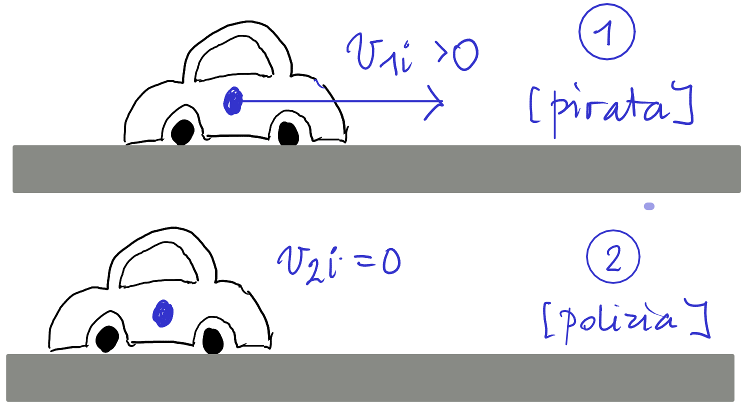
CINEMATICA

Moto dei corpi, indipendentemente dalle cause del moto → particelle

CINEMATICA 1D



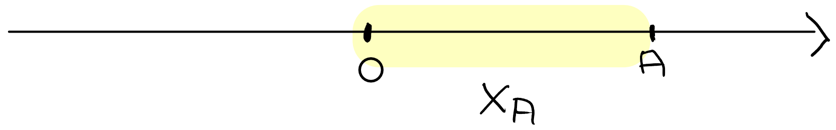
velocità costante

accelerazione costante

→ Dopo quanto tempo 2 raggiunge 1 ?

Evento < dove? quando? (x, t) → Origine

Definizione operativa: metro + cronometro

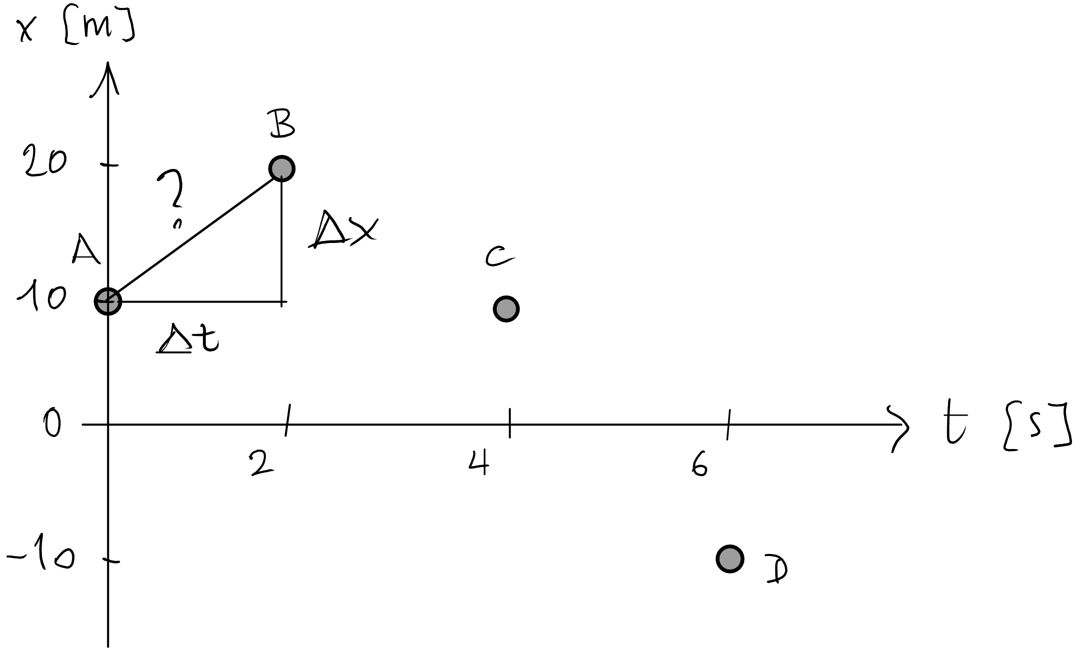


Posizione: distanza dall'origine, con segno − se A è dalla parte opposta della freccia

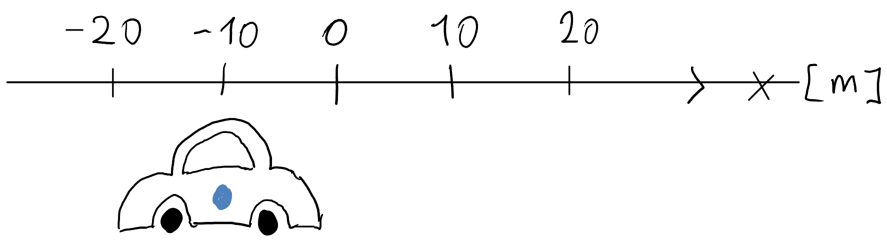
Istante di tempo: tempo trascorso dall'origine → t_A

Spostamento: $\Delta x = x_B - x_A$ **Intervallo di tempo**: $\Delta t = t_B - t_A$

Velocità : quanto rapidamente varia la posizione ?



	t [s]	x [m]
A	0.0	10
B	2.0	20
C	4.0	10
D	6.0	-10

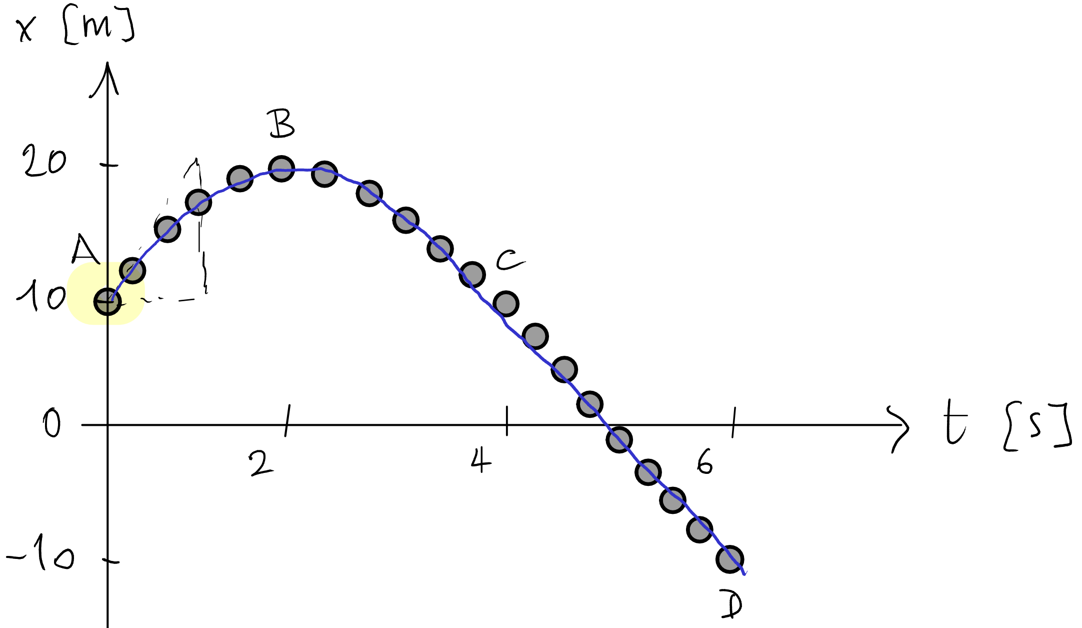


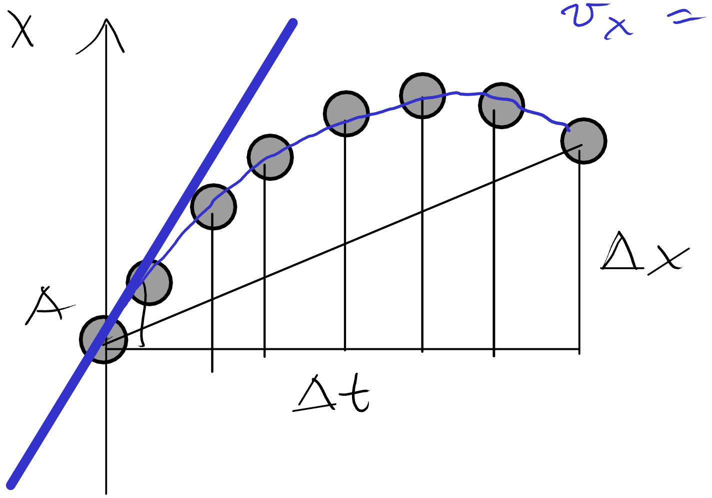
$$v_{xm} = \frac{\Delta x}{\Delta t}$$ **velocità media**

$$[v_{xm}] = \frac{L}{T} \qquad SI: \frac{m}{s} \qquad \pm$$

$A \to B$ $\begin{cases} \Delta t = 2\,s \\ \Delta x = 10\,m \end{cases} \to v_{xm} = \dfrac{10\,m}{2\,s} = 5\,\dfrac{m}{s}$

