

DINAMICA NEWTONIANA

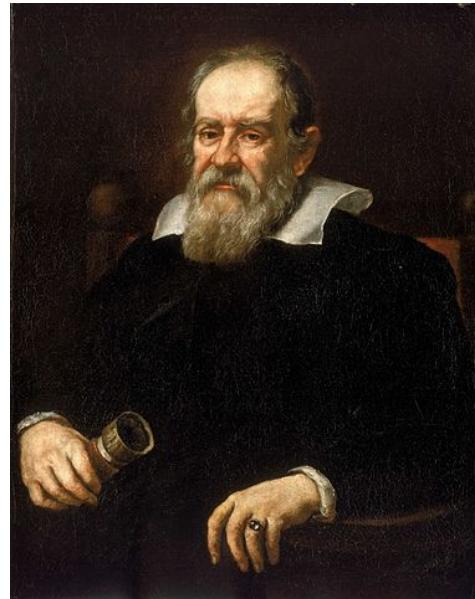
$$y = a x^b$$

$$\log y = \log a + b \log x$$

Cause del moto + cinematica

PRINCIPI DELLA DINAMICA

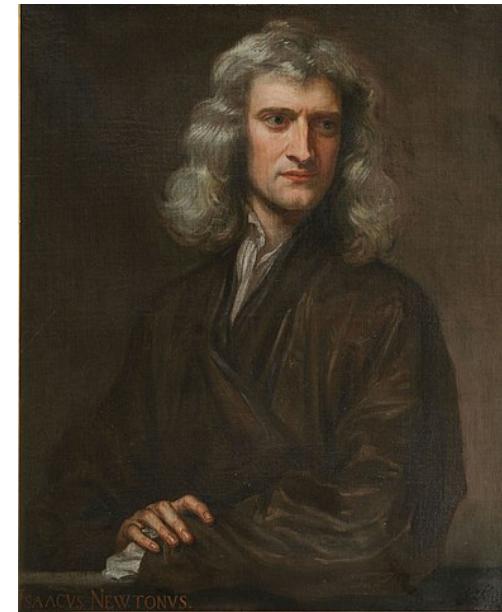
1600 : Galileo → metodo sperimentale, caduta dei gravi → g
Keplero → orbite dei pianeti : 3 leggi di Keplero → III legge Keplero
Newton : 3 principi → universale entro limiti di validità



Galileo Galilei
1564 - 1642



Johannes Kepler
1571 - 1630



Isaac Newton
1642 - 1727

$$\tau \sim R^{3/2}$$

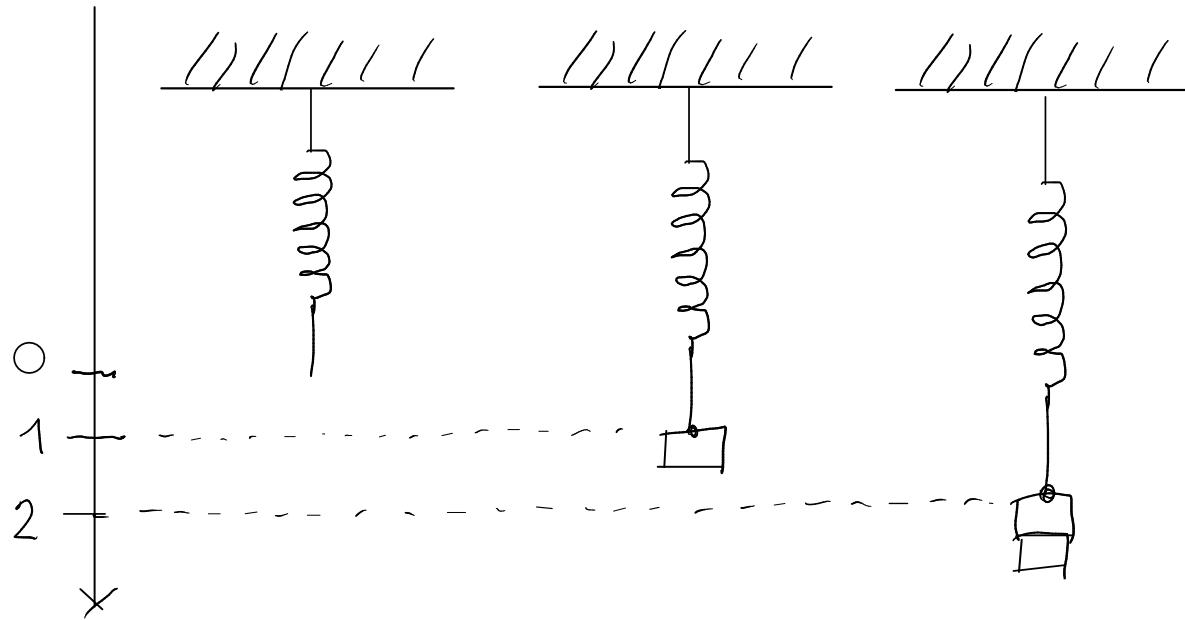
$$\tau^2 \sim R^3$$

1900 : velocità $\approx c$ → relatività speciale
lunghezze $\approx \lambda$ → meccanica quantistica } ⇒ limiti di validità

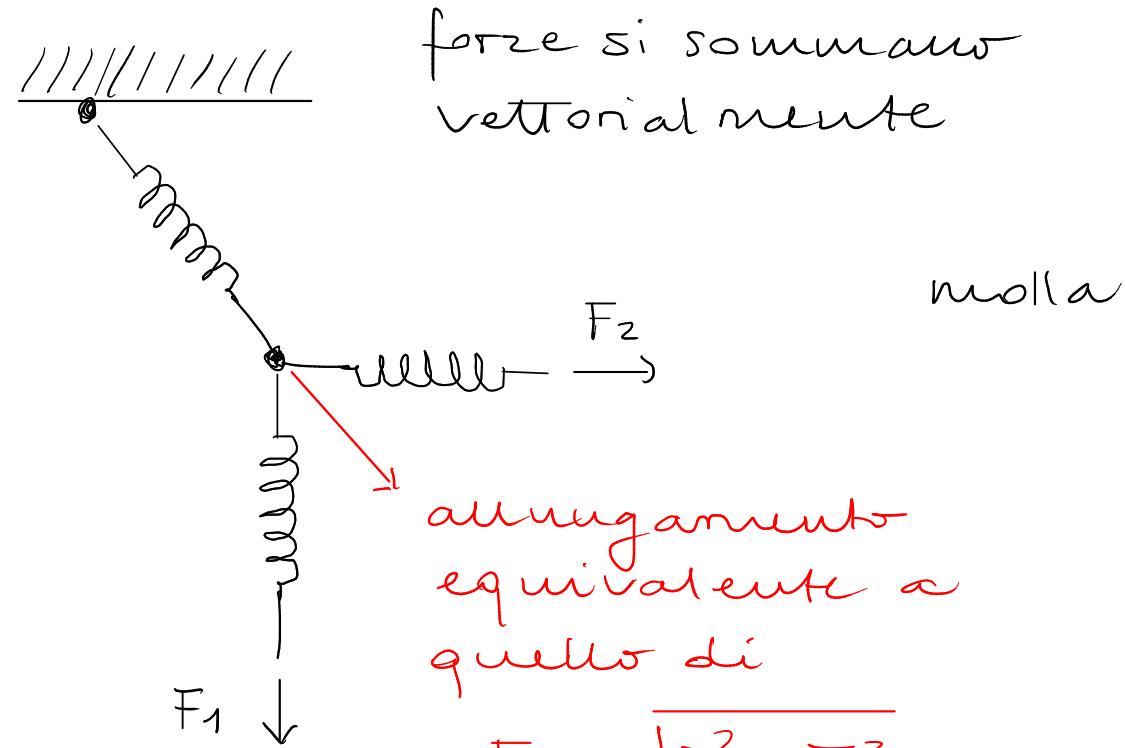
Definizione operativa di forza

Macro: deformazione di un corpo \rightarrow molla

dinamometro



La forza è una grandezza fisica vettoriale che posso misurare con un dinamometro.



forze si sommano vettorialmente

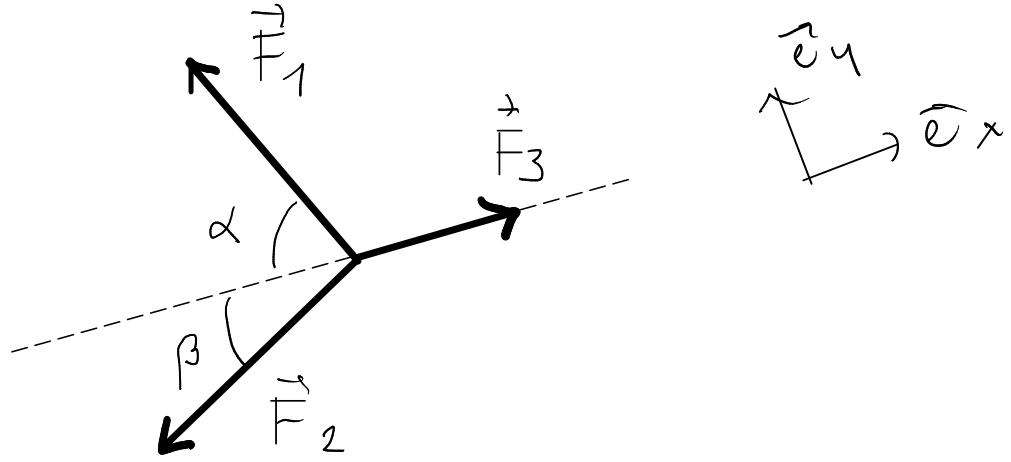
molla

allungamento equivalente a quello di

$$F = \sqrt{F_1^2 + F_2^2}$$



Es. dati $|\vec{F}_1|, |\vec{F}_2|, |\vec{F}_3|, \alpha, \beta$ determina $\sum_i \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$ in una base cartesiana $\{\vec{e}_x, \vec{e}_y\}$



I principio della dinamica (principio di inerzia, Galileo)

Se un corpo non interagisce con altri corpi, esso mantiene una velocità costante.

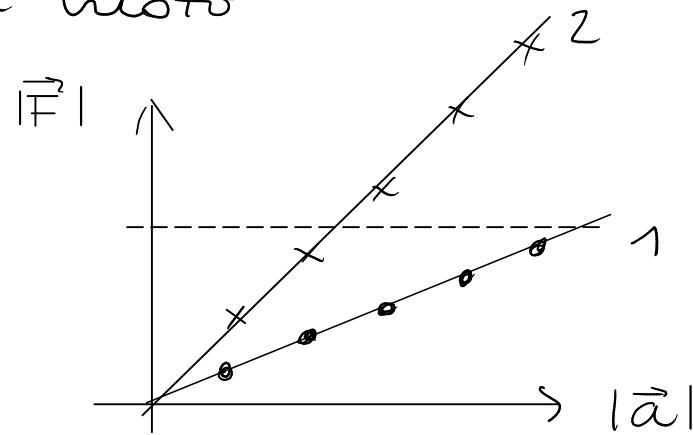
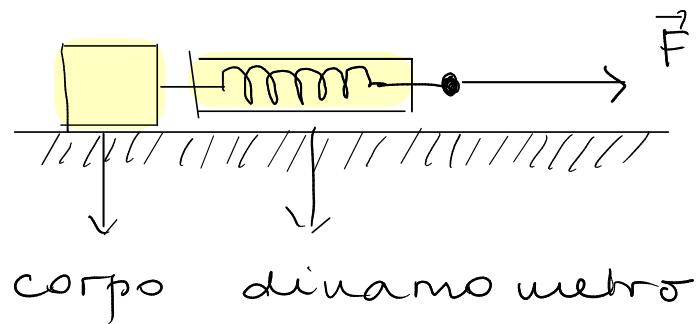


↓
no forze dovute
all'ambiente esterno

accelerazione nulla

II principio della dinamica (principio fondamentale della dinamica)

Forza \Rightarrow variazione del moto



$$|\vec{F}| \sim |\vec{a}|$$

$$m = \frac{|\vec{F}|}{|\vec{a}|}$$

massa inerziale

↓
massa
SI: kg

$$\vec{F}_1, \dots, \vec{F}_N$$

$$\sum_{i=1}^N \vec{F}_i = \underline{\underline{\sum \vec{F} = m \vec{a}}}$$

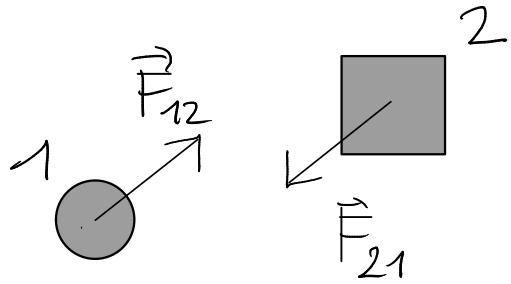
$$\left\{ \begin{array}{l} \sum F_x = m a_x \\ \sum F_y = m a_y \end{array} \right.$$

$$|\vec{F}| = m |\vec{a}|$$

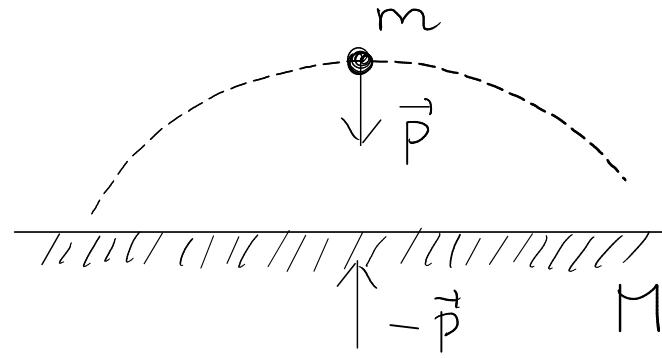
↓
SI: $\text{kg} \cdot \frac{\text{m}}{\text{s}^2} = \text{N}$ (Newton)

