B''} = V_{B'}$ da cui



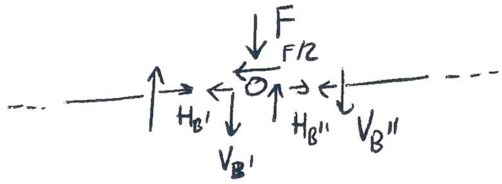
Soluz. esercizio precedente



Quando la cerniera è caricata $H_{B'} \neq H_{B''}$; $V_{B'} \neq V_{B''}$: le incognite del problema diventano 8:



$V_A, H_A, V_C, H_C, H_{B'}, V_{B'}, H_{B''}, V_{B''}$



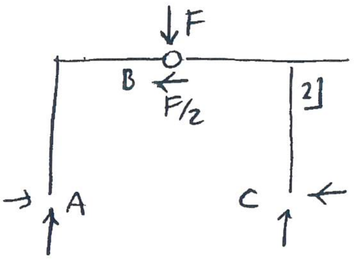
$\Longrightarrow$

Le equazioni a disposizione sono però 8:

3 C.R. $\boxed{1}$, 3 C.R $\boxed{2}$, 2 equilibrio cerniera:

$\xrightarrow{+}: \; H_{B'} + \frac{F}{2} - H_{B''} = 0$; $\quad +\uparrow: \; -V_{B'} - F + V_{B''} = 0$

Per risolvere l'esercizio si possono scrivere 3 eq. eq. globale + 1 ausiliaria per ottenere V_A, H_A, V_C, H_C:



$\xrightarrow{+}: \; H_A - F/2 - H_C = 0$

$+\uparrow: \; V_A + V_C - F = 0$

$\overset{\curvearrowleft}{A}: \; V_C \, 2\cancel{\ell} + \frac{F}{2}\cancel{\ell} - F\cancel{\ell} = 0$

$M_B^{\boxed{2}}: \; V_C \ell - H_C \ell = 0$

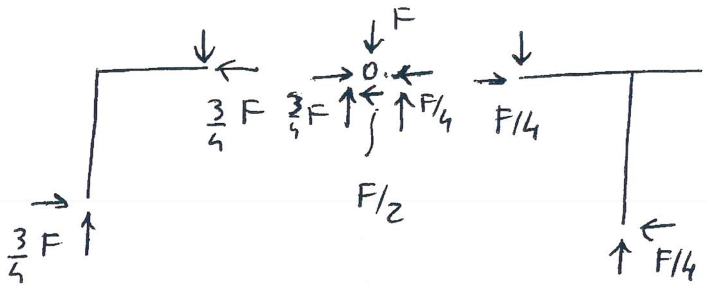
$H_A = 3/4 \, F$

$V_A = F - V_C = \frac{3}{4} F$

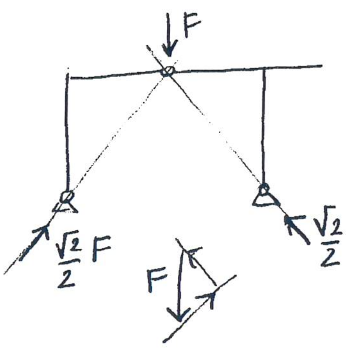
$V_C = \frac{F}{4}$

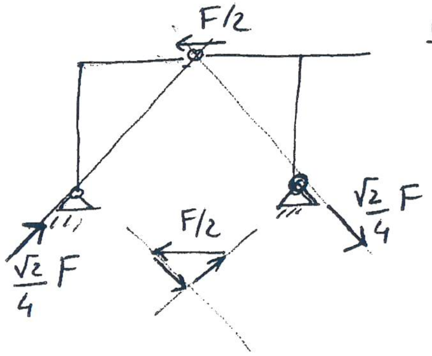
$H_C = \frac{F}{4}$

Poi si equilibrano i corpi $\boxed{1}$ e $\boxed{2}$ e si verifica che la cerniera sia anch'essa in equilibrio:



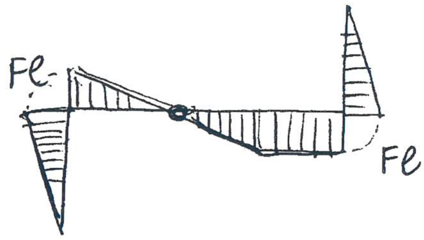
Graficamente:

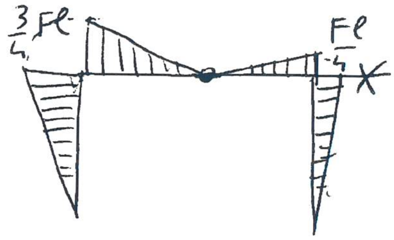




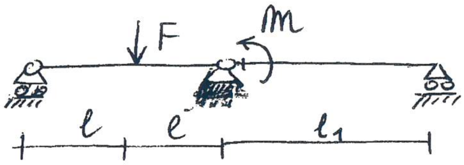
La somma dei 2 schemi corrisponde al calcolo analitico.

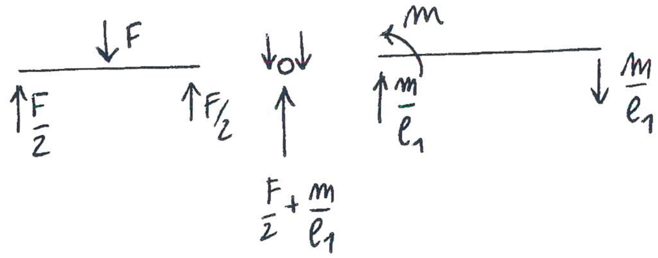
Diagrammi dei momenti delle strutture Ⓐ e Ⓑ

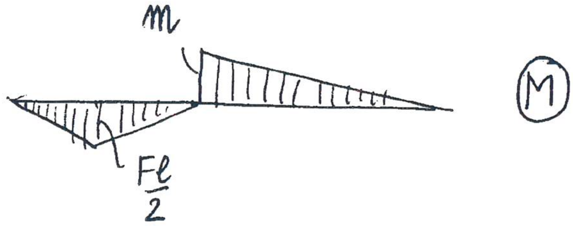


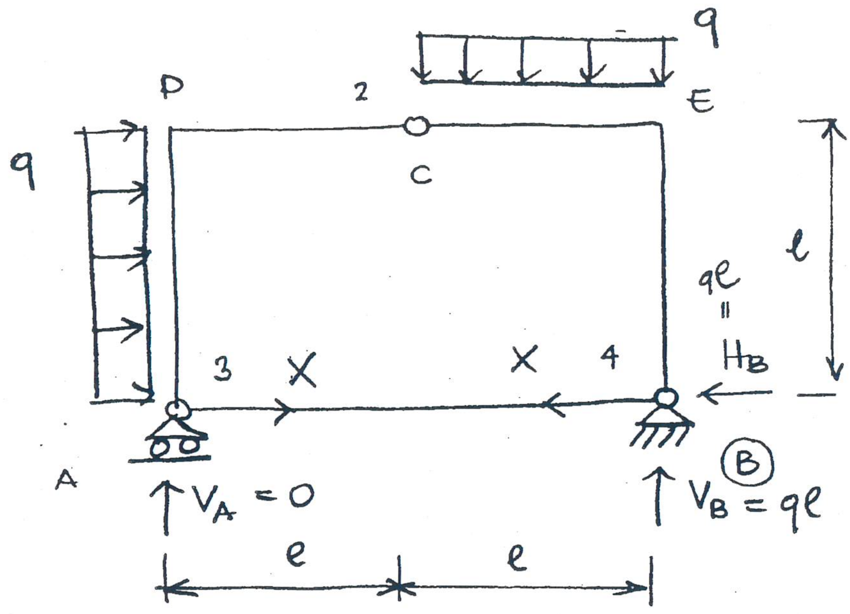


Altro esercizio semplificativo: trave gerber con vincolo interno/esterno:









$g = 3 \cdot 3 = 9$

$m = 3+4+2 = 9 = g$

<u>struttura isostatica</u>

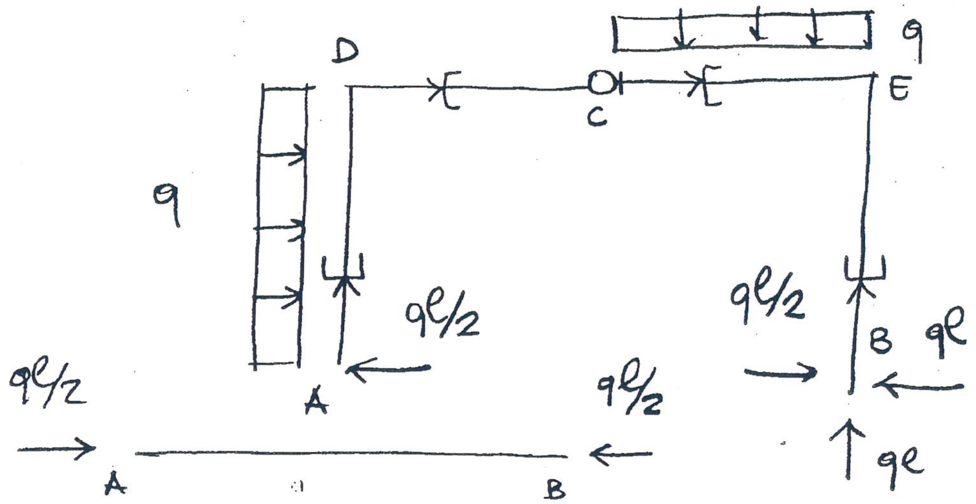
Troviamo le reazioni vincolari

$$\begin{cases} -H_B + ql = 0 & H_B = ql \\ V_A + V_B - ql = 0 & V_B = ql \\ -V_A\, 2l + \dfrac{ql^2}{2} + \dfrac{ql^2}{2} = 0 & \Rightarrow \quad V_A = 0 \end{cases}$$

Apriamo la struttura, sostituendo al tratto AB l'azione interna incognita X. Per determinare X scriviamo un equazione ausiliaria per il tratto CA, ovvero $M_{CA} = 0$.

$$Xl + \frac{ql^2}{2} = 0 \quad \Rightarrow \quad X = -\frac{ql}{2}$$

(verso opposto a quello ipotizzato)



AB $(0 \leq x_3 \leq 2\ell)$

$$\begin{cases} N = -\dfrac{q\ell}{2} \\ T = M = 0 \end{cases}$$

AD $(0 \leq x_3 \leq \ell)$

$$\begin{cases} N = 0 \\ T = \dfrac{q\ell}{2} - q x_3 \\ M = \dfrac{q\ell}{2} x_3 - \dfrac{q x_3^2}{2} \end{cases}$$