$C \to D$ $\begin{cases} \Delta t = 2\,s \\ \Delta x = x_D - x_C = -10\,m - 10\,m = -20\,m \end{cases} \to v_{xm} = \dfrac{-20\,m}{2\,s} = -10\,\dfrac{m}{s}$

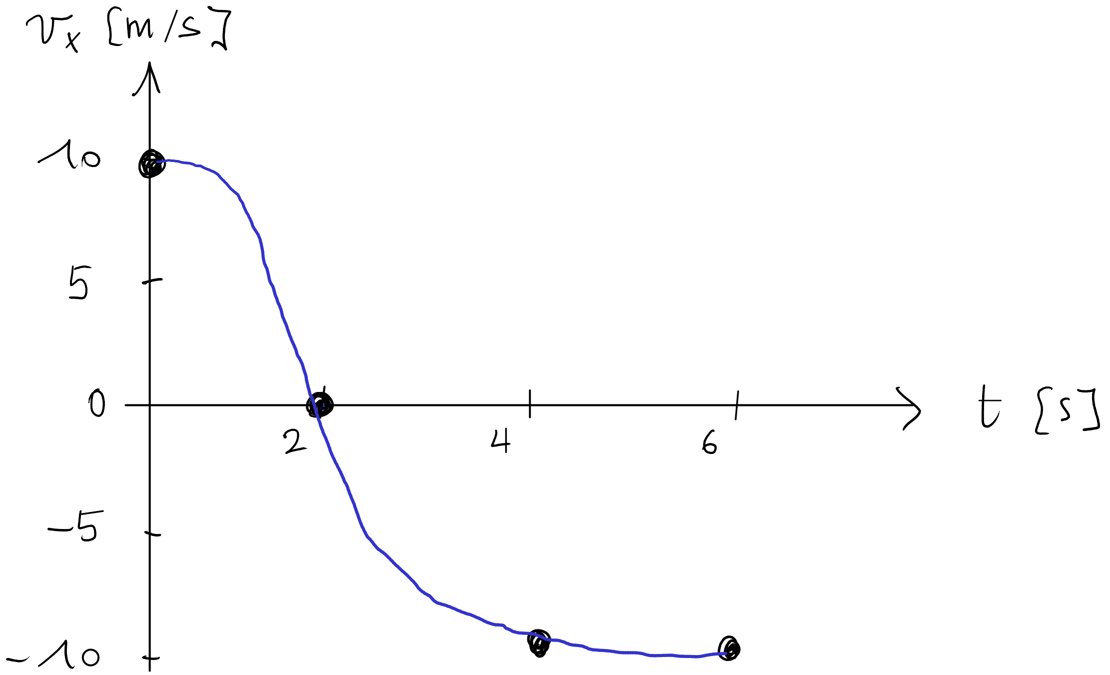




v_x = coefficiente angolare della retta tangente alla curva $x(t)$ in A

$$\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = v_x$$

velocità istantanea



$$\frac{dx}{dt} = v_x$$

derivata della posizione rispetto al tempo

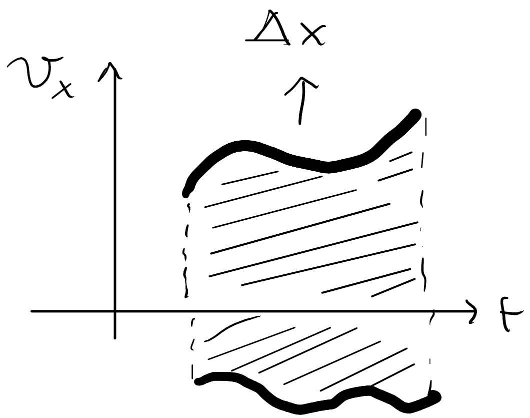
$$\begin{cases} x(t) \\ v_x(t) \end{cases}$$

leggi orarie del moto

Moto rettilineo uniforme

Particella, 1d. Per ogni $t \in [t_i, t_f]$

$$v_x = \text{costante} = v_{xm}$$

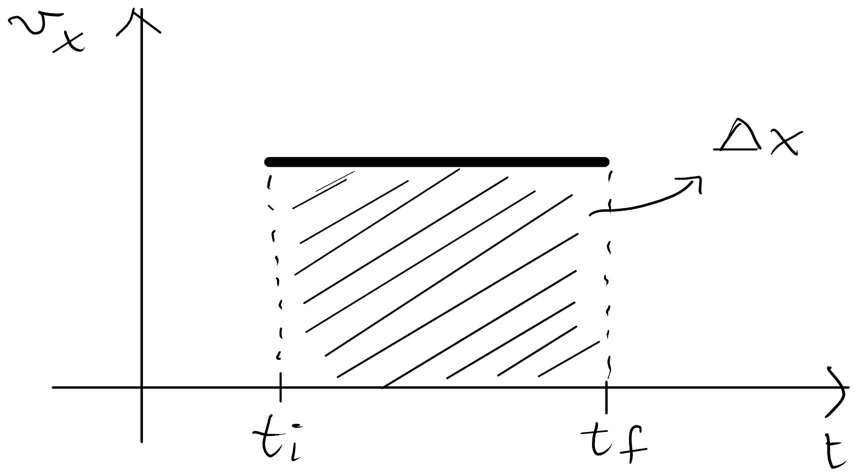


① $v_x = v_{xm} = \dfrac{x_f - x_i}{t_f - t_i}$

$x_f - x_i = v_x (t_f - t_i)$ $\qquad \Delta x = v_x \Delta t$

$x_f = x_i + v_x (t_f - t_i)$

$x = x_i + v_x (t - t_i)$ $\qquad v_x = \text{cost}$ $\qquad$ leggi orarie



② Calcolo integrale:

$\dfrac{dx}{dt} = v_x = \text{cost}$ $\qquad dx = v_x \, dt$

$$\int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} v_x \, dt \rightarrow \overbrace{x_f - x_i}^{\Delta x} = v_x (t_f - t_i) \rightarrow x = x_i + v_x (t - t_i)$$

$$\int_{t_i}^{t_f} \underbrace{\frac{dx}{dt}\, dt}_{dx} = \int_{t_i}^{t_f} v_x \, dt$$

cambiamento di variabile : $\displaystyle\int_a^b f(g(x))\, \frac{dg}{dx}\, dx \overset{y = g(x)}{=} \int_{g(a)}^{g(b)} f(y)\, dy$

$$\int_{x_i}^{x_f} dx = v_x (t_f - t_i) \quad \text{con } x_f = x(t_f) \text{ e } x_i = x(t_i)$$

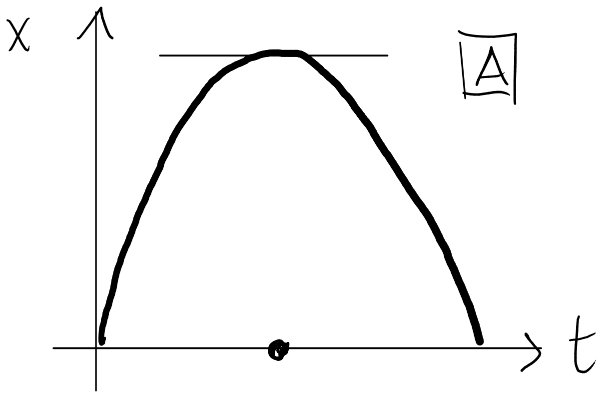
$$x_f - x_i = v_x (t_f - t_i)$$

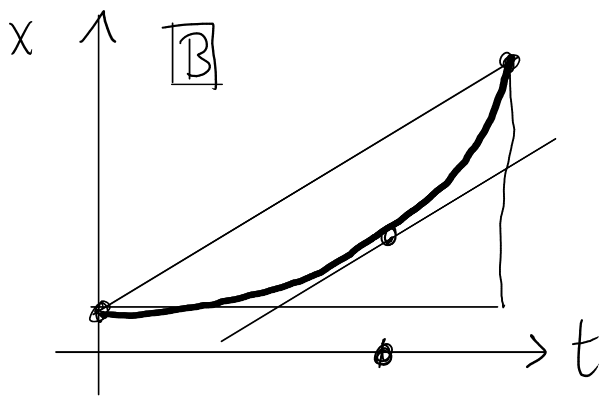
Esercizio: velocità media e istantanea

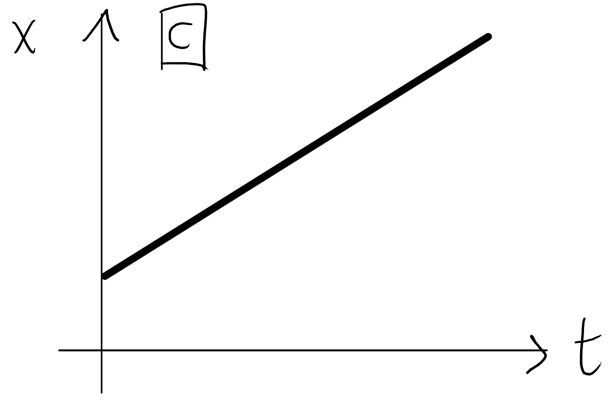
[A] palla lanciata in aria che ricade a terra