III principio della dinamica (principio di azione e reazione)

Se due corpi 1 e 2 interagiscono, la forza \vec{F}_{12} esercitata da 2 su 1 è l'opposto della forza \vec{F}_{21} esercitata da 1 su 2



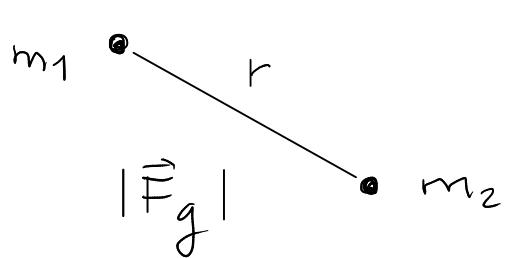
$$\vec{F}_{12} = -\vec{F}_{21}$$



Limiti di validità \rightarrow sistemi inerziali

INTERAZIONI FONDAMENTALI

1) Interazione gravitazionale : attrattive, "deboli"

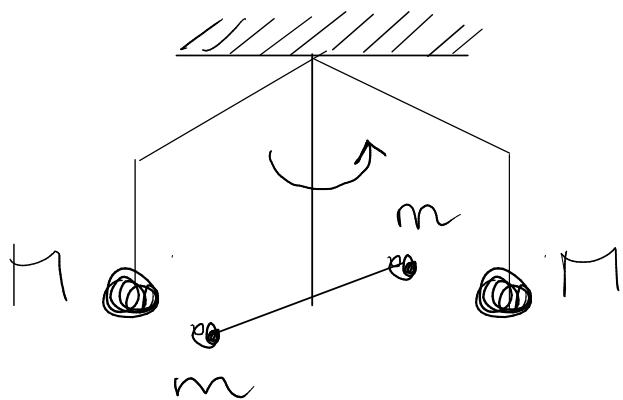


$$\left. \begin{aligned} |\vec{F}_g| &\sim \frac{1}{r^2} \\ |\vec{F}_g| &\sim m_1 m_2 \end{aligned} \right\} |\vec{F}_g| \sim \frac{m_1 m_2}{r^2}$$

costante di gravitazione universale

$$G = 6.674 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$$

bilancia di torsione



$$|\vec{F}_g| = G \frac{m_1 m_2}{r^2}$$

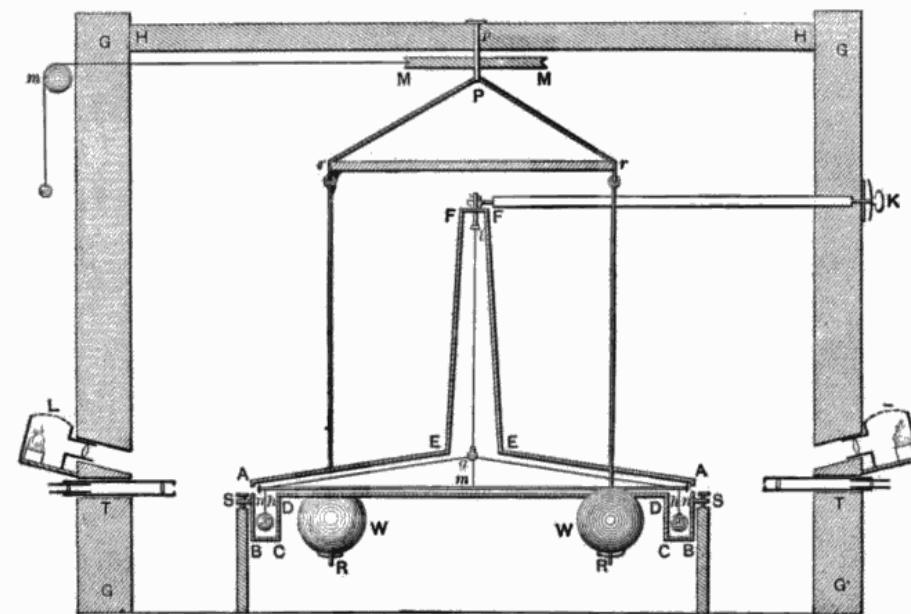
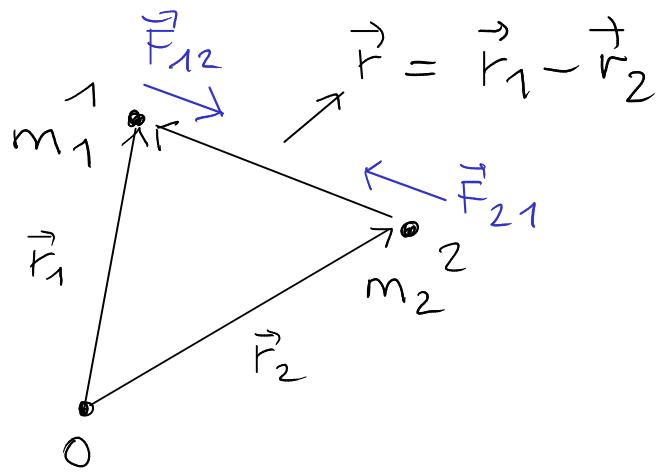


Fig. 1

Esperimento di Cavendish
1798

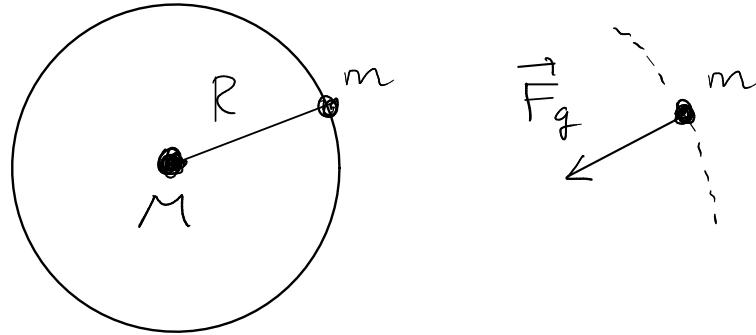


$$\vec{F}_{12} = -G \frac{m_1 m_2}{r^2} \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{F}_{21} = +G \frac{m_1 m_2}{r^2} \frac{\vec{r}}{|\vec{r}|}$$

Applicazione : **III legge di Keplero**

$$T^2 \sim R^3 \quad \text{circolare}$$



II Newton :

$$\sum \vec{F} = m \vec{a}$$

$$\vec{F}_g = m \vec{a}$$

$$|\vec{F}_g| = m |\vec{a}|$$

$$G \frac{m M}{R^2} = m \frac{v^2}{R} = m \frac{4\pi^2 R}{R T^2}$$

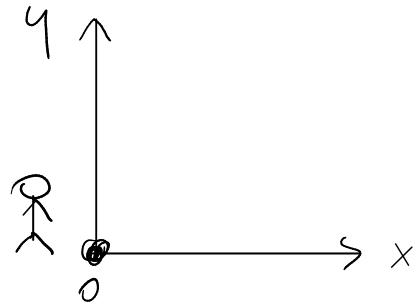
moto circolare uniforme

$$|\vec{a}| = a_c = \frac{v^2}{R}$$

$$\text{periodo : } 2\pi R = v T$$

$$T^2 = \frac{4\pi^2}{MG} R^3 \Rightarrow T^2 \sim R^3 \quad \square \checkmark$$

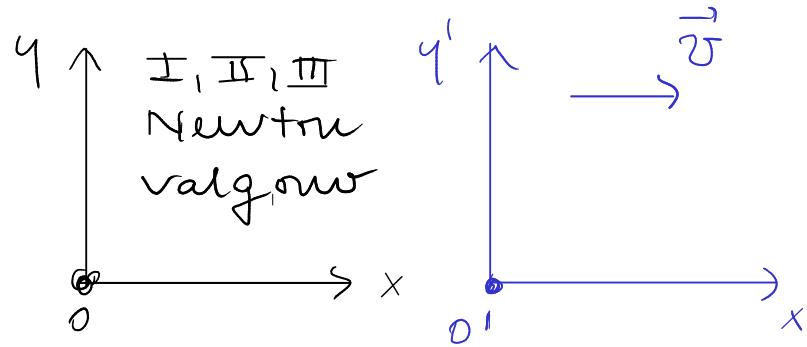
Sistemi di riferimento



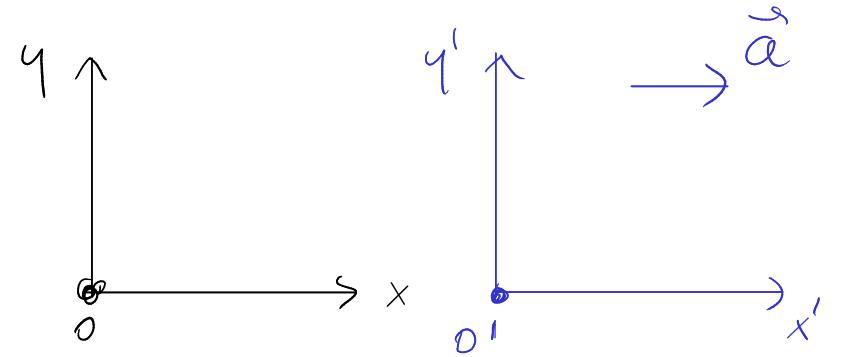
osservatore
in quiete
all'origine



sistema di riferimento



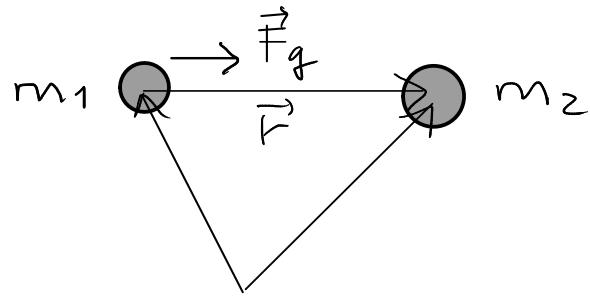
sistemi di
riferimento
inerziali



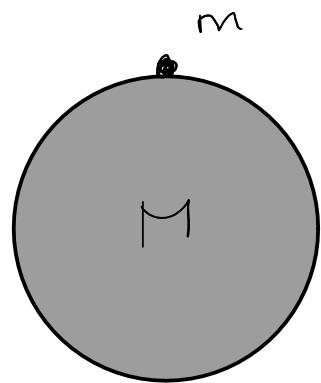
sistemi di
riferimento
non inerziali

Modello di punto materiale

Legge di gravitazione universale



$$\vec{F}_g = G \frac{m_1 m_2}{|\vec{F}|^2} \frac{\vec{F}}{|\vec{F}|}$$



m
•
M

Teorema di Gauss ci permette di concludere che

Nel caso di un corpo sferico, l'attrazione tra i corpi è la stessa che si avrebbe se la massa della sfera fosse concentrata nel suo centro

⇒ modello di particella o punto materiale

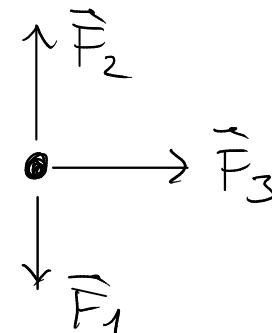
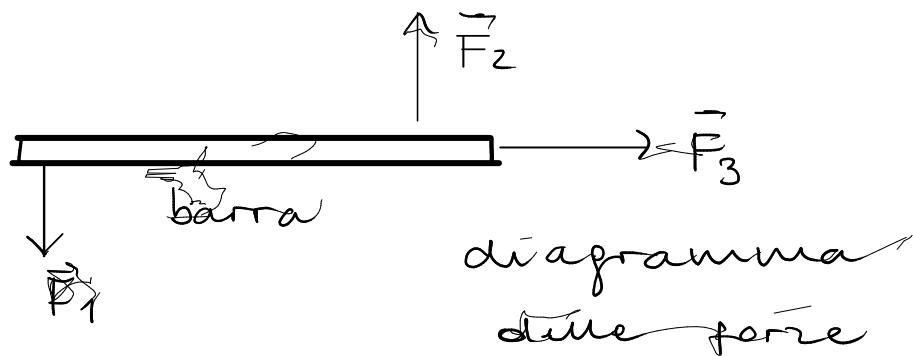
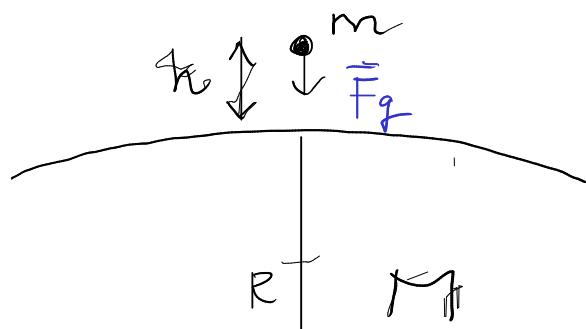


diagramma di
corpo libero

Applicazione: accelerazione di gravità terrestre



II Newton: $\Sigma \vec{F} = m\vec{a}$ (corpo di massa m)

$$\vec{F}_g = m\vec{a}$$

$$G \frac{Mm}{(R+h)^2} = m|\vec{a}| \Rightarrow |\vec{a}| = \frac{GM}{(R+h)^2}$$

$h \ll R$ es: $h = 1 \text{ km}$, $R \approx 6000 \text{ km}$

Sviluppo in serie di Taylor in h/R

$$g = \frac{GM}{R^2} \left(\frac{1}{1+h/R} \right)^2$$

Ordine 0:

$$g \approx \frac{GM}{R^2} \rightarrow g = 9,81 \frac{\text{m}}{\text{s}^2}$$

Ordine 1: dipendenza da h

$$g \approx \frac{GM}{R^2} \left(1 - 2\frac{h}{R} \right) = g(h)$$

$$(1+x)^\alpha \approx 1 + \alpha x \Rightarrow \Delta g = g(h) - g(0)$$

$$\frac{\Delta g}{g} \approx -2 \frac{h}{R} \quad \frac{h}{R} = \frac{1}{6000} \quad \frac{\Delta g}{g} \approx -\frac{2}{6000} = -\frac{1}{3000} \quad \Rightarrow \quad g \approx 9,81 \frac{\text{m}}{\text{s}^2} \quad \text{OK}$$

