GRANDEZZE FISICHE FONDAMENTALI

LUNGHEZZA m $c \cong 3 \cdot 10^8 \frac{m}{s} = 30 \frac{cm}{ns}$

$= 299\,792\,458 \frac{m}{s}$

TEMPO s $\Delta t = \frac{1}{299\,792\,458}$ s

MASSA Kg

TEMPERATURA K (Kelvin)

Quantità di materia mol

Intensità di corrente elettrica A (Ampere)

Intensità luminosa cd (candela)

DIMENSIONI DELLE GRANDEZZE FISICHE

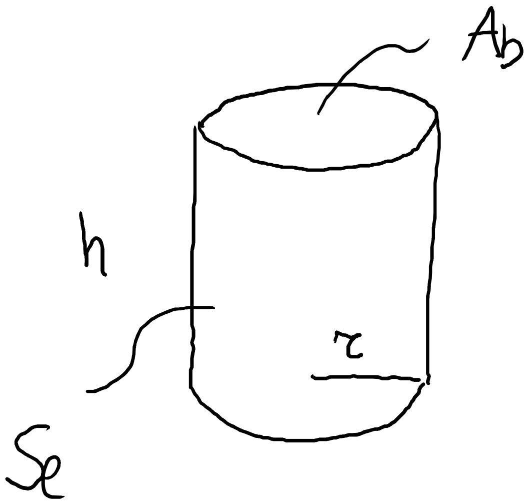
$$[d] = [L]$$

$$[a] = [L][t^{-2}]$$

$$[E_{cin}] = [M][L^2][t^{-2}]$$

$$[G] = [L^\alpha][M^\beta][t^\gamma]\cdot[T^\delta]\cdots\cdots[H^\eta]$$

G = generica grandezza fisica



$$r = 2{,}0 \text{ cm}$$
$$h = 5{,}0 \text{ cm}$$

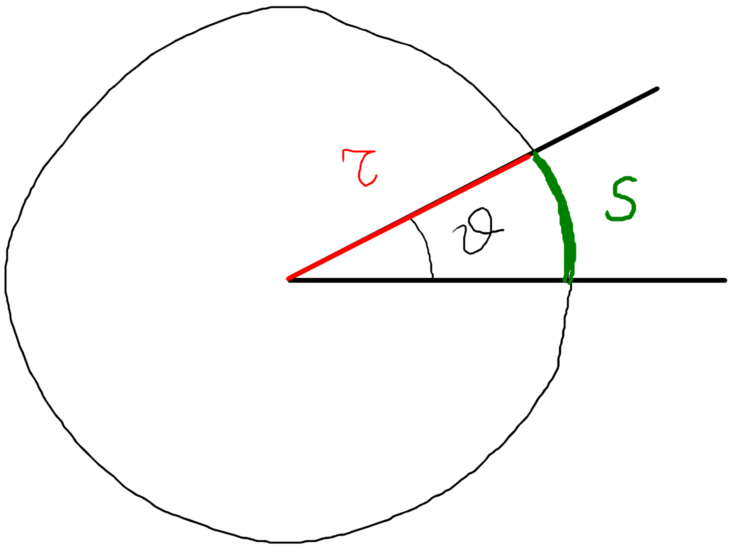
$$A_b = \pi \cdot r^2 = \pi\,(2{,}0 \text{ cm})^2 = \pi\; 4{,}0 \text{ cm}^2$$

$$S_e = 2\pi r h = 2\pi\,(2{,}0 \text{ cm})(5{,}0 \text{ cm}) =$$
$$\overset{!}{=} 20\pi \text{ cm}^2$$

$$V = \pi r^2 \cdot h = \pi\,(2{,}0 \text{ cm})^2 \cdot (5{,}0 \text{ cm})$$
$$\overset{!}{=} 20\pi \text{ cm}^3$$

