Fisica Generale 1 – 018IN

Unità e vettori

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Grandezze fisiche

* Definizione operativa

Una grandezza fisica è definita solo dalle operazioni necessarie per misurarla

* Le grandezze fisiche si esprimono in termini di campione, chiamato unità

Grandezze fondamentali (solo 3 per noi)

* tempo: Unità: secondo (s)

da 1967: 9 192 631 770 volte il periodo di oscillazione di una risonanza del atom ^{133}Ce

* lunghezza: Unità metro (m)

$1/299\,792\,458$ della distanza percorsa dalla luce in 1s.

Dimensioni e unità di misura

* massa : Unità : chilogrammo (kg)

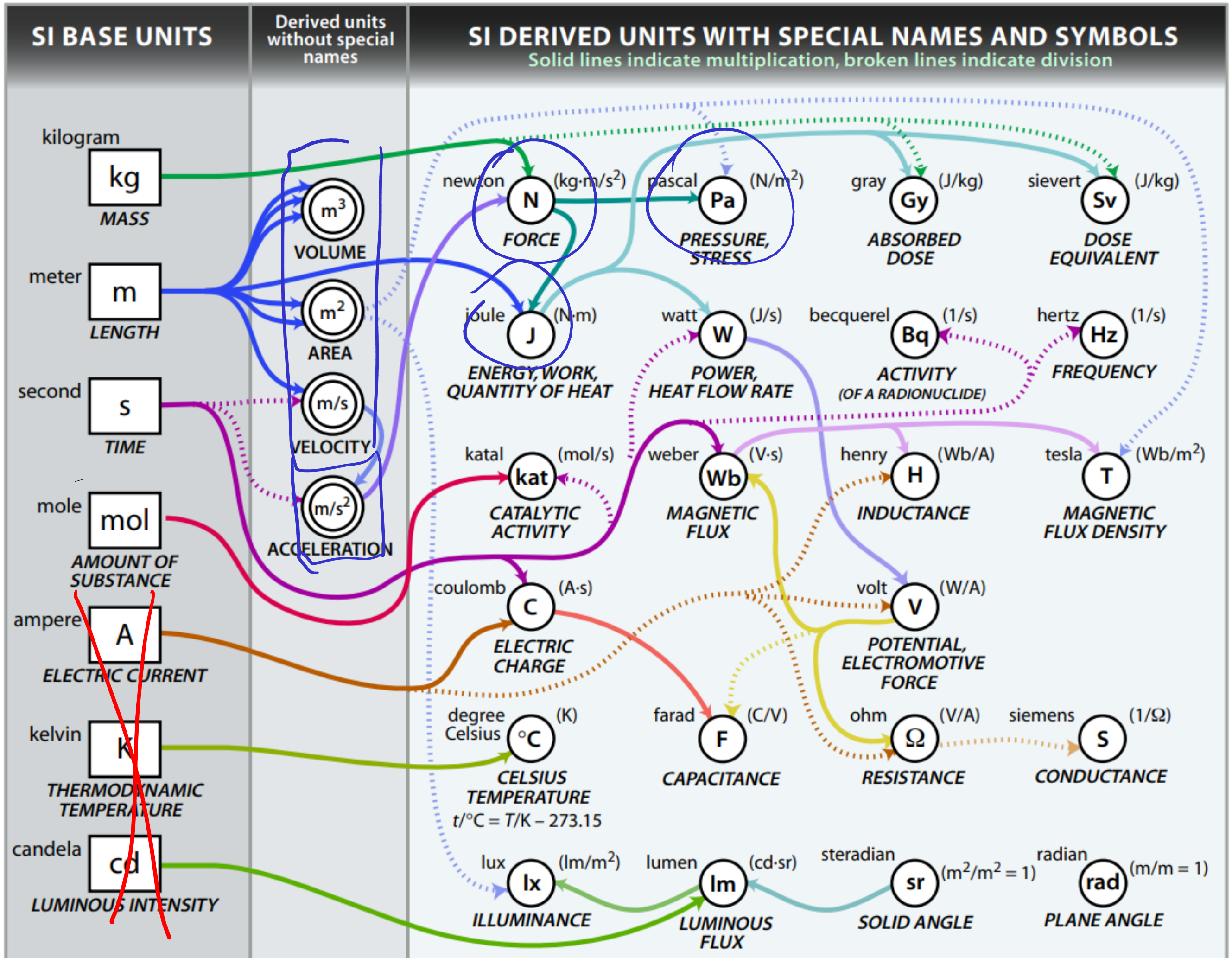
$$h = 6.626\,070\,15 \times 10^{-34}\, kg\, m^2 s^{-1}$$

costante di Planck

kg viene da questa costante

Campione chilogramma mandato dalla Francia agli Stati Uniti in 1794, ma molto probabilmente rubato da pirati nei Caraibi.

Unità derivate



Cifre significative e incertezza

In realtà tutte le misure hanno un livello di incertezza

e.g.