Es.: stimare la differenza percentuale $\frac{\Delta g}{g}$ tra l'accelerazione di gravità terrestre ai poli e quella all'equatore

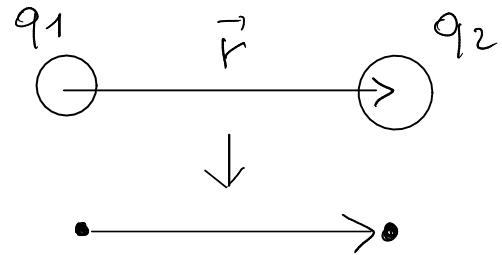
$$R_0 = 6,378 \times 10^6 \text{ m} \quad \text{equatore}$$

$$R_1 = 6,356 \times 10^6 \text{ m} \quad \text{poli}$$

2. Interazioni elettrostatiche (elettiche)

Stabilità della materia

~ 1700 Franklin → cariche di segni opposti
Coulomb SI: C (Coulomb)



modello di particella

$$|\vec{F}_e| \sim \frac{|q_1 q_2|}{|\vec{r}|^2} = k_e \frac{|q_1 q_2|}{|\vec{r}|^2} \sim \frac{1}{|\vec{r}|^2}$$

$$\vec{F}_{12} \leftarrow q_1 \xrightarrow{\vec{r}} q_2 \rightarrow \vec{F}_{21} \quad q_1 q_2 > 0 \text{ repulsiva}$$

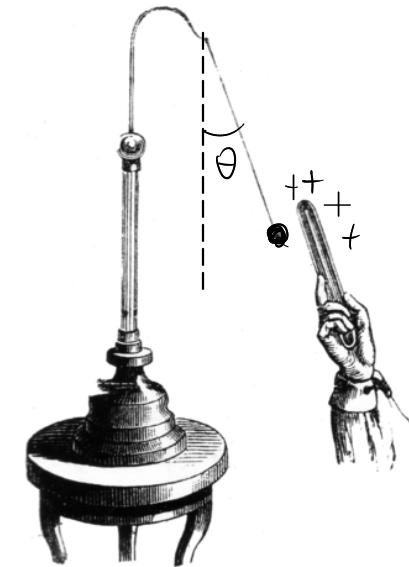
$$\vec{F}_{12} = -k_e \frac{q_1 q_2}{|\vec{r}|^2} \frac{\vec{r}}{|\vec{r}|}$$

$$q_1 \xrightarrow{\vec{r}} q_2 \leftarrow \vec{F}_{21} \quad q_1 q_2 < 0 \text{ attrattiva}$$

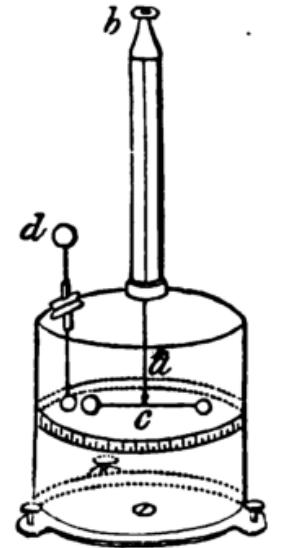
$$\vec{F}_{12} = -k_e \frac{q_1 q_2}{|\vec{r}|^2} \frac{\vec{r}}{|\vec{r}|}$$

costante di Coulomb

$$k_e = 8,99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$$



elektroskopio



elektrometro

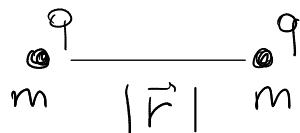
$$\vec{F}_{12} = -k_e \frac{q_1 q_2}{|\vec{r}|^2} \frac{\vec{r}}{|\vec{r}|} = -\vec{F}_{21}$$

Protone : $m = 1.672 \times 10^{-27} \text{ kg}$ $q = + 1.602 \times 10^{-19} \text{ C}$

Elettrone : $m = 9.109 \times 10^{-31} \text{ kg}$ $q = - 1.602 \times 10^{-19} \text{ C}$

Neutrone : $m = 1.674 \times 10^{-27} \text{ kg}$ $q = 0 \text{ C}$

Es.: calcola $\frac{|\vec{F}_e|}{|\vec{F}_g|}$ tra 2 protoni



$$\frac{|\vec{F}_e|}{|\vec{F}_g|} = \frac{k_e q^2}{|\vec{r}|^2} \frac{|\vec{r}|^2}{G m^2} = \frac{k_e q^2}{G m^2} = \frac{9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \times (1.6 \times 10^{-19})^2 \text{C}^2}{6 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \times (1.7 \times 10^{-27})^2 \text{kg}^2}$$

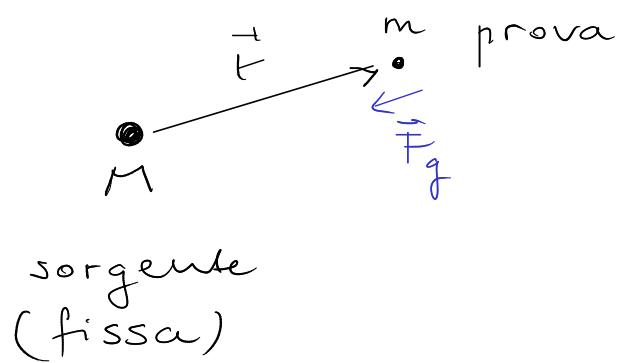
$$\approx \frac{10^9 \times 10^{-38}}{10^{-11} \times 10^{-54}} = \underline{\underline{10^{36}}}$$

Campi di forze

~1800 Maxwell → campo di forze

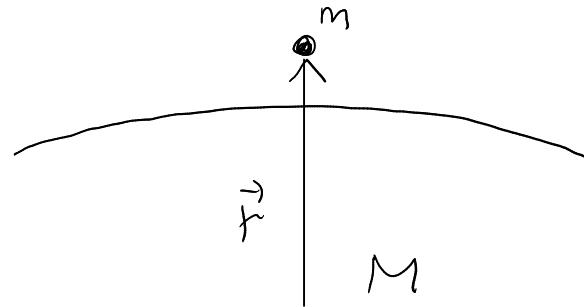
Campo: grandezza fisica che varia nello spazio

Campo gravitazionale



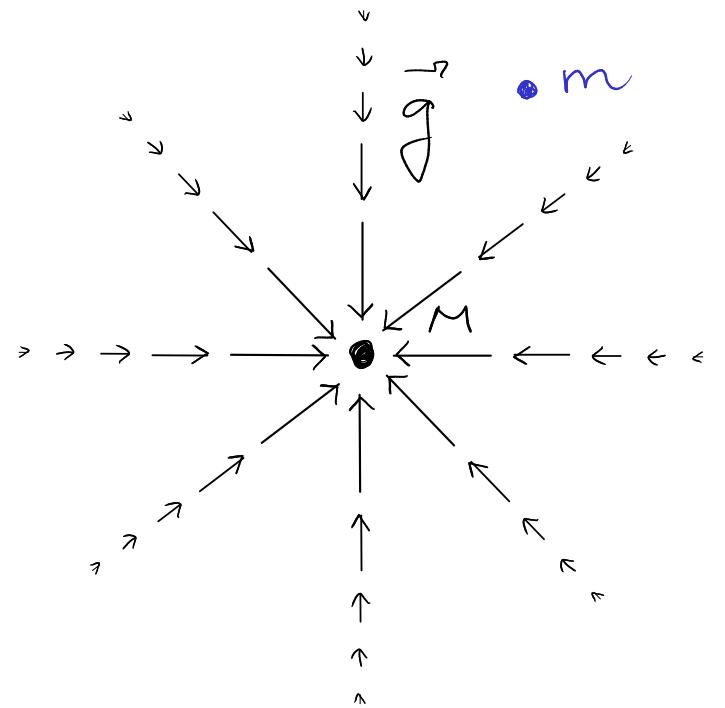
$$\vec{F}_g = -m \frac{GM}{|\vec{r}|^2} \frac{\vec{r}}{|\vec{r}|}$$
$$\vec{F}_g = m\vec{a} \rightarrow \vec{a} = \vec{g}$$
$$\vec{g} = \frac{\vec{F}_g}{m}$$

campo gravitazionale generato da M

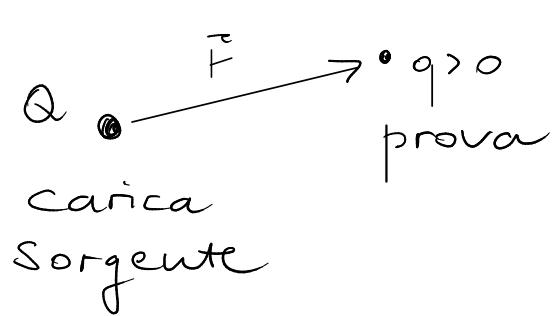


$$\vec{g} = -\frac{GM}{|\vec{r}|^2} \frac{\vec{r}}{|\vec{r}|}$$

$$SI: \frac{N}{kg}$$



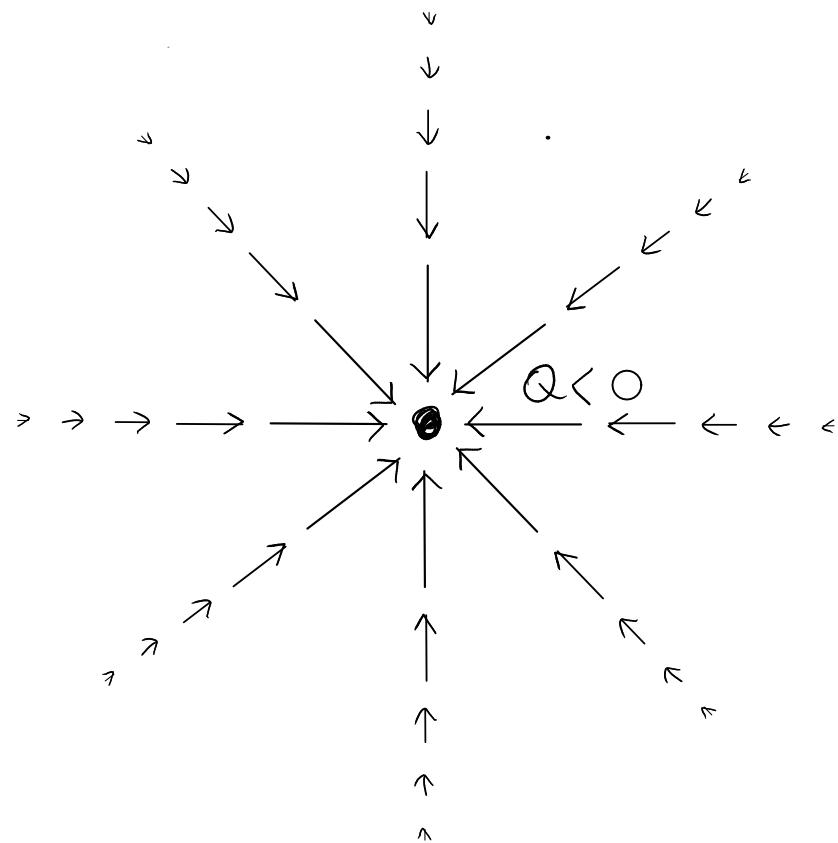
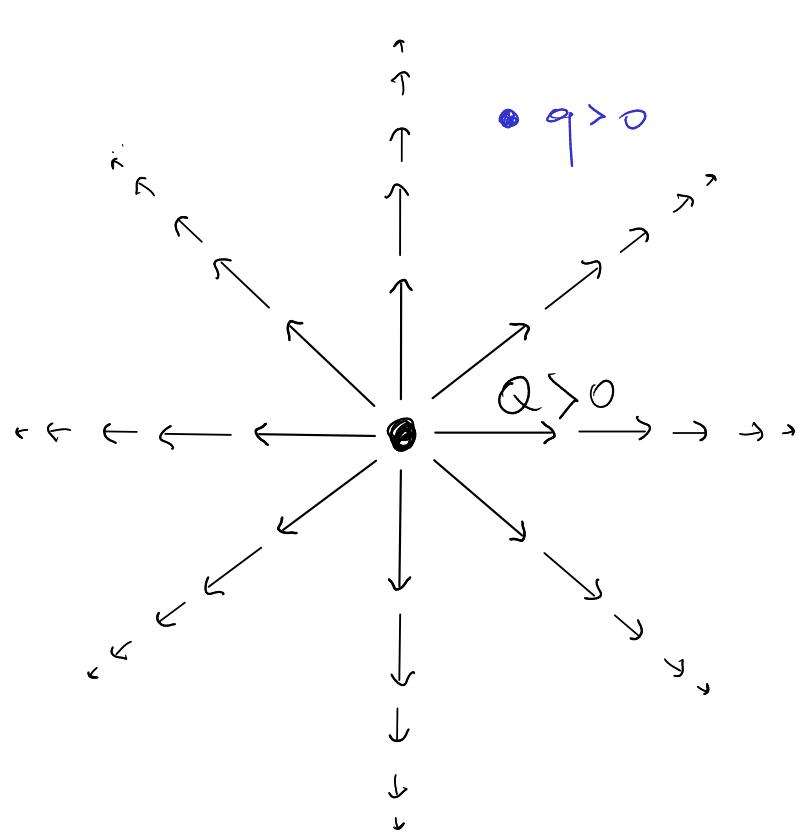
Campo elettrostatico (elettrico)