$S_e + V =$ New Cuyama

ANGOLI

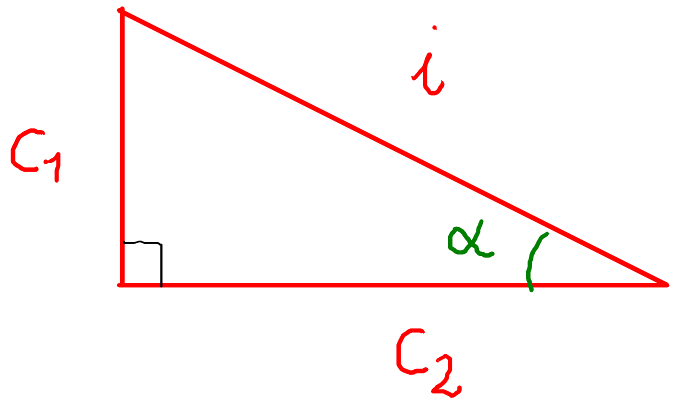


$$\theta = \frac{s}{r}$$

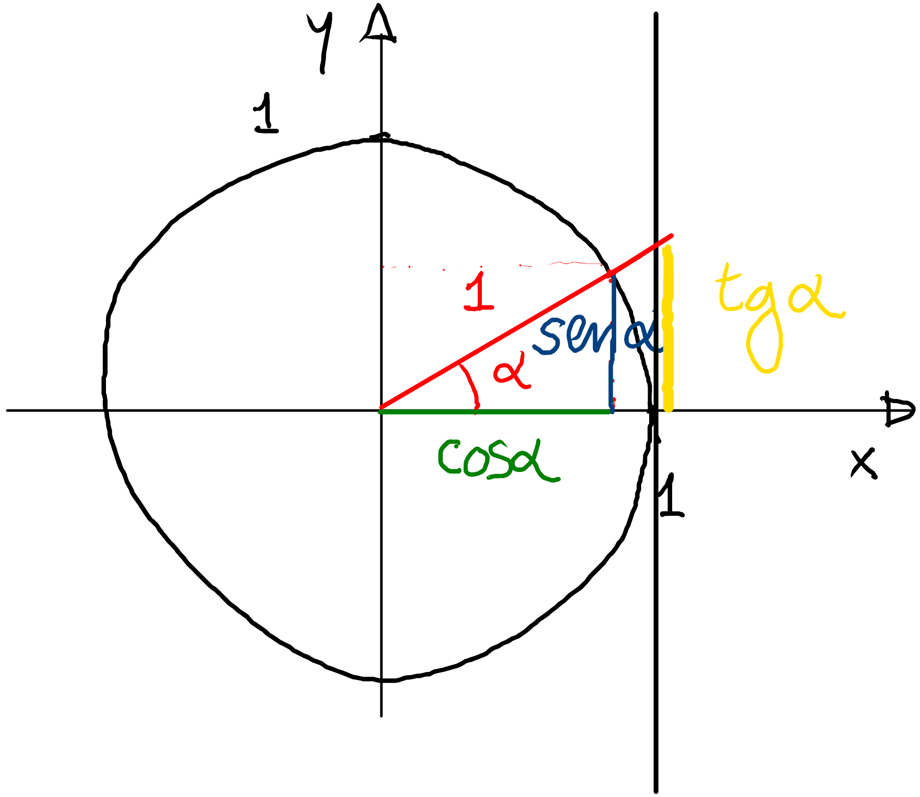
$$[\theta] = \frac{[L]}{[L]}$$ é adimensioni !

Angolo	Misura in gradi	Misura in radianti
giro	360°	$\frac{2\pi r}{r} = 2\pi$
piatto	180°	$\frac{\pi r}{r} = \pi$
retto	90°	$\frac{\pi}{2}$
	60°	$\frac{\pi}{3}$
	45°	$\frac{\pi}{4}$
	30°	$\frac{\pi}{6}$

TRIGONOMETRIA



$$C_1 = i \cdot \operatorname{sen}\alpha$$
$$C_2 = i \cos\alpha$$
$$\frac{C_1}{C_2} = \frac{\cancel{i}\ \operatorname{sen}\alpha}{\cancel{i}\ \cos\alpha} = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \operatorname{tg}\alpha$$
$$C_1 = C_2 \operatorname{tg}\alpha$$



sen α e cos α
agiscono su angolo α
e forniscono un valore $\in [-1, 1]$

α	$\cos\alpha$	$\operatorname{sen}\alpha$
0	1	0
$\pi/2$	0	1
π	-1	0
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$

CIFRE SIGNIFICATIVE

800 ± 1

800 ± 10

800 ± 100

$0{,}0034 = 3{,}4 \cdot 10^{-3} \longrightarrow (3{,}4 \pm 0{,}1) \cdot 10^{-3}$

$1{,}0034 \pm 0{,}0001$

$\rightarrow \cdot \ / \Rightarrow \quad 142 \cdot 51{,}3 = 7284{,}6$

7280

$142 \cdot 5{,}1 = 724{,}2$

720

$\rightarrow + \ - \Rightarrow$

$$\begin{array}{r} 142 \ + \\ 5{,}1 \ = \\ \hline 147{,}1 \end{array} \rightarrow 147 \qquad \begin{array}{r} 142 \ + \\ 0{,}0034 \\ \hline 142 \end{array}$$

$$K = 1022{,}751 \ \frac{N}{m} = 1000 \ \frac{N}{m}$$

I VETTORI

GRANDEZZE SCALARI: numero ed unità di misura

Es: massa, energia...

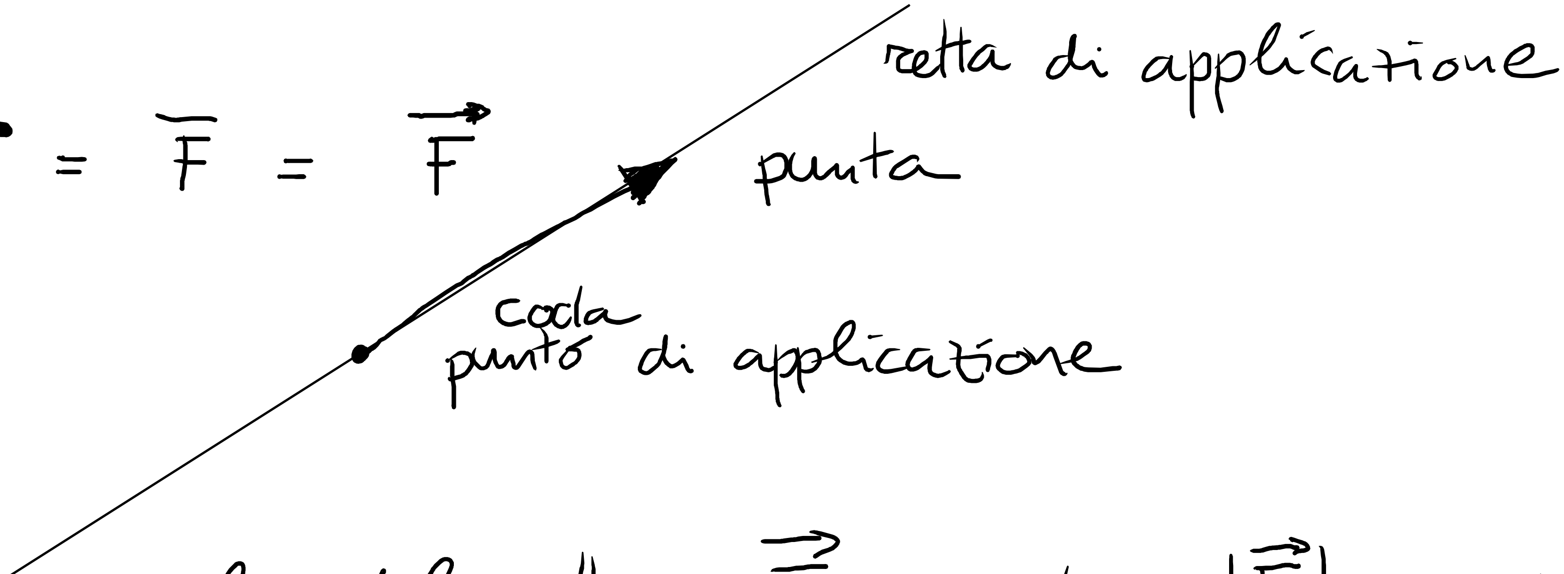
GRANDEZZE VETTORIALI: numero e unità di misura determinano il MODULO (o l'intensità) della grandezza fisica

ma servono anche: DIREZIONE

VERSO

Es: velocità, forza, etc...