DC $(0 \leq x_3 \leq \ell)$

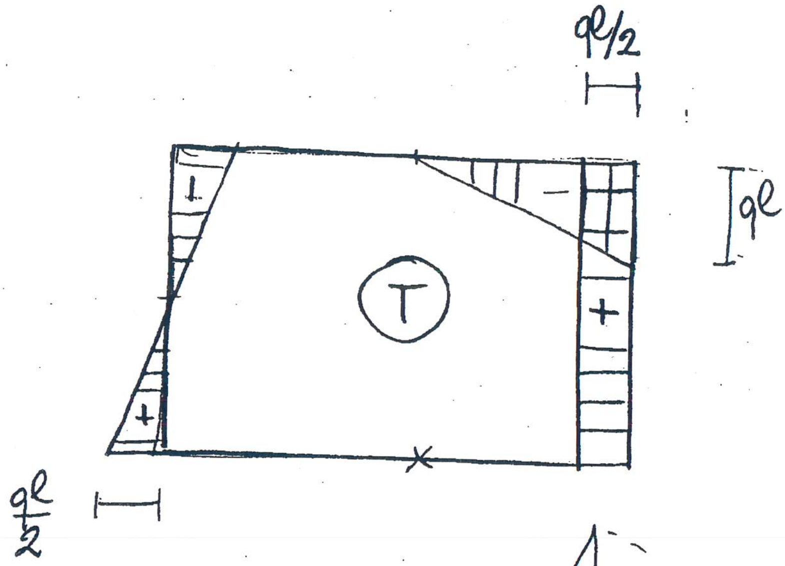
$$\begin{cases} N = -\dfrac{q\ell}{2} \\ T = 0 \\ M = \dfrac{q\ell}{2}\ell - q\ell\dfrac{\ell}{2} = 0 \end{cases}$$

CE $(0 \leq x_3 \leq \ell)$

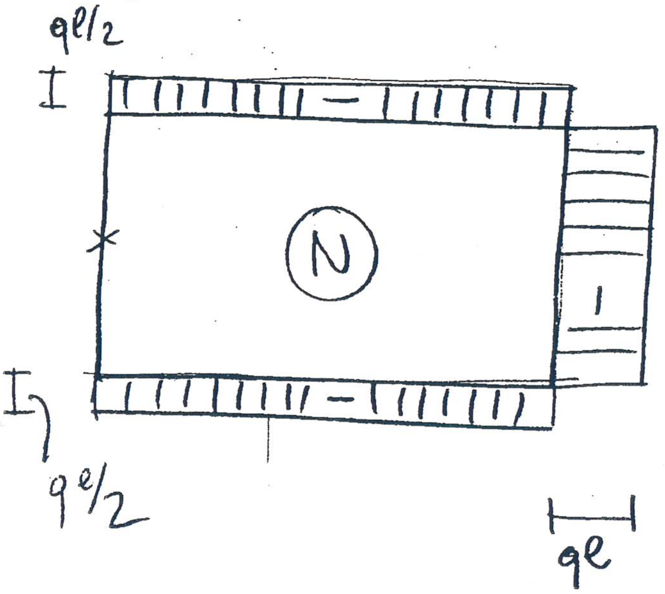
$$\begin{cases} N = -\dfrac{q\ell}{2} \\ T = -q x_3 \\ M = -q\dfrac{x_3^2}{2} \end{cases}$$