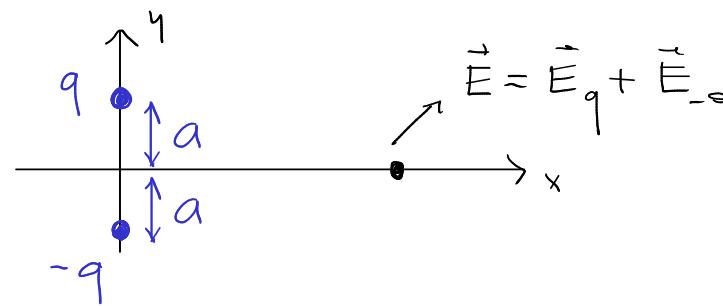
$$\vec{F}_e = q \frac{k_e Q}{|\vec{r}|^2} \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{E} = \frac{\vec{F}_e}{q} = k_e \frac{Q}{|\vec{r}|^2} \frac{\vec{r}}{|\vec{r}|} \quad \text{SI: } \frac{\text{N}}{\text{C}}$$

campo elettrostatico generato da Q



Es.: dipolo elettrico

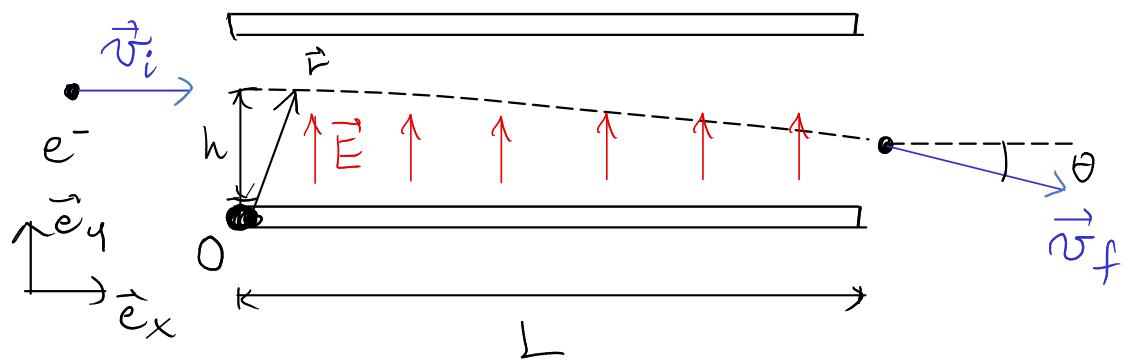


- 1) Determina $|\vec{E}|$ per $x \gg a$ e mostra che

$$|\vec{E}| \sim \frac{1}{x^\alpha} \quad \alpha = ?$$

- 2) Misura $|\vec{E}|$ in funzione di x su PHET e confronta con 1)

Esempio: moto di una particella carica in un campo elettrico costante



elettrone $q = -e$

$$\vec{E} = \text{cost}$$

$$\theta = ?$$

def. operativa

$$\frac{\vec{F}_e}{q} = \vec{E}$$

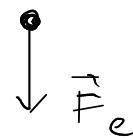
Forze: $\vec{F}_e = q \vec{E} = -e \vec{E}$

II Newton: $\Sigma \vec{F} = m \vec{a}$

$$-e \vec{E} = m \vec{a}$$

$$\vec{a} = -\frac{e}{m} \vec{E} = \text{cost} \rightarrow \text{moto uniformemente accelerato}$$

Diagramma di corpo libero:



Condizioni iniziali: $t_i = 0, \vec{F}_i, \vec{v}_i$

Leggi orarie: \triangle

$$\vec{v} = \vec{v}_i + \vec{a}t$$

$$\vec{r} = \vec{F}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

Base cartesiana + sist. di coordinate

$$\vec{E} = |\vec{E}| \vec{e}_y$$

$$(\vec{E} = E \vec{e}_y, E > 0)$$

$$\vec{v}_i = v_{ix} \vec{e}_x$$

$$\vec{F}_i = h \vec{e}_y$$

$$v_{ix} > 0$$

$$\begin{cases} \vec{v} = \underbrace{v_{ix}} \vec{e}_x - \frac{e}{m} |\vec{E}| t \vec{e}_y \\ \vec{r} = \underbrace{h} \vec{e}_y + \underbrace{v_{ix} t}_{\text{}} \vec{e}_x - \frac{1}{2} \frac{e}{m} |\vec{E}| t^2 \vec{e}_y \end{cases}$$

$$\begin{cases} \vec{v} = \underbrace{v_x} \vec{e}_x + \underbrace{v_y} \vec{e}_y \\ \vec{r} = \underbrace{x} \vec{e}_x + \underbrace{y} \vec{e}_y \end{cases}$$

$$\begin{cases} v_x = v_{ix} \\ v_y = -\frac{e}{m} |\vec{E}| t \end{cases}$$

$$\begin{cases} x = v_{ix} t & x = x(t) \\ y = h - \frac{1}{2} \frac{e}{m} |\vec{E}| t^2 & y = y(t) \end{cases}$$

Tempo di uscita: $t_f \leftarrow L = x(t_f) + y(t_f) > 0$

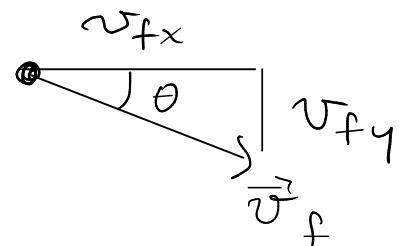
$$L = v_{ix} t_f \Rightarrow t_f = \frac{L}{v_{ix}}$$

$$\begin{cases} v_{fx} = v_{ix} \\ v_{fy} = -\frac{e}{m} |\vec{E}| \frac{L}{v_{ix}} \end{cases}$$

$$\Rightarrow \tan \theta = \frac{v_{fy}}{v_{fx}} = -\frac{e |\vec{E}| L}{m v_{ix}^2}$$

$$\theta = \arctan \left(-\frac{e |\vec{E}| L}{m v_{ix}^2} \right) \quad \checkmark$$

$$\underline{es.} \left[\frac{e |\vec{E}| L}{m v_{ix}^2} \right] = 1$$



3. Interazioni magnetiche

~ 1200 poli $+/-$ (N, S)

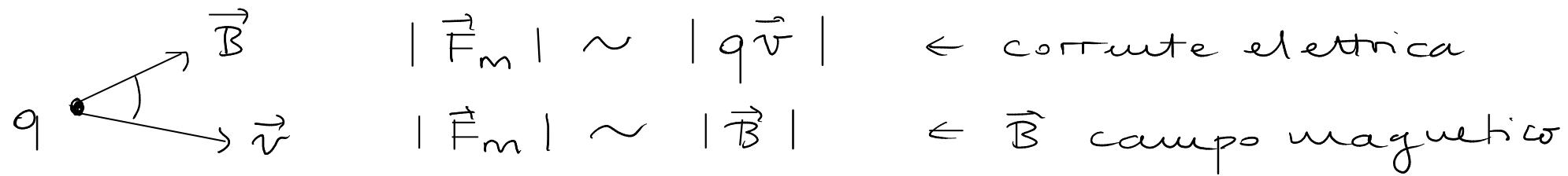
~ 1800 elettricità \Leftrightarrow magnetismo

Oersted, Faraday

campo magnetico

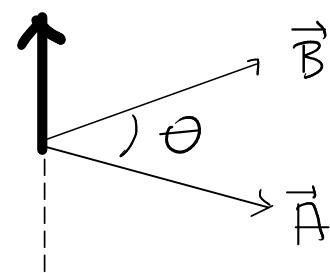
Maxwell: campo elettromagnetico

Carica in moto q



Prodotto vettoriale $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

Prodotto scalare

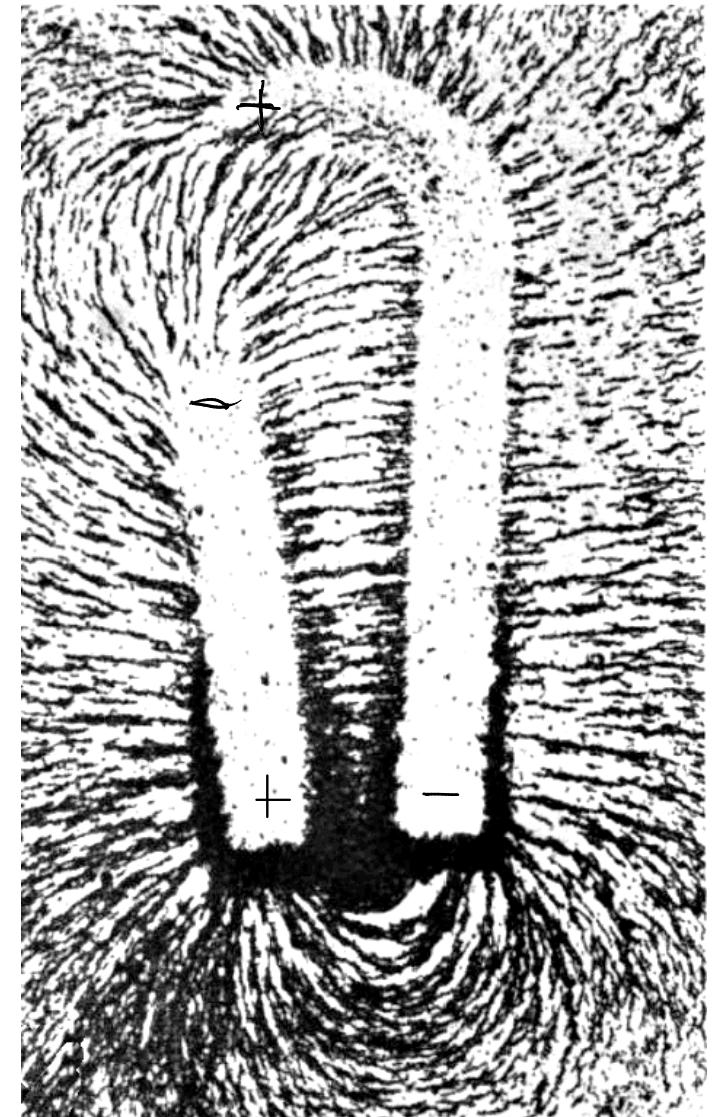


- direzione: $\perp \vec{A}, \vec{B}$

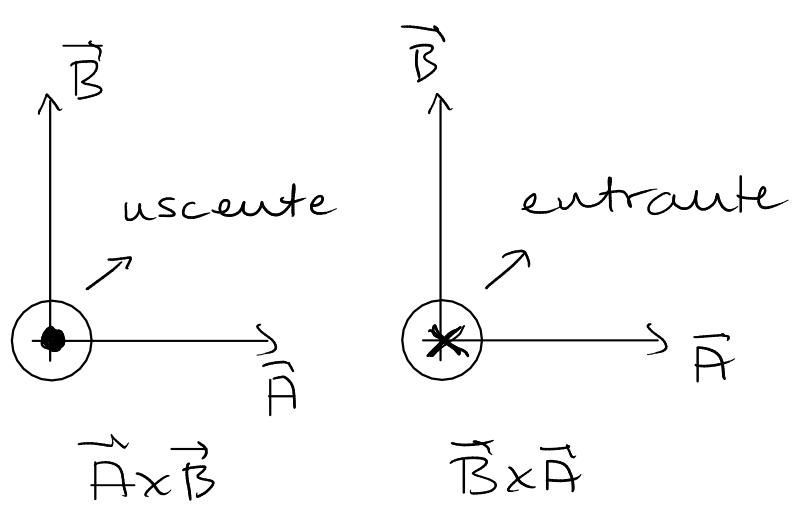
$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

- verso: regola della mano destra

- modulo: $|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta$



Aghi di ferro deflessi dal campo magnetico di un ferro di cavallo



campo magnetico

$$SI : \frac{Ns}{Cm} = T \quad (\text{Tesla})$$

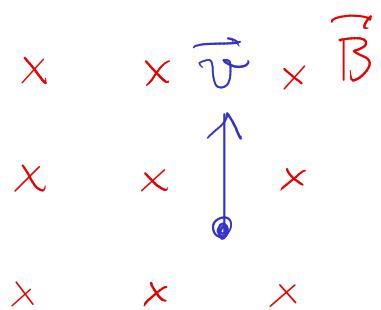
$$\vec{F}_m = q\vec{v} \times \vec{B} \quad \text{def. operativa di } \vec{B}$$

$$|\vec{F}_m| = |q| |\vec{v}| |\vec{B}| \sin\theta$$

$$\vec{v} \parallel \vec{B} \rightarrow |\vec{F}_m| = 0$$

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \quad \text{forza di Lorentz}$$

Esempio: moto di una particella carica in un campo magnetico costante



$q > 0$
 $\vec{B} = \text{cost}$
 moto?

