$$Y(s) = \frac{4}{(s+2)(s^2+4)}$$

Determinare $y(t)$ tramite la formula dei residui

$$y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

Sviluppo in fratti semplici:

$$Y(s) = \frac{4}{(s+2)(s+j2)(s-j2)} = \frac{c_1}{s+2} + \frac{c_2}{s+j2} + \frac{c_2^*}{s-j2}$$

$$c_1 = \lim_{s\to -2} Y(s)(s+2) = \lim_{s\to -2} \frac{4}{(s+j2)(s-j2)} =$$

$$= \frac{4}{(-2+j2)(-2-j2)} = \frac{4}{(-2)^2-(j2)^2} = \frac{4}{8} = \frac{1}{2}$$

$$c_2 = \lim_{s\to -j2} Y(s)(s+j2) = \lim_{s\to -j2} \frac{4}{(s+2)(s-j2)} =$$

$$= \frac{4}{(2-j2)(-j2-j2)} = \frac{\cancel{4}\,1}{-j\cancel{4}_{\,1}(2-j2)} = \rightarrow$$

$$C_2 = \frac{\cancel{4}\,1}{-j\cancel{4}\,1\,(2-j2)} \cdot \frac{j}{j} = \frac{j}{(-j)j(2-j2)} =$$

$+1$

$$= \frac{j}{(2-j2)} \cdot \frac{(2+j2)}{(2+j2)} = \frac{j(2+j2)}{4+4} = \frac{\cancel{2}\,1\; j(1+j)}{\cancel{8}\,4} =$$

$$= \frac{1}{4}j(1+j) = -\frac{1}{4} + j\frac{1}{4}$$

$$C_2^* = \lim_{s \to j2} Y(s)(s-j2) = -\frac{1}{4} - j\frac{1}{4}$$

A questo punto:

$$Y(s) = \frac{C_1}{s+2} + \frac{C_2}{s+j2} + \frac{C_2^*}{s-j2} =$$

$$= \frac{1}{2} \cdot \frac{1}{s+2} + \frac{(-1+j)/4}{(s+j2)} + \frac{(-1-j)/4}{(s-j2)}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s+2}\right) = e^{-2t} \cdot 1(t)$$

$$\mathcal{L}^{-1}\left(\frac{1}{s-j2}\right) = e^{+2jt} \cdot 1(t)$$

$$\mathcal{L}^{-1}\left(\frac{1}{s+j2}\right) = e^{-2jt} \cdot 1(t)$$

Quindi:

$$y(t) = \frac{1}{2} e^{-2t} \cdot 1(t) + \frac{1}{4}(-1+j)\, e^{-2jt} \cdot 1(t) + $$
$$+ \frac{1}{4}(-1-j)\, e^{+2jt} \cdot 1(t)$$

riassemblo questi 2 termini, ricordando che

$$\cos(\omega t) \cdot 1(t) = \frac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right) \cdot 1(t)$$

$$\operatorname{sen}(\omega t) \cdot 1(t) = \frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right) \cdot 1(t)$$

$$\left(-\frac{1}{4}\right)\left[e^{-2jt} + e^{+2jt}\right] \cdot 1(t) +$$

$$+ \left(-\frac{1}{4} j\right)\left[e^{2jt} - e^{-2jt}\right] \cdot 1(t)$$

$$-\frac{1}{2} \cdot \frac{e^{-2jt} + e^{2jt}}{2} \cdot 1(t)$$

$$-\frac{1}{4} j = +\frac{1}{4j} \Longrightarrow +\frac{1}{2} \cdot \frac{e^{2jt} - e^{-2jt}}{2j} \cdot 1(t)$$

In definitiva:

$$y(t) = +\frac{1}{2}e^{-2t}\cdot 1(t) +$$

$-\frac{1}{2}\cos(2t)\cdot 1(t)$ $$-\frac{1}{2}\cdot\frac{e^{-2jt}+e^{2jt}}{2}\cdot 1(t) +$$

$\frac{1}{2}\cdot \operatorname{sen}(2t)\cdot 1(t)$ $$+\frac{1}{2}\cdot\frac{e^{2jt}-e^{-2jt}}{2j}\cdot 1(t)$$

$$y(t) = \frac{1}{2}e^{-2t}\cdot 1(t) - \frac{1}{2}\cos(2t)\cdot 1(t) +$$
$$+\frac{1}{2}\operatorname{sen}(2t)\cdot 1(t)$$

Antitrasformare $Y(s) = \dfrac{4}{(s+2)(s^2+4)}$

utilizzando il

principio di identità dei polinomi:

$$Y(s) = \frac{A}{s+2} + \frac{B}{s-j2} + \frac{C}{s+j2}$$

$$= \frac{A(s^2+4) + B(s+2)(s+j2) + C(s+2)(s-j2)}{(s+2)(s^2+4)}$$

$$= \frac{As^2 + 4A + (s+2)[Bs + j2B + Cs - 2jC]}{(s+2)(s^2+4)}$$

Solo il numeratore:

$$As^2 + 4A + (s+2)[(B+C)s + 2(B-C)j] = 4$$

$$(A+B+C)s^2 + [2(B-C)j + 2(B+C)]s +$$
$$+ 4A + 4(B-C)j = 4$$

Uguagliando i coefficienti dei monomi col medesimo grado si ottiene:

$$\begin{cases} A+B+C=0 \\ 2(B+C)+2j(B-C)=0 \\ A+(B-C)j=1 \end{cases}$$

$B=b+jd$

$C=b-jd$

$b,d\in\mathbb{R}$

$$\boxed{\begin{array}{c} \mathcal{B}\quad B^*=C \\ B,C\in\mathbb{C} \end{array}}$$

$B+C=2b$

$B-C=2jd$

$$\begin{cases} A+2b=0 \\ 4b+2j(2jd)=0 \\ A+(2jd)j=1 \end{cases}\qquad \begin{cases} A=-2b \\ 4b-4d=0 \\ A-2d=1 \end{cases}$$

$$\begin{cases} A=-2b \\ b=d \\ -4b=1 \end{cases}\qquad \begin{cases} b=-\frac{1}{4} \\ d=-\frac{1}{4} \end{cases}\qquad A=(-2)\left(-\frac{1}{4}\right)=\frac{1}{2}$$

$$\begin{cases} A = +\frac{1}{2} \\ B = -\frac{1}{4} - \frac{1}{4}j \\ C = -\frac{1}{4} + \frac{1}{4}j \end{cases}$$