$L = 1{,}82 \pm 0{,}02$ m incertezza : 1 cifre

$m = 3{,}5 \pm 0{,}1$ kg

Cifre significative → indicare il livello di precisione

$$\boxed{\begin{array}{l} L = 1{,}82\,\text{m} \\ m = 3{,}5\,\text{kg} \end{array}}$$

$\Longrightarrow 1{,}82 \pm 0{,}01$ m

$\rightarrow 3{,}5 \pm 0{,}1$ kg

Cifre significative e incertezza

Operazioni:

* prodotto, divisione: tenere il numero più basso di cifre significative

e.g. $\underbrace{(1{,}1\,m)}_{2\text{ c.f.}} \times \underbrace{(3{,}45\,m)}_{3\text{ c.f.}} = \underbrace{3{,}\cancel{795}}_{4\text{ c.f}}\ m^2 = 3{,}8\ m^2$

$(1{,}1 \pm 0{,}1\,m) \times (3{,}45 \pm 0{,}01\,m) = (3{,}795 \pm 0{,}001\ m^2)$ (non va bene)

* addizioni, sostrazioni: tenere il numero più basso di decimali

e.g. $1{,}1\,m - 12\ cm = 1{,}1 - 0{,}12\ m$

$= 0{,}98\ m \longrightarrow 1{,}0\ m$

$1{,}1\,m \neq 1{,}10\ m$

Ordini di grandezza

that looked like a conglomeration of flames that promptly started rising. After a few seconds the rising flames lost their brightness and appeared as a huge pillar of smoke with an expanded head like a gigantic mushroom that rose rapidly beyond the clouds probably to a height of the order of 30,000 feet. After reaching its full height, the smoke stayed stationary for a while before the wind started dispersing it.

About 40 seconds after the explosion the air blast reached me. I tried to estimate its strength by dropping from about six feet small pieces of paper before, during and after the passage of the blast wave. Since, at the time, there was no wind I could observe very distinctly and actually measure the displacement of the pieces of paper that were in the process of falling while the blast was passing. The shift was about 2½ meters, which, at the time, I estimated to correspond to the blast that would be produced by ten thousand tons of T.N.T.

FINAL DETERMINATION
UNCLASSIFIED
L. M. Redman
JUL 17, 1979

Classification changed to UNCLASSIFIED
by authority of the U. S. Atomic Energy Commission,
Per R. D. Krohn 1-31-73
(Person authorizing change in classification) (Date)

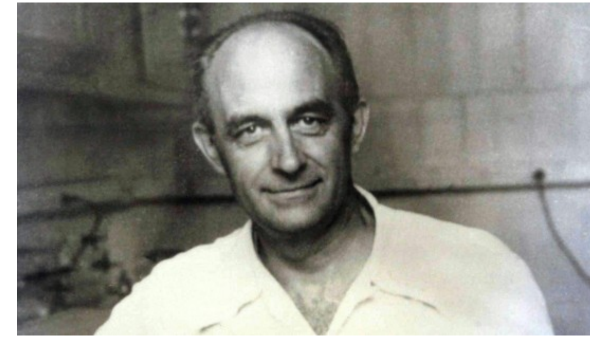
[redacted]

This document [redacted] National Defense [redacted] meaning of the [redacted] Its transmission [redacted] any manner to an unauthorized person [redacted] by law.

H. K. Bramlett
[redacted] making the change, and date)
4-3-75

SPECIAL RE-REVIEW
FINAL DETERMINATION
UNCLASSIFIED, DATE: JUL 9 1981

PUBLICLY RELEASABLE
LANL Classification Group
J.A. Brown 4/25/01



Enrico Fermi (1901-1954)

Esplosione "Trinity",
16 Luglio 1945

Ordini di grandezza

Scopo: calcolo veloce per una stima

ordine di grandezza: ritenere la potenza di 10 più vicina

Esempio: Un ingegnere deve fabricare un nuovo pacemaker.

quanti battiti di cuore deve fare senza malfunzionamento?

$$1 \text{ battito}/s \times 60 \text{ anni} \times \pi \times 10^7 \text{ s/anno} = 2 \times 10^9 \text{ battiti}$$

Analisi dimensionale

"$A = B$" non può essere valido se A e B hanno unità diverse.

↳ utile per validare soluzioni

e.g. quali sono le unità della costante di richiamo di una molla?

$$F = -kx \quad \leftarrow m$$

$$N = kg\,m\,s^{-2}$$

$$[k] = \frac{[F]}{[x]} = [k] = \frac{kg\,m\,s^{-2}}{m} = kg/s^2$$

il periodo di oscillazione di un sistema molla + mass dipende solo dalla massa m e da k $\Rightarrow$ periodo: $\propto \sqrt{\frac{m}{k}}$