$$\vec{F}_m = q \vec{v} \times \vec{B}$$

II Newton: $\vec{F}_m = m \vec{a}$
 $\vec{a} \perp \vec{v}$

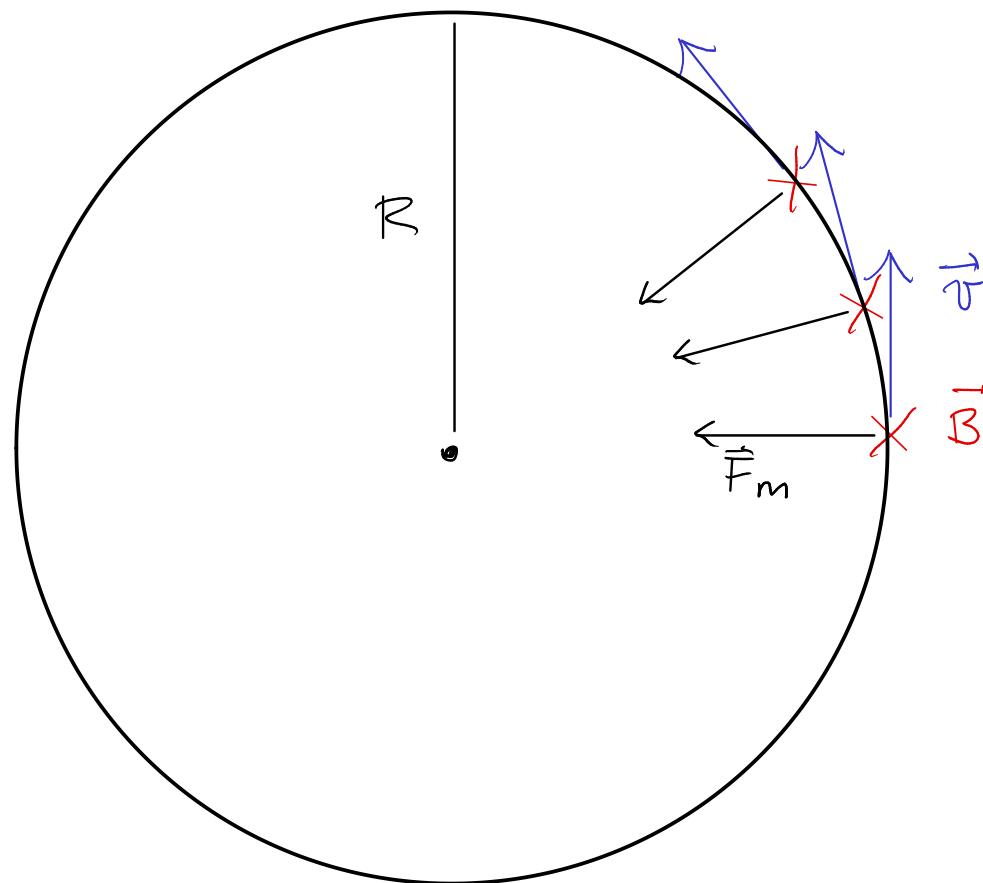
Moto circolare uniforme: $|\vec{v}| = \text{cost}$
 $\vec{a} \perp \vec{v}$

$$\begin{cases} |\vec{a}| = \frac{|\vec{F}_m|}{m} = \frac{q |\vec{v}| |\vec{B}|}{m} \\ |\vec{a}| = \frac{v^2}{R} = \frac{|\vec{v}|^2}{R} = \frac{q |\vec{v}| |\vec{B}|}{m} \end{cases}$$

$$R = \frac{m |\vec{v}|}{q |\vec{B}|}$$

$$2\pi R = v \tau \Rightarrow \tau = \frac{2\pi R}{|\vec{v}|}$$

$$= \frac{2\pi m |\vec{v}|}{q |\vec{B}| |\vec{v}|} = \frac{2\pi m}{q |\vec{B}|} \quad \square$$



4. Interazione forte → responsabile della stabilità dei nuclei

nucleo : protoni + neutroni

$$q=e$$

$$q=0$$

5. Interazione debole → responsabile di fenomeni radioattivi

Unificazione delle interazioni fondamentali

1. Gravitazione

2. Elettricità

3. Magnetismo

4. Debole

5. Forte

}
}

Elettromagnetismo

}
}

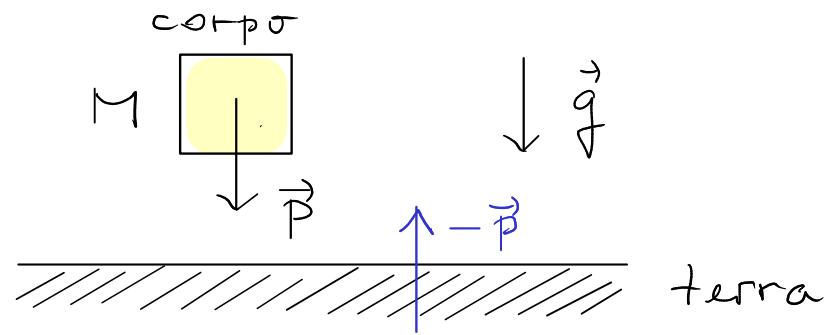
Elettro-debole

FORZE MACROSCOPICHE

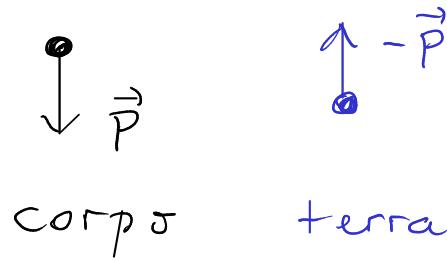
Macroscopico: scala di lunghezza $\gtrsim 0.1 \text{ mm} \sim 1 \text{ m}$

Corpo macroscopico: N costituenti elementari $N \approx 6 \times 10^{23}$

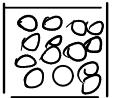
1) Peso



$$\vec{P} = M \vec{g}$$



Interpretazione micro:


$$\vec{P} = \sum_{i=1}^N m \vec{g} = \underbrace{\left(\sum_{i=1}^N m \right)}_M \vec{g}$$

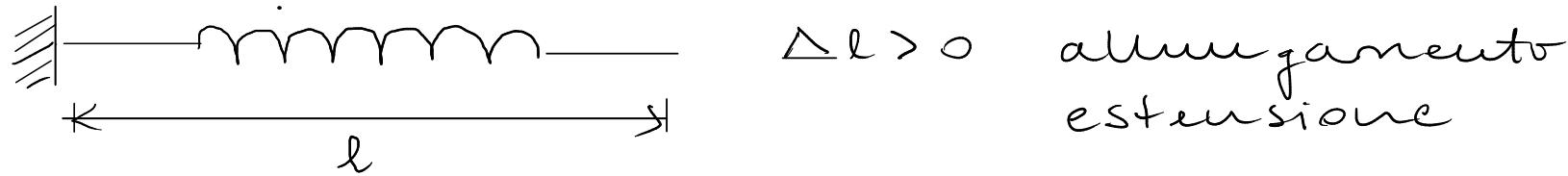
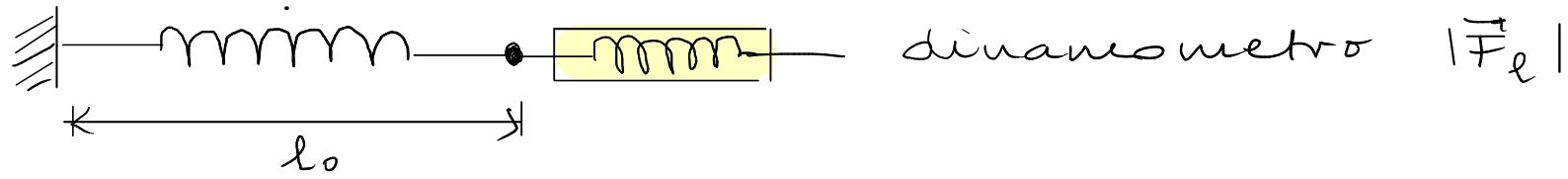
N m massa atomo M massa totale

III Newton

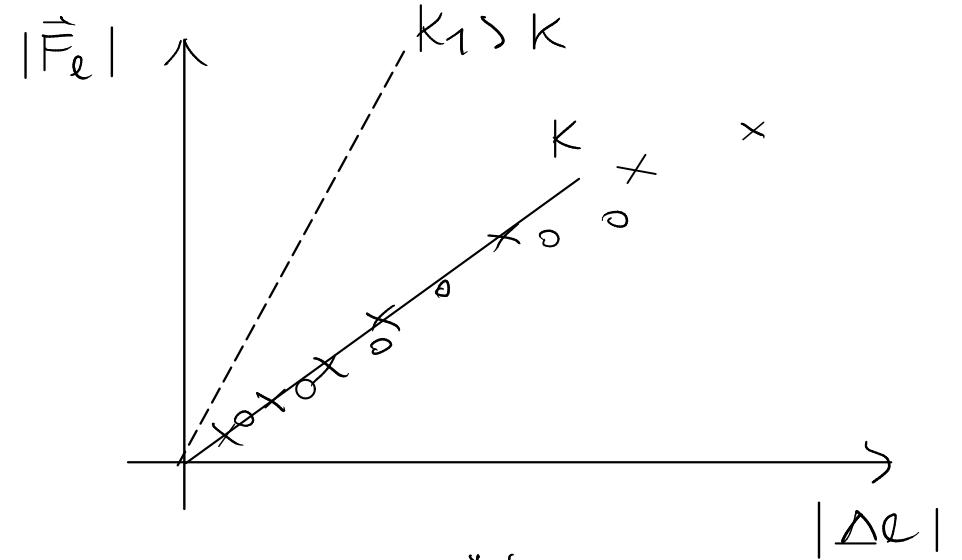
↓
forze macro = risultante moltitudine
forze micro

2. Forza elastica : risposta di un corpo solido a una "piccola" deformazione

Es.: molle $l_0 =$ lunghezza a riposo $\Delta l = l - l_0$



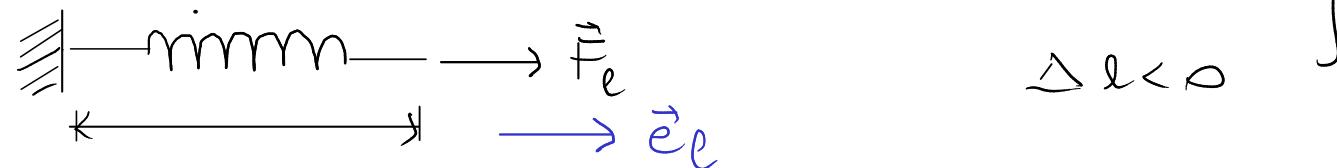
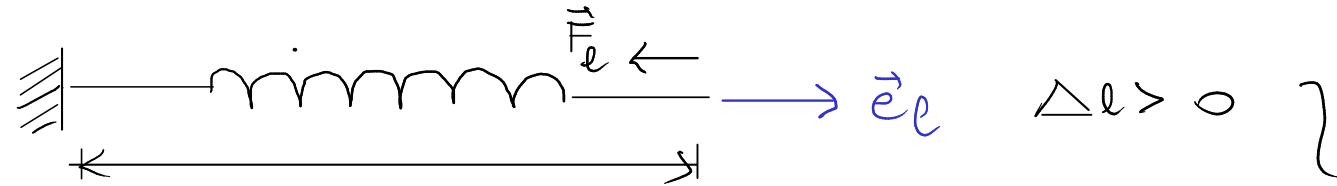
$$|\vec{F}_e| \sim |\Delta l| \quad |\vec{F}_e| = k |\Delta l|$$



SI: $\frac{N}{m}$

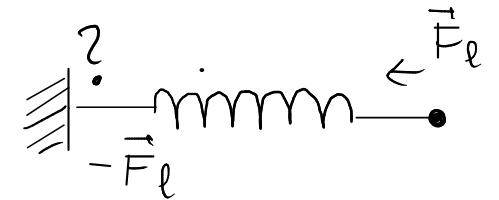
Natura vettoriale: $|\vec{e}_e| = 1$

costante elastica \rightarrow rigidità



$$\vec{F}_e = -k \Delta l \vec{e}_e$$

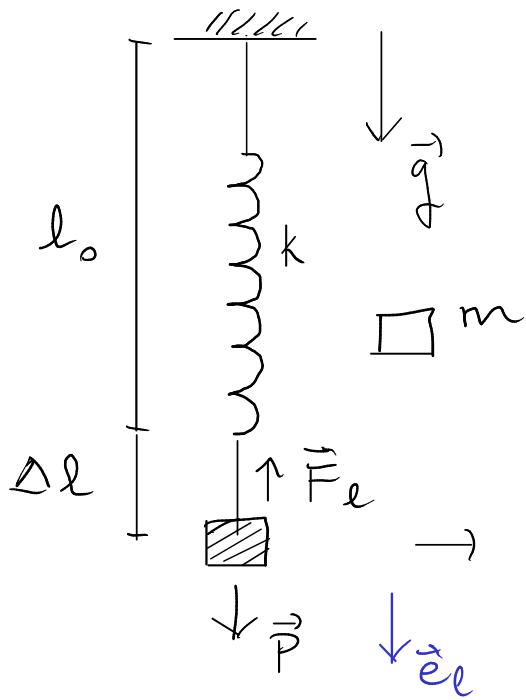
legge di Hooke



Modello di molla ideale :