[B] auto che aumenta la sua velocità da 0 km/h a 100 km/h

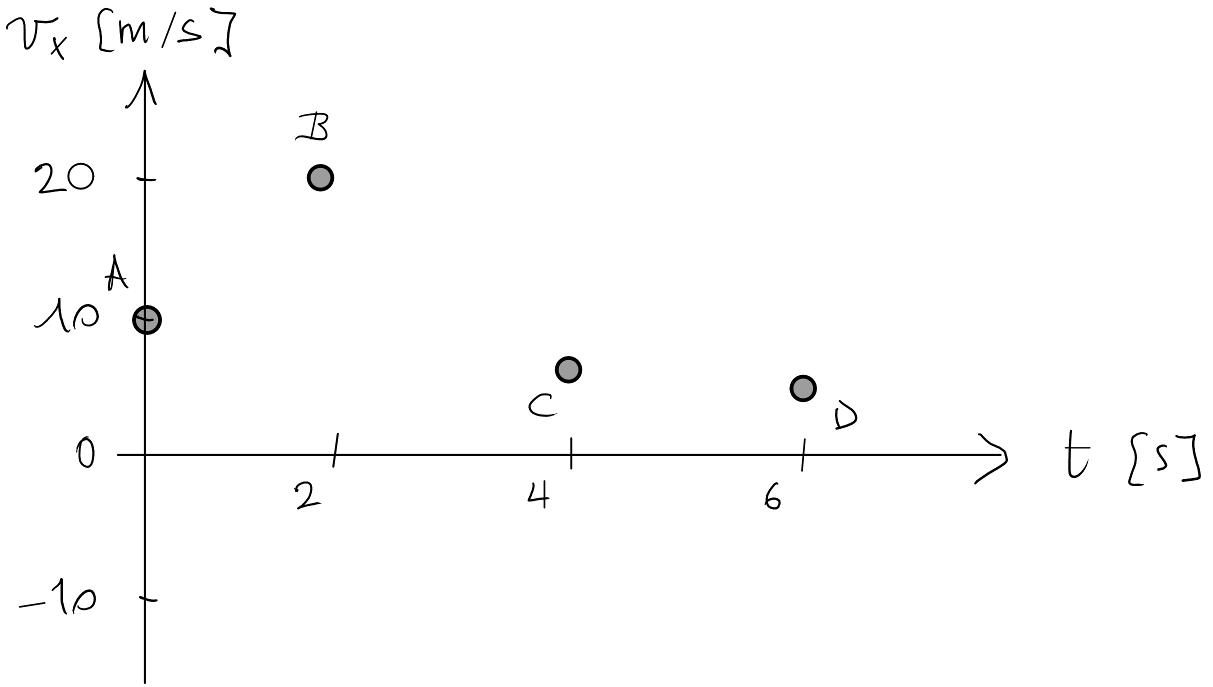
[C] sonda spaziale che si muove nel vuoto a velocità costante







Accelerazione : quanto rapidamente varia la velocità istantanea ?



	t[s]	v_x [m/s]
A	0.0	10
B	2,0	20
C	4.0	6
D	6.0	5

$A \to B \quad \begin{cases} \Delta t = 2\,s \\ \Delta v_x = v_{xB} - v_{xA} = 20\,m/s - 10\,m/s = 10\,m/s \end{cases}$

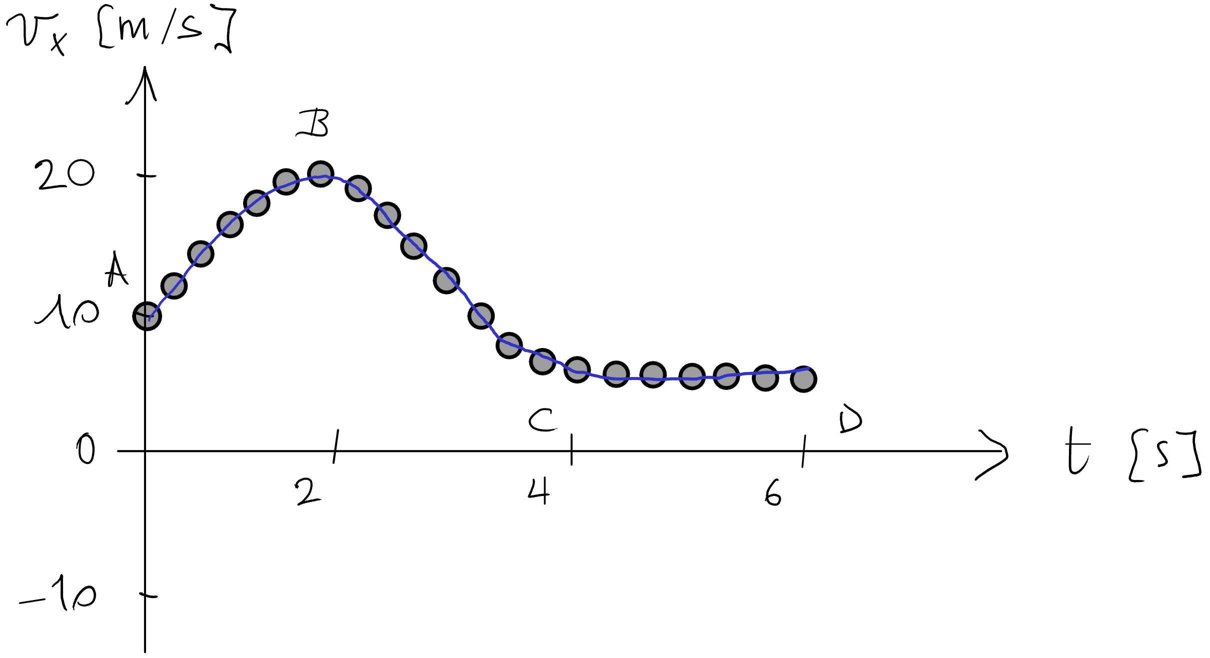
$C \to D \quad \begin{cases} \Delta t = 2s \\ \Delta v_x = -1\ m/s \end{cases}$

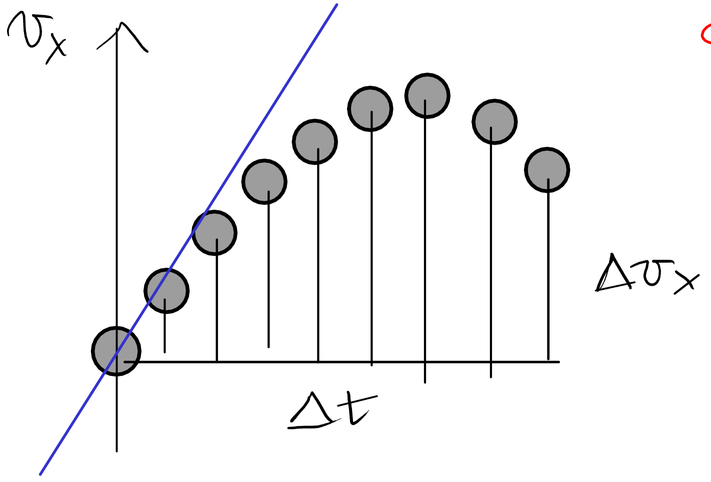
$a_{xm} = \dfrac{\Delta v_x}{\Delta t}$ accelerazione media