$\mathbf{F} = \bar{F} = \vec{F}$



il modulo del vettore $\vec{F}$ si indica $|\vec{F}|$ o sempl. F

Due vettori sono uguali quando hanno

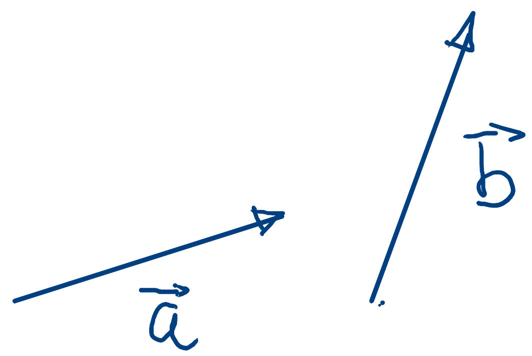
stesso modulo

stessa direzione

stesso verso

Dal punto di vista fisico talvolta sono importanti anche la retta di applicazione ed il punto di applicazione

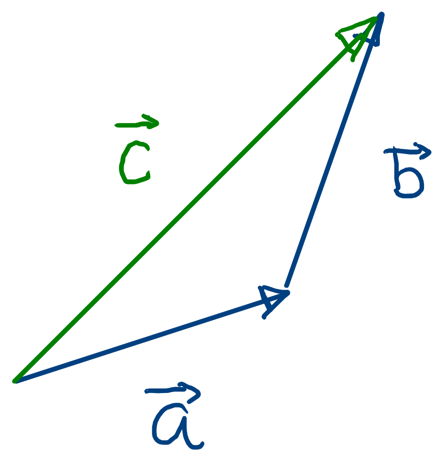
SOMMA DI VETTORI



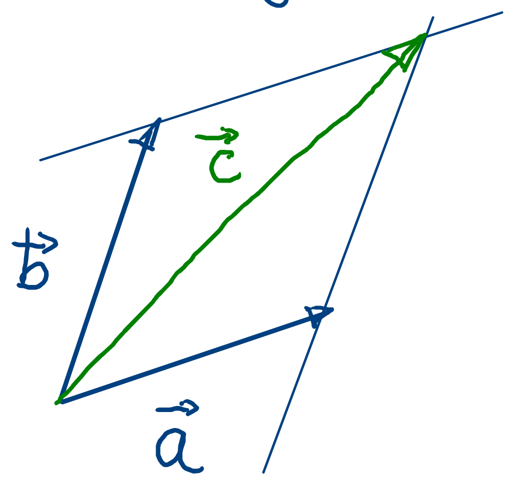
$\vec c$ è la somma o composizione di $\vec a$ e $\vec b$

$$\vec a + \vec b = \vec c$$

(1) Punta-coda



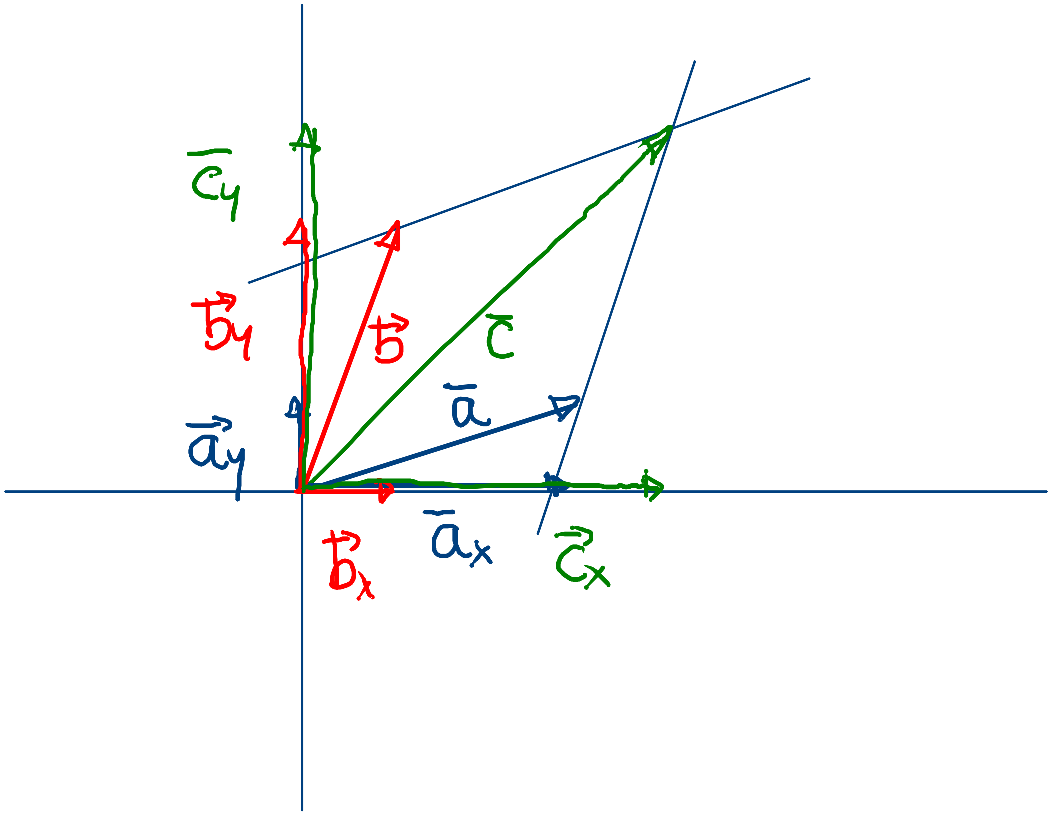
(2) Parallelogramma



COMMUTATIVA: $\vec a + \vec b = \vec b + \vec a = \vec c$

ASSOCIATIVA: $\vec a + (\vec b + \vec e) = (\vec a + \vec b) + \vec e \quad \forall\, \vec a, \vec b, \vec e$