- massa trascurabile ($m \approx 0$)
- legge di Hooke

Es. corpo appeso all'estremità di una molla nel campo gravitazionale terrestre



$$\vec{g} = \text{cost} = g \vec{e}_e$$

molla ideale

Equilibrio

- equilibrio statico: corpo immobile $\vec{v} = \vec{0}$

- equilibrio meccanico: $\sum \vec{F} = \vec{0}$

statico \Rightarrow meccanico

$$\text{Equilibrio statico} \Rightarrow \sum \vec{F} = \vec{0}$$

$$m\vec{g} + \vec{F}_e = \vec{0}$$

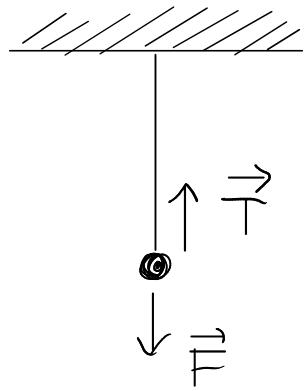
$$m\vec{g} - k\Delta l \vec{e}_e = \vec{0}$$

$$mg\vec{e}_e - k\Delta l \vec{e}_e = \vec{0}$$

$$mg = k\Delta l$$

$$\Delta l = \frac{mg}{k} \quad \square$$

3. Tensione risposta elastica "estrema" di un filo in estensione

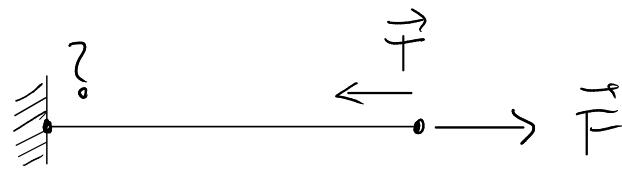


\vec{T} tensione

$$\vec{F} + \vec{T} = \vec{0} \quad (\text{immobilit\`a})$$

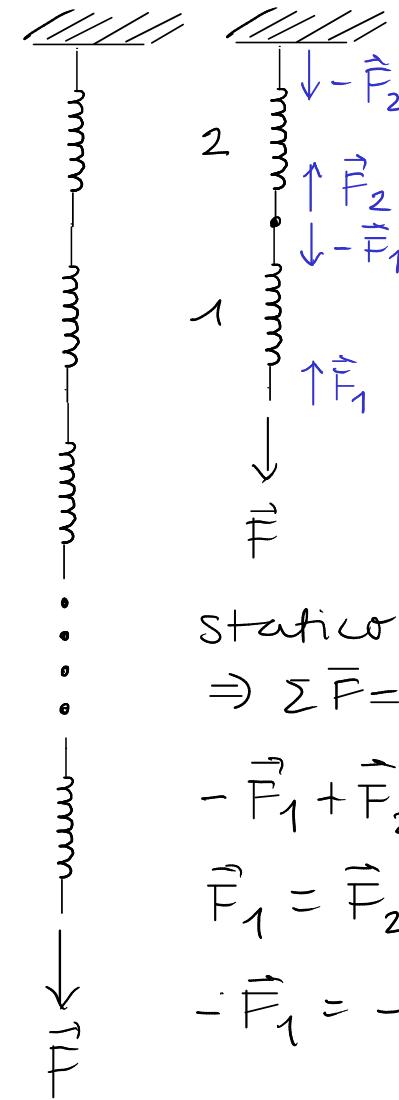
$$\vec{T} = -\vec{F}$$

otteniamo \vec{T} a posteriori
dalla condizione di
equilibrio statico



Modello di filo ideale:

- massa trascurabile ($m \approx 0$)
- inestensibile
- tensione $|\vec{T}|$ è la stessa in ogni punto



statico

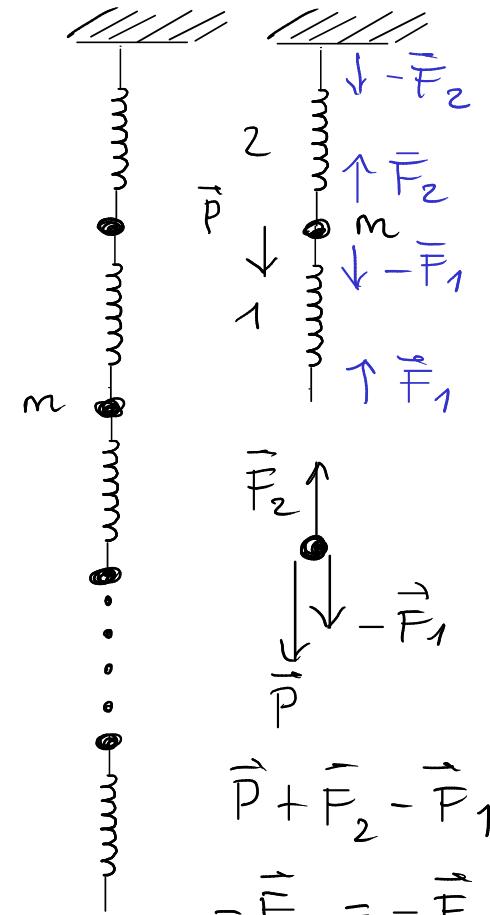
$$\Rightarrow \sum \vec{F} = \vec{0}$$

$$-\vec{F}_1 + \vec{F}_2 = \vec{0}$$

$$\vec{F}_1 = \vec{F}_2$$

$$-\vec{F}_1 = -\vec{F}_2$$

massa
trascurabile

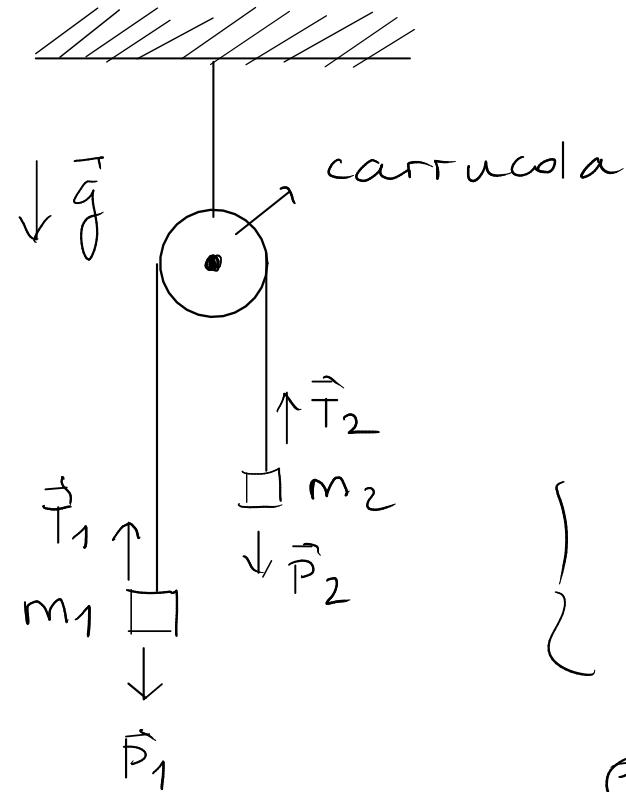
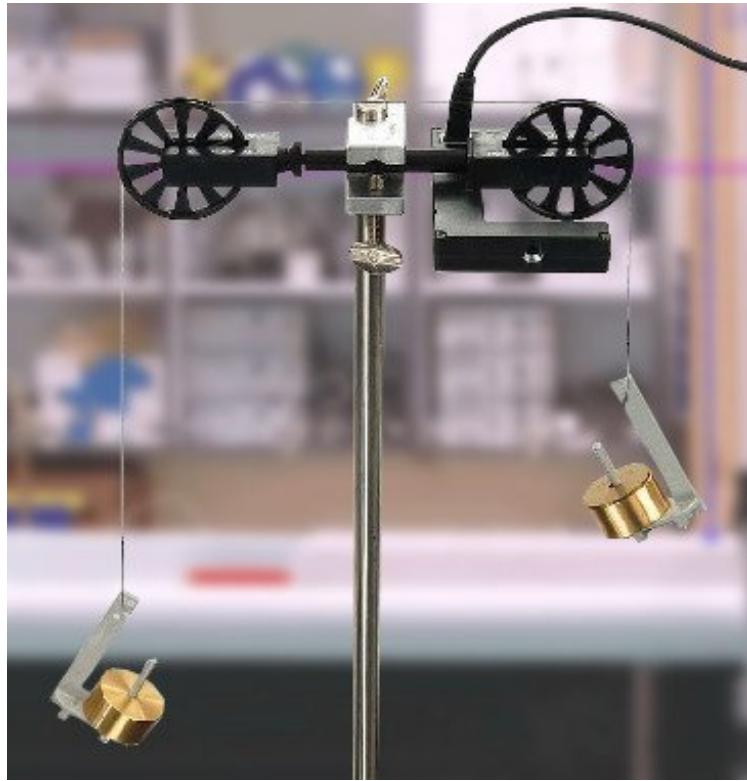


$$\vec{P} + \vec{F}_2 - \vec{F}_1 = \vec{0}$$

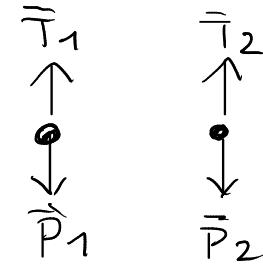
$$-\vec{F}_2 = -\vec{F}_1 + \vec{P}$$

massa
 $\neq 0$

Es.: macchina di Atwood



Filo ideale \rightarrow inestensibile



$$\vec{T}_1 = \vec{T}_2 = \vec{T}$$

II Newton: $\sum \vec{F} = m \vec{a}$

$$\begin{cases} \vec{T} + m_1 \vec{g} = m_1 \vec{a}_1 & \textcircled{1} \quad \text{filo inestens.} \\ \vec{T} + m_2 \vec{g} = -m_2 \vec{a}_1 & \textcircled{2} \quad \vec{a}_2 = -\vec{a}_1 \end{cases}$$

$$\textcircled{1} - \textcircled{2}$$

$$m_1 \vec{g} - m_2 \vec{g} = m_1 \vec{a}_1 + m_2 \vec{a}_1$$

$$(m_1 - m_2) \vec{g} = (m_1 + m_2) \vec{a}_1$$

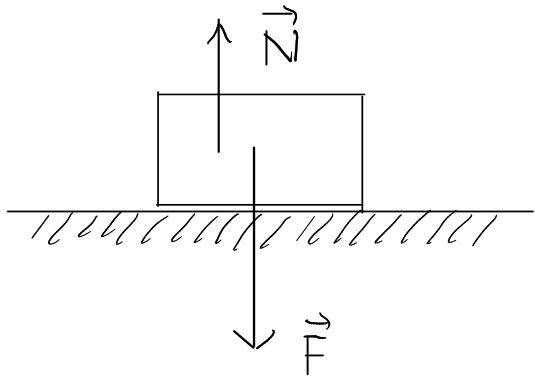
$$\vec{a}_1 = \frac{m_1 - m_2}{m_1 + m_2} \vec{g} \quad \square \quad \vec{g} = \frac{m_1 + m_2}{m_1 - m_2} \vec{a}_1$$

$$m_1 = m_2 : \quad \vec{a}_1 = \vec{a}_2 = \vec{0}$$

$$m_1 > m_2 : \quad \vec{a}_1 \text{ stesso verso di } \vec{g}$$

$$m_1 \gg m_2 : \quad \vec{a}_1 \approx \frac{m_1}{m_1} \vec{g} = \vec{g}$$

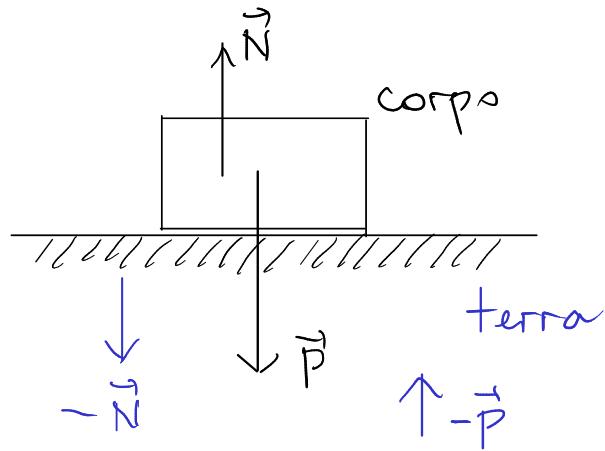
4. Reazione normale : risposta elastica "estrema" di un corpo solido in compress.



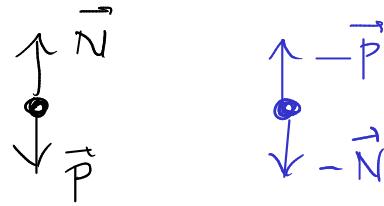
\vec{N} = reazione normale

$$\vec{F} + \vec{N} = \vec{0} \Rightarrow \vec{N} = -\vec{F}$$

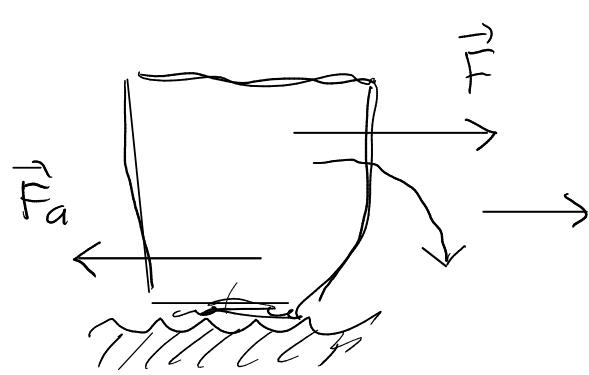
determinata a posteriori
dalla condizione di eq.
statico



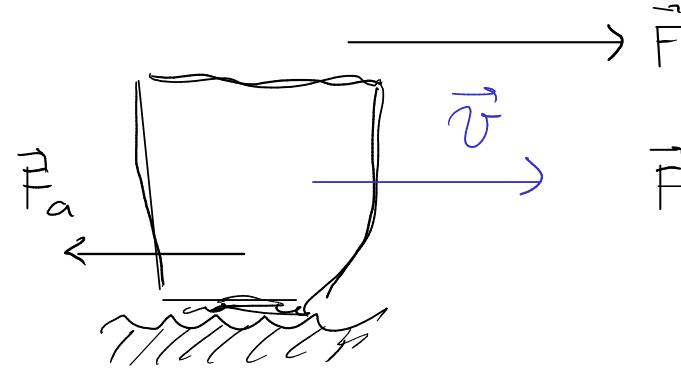
III Newton



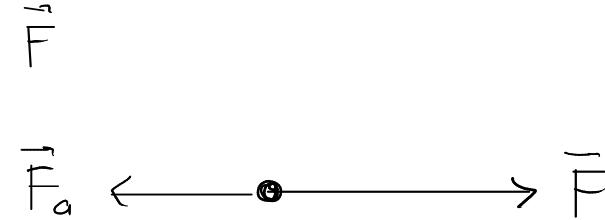
5. Attrito radente : forza a corto raggio tra corpi solidi // piano di contatto