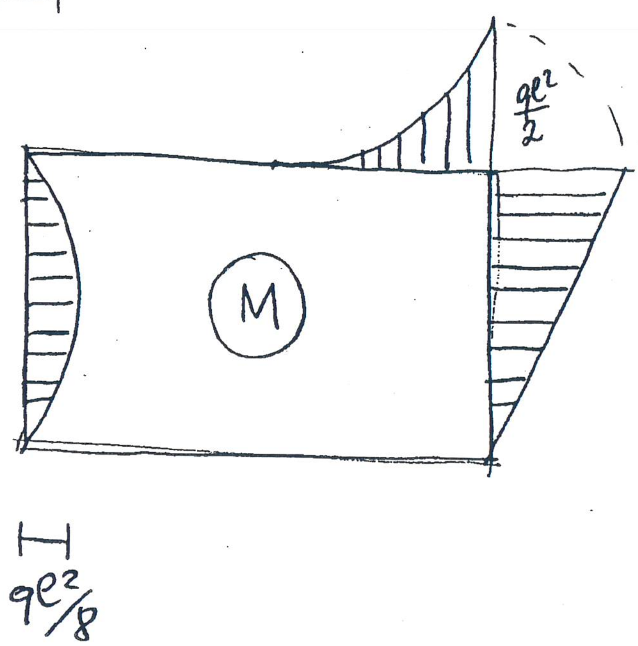
BE $(0 \leq x_3 \leq \ell)$

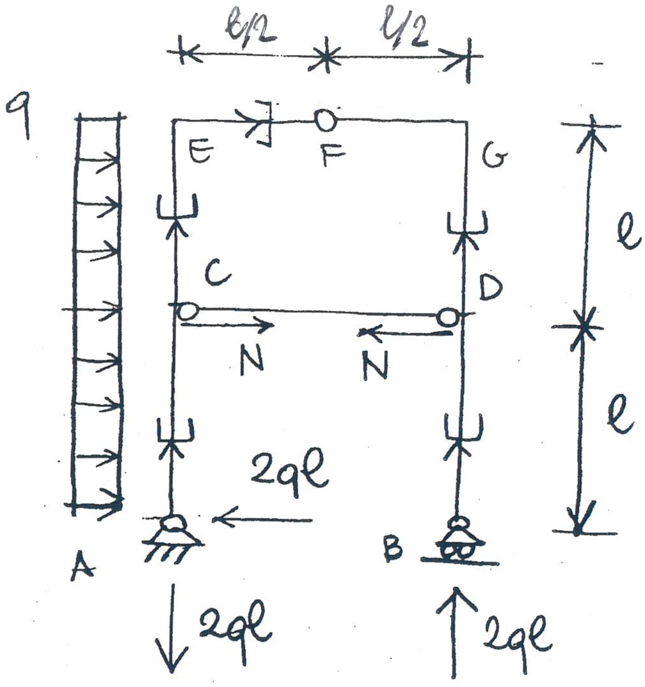
$$\begin{cases} N = -q\ell \\ T = q\ell/2 \\ M = \dfrac{q\ell}{2} x \end{cases}$$



diagrammi:







Per aprire la struttura è conveniente sostituire la biella con la forza N da essa trasmessa alla struttura (1 sola incognita)

e.e.a:

$$M_{FB} = 2q\frac{l^{\cancel{2}}}{2} - N\cancel{l} = 0 \qquad N = ql$$

AC $(0 \leq x_3 \leq l)$:

$$\begin{cases} N = 2ql \\ T = 2ql - qx_3 \\ M = 2qlx_3 - \dfrac{qx_3^2}{2} \end{cases}$$

CE $(0 \leq x_3 \leq l)$

$$\begin{cases} N = 2ql \\ T = -qx_3 \\ M = 2ql\left(l + \dfrac{x_3}{2}\right) - \cancel{qlx_3} - \dfrac{q(l+x_3)}{2} \end{cases}$$

$$= \frac{3}{2}ql^2 - \frac{qx_3^2}{2}$$

EG $(0 \leq x_3 \leq l)$:

$$\begin{cases} N = -ql \\ T = -2ql \\ M = 2ql(l - x_3) - ql^2 \\ \quad = ql^2 - 2qlx_3 \end{cases}$$