③ Mediante componenti



$$\vec{a} + \vec{b} = \vec{c}$$

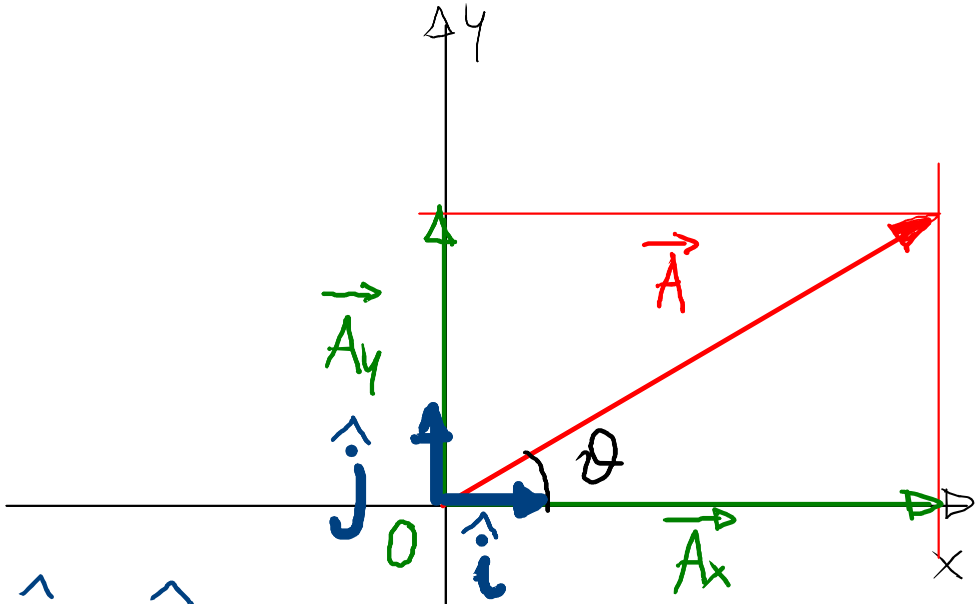
$$\begin{cases} \vec{a}_x + \vec{b}_x = \vec{c}_x \\ \vec{a}_y + \vec{b}_y = \vec{c}_y \end{cases}$$

$$\begin{cases} a_x \cancel{\hat{i}} + b_x \cancel{\hat{i}} = c_x \cancel{\hat{i}} \\ a_y \cancel{\hat{j}} + b_y \cancel{\hat{j}} = c_y \cancel{\hat{j}} \end{cases}$$

$$\begin{cases} c_x = a_x + b_x \\ c_y = a_y + b_y. \end{cases}$$

$$\left(c_z = a_z + b_z \right)$$

COMPONENTI



$\hat{i}$ e $\hat{j}$ sono versori

$|\hat{i}| = |\hat{j}| = 1$

$$\vec{A}_x = |\vec{A}_x| \cdot \hat{i} = A_x \hat{i}$$

$$\vec{A}_y = |\vec{A}_y| \cdot \hat{j} = A_y \cdot \hat{j}$$

componenti scalari

componenti vettoriali

$$\vec{A}_x + \vec{A}_y = \vec{A}$$

$$|\vec{A}_x| = |\vec{A}| \cos\theta$$

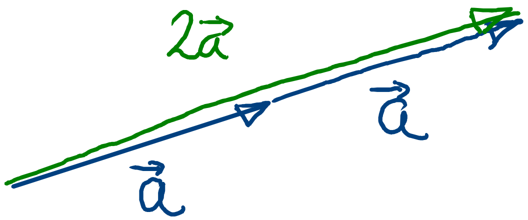
$$|\vec{A}_y| = |\vec{A}| \operatorname{sen}\theta$$

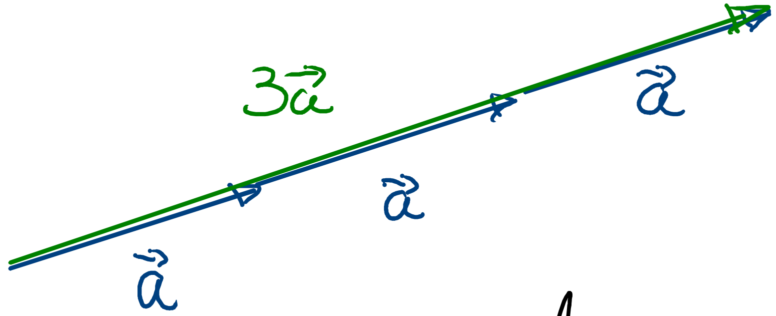
$$\frac{|\vec{A}_y|}{|\vec{A}_x|} = \operatorname{tg}\theta$$

$$\sqrt{|A_x|^2 + |A_y|^2} = |\vec{A}|$$

PRODOTTO DI UN VETTORE PER UNO SCALARE

$\vec{a} + \vec{a} = 2\vec{a}$ $\qquad\qquad$ $\vec{a} + \vec{a} + \vec{a} = 3\vec{a}$