Scalari e vettori

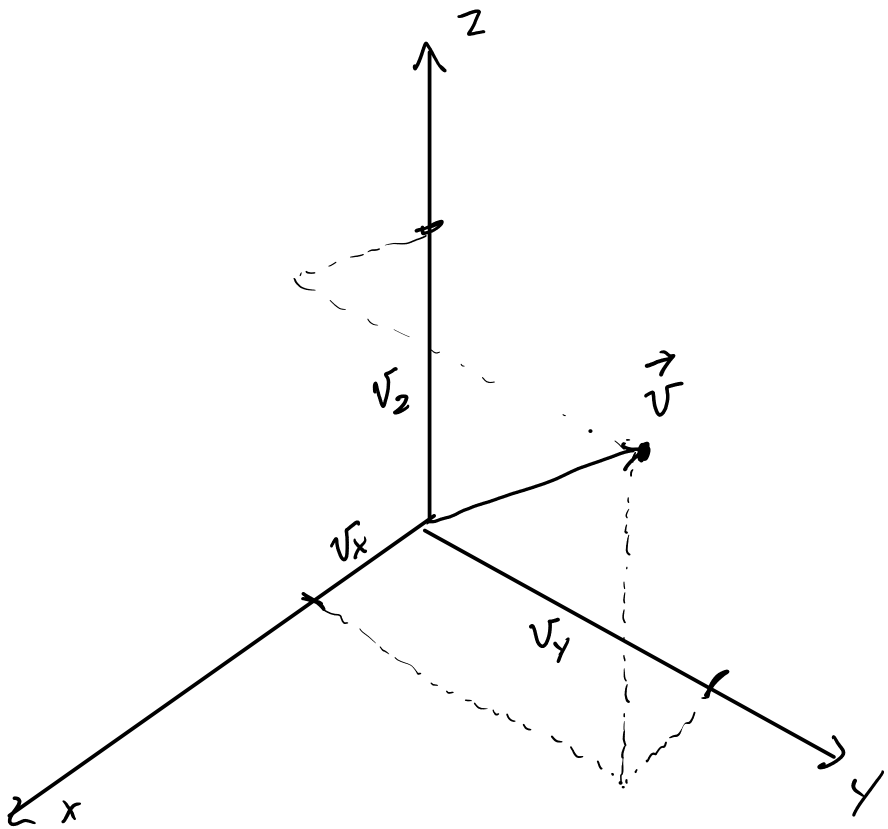
Scalare: una grandezza specificata da un numero + unità

e.g. lunghezza, massa, energia, ...

Vettori: una quantità definita da un valore e una direzione (e un verso)

oppure: una quantità con più valori (una lista di numeri)

Vettori



$\vec{v} = (v_x, v_y, v_z)$

* Anche se questa definizione sembra identica alla definizione del punto, sono diversi:

  - un punto non ha una lunghezza
  - non si può fare la somma di due punti
  - ...