$[a_{xm}] = \dfrac{L}{T^2}$ SI : $\dfrac{m}{s^2}$ $\pm$

$A \to B$: $a_{xm} = \dfrac{10\ m/s}{2s} = 5\ \dfrac{m}{s^2}$

$C - D$: $a_{xm} = \dfrac{-1\ m/s}{2s} = -0,5\ \dfrac{m}{s^2}$

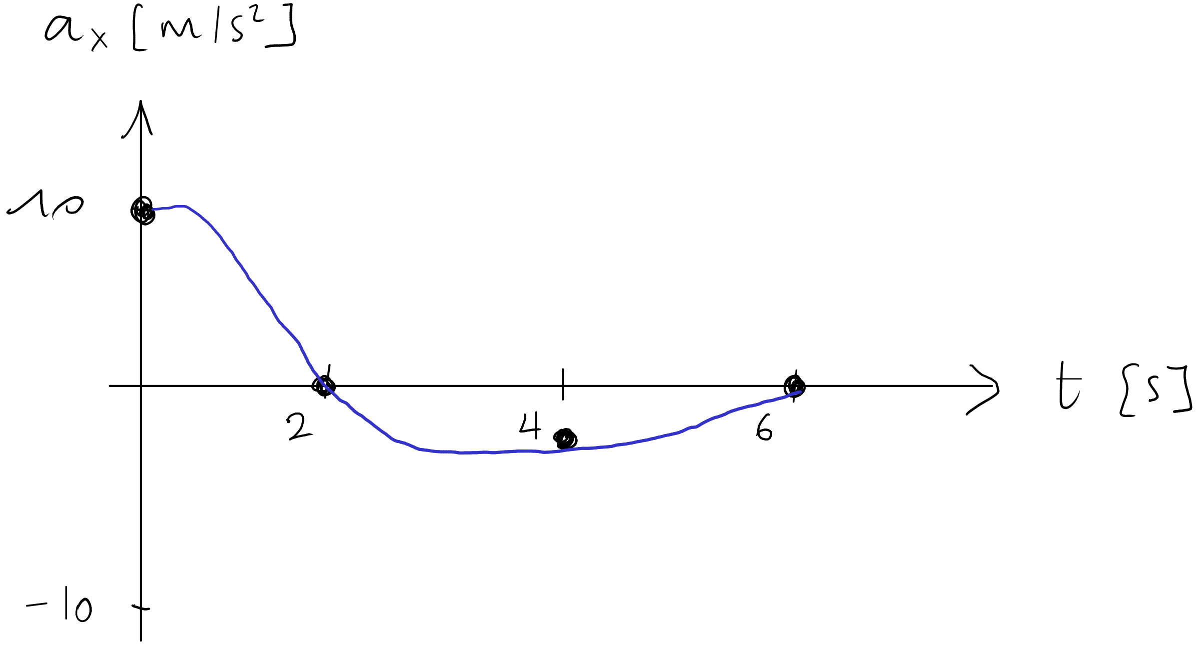




coefficiente angolare della retta tangente

$$\lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = a_x$$ accelerazione istantanea

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$



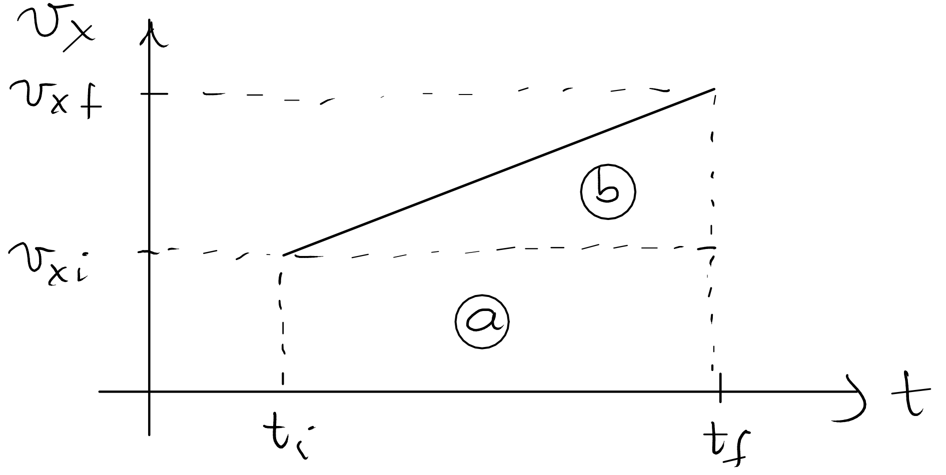
$\left.\begin{array}{l} x(t) \\ v_x(t) \\ a_x(t) \end{array}\right\}$ leggi orarie del moto

Moto uniformemente accelerato

Particella, 1d. Per ogni $t \in [t_i, t_f]$

$$a_x = \text{cost} = a_{xm}$$

① Approccio "geometrico"



$$a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

$$v_{xf} - v_{xi} = a_x (t_f - t_i)$$

$$v_{xf} = v_{xi} + a_x (t_f - t_i)$$

$$v_x = v_{xi} + a_x (t - t_i) \quad , \quad a_x = \text{cost}$$

$$\Delta x = x_f - x_i = a + b = v_{xi} (t_f - t_i) + \frac{1}{2} (v_{xf} - v_{xi}) \cdot (t_f - t_i)$$

$$x_f = x_i + v_{xi} (t_f - t_i) + \frac{1}{2} a_x (t_f - t_i)^2$$

$$x = x_i + v_{xi} (t - t_i) + \frac{1}{2} a_x (t - t_i)^2$$

② Calcolo integrale

$$\frac{dv_x}{dt} = a_x = \text{cost}$$

$$dv_x = a_x\, dt$$

$$\int_{v_{xi}}^{v_{xf}} dv_x = \int_{t_i}^{t_f} a_x\, dt = a_x \int_{t_i}^{t_f} dt$$

$$v_{xf} = v_{xi} + a_x (t_f - t_i)$$

$$v_x = v_{xi} + a_x (t - t_i)$$

$$\frac{dx}{dt} = v_x$$

$$dx = v_x dt = v_{xi} dt + a_x (t - t_i)\, dt$$

$$\int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} v_{xi}\, dt + \int_{t_i}^{t_f} a_x (t - t_i)\, dt$$

$$x_f = x_i + v_{xi}(t_f - t_i) + a_x \int_{t_i}^{t_f} (t - t_i)\, dt$$

cambiamento di variabile

$$\int_{t_i}^{t_f} (t - t_i)\, dt = \int_0^{t_f - t_i} t'\, dt' = \frac{1}{2}(t_f - t_i)^2$$

$$t' = t - t_i \qquad dt = dt'$$

$$= x_i + v_{xi}(t_f - t_i) + a_x \left[\frac{1}{2}(t - t_i)^2 \right]_{t_i}^{t_f}$$

$$= x_i + v_{xi}(t_f - t_i) + \frac{1}{2} a_x \left[(t_f - t_i)^2 - 0 \right]$$

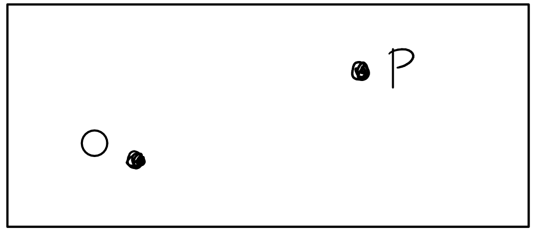
$$= x_i + v_{xi}(t_f - t_i) + \frac{1}{2} a_x (t_f - t_i)^2$$

$$x = x_i + v_{xi}(t - t_i) + \frac{1}{2} a_x (t - t_i)^2$$

CINEMATICA 2D

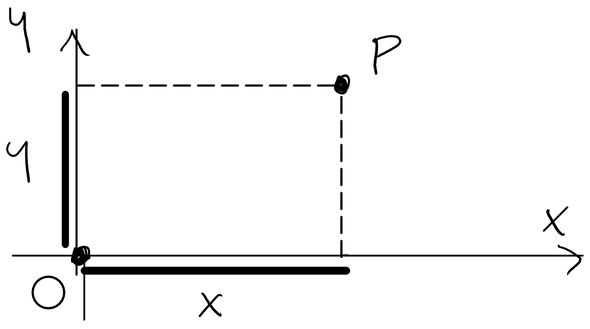
Moto di un corpo sul piano (2d) $\longrightarrow$ corpo = particella

Sistema di coordinate



1. punto di riferimento (origine)
2. 2 assi
3. protocollo per determinare il punto P in cui si trova la particella

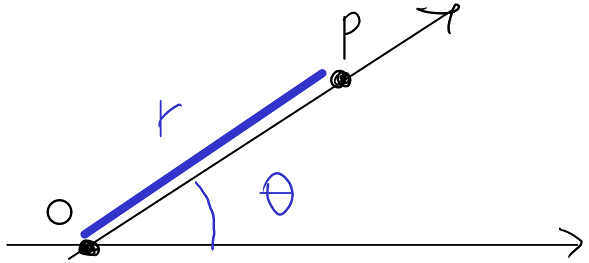
Coordinate cartesiane : (x, y)



x (x_p) : lunghezza del segmento tra O e la proiezione di P su asse x con segno − se proiezione è dalla parte opposta della freccia rispetto a O

y (y_p) : ..... su asse y ....