$m\vec{a} \qquad m \in \mathbb{R}$

valore assoluto di m

modulo di $\vec{a}$

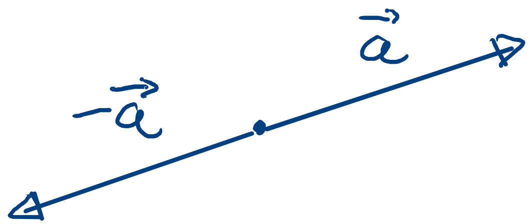
modulo $\quad |m\vec{a}| = |m||\vec{a}|$

direzione $\quad$ la stessa di $\vec{a}$

verso $\quad$ lo stesso di $\vec{a}$ $\quad$ se $m > 0$

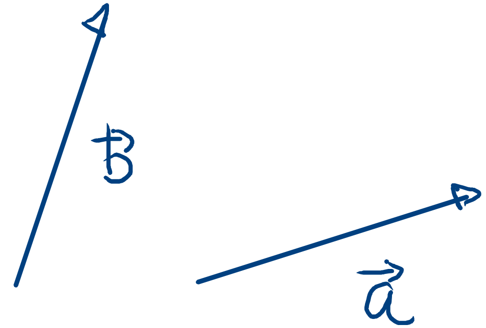
opposto $\quad$ se $m < 0$

In particolare $\quad -1 \cdot \vec{a} = -\vec{a}$

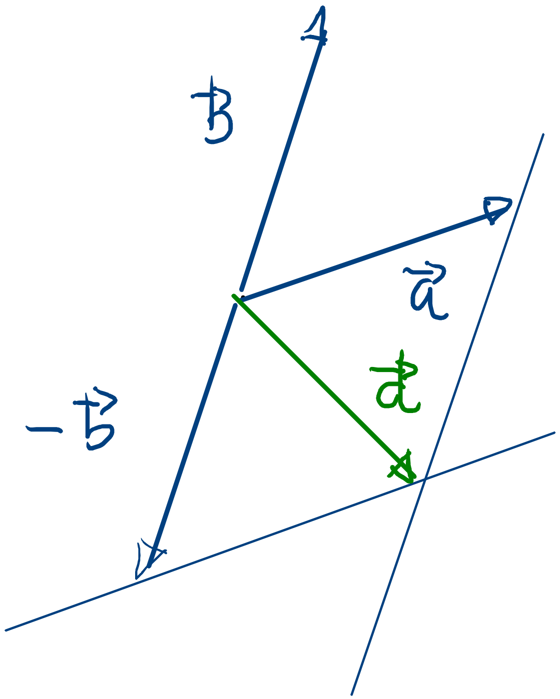


DIFFERENZA TRA VETTORI

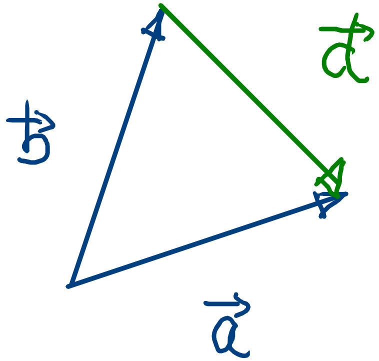
$$\vec{d} = \vec{a} - \vec{b} \stackrel{!}{=} \vec{a} + (-\vec{b})$$



①



②

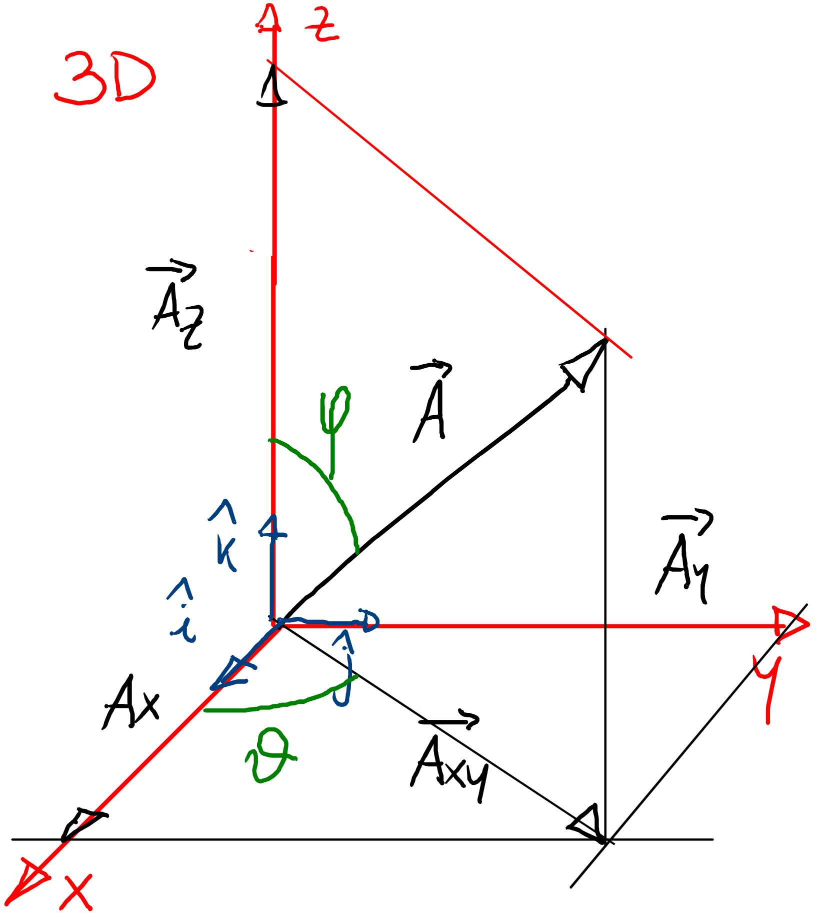


$\vec{d}$ va dalla punta di $\vec{b}$ alla punta di $\vec{a}$

③ Per componenti:

$$d_x = a_x - b_x$$
$$d_y = a_y - b_y$$
$$(d_z = a_z - b_z)$$

In 3D



$$\vec{A} = \vec{A}_{xy} + \vec{A}_z = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

$$\vec{A}_{xy} = \vec{A}_x + \vec{A}_y$$

$$|\vec{A}_{xy}| = |\vec{A}| \operatorname{sen}\varphi$$

$$|\vec{A}_x| = |\vec{A}_{xy}| \cos\vartheta = |\vec{A}| \operatorname{sen}\varphi \cos\vartheta$$

$$|\vec{A}_y| = |\vec{A}_{xy}| \operatorname{sen}\vartheta = |\vec{A}| \operatorname{sen}\varphi \operatorname{sen}\vartheta$$