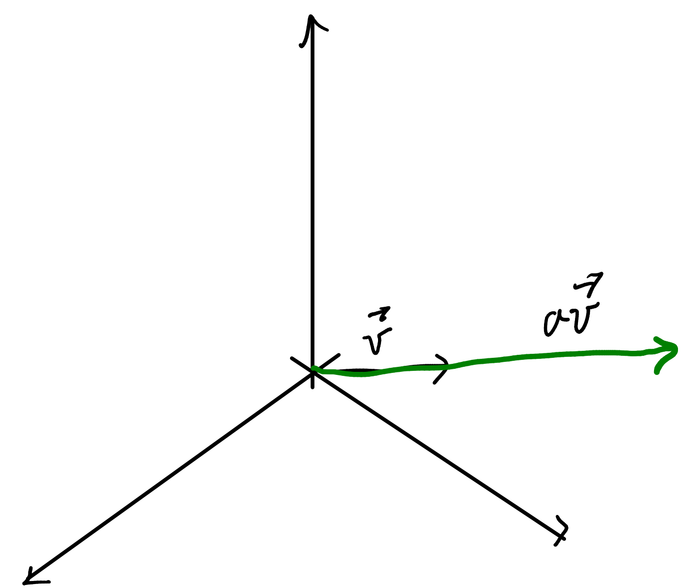
Prodotto con un scalare

$\vec{v} = (v_x, v_y, v_z)$

$a\vec{v} = (av_x, av_y, av_z)$

allungamento / "compressione"
del vettore

se $a < 0 \rightarrow$ verso opposto

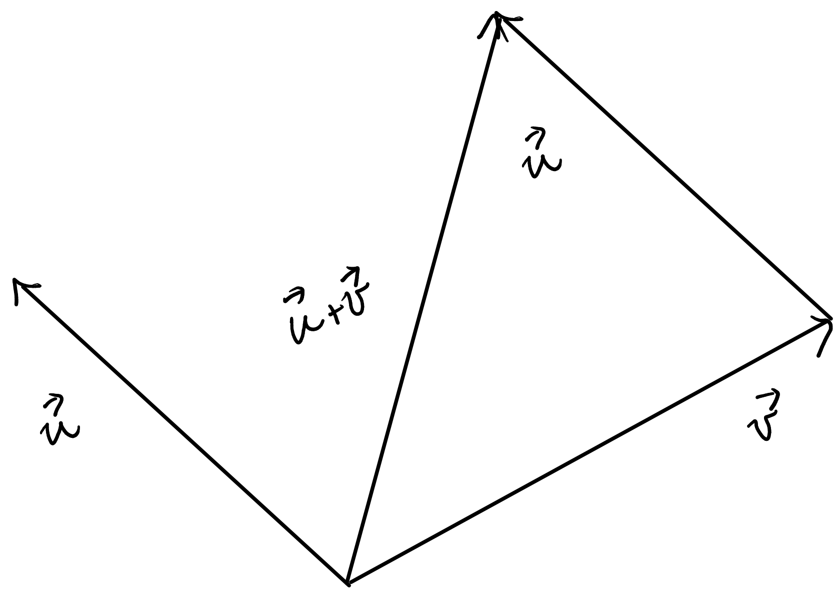


Somma vettoriale

$$\vec{u} + \vec{v} = (u_x + v_x, u_y + v_y, u_z + v_z)$$

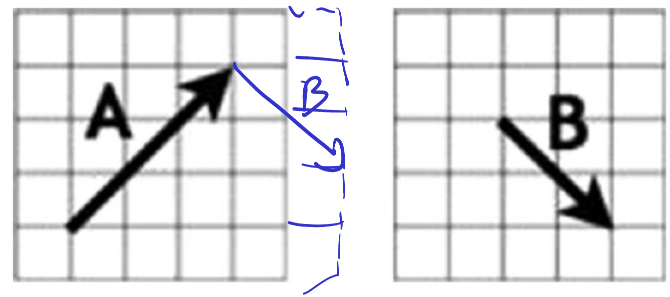
"punta - coda"

somma grafica

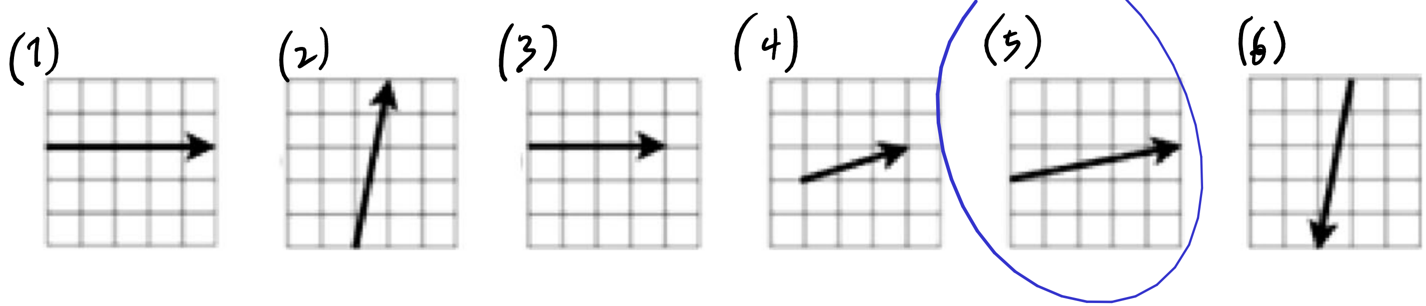


Somma grafica

1. Vectors **A** and **B** are shown.



Which of the following options represents the *vector sum*, **A** + **B**? Please circle your choice.



Fonte: Mikula et al., Phys. Rev. Phys. Ed. Res **13** 010122 (2017)
DOI: 10.1103/PhysRevPhysEducRes.13.010122

Somma grafica

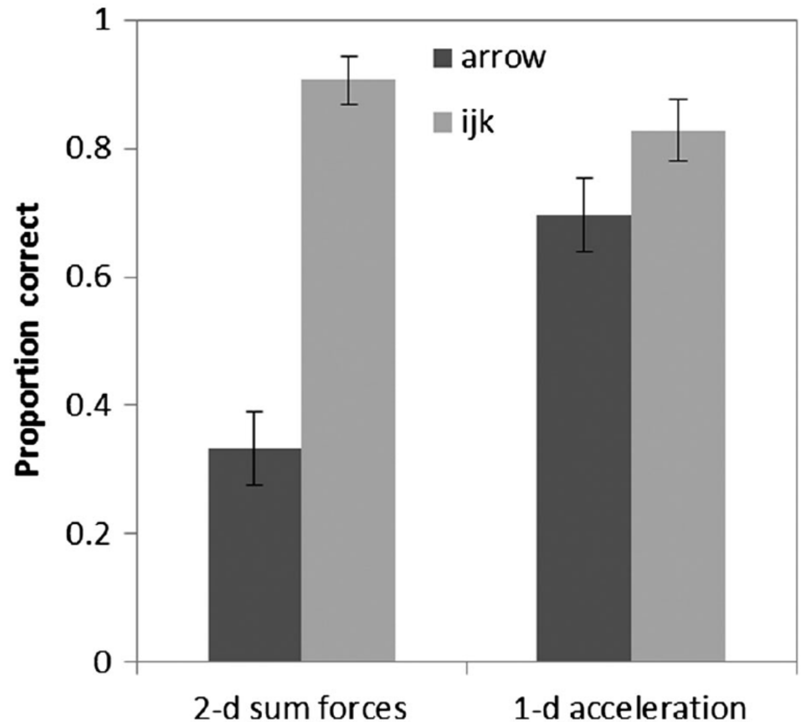


FIG. 8. Student performance ($N = 130$) on two-dimensional (Fig. 6) and one-dimensional (Fig. 7) physics context questions in both the arrow (grid) and ijk formats. Error bars indicate ± 1 SE.