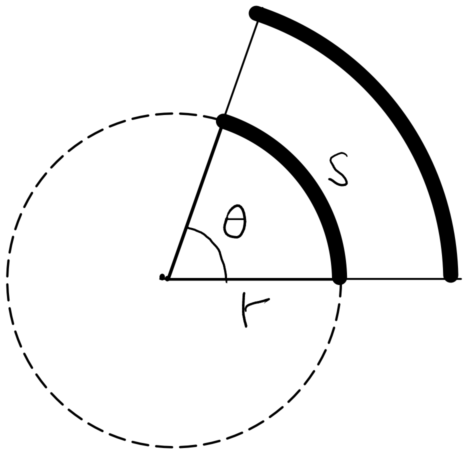
Coordinate polari : (r, θ)



r : lunghezza segmento tra O e P

θ : angolo tra assi in senso antiorario

Richiami di trigonometria



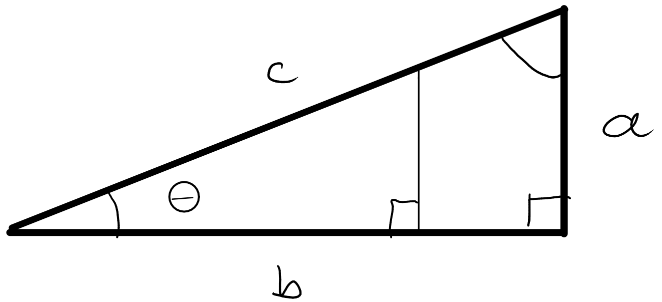
$$\theta = \frac{s}{r} \qquad [\theta] = 1$$

unità di misura:

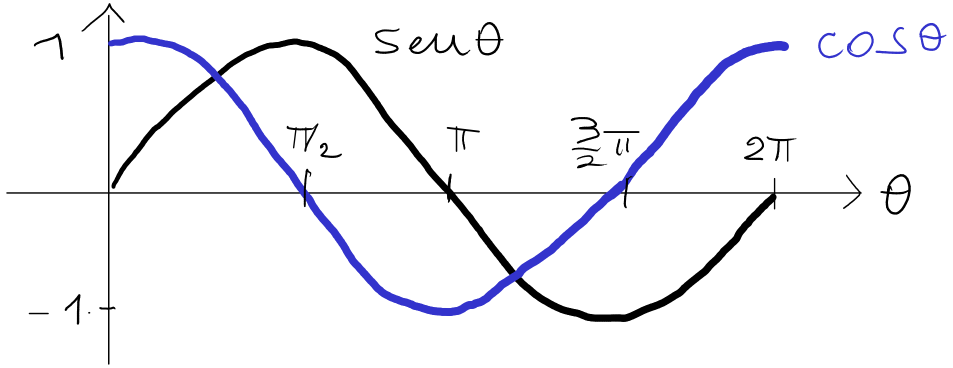
rad

(radiante)

$s = 2\pi r$	$\theta = 2\pi$	$\theta = 360°$
$s = \pi r$	$\theta = \pi$	$\theta = 180°$
$s = \frac{\pi}{2} r$	$\theta = \frac{\pi}{2}$	$\theta = 90°$
	...	...
$s = \frac{2\pi r}{360}$	$\theta \approx 0.0174$	$\theta = 1°$



$$\begin{cases} \operatorname{sen}\theta = \frac{a}{c} \\ \cos\theta = \frac{b}{c} \\ \tan\theta = \frac{a}{b} \end{cases} \Rightarrow \begin{cases} \operatorname{sen}^2\theta + \cos^2\theta = 1 \\ \tan\theta = \frac{\sin\theta}{\cos\theta} \end{cases}$$

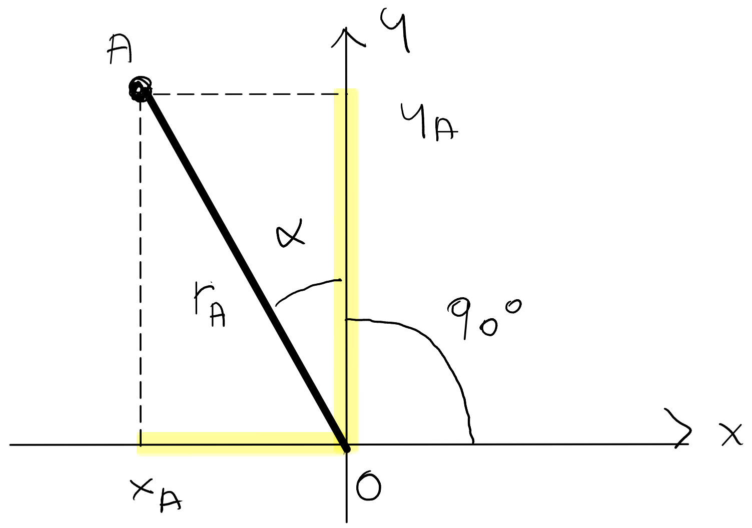


	0°	30°	45°	60°	90°
sen	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

$$\begin{cases} x = r\cos\theta \\ y = r\,\mathrm{sen}\,\theta \end{cases} \qquad \begin{cases} r = \sqrt{x^2+y^2} \\ \theta = \arctan\left(\frac{y}{x}\right) \end{cases}$$

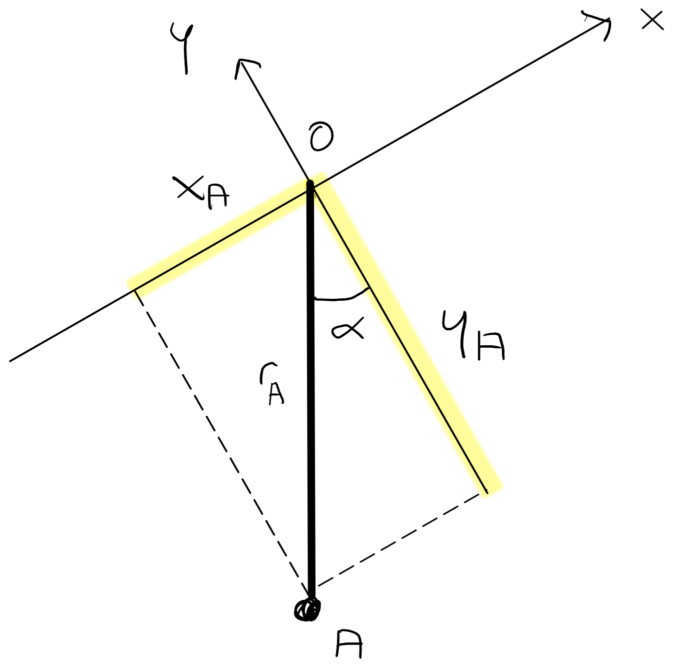
Esempi

a)



$$\begin{cases} x_A = r_A\cos(90° + \alpha) = -\,r_A\sin\alpha \\ y_A = r_A\sin(90° + \alpha) = r_A\cos\alpha \end{cases}$$

b)



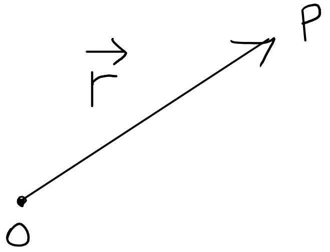
(es.)

Vettori

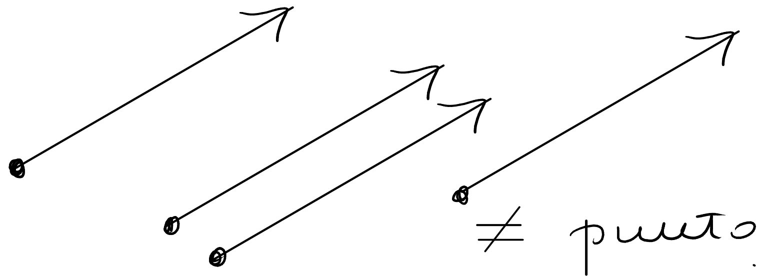
Grandezze SCALARI → 1 numero + unità di misura (es. L, T, M, ...)