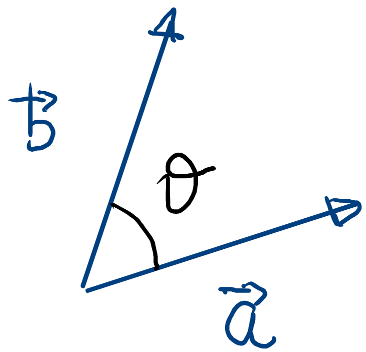
$$|\vec{A}_z| = |\vec{A}| \cos\varphi$$

$$|\vec{A}| = \sqrt{|\bar{A}_x|^2 + |\bar{A}_y|^2 + |\bar{A}_z|^2}$$

$$\vec{A} = |\vec{A}| \operatorname{sen}\varphi \cos\vartheta \, \hat{i} + |\vec{A}| \operatorname{sen}\varphi \operatorname{sen}\vartheta \, \hat{j} + |\vec{A}| \cos\varphi \, \hat{k}$$

PRODOTTO SCALARE TRA DUE VETTORI

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\theta$$



$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y \; (+ \; a_z b_z)$$ è commutativo

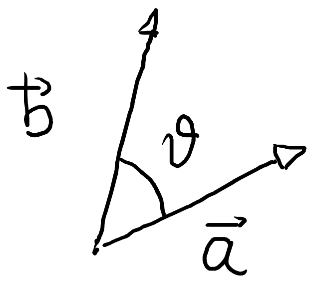
È associativo? La domanda non ha senso in quanto il prodotto scalare dà come risultato uno scalare e non un vettore

$$\underbrace{(\vec{a} \cdot \vec{b})}_{\text{numero}} \; \underbrace{\cdot \, \vec{c}}_{\text{vettore}} \rightarrow$$ scrittura priva di senso

PRODOTTO VETTORIALE

$$\vec{a} \times \vec{b} = \vec{v}$$

modulo : $|\vec{v}| = |\vec{a}| \cdot |\vec{b}| \operatorname{sen}\theta$

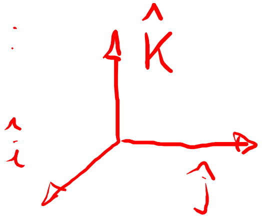


direzione : ortogonale ($\perp$) al piano individuato da $\vec{a}$ e $\vec{b}$

verso : regola della mano dx

Mediante componenti :
$$\begin{cases} v_x = + a_y b_z - b_y a_z \\ v_y = - a_x b_z + b_x a_z \\ v_z = + a_x b_y - b_x a_y \end{cases}$$

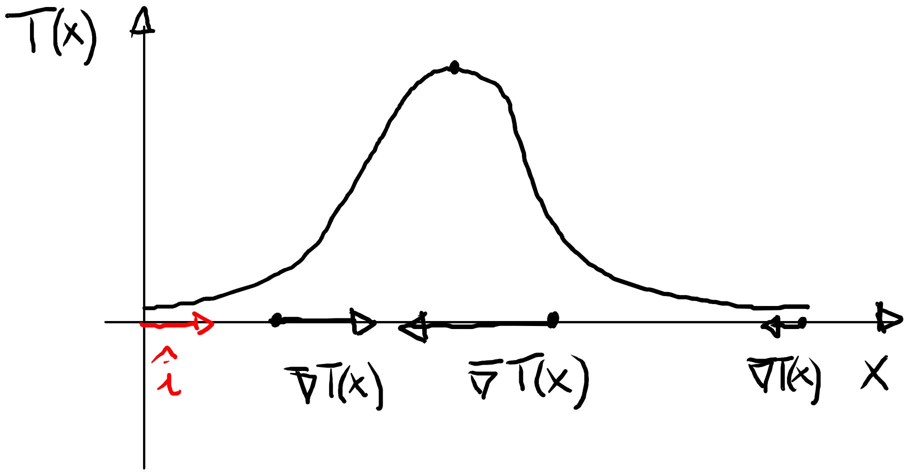
Esempio :



$$\hat{i} \times \hat{j} = \hat{k}$$

$$\begin{cases} v_x = 0 \\ v_y = 0 \\ v_z = 1 \cdot 1 - 0 \cdot 0 = 1 \end{cases} \Rightarrow \hat{k}$$

GRADIENTE (1D)



$$\overline{\text{grad}}\, T(x) \equiv \bar{\nabla}\, T(x) \quad \text{vettore:}$$

$$|\nabla T(x)| = \frac{dT(x)}{dx}$$

direzione: $\hat{i}$

verso: $\dfrac{\Delta T(x)}{\Delta x} > 0$

In generale (3D)