Fonte: Mikula et al., Phys. Rev. Phys. Ed. Res **13** 010122 (2017)

DOI: 10.1103/PhysRevPhysEducRes.13.010122

Versori

Definiamo:

$$\hat{i} = (1,0,0)$$
$$\hat{j} = (0,1,0)$$
$$\hat{k} = (0,0,1)$$

$$\vec{v} = (v_x, v_y, v_z) = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

v_x, v_y, v_z : le componenti di $\vec{v}$ in direzione $\hat{i}, \hat{j}, \hat{k}$

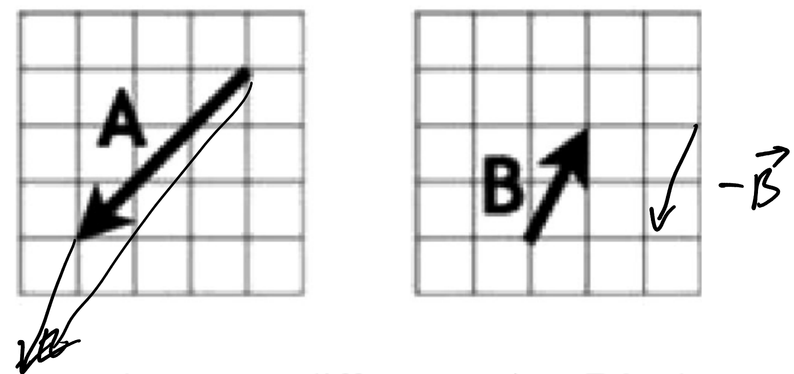
perché $\hat{i}$ e no $\vec{i}$? Perché $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

versori = "vettori unità"

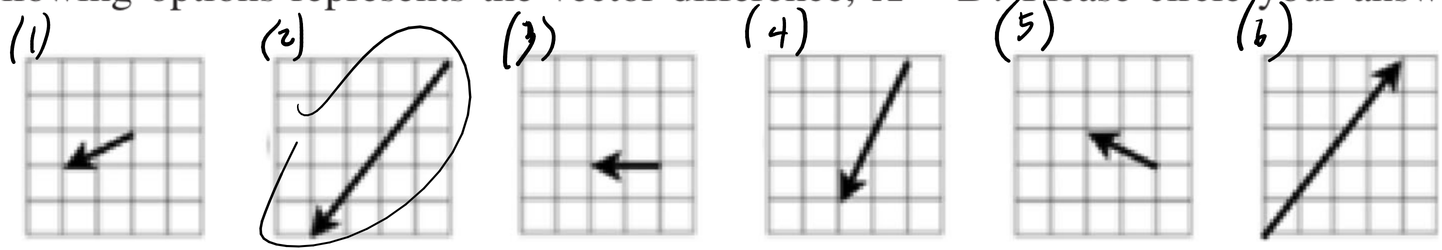
Somma vettoriale

18. Vectors **A** and **B** are shown.

$\vec{A} - \vec{B} = A + (-\vec{B})$



Which of the following options represents the vector difference, **A** − **B**? Please circle your answer.

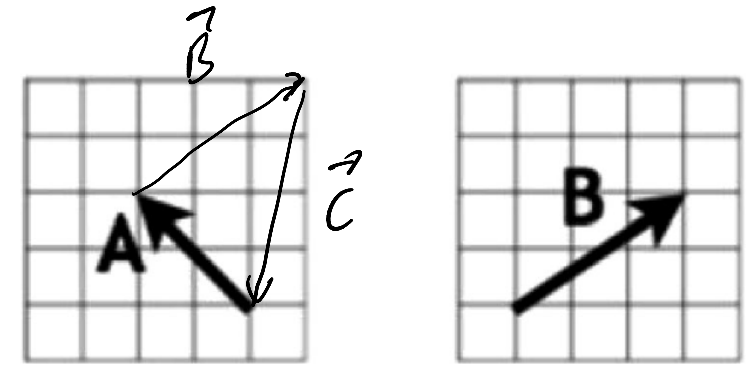


$\vec{A} = -3\hat{i} - 3\hat{j}$ $\qquad$ $\vec{B} = 1\hat{i} + 2\hat{j}$

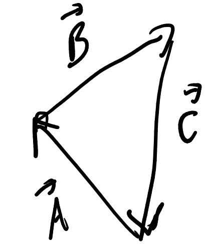
$$\vec{A} - \vec{B} = (-3-1)\hat{i} + (-3-2)\hat{j} = -4\hat{i} - 5\hat{j}$$