Grandezze VETTORIALI → 2 o + numeri + -//- (es. (x, y))

Def. geometrica: grandezza caratterizzata da $\vec{r}, \bar{r}, \underline{r}, \mathbf{r}$

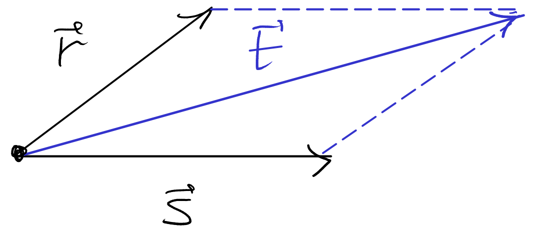


- direzione
- verso
- modulo (norma)



≠ punto di applicazione

* addizione $\vec{t} = \vec{r} + \vec{s}$



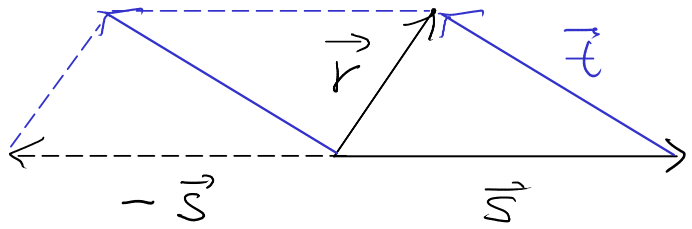
regola del parallelogramma

* opposto $-\vec{r}$

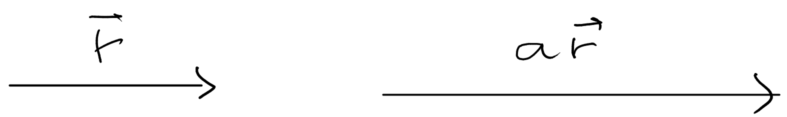
$$\vec{r} + (-\vec{r}) = \vec{0}$$

↑ Vettore di modulo nullo

* sottrazione $\vec{t} = \vec{r} - \vec{s}$



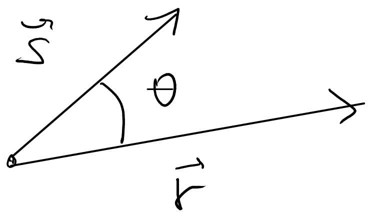
* Moltiplicazione per scalare $a\vec{r}$ $\quad a \in \mathbb{R}$



modulo di $a\vec{r}$: $|a| \cdot |\vec{r}|$ direzione: invariata

verso: opposto se $a < 0$

* Prodotto scalare $\vec{r} \cdot \vec{s}$



$$\vec{r} \cdot \vec{s} = |\vec{r}|\,|\vec{s}|\cos\theta$$

$// \Rightarrow \cos\theta = 1 \qquad \perp \Rightarrow \cos\theta = 0$

* Proprietà

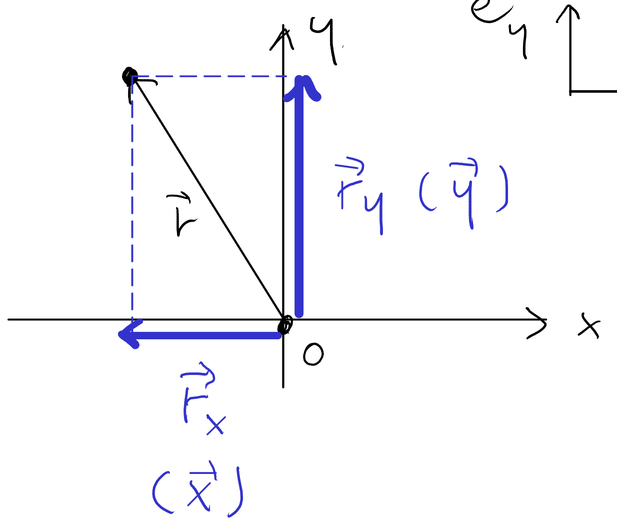
— commutativa $\quad \vec{r} + \vec{s} = \vec{s} + \vec{r}$

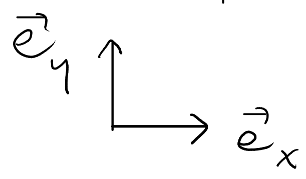
— associativa $\quad (\vec{r} + \vec{s}) + \vec{t} = \vec{r} + (\vec{s} + \vec{t})$

— distributiva $\quad a(\vec{r} + \vec{s}) = a\vec{r} + a\vec{s}$

$$\vec{t} \cdot (\vec{r} + \vec{s}) = \vec{t} \cdot \vec{r} + \vec{t} \cdot \vec{s}$$

Definizione in componenti





$$\vec r = \underbrace{\vec r_x + \vec r_y}_{\text{vettori componenti}} \qquad \underbrace{|\vec e_x| = |\vec e_y| = 1}_{\text{vettori "unitari" VERSORI}} \qquad \vec e_x \cdot \vec e_y = 0$$

ortogonali

$$\begin{cases} \vec r_x = r_x \vec e_x \\ \vec r_y = r_y \vec e_y \end{cases}$$ componenti cartesiane di $\vec r$ $\quad \left[\begin{cases} \vec x = x \vec e_x \\ \vec y = y \vec e_y \end{cases} \right]$

$$\vec r = r_x \vec e_x + r_y \vec e_y$$ $\{\vec e_x, \vec e_y\}$ base cartesiana

Modulo:

$$|\vec r| = \sqrt{r_x^2 + r_y^2} \qquad \left[= \sqrt{x^2 + y^2} \right]$$

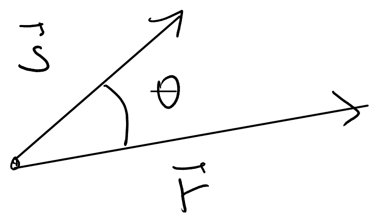
Somma:

$$\vec t = \vec r + \vec s = \underbrace{r_x \vec e_x + r_y \vec e_y} + \underbrace{s_x \vec e_x + s_y \vec e_y} = \underbrace{(r_x + s_x)}_{t_x} \vec e_x + \underbrace{(r_y + s_y)}_{t_y} \vec e_y$$

Moltiplicazione per scalare $a \in \mathbb{R}$

$$\vec{s} = a\vec{r} = a(r_x \vec{e}_x + r_y \vec{e}_y) = \underbrace{a r_x}_{s_x} \vec{e}_x + \underbrace{a r_y}_{s_y} \vec{e}_y$$

Prodotto scalare



$$\vec{r} \cdot \vec{s} = |\vec{r}|\,|\vec{s}| \cos\theta$$

$$= (r_x \vec{e}_x + r_y \vec{e}_y) \cdot (s_x \vec{e}_x + s_y \vec{e}_y)$$

$$= r_x \vec{e}_x \cdot s_x \vec{e}_x + r_y \vec{e}_y \cdot s_y \vec{e}_y$$

$$+ \cancel{r_x \vec{e}_x \cdot s_y \vec{e}_y} + \cancel{r_y \vec{e}_y \cdot s_x \vec{e}_x}$$

$$\begin{cases} \vec{e}_x \cdot \vec{e}_x = 1 \\ \vec{e}_x \cdot \vec{e}_y = 0 \end{cases} \longrightarrow \; = r_x s_x + r_y s_y$$

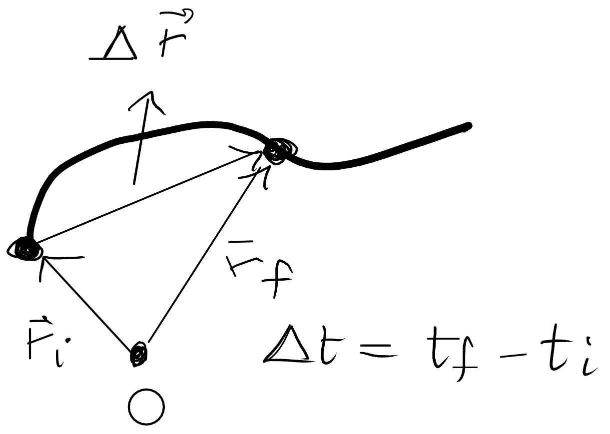
$$|\vec{r}|^2 = \vec{r} \cdot \vec{r} \;\rightarrow\; |\vec{r}| = \sqrt{\vec{r} \cdot \vec{r}}$$