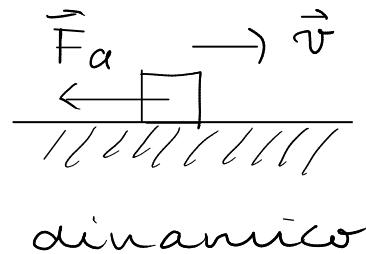
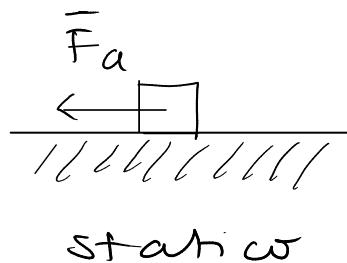
regime statico



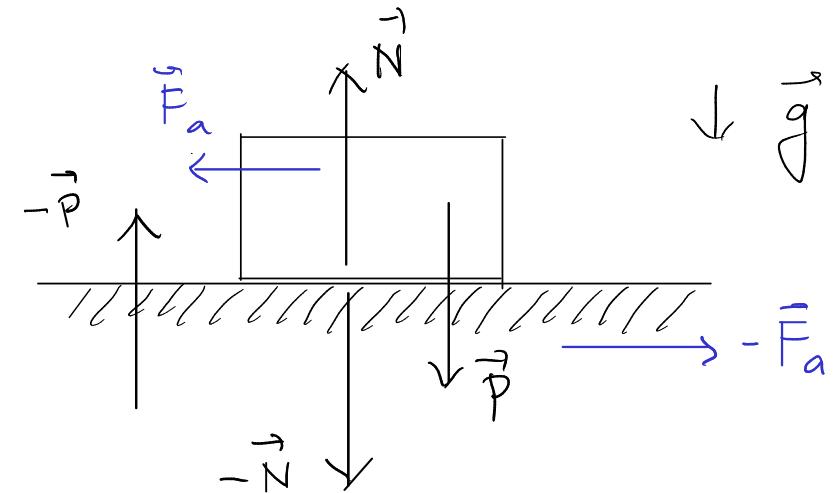
regime dinamico



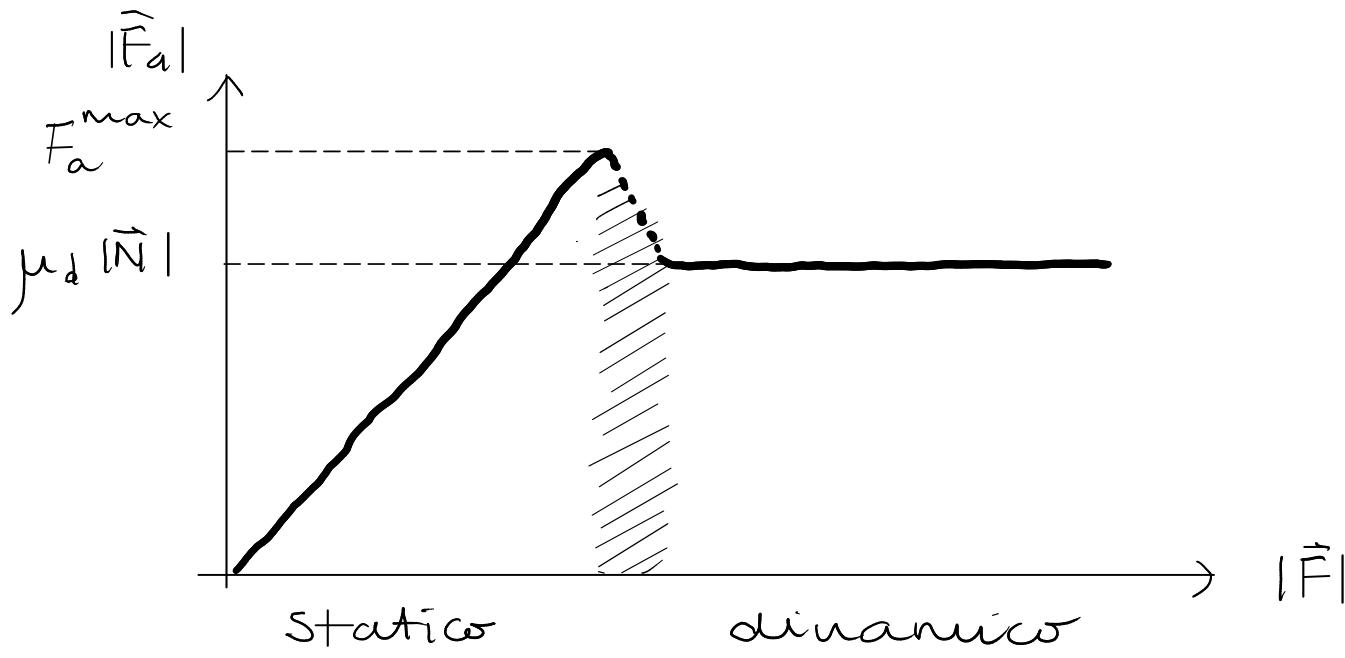
- direzione: // piano contatto
- verso: opposto al $\begin{cases} \text{moto imminente (statico)} \\ \text{moto (dinamico)} \end{cases}$



III Newton



Coefficienti di attrito



F_a^{\max} : massimo valore del modulo della \vec{F}_a in regime statico

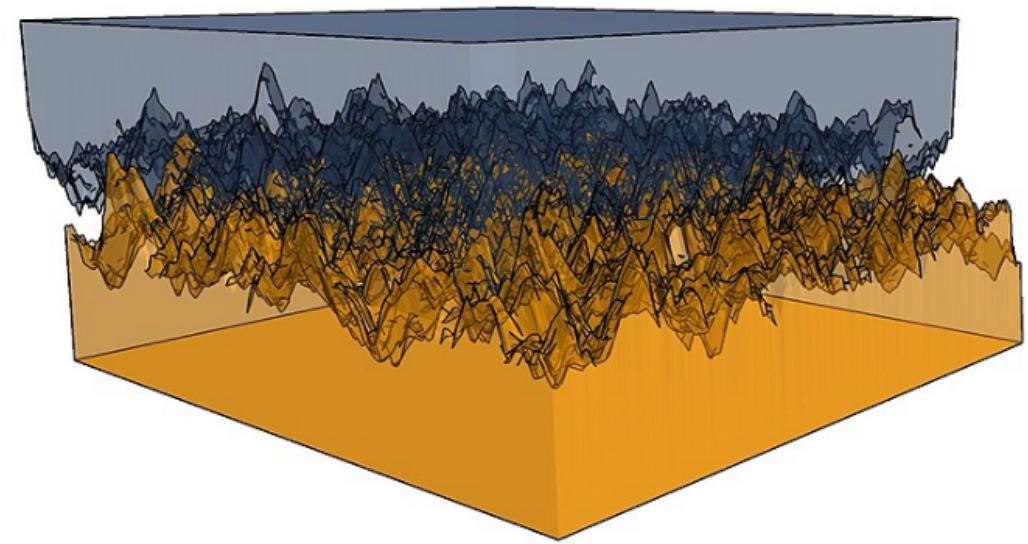
Regime statico: $|\vec{F}_a| \leq F_a^{\max} = \mu_s |\vec{N}|$

coeff. attrito statico
SI: adimensionale

Regime dinamico: $|\vec{F}_a| = \mu_d |\vec{N}|$

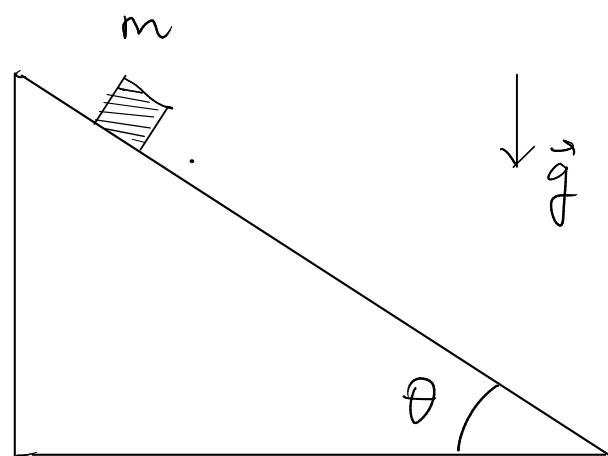
coeff. attrito dinamico
SI: adimensionale

$\mu_d \lesssim \mu_s \lesssim 1$

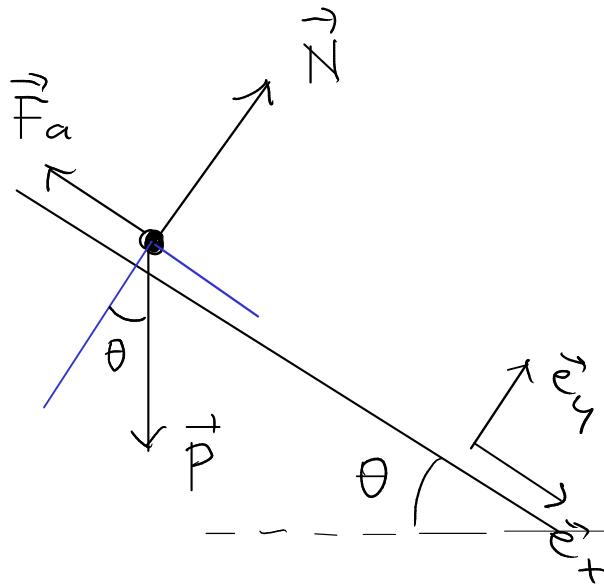


$\rightsquigarrow \vec{F}_a$

Es.: misura di μ_s e μ_d



piano inclinato



Regime statico : immobile

$$\Rightarrow \sum \vec{F} = \vec{0}$$

$$\vec{P} + \vec{N} + \vec{F}_a = \vec{0}$$

Base cartesiana \vec{e}_x, \vec{e}_y

$$\vec{P} = m\vec{g} = mg \sin\theta \vec{e}_x - mg \cos\theta \vec{e}_y$$

$$mg \sin\theta \vec{e}_x - mg \cos\theta \vec{e}_y + |\vec{N}| \vec{e}_y - |\vec{F}_a| \vec{e}_x = \vec{0}$$

$$\begin{cases} mg \sin\theta - |\vec{F}_a| = 0 \\ -mg \cos\theta + |\vec{N}| = 0 \end{cases} \rightarrow \begin{cases} mg \sin\theta = |\vec{F}_a| \leq F_a^{\max} = \mu_s |\vec{N}| = \mu_s mg \cos\theta \\ \sin\theta \leq \mu_s \cos\theta \end{cases}$$

$$\tan\theta \leq \mu_s \quad \theta \leq \theta_{\max}$$

$$\tan\theta_{\max} = \mu_s$$

$$\text{Es: } \theta_{\max} = 20^\circ \\ \mu_s = 0.36$$

Regime dinámico: $\sum \vec{F} = m\vec{a}$

$$\underbrace{mg \sin \theta}_{\vec{e}_x} - \underbrace{mg \cos \theta}_{\vec{e}_y} + \underbrace{|\vec{N}|}_{\vec{e}_y} - \underbrace{|\vec{F}_a|}_{\vec{e}_x} = m\vec{a} = \underbrace{m a_x}_{\vec{e}_x} + \underbrace{0}_{\vec{e}_y}$$

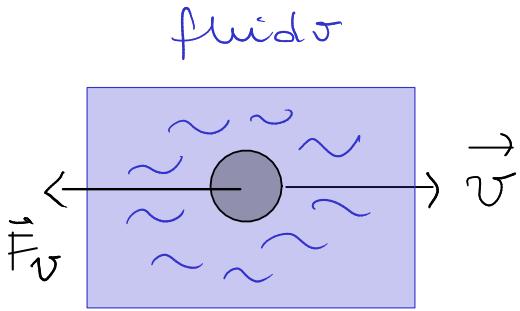
$$\begin{cases} mg \sin \theta - \mu_d |\vec{N}| = m a_x \\ -mg \cos \theta + |\vec{N}| = 0 \end{cases}$$

$$m a_x = mg \sin \theta - \mu_d mg \cos \theta$$

$$a_x = g (\sin \theta - \mu_d \cos \theta)$$

$$v = \text{const} \Rightarrow \mu_d = \tan \theta \quad \square$$

6. Attrito viscoso : forza su un corpo solido in moto in un fluido



“Bassa” velocità

$$\vec{F}_v = -\zeta \vec{v}$$

↑
coeff. attrito viscoso

“Alta” velocità

$$|\vec{F}_v| \sim |\vec{v}|^2$$

$$|\vec{F}_v| = \zeta |\vec{v}|$$

$$M \cdot \frac{L}{T^2} = [\zeta] \frac{L}{T} \Rightarrow [\zeta] = \frac{M}{T} \quad \text{SI: } \frac{kg}{s}$$