Somma vettoriale

16. Vectors **A** and **B** are shown.

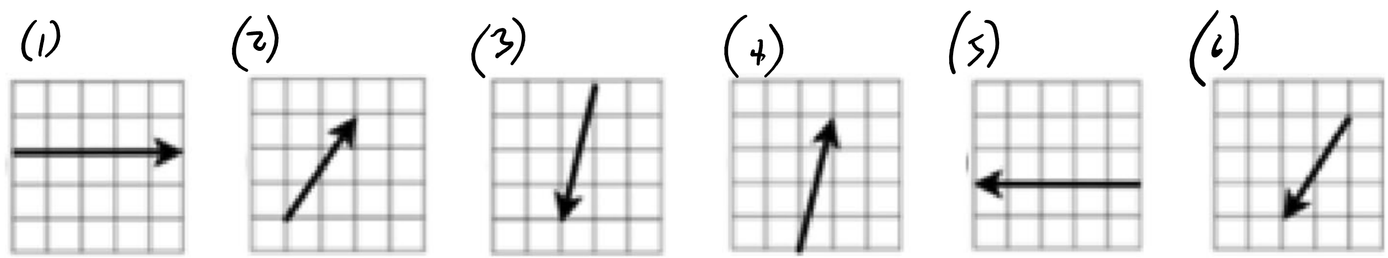


$\vec{A} + \vec{B} + \vec{C} = 0$



The *vector sum* of vectors **A**, **B**, and **C** is zero. Which of the options below represents vector **C**? Please circle your answer.

$\vec{C} = ?$



$\vec{C} = -(\vec{A} + \vec{B}) = -1\hat{i} - 4\hat{j}$

Somma vettoriale

13. Consider the two vectors below.

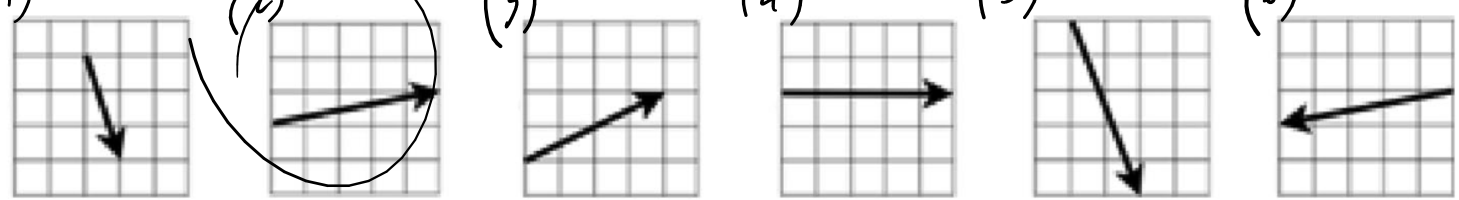
$\mathbf{A} = 1\mathbf{i} + 3\mathbf{j}$

$\mathbf{B} = -4\mathbf{i} + 2\mathbf{j}$

Which of the following options represents the vector difference, $\mathbf{A} - \mathbf{B}$? Please circle your answer.

Note: $\mathbf{i}$ and $\mathbf{j}$ represent the unit vectors in the x and y directions, respectively.

(1) (2) (3) (4) (5) (6)



$\vec{A} - \vec{B} = 5\hat{i} + 1\hat{j}$

Modulo e direzione

Il modulo di un vettore è la sua lunghezza geometrica

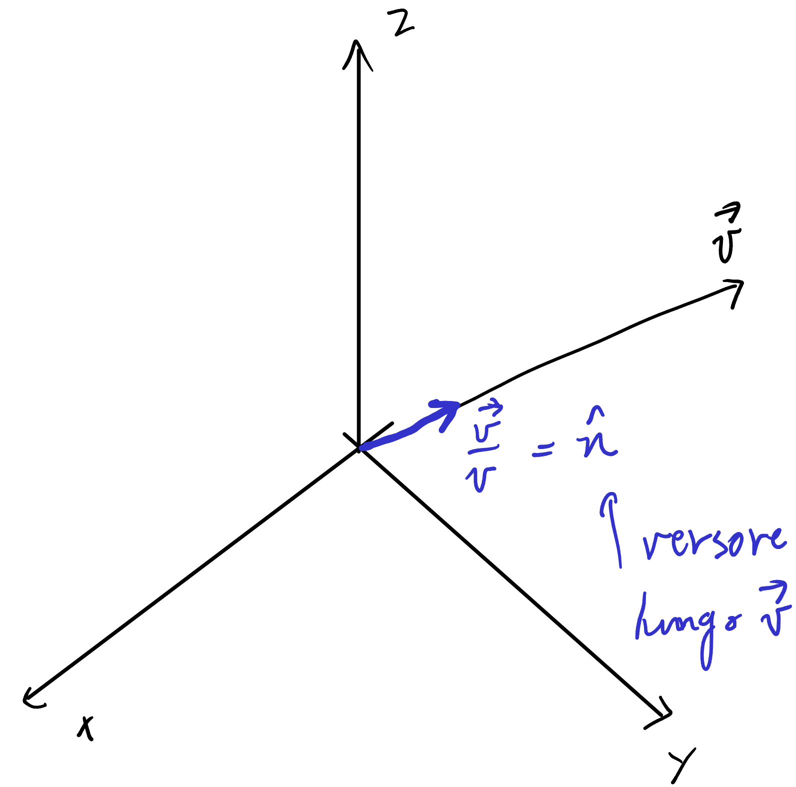
$$v = |\vec{v}|$$

In termini dei componenti:

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

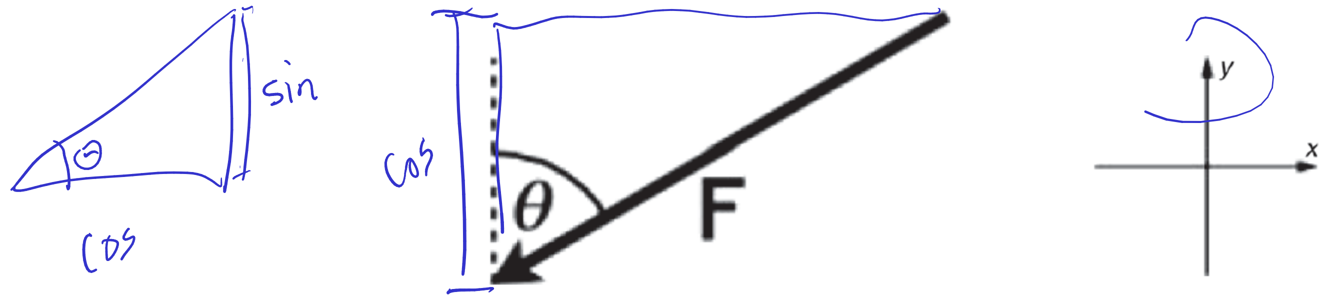
Qual è il modulo di $\vec{v}/v$?

$$\left|\frac{\vec{v}}{v}\right| = \frac{1}{|v|}|\vec{v}| = \frac{v}{v} = 1$$



Componenti

2. A vector with magnitude F is shown. Find the component of vector F in the *y* direction.
Note: The coordinate system provided specifies the positive directions for *x* and *y*.

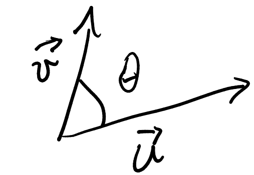


a) F*cos(θ)
b) -F*cos(θ)
c) F*sin(θ)
d) -F*sin(θ)
e) F*cos(90º-θ)
f) -F*cos(90º-θ)
g) F*sin(90º-θ)
h) -F*sin(90º-θ)
i) F*tan(θ)
j) -F*tan(θ)
k) Zero

Prodotto scalare

$$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z$$ ← scalare

$$\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}| \cos\theta$$



Osservazioni:

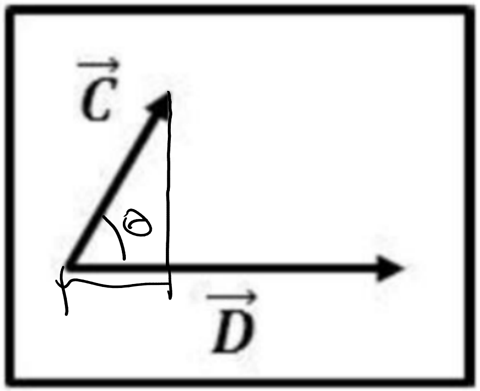
* $\vec{v} \cdot \vec{v} = v_x^2 + v_y^2 + v_z^2 = |\vec{v}|^2$
* $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$
* $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{j} = 0$

$\vec{v} \cdot \hat{n}$: componente di $\vec{v}$ in direzione $\hat{n}$ = proiezione di $\vec{v}$ sulla rete $\hat{n}$

$$\vec{v} \cdot \hat{i} = (v_x \hat{i} + v_y \hat{j} + v_z \hat{k}) \cdot \hat{i} = (v_x \hat{i} \cdot \hat{i} + v_y \hat{j} \cdot \hat{i} + v_z \hat{k} \cdot \hat{i}) = v_x$$

Prodotto scalare

3. The figure below shows vectors $\vec{C}$ and $\vec{D}$. Which option is the best interpretation of the dot product $(\vec{C}\cdot\vec{D})$?



(A) The magnitude of a vector between $\vec{C}$ and $\vec{D}$ pointing up to the right.

(B) The projection of vector $\vec{C}$ onto vector $\vec{D}$ multiplied by the magnitude of vector $\vec{D}$.

(C) A vector between $\vec{C}$ and $\vec{D}$ pointing up to the right.

(D) A vector perpendicular to both vectors.

(E) A vector in the direction of $\vec{D}$.