Cinematica sul piano

$\vec{r} = x\,\vec{e}_x + y\,\vec{e}_y$ posizione

$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$ spostamento

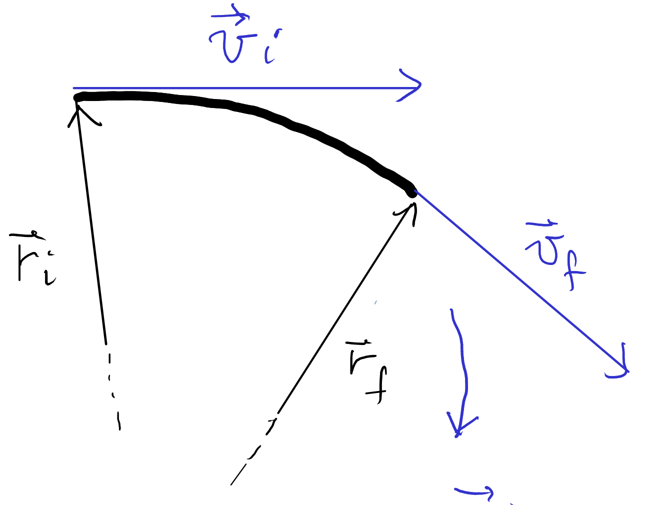
$\vec{v}_m = \dfrac{\Delta\vec{r}}{\Delta t}$ velocità media



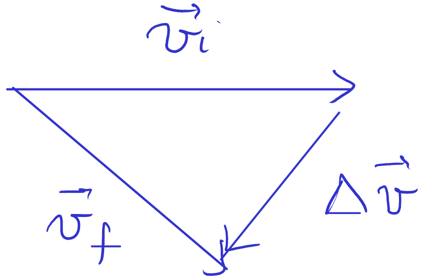
$$\vec{v}_m = \frac{1}{\Delta t}\Delta\vec{r} = \frac{1}{\Delta t}\left[\Delta x\,\vec{e}_x + \Delta y\,\vec{e}_y\right] = \frac{\Delta x}{\Delta t}\vec{e}_x + \frac{\Delta y}{\Delta t}\vec{e}_y$$

$$\lim_{\Delta t \to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[x\,\vec{e}_x + y\,\vec{e}_y\right] = \frac{d}{dt}(x\,\vec{e}_x) + \frac{d}{dt}(y\,\vec{e}_y)$$

$$\vec{v} = \frac{dx}{dt}\vec{e}_x + \frac{dy}{dt}\vec{e}_y = v_x\,\vec{e}_x + v_y\,\vec{e}_y$$ velocità



$\vec{a}_m = \dfrac{\Delta\vec{v}}{\Delta t}$ accelerazione media



accelerazione

$$\lim_{\Delta t \to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = \frac{d^2x}{dt^2}\vec{e}_x + \frac{d^2y}{dt^2}\vec{e}_y = a_x\vec{e}_x + a_y\vec{e}_y = \vec{a}$$

Moto di un proiettile

	Con attrito
Massa e diametro non influenzano la traiettoria	✗
Altezza massima proporzionale angolo iniziale	✓
Gittata diminuisce con attrito	
Gittata massima per 45°	?
Gittata dipende in modo non lineare da v_i	✓
Gittata simmetrica rispetto a 45°	

Moto uniformemente accelerato in 2d

Corpo = particella

Accelerazione costante : $\vec{a} = \text{cost}$ (no attrito)

Base cartesiana : $\{\vec{e}_x, \vec{e}_y\}$ → sistema di coordinate

Condizioni iniziali : $\vec{r}_i = x_i \vec{e}_x + y_i \vec{e}_y$ $\quad \vec{v}_i = v_{ix} \vec{e}_x + v_{iy} \vec{e}_y$ $\quad t_i = 0$

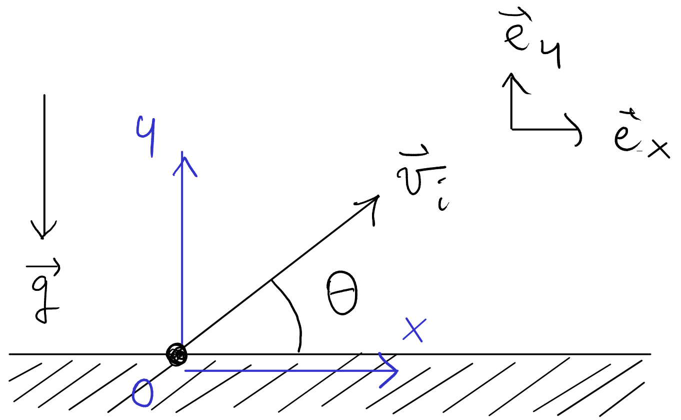
$$\vec{a} = \frac{d^2x}{dt^2} \vec{e}_x + \frac{d^2y}{dt^2} \vec{e}_y = a_x \vec{e}_x + a_y \vec{e}_y \quad (a_x, a_y \text{ costanti})$$

$$\begin{cases} \dfrac{d^2x}{dt^2} = a_x \\ \dfrac{d^2y}{dt^2} = a_y \end{cases} \Rightarrow \begin{cases} v_x = v_{xi} + a_x t \\ v_y = v_{yi} + a_y t \end{cases} \Rightarrow \begin{cases} x = x_i + v_{ix} t + \frac{1}{2} a_x t^2 \\ y = y_i + v_{iy} t + \frac{1}{2} a_y t^2 \end{cases}$$

$$\vec{a} = \text{cost} \qquad \underbrace{\vec{v} = \vec{v}_i + \vec{a} t \qquad \vec{r} = \vec{r}_i + t \vec{v}_i + \frac{1}{2} t^2 \vec{a}}_{\text{leggi orarie}}$$

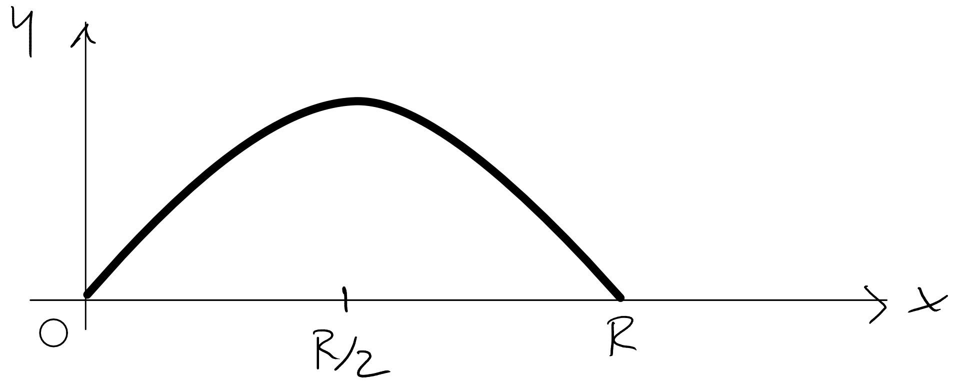
$g = 9{,}81 \ \frac{m}{s^2}$ $\quad \vec{a} = \vec{g} = -g\,\vec{e}_y$ $\quad \vec{r}_i = \vec{0}$ $\quad \vec{v}_i = v_{ix}\,\vec{e}_x + v_{iy}\,\vec{e}_y$

$$\vec{v}_i = |\vec{v}_i| \cos\theta \ \vec{e}_x + |\vec{v}_i| \operatorname{sen}\theta \ \vec{e}_y$$



$$\begin{cases} x = |\vec{v}_i| \cos\theta \ t \\ y = |\vec{v}_i| \operatorname{sen}\theta \ t - \frac{1}{2} g t^2 \end{cases}$$

$$\vec{r} = \underset{\sim\sim}{v_{ix} t}\, \vec{e}_x + \underline{v_{iy} t}\, \vec{e}_y \underline{- \frac{1}{2} g t^2}\, \vec{e}_y = \underset{\sim}{x}\, \vec{e}_x + \underline{y}\, \vec{e}_y$$



Traiettoria : $y = y(x)$

$$t = \frac{x}{|\vec{v}_i| \cos\theta}$$

$$y = \frac{\operatorname{sen}\theta}{\cos\theta}\, x - \frac{1}{2} \frac{g}{|\vec{v}_i|^2 \cos^2\theta}\, x^2$$

$$y = \tan\theta \ x - \frac{1}{2} \frac{g}{|\vec{v}_i|^2 \cos^2\theta}\, x^2 \Rightarrow \text{parabola} \ \boxed{V}$$