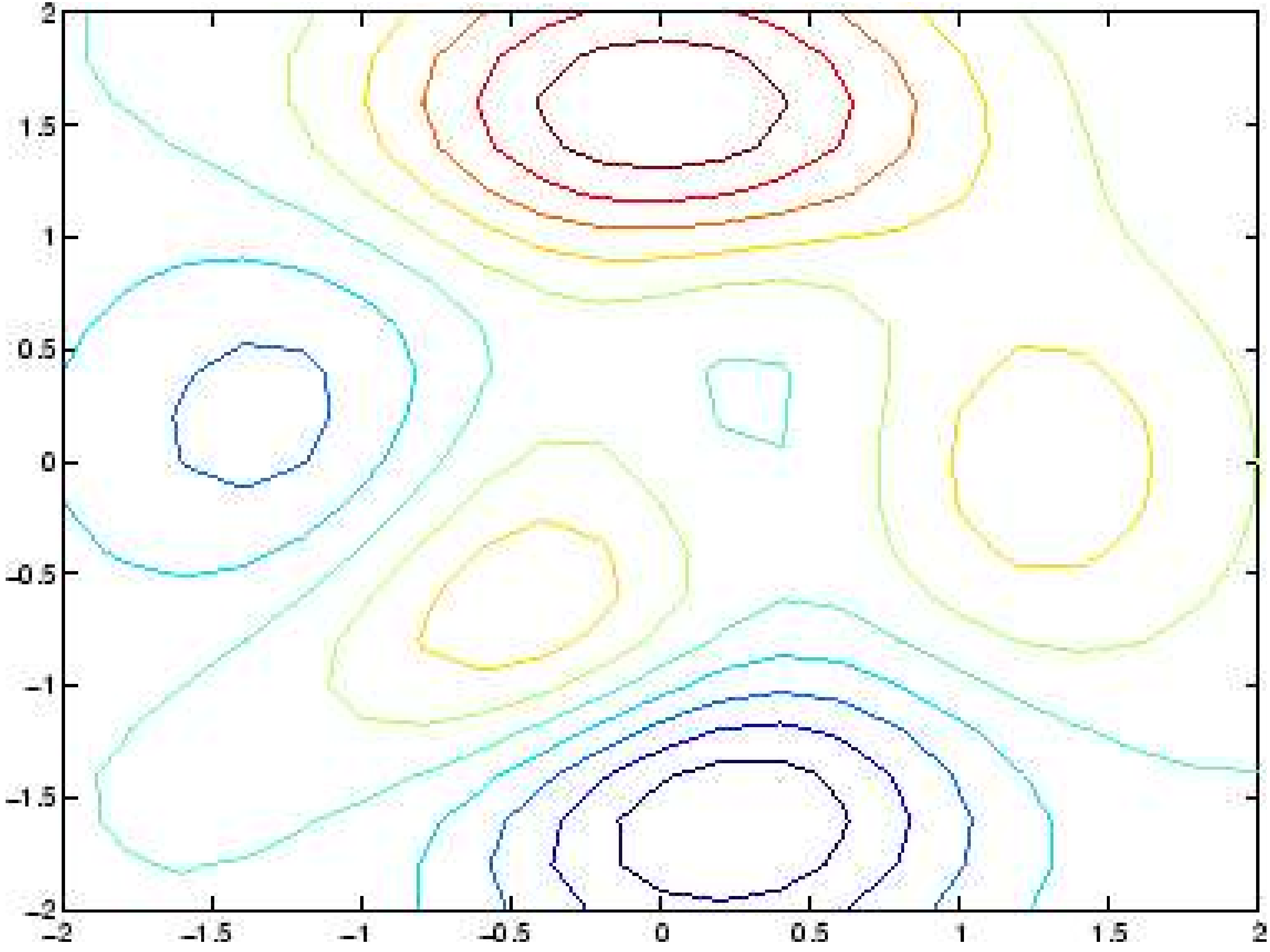
$T(x,y,z) \leftarrow$ campo scalare

$$\nabla T(x,y,z) = \frac{\partial}{\partial x} T(x,y,z)\, \hat{i} + \frac{\partial}{\partial y} T(x,y,z)\, \hat{j} + \frac{\partial}{\partial z} T(x,y,z)\, \hat{k}$$