DG $(0 \leq x_3 \leq l)$

$$\begin{cases} N = -2ql \\ T = ql \\ M = qlx_3 \end{cases}$$

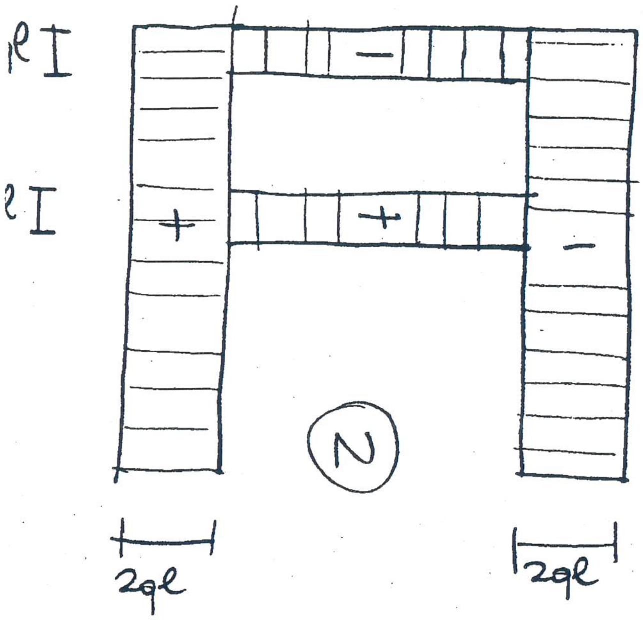
BD

$$\begin{cases} N = -2ql \\ M = T = 0 \end{cases}$$

CD

$$\begin{cases} N = ql \\ M = T = 0 \end{cases}$$

Diagrammi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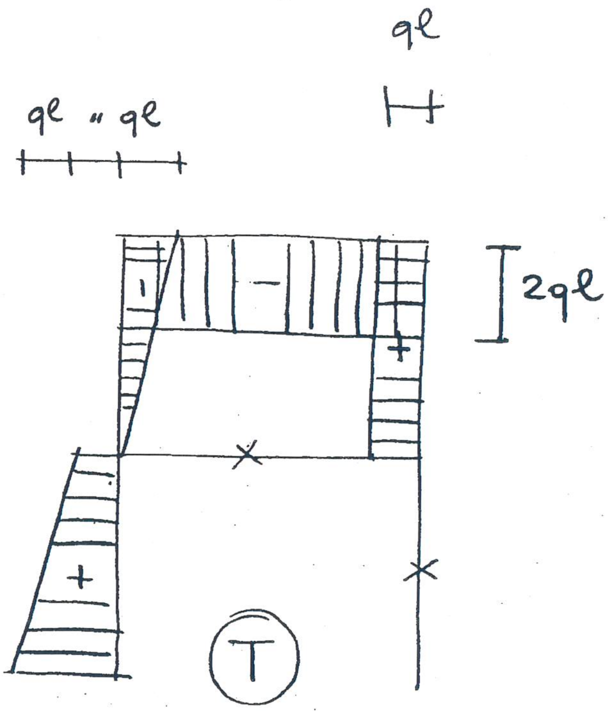
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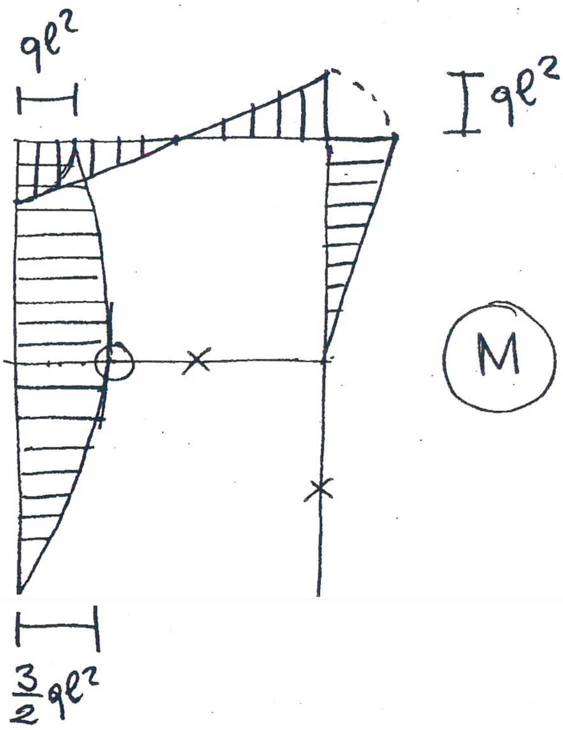
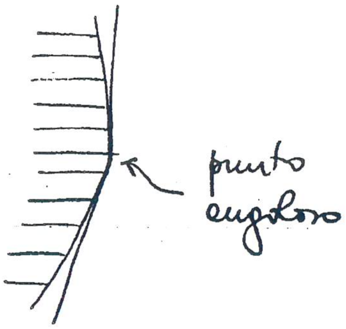

$q\ell^2$
$q\ell^2$
M
$\frac{3}{2}q\ell^2$


punto angoloso