Gittata : $0 = y(R)$ $\quad (R \neq 0)$

$$0 = \tan\theta \cdot \cancel{R} - \frac{1}{2}\frac{g}{|\vec{v}_i|^2\cos^2\theta} R^{\cancel{2}} \Rightarrow R = \tan\theta \frac{2|\vec{v}_i|^2\cos^2\theta}{g}$$

$$R = 2\sin\theta\cos\theta \frac{|\vec{v}_i|^2}{g} = \sin(2\theta)\frac{|\vec{v}_i|^2}{g}$$

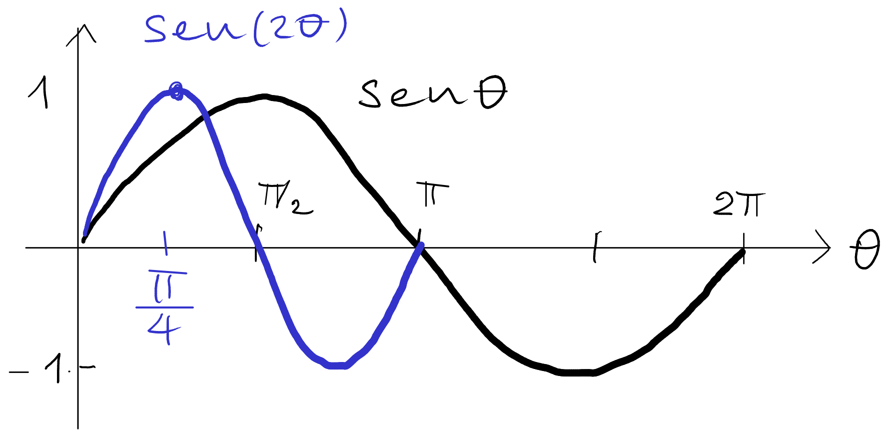
$$R \sim \underset{\boxed{V}}{|\vec{v}_i|^2} \sim \frac{1}{g} \sim \sin(2\theta)$$

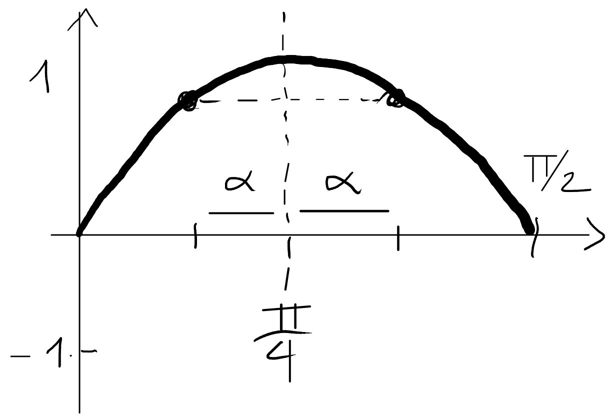
$\Downarrow$

gittata max

$\theta = \pi/4$

$\boxed{✓}$





simmetria rispetto a $\theta = \pi/4$

$\boxed{V}$

Altezza massima : $H = y\left(\frac{R}{2}\right)$

$$\begin{cases} v_x = |\vec{v}_i|\cos\theta \\ v_y = |\vec{v}_i|\,\mathrm{sen}\,\theta - gt \end{cases}$$

$v_y(t^*) = 0 \Rightarrow H = y(t^*)$

$t^* = \frac{|\vec{v}_i|\,\mathrm{sen}\,\theta}{g} \Rightarrow H = |\vec{v}_i|\,\mathrm{sen}\,\theta\, t^* - \frac{1}{2}g(t^*)^2$

$$H = \frac{|\bar{v}_1|^2 \operatorname{sen}^2\theta}{g} - \frac{1}{2}\cancel{g}\,\frac{|\bar{v}_1|^2 \operatorname{sen}^2\theta}{g^{\cancel{2}}} = \frac{1}{2}\,\frac{|\bar{v}_1|^2 \operatorname{sen}^2\theta}{g}$$

$$\left.\begin{array}{l} H \sim |\bar{v}_i|^2 \\ H \sim \operatorname{sen}^2\theta \end{array}\right\} \quad H \sim v_{iy}^2 \qquad \boxed{\checkmark}$$

$$H \sim \frac{1}{g}$$

Moto rettilineo uniforme sul piano

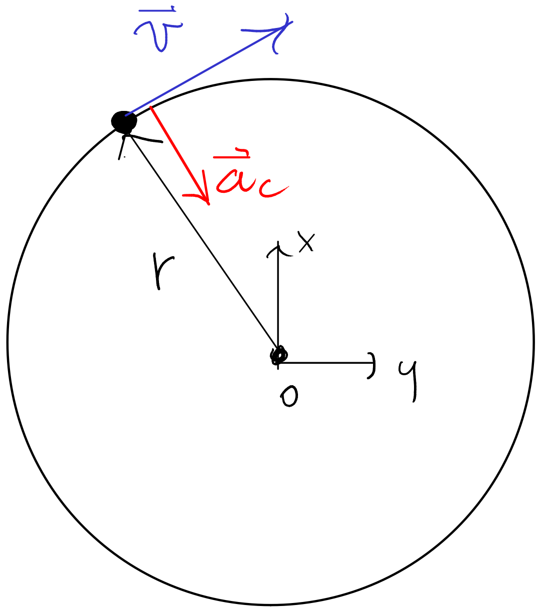
$$\begin{cases} \vec{a} = \vec{0} \\ \vec{v} = \vec{v}_i \\ \vec{r} = \vec{r}_i + \vec{v}_i\,(t - t_i) \end{cases}$$

Moto circolare uniforme

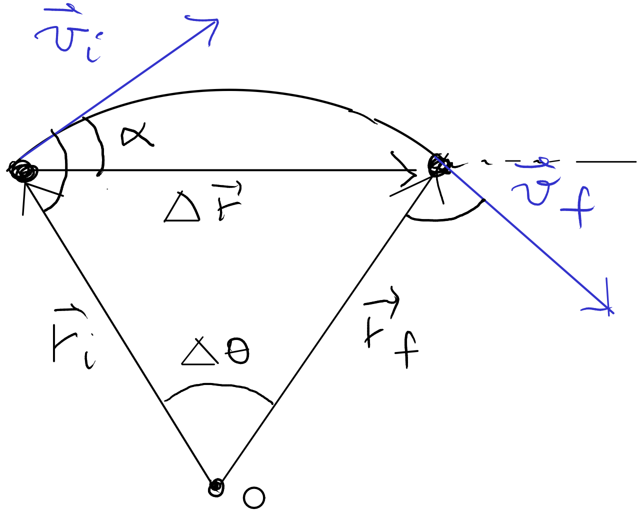
$|\vec{r}| = r = \text{cost} \qquad |\vec{v}| = v = \text{cost}$

$$\lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \vec{a}$$

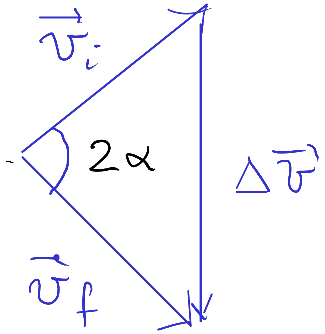
$\Rightarrow$ è accelerato !



$\vec{r} \perp \vec{v}$



$\Delta t = t_f - t_i$



$$\pi = \Delta\theta + 2\left(\frac{\pi}{2} - \alpha\right)$$
$$= \Delta\theta + \pi - 2\alpha$$
$$\Delta\theta = 2\alpha$$

$\Delta t \to 0$

$$\frac{|\Delta \vec{r}|}{|\vec{r}|} = \frac{|\Delta \vec{v}|}{|\vec{v}|} \qquad \frac{|\Delta \vec{r}|}{\Delta t}\,\frac{1}{r} = \frac{|\Delta \vec{v}|}{\Delta t}\,\frac{1}{v} \qquad \underbrace{\left|\frac{\Delta \vec{r}}{\Delta t}\right|}_{\to |\vec{v}|}\frac{1}{r} = \underbrace{\left|\frac{\Delta \vec{v}}{\Delta t}\right|}_{\to |\vec{a}|}\frac{1}{v}$$

$$\frac{v}{r} = \frac{|\vec{a}|}{v} \Rightarrow |\vec{a}| = \frac{v^2}{r}$$

accelerazione centripeta $\quad \boxed{a_c = \frac{v^2}{r}}$ ☑

Periodo: $2\pi r = v\tau \Rightarrow \tau = \dfrac{2\pi r}{v}$

$\uparrow$ periodo