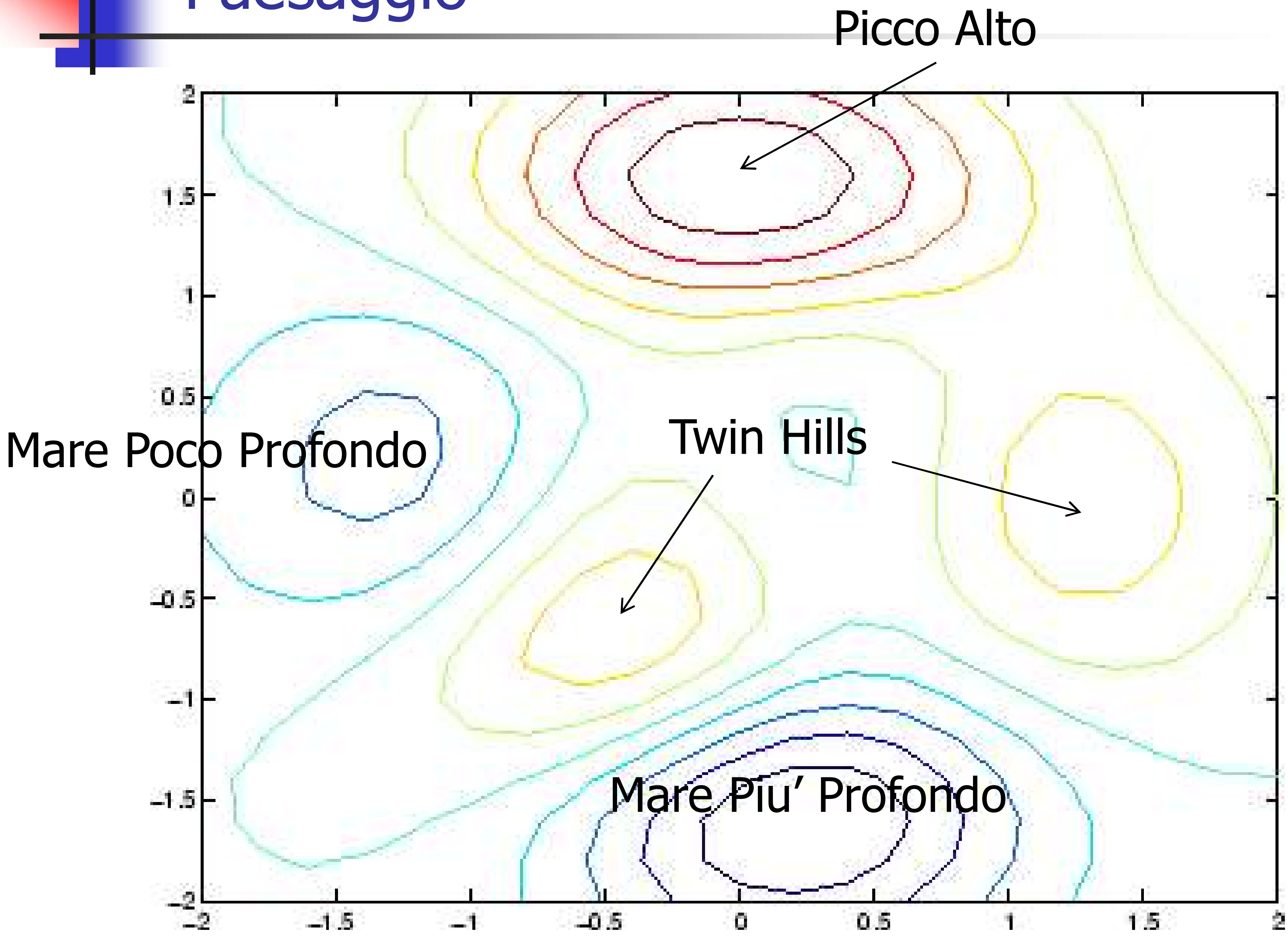
$\uparrow$ campo vettoriale

derivata parziale.
si deriva lungo z
tenendo x e y costanti

Paesaggio

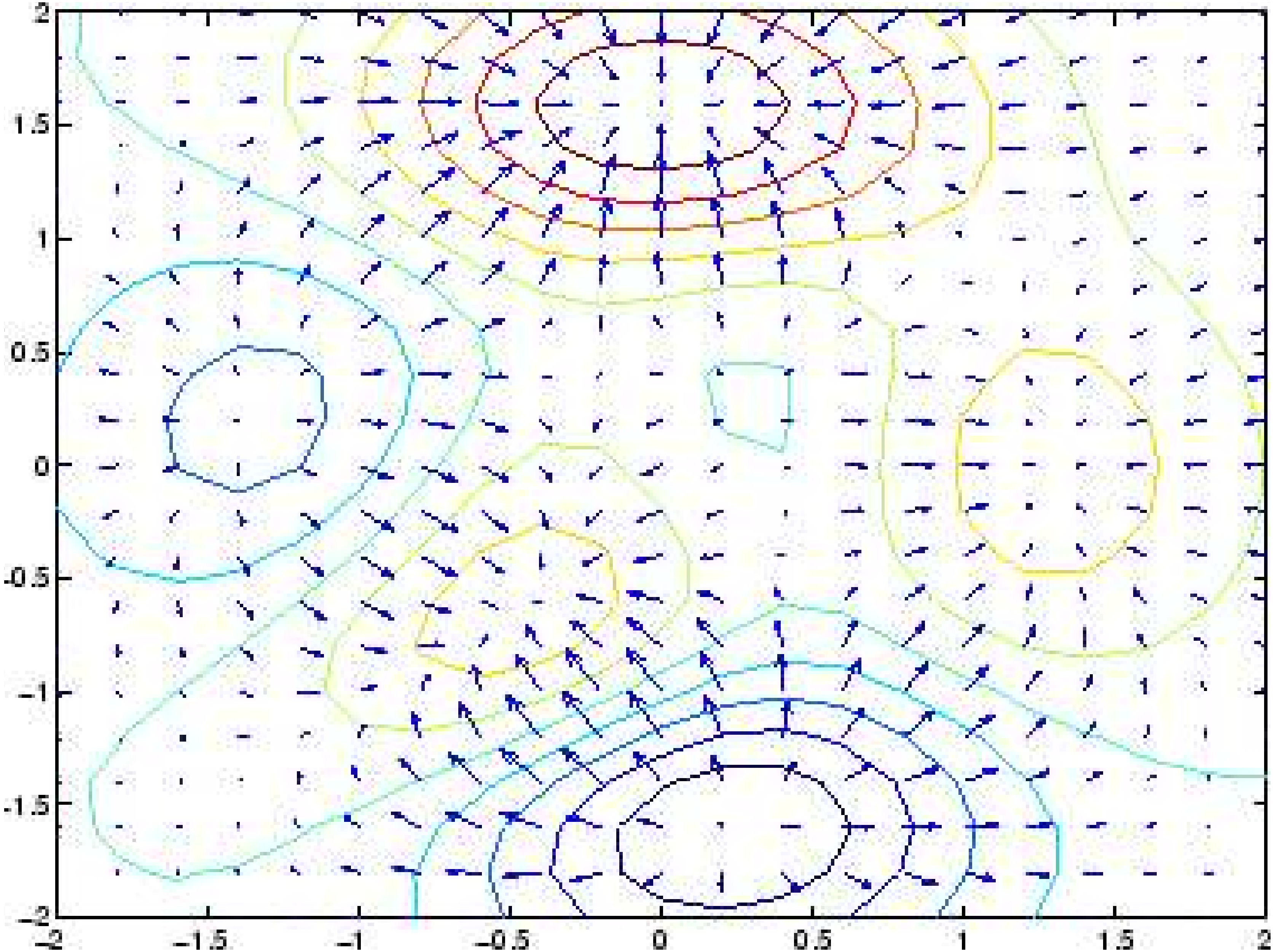


Paesaggio



Picco Alto
Mare Poco Profondo
Twin Hills
Mare Piu' Profondo
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

Paesaggio con Gradiente



Esempio

$$f(x,y) = x^3 - 3x - 2y^2$$

$$\nabla f(x,y) = \frac{\partial}{\partial x} f(x,y)\,\hat{i} + \frac{\partial}{\partial y} f(x,y)\,\hat{j}$$

$$\frac{\partial}{\partial x} f(x,y) = 3x^2 - 3 = 3(x^2-1)$$

$$\frac{\partial}{\partial y} f(x,y) = -4y$$

$$\nabla f(x,y) = 3(x^2-1)\,\hat{i} - 4y\,\hat{j}$$

$$\nabla f(x,y)\Big|_{(0,0)} = -3\,\hat{i}$$

Gradiente di $f(x,y)=x^3-3x-2y^2$