$$|C|\cos\theta \cdot |D|$$

$$\vec{C} \cdot \frac{\vec{D}}{\cancel{|\vec{D}|}} \cdot \cancel{|D|}$$

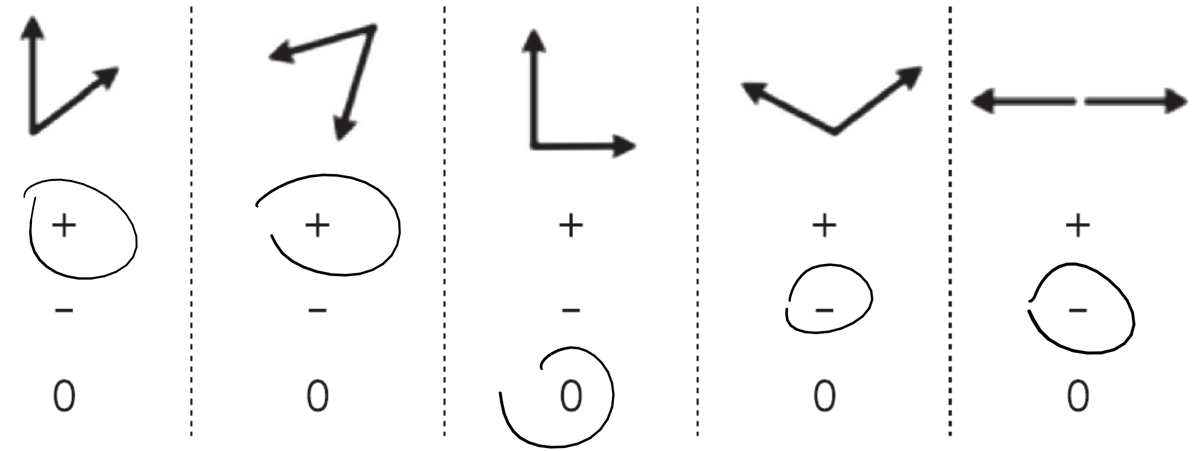
Fonte: Barniol et al, Eurasia Journal of Mathematics, Science & Technology Education 12, 2387-2398 (2016)
doi: 10.12973/eurasia.2016.1271a

Prodotto scalare

Prodotto scalare

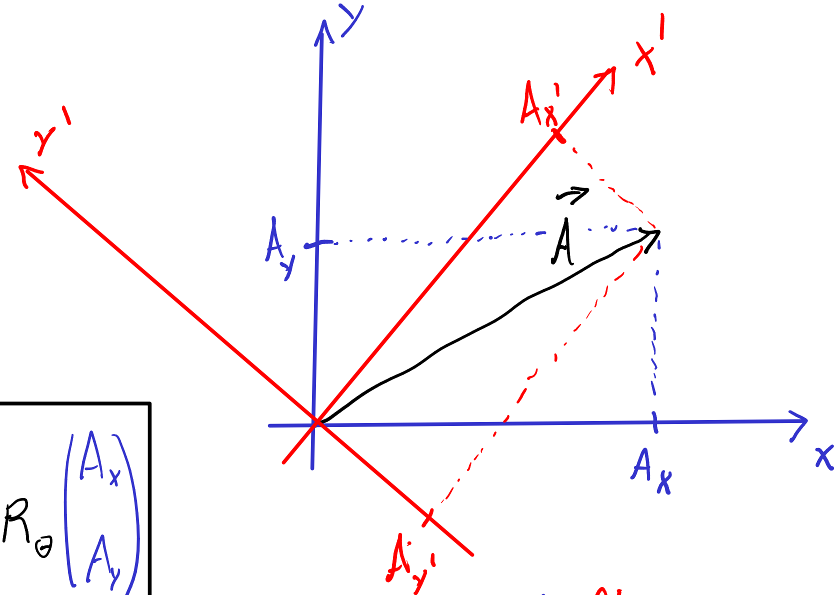
11. For each of the pairs of vectors below, determine whether their dot product is positive (+), negative (−), or zero (0). Please circle your answers. (5 points)

$\vec{A} \cdot \vec{B} = A B \cos\theta$



Cambio di base

Sarà utile a volte cambiare l'orientazione delle assi

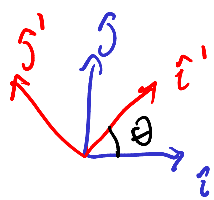


$$\vec{A} = A_x\hat{i} + A_y\hat{j} = A_{x'}\hat{i}' + A_{y'}\hat{j}'$$

tipicamente: cerchiamo $A_{x'}$ dati A_x e A_y

$$\begin{pmatrix} A_{x'} \\ A_{y'} \end{pmatrix} = R_\theta \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

$$R_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$



$$\hat{i}\cdot\hat{i}' = \cos\theta$$

$$\hat{j}\cdot\hat{i}' = \operatorname{sen}\theta$$

$$A_{x'} = \vec{A}\cdot\hat{i}' = (A_x\hat{i} + A_y\hat{j})\cdot\hat{i}'$$

$$= A_x(\hat{i}\cdot\hat{i}') + A_x(\hat{j}\cdot\hat{i}')$$

$$= A_x\cos\theta + A_y\operatorname{sen}\theta$$

(algebra lineare)

Riassunto

* Vettore: quantità con modulo e direzione (+ verso)

* Componente in direzione $\hat{n}$: $\vec{v} \cdot \hat{n}$ dove $\hat{n}$ è un versore $|\hat{n}| = 1$

* Prodotto scalare: $\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}| \cos\theta$ angolo tra $\vec{u}$ e $\vec{v}$

* Base ortogonale $\hat{i}, \hat{j}, \hat{k}$: $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$, $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

$$\vec{v} = (\vec{v} \cdot \hat{i})\,\hat{i} + (\vec{v} \cdot \hat{j})\,\hat{j} + (\vec{v} \cdot \hat{k})\,\hat{k} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

* Prodotto scalare in termini dei componenti: $\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z$