“Unificazione” delle forze macroscopiche

1. Peso

2. Forza elastica

3. Trusione

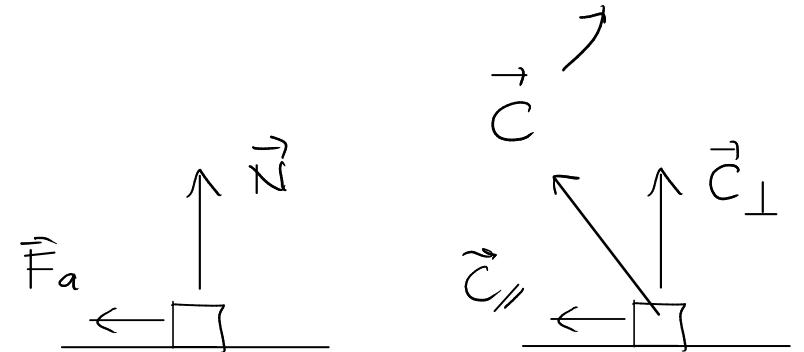
4. Reazione normale

5. Attrito radente

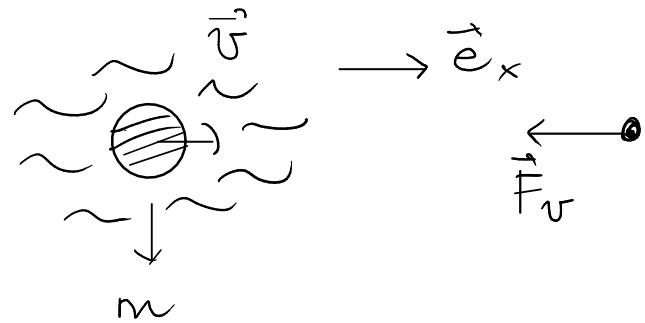
6. Attrito viscoso

} elasticità }
} contatto }

forza di contatto



Es.: moto di un corpo solido in un fluido



II Newton: $\Sigma \vec{F} = m\vec{a}$
 $\vec{F}_v = m\vec{a}$
 $-\zeta \vec{v} = m\vec{a}$

$$\begin{cases} \vec{v} = v \vec{e}_x \\ \vec{a} = a \vec{e}_x \end{cases} \quad -\zeta v = ma \rightarrow m \frac{dv}{dt} = -\zeta v \rightarrow \frac{dv}{dt} = -\frac{\zeta}{m} v$$

$\left[\frac{dx}{dt} = \text{cost} \rightarrow x(t) \quad ; \quad \frac{dv}{dt} = \text{cost} \rightarrow v(t) \rightarrow x(t) \right] \rightarrow$ equazioni differenziali ordinarie

$\left[\frac{dv}{dt} = v \quad v = \exp(t) \text{ è soluzione} \quad v = A \exp(t) \text{ anche soluzione} \right]$

$$\frac{dv}{dt} = -\frac{\zeta}{m} v \rightarrow \frac{dv}{dt} = B v \quad B = -\frac{\zeta}{m}$$

1) Soluzione: $v = A \exp(Bt)$

$$\frac{dv}{dt} = AB \exp(Bt) = B \cdot (A \exp(Bt)) = Bv \quad \square$$

$$v(t) = A \exp\left(-\frac{\xi}{m}t\right)$$

Condizioni iniziali: $v(t=0) = v_0$

$$v(0) = A \exp(0) = A \Rightarrow A = v_0$$

$$v(t) = v_0 \exp\left(-\frac{\xi}{m}t\right)$$

2) Separazione delle variabili

$$\frac{dv}{dt} = -\frac{\xi}{m}v$$

$$\frac{dv}{v} = -\frac{\xi}{m}dt$$

$$\int_{v_i}^{v_f} \frac{dv}{v} = -\frac{\xi}{m} \int_{t_i}^{t_f} dt = -\frac{\xi}{m}(t_f - t_i)$$

$$\ln v_f - \ln v_i = -\frac{\xi}{m} (t_f - t_i)$$

$$\ln \left(\frac{v_f}{v_i} \right) = -\frac{\xi}{m} (t_f - t_i)$$

$$\frac{v_f}{v_i} = \exp \left[-\frac{\xi}{m} (t_f - t_i) \right]$$

$$v_f = v_i \exp \left[-\frac{\xi}{m} (t_f - t_i) \right]$$

$$t_i = 0, \quad v = v_f \quad \Rightarrow \quad v = v_i \exp \left(-\frac{\xi}{m} t \right)$$

Verifica di dipendenze esponenziali

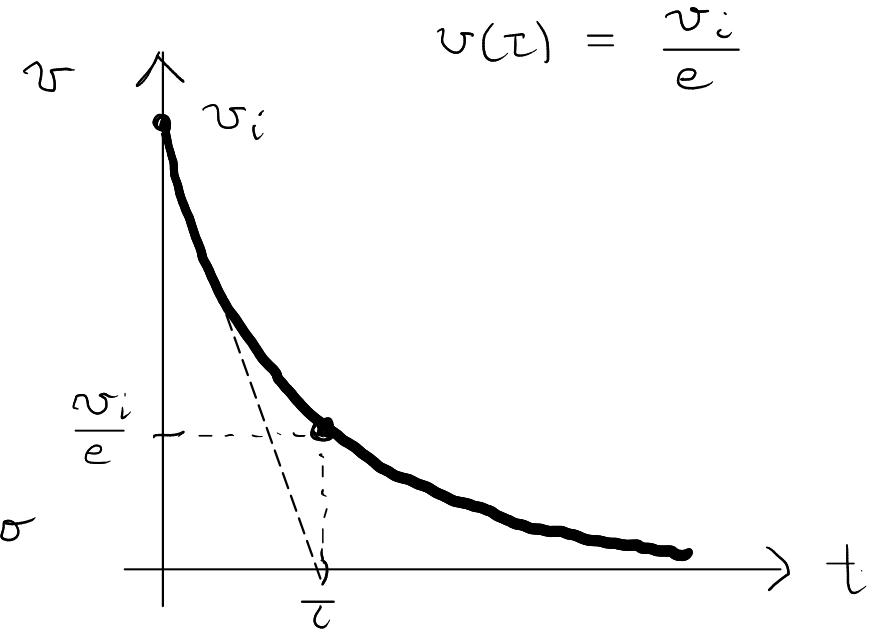
Moto di un corpo solido in un fluido

$$\frac{d\vec{v}}{dt} = -\frac{\xi}{m} \vec{v} \quad \rightarrow \quad v(t) = v_i \exp\left(-\frac{\xi}{m} t\right) = v_i \exp\left(-\frac{t}{\tau}\right)$$

$$\lim_{t \rightarrow \infty} v(t) = 0$$

$$\tau = \frac{m}{\xi}$$

tempo di rilassamento



Linearizzazione:

$$y = a \exp(-bt)$$

$$\log y = \log a + \log [\exp(-bt)] = \log a - bt$$

$$\begin{array}{cc} \uparrow & \uparrow \\ \text{cost} & \text{cost} \end{array}$$

$\Rightarrow \log y$ vs. t

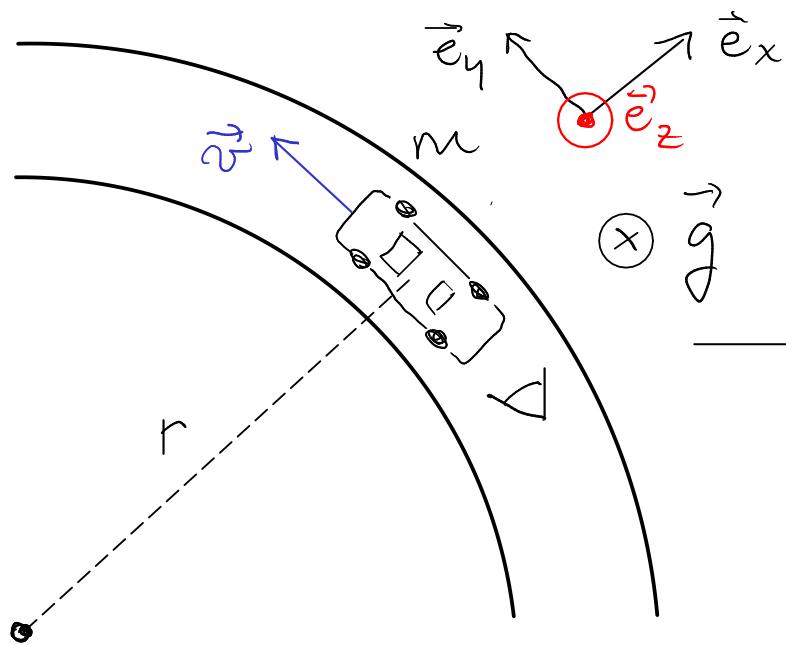
Scala semi-logaritmica
(log sull'asse y)

$$y = a + b \cdot \log x$$

Es.: $N \sim \exp[a(t-t_0)]$

$$\log N \sim a(t-t_0)$$

Es.: aderenza in curva



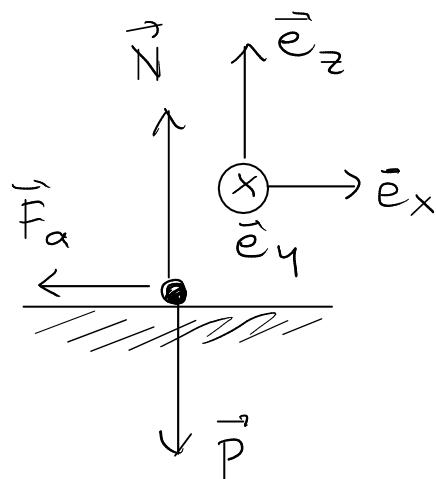
$$r = 30 \text{ m} \quad m = 10^3 \text{ kg}$$

$$|\vec{v}| = \text{cost}$$

$$\mu_s = 1, \quad \mu_d = 0.8$$

velocità massima?

$$m \frac{v^2}{r} \leq \mu_s m g \quad v^2 \leq \mu_s r g \quad v \leq \sqrt{\mu_s r g} = \sqrt{30 \text{ m} \times 10 \frac{\text{m}}{\text{s}^2}} \approx 17 \frac{\text{m}}{\text{s}} \approx 62 \frac{\text{km}}{\text{h}} \quad \square$$



$\vec{F}_a \rightarrow$ statico

$$|\vec{F}_a| \leq \mu_s |\vec{N}|$$

II Newton:

$$\vec{P} + \vec{N} + \vec{F}_a = m \vec{a}$$

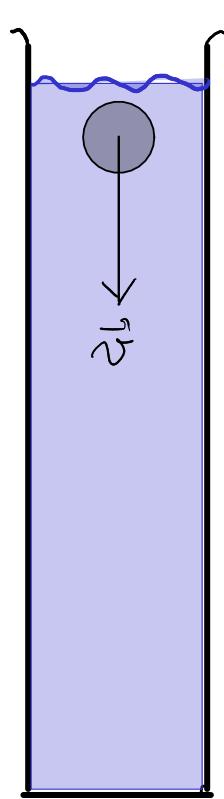
$$-mg \vec{e}_z + |\vec{N}| \vec{e}_z - |\vec{F}_a| \vec{e}_x = m a \vec{e}_x$$

$$\begin{cases} |\vec{N}| = mg \\ -|\vec{F}_a| = ma < 0 \end{cases}$$

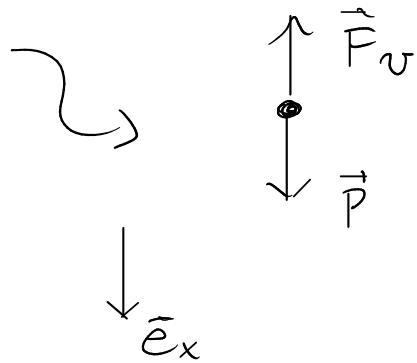
Moto circolare uniforme: $|a| = a_c = \frac{v^2}{r}$

$$|\vec{F}_a| = m |a| = m \frac{v^2}{r} \leq \mu_s |\vec{N}| = \mu_s mg$$

Es.: velocità limite di una biglia in un fluido viscoso



- fluido = gas (densità trascurabile rispetto a quella del corpo)
- bassa velocità $|\vec{F}_v| \sim |\vec{v}|$



II Newton: $\sum \vec{F} = m\vec{a}$

$$\vec{P} + \vec{F}_v = m\vec{a}$$

$$mg\vec{e}_x - \xi v\vec{e}_x = ma\vec{e}_x$$

$$m \frac{dv}{dt} = mg - \xi v$$

$$\frac{dv}{dt} = g - \frac{\xi}{m} v$$

$$\vec{v} = v\vec{e}_x \quad \vec{a} = a\vec{e}_x$$

$$\vec{F}_v = -\xi \vec{v} = -\xi v\vec{e}_x$$

Velocità limite:

$$a = 0 \Rightarrow v = v_l = \text{cost}$$

$$0 = g - \frac{\xi}{m} v_l \Rightarrow \underline{v_l = \frac{mg}{\xi}} \quad v_l \sim \frac{1}{\xi} \quad v_l \sim m$$

Legge oraria : $v(t)$?

$$\frac{dv}{dt} = \left(g - \frac{\Sigma}{m} v \right)$$

Separazione delle variabili

$$\frac{dv}{g - \frac{\Sigma}{m} v} = dt$$

$$\int \frac{dv}{g - \frac{\Sigma}{m} v} = \int dt$$

$$v' = g - \frac{\Sigma}{m} v \quad \frac{dv'}{dv} = -\frac{\Sigma}{m}$$

$$\left(-\frac{m}{\Sigma} \right) \int \frac{1}{g - \frac{\Sigma}{m} v} \left(-\frac{\Sigma}{m} \right) dv = \int dt$$

$$-\frac{m}{\Sigma} \int \frac{dv'}{v'} = \int dt$$

Condizioni iniziali : $v(t=0) = v_i$

$$v' = g - \frac{\Sigma}{m} v \quad \frac{dv'}{dt} = \frac{dv}{dt}$$

Integrazione con cambio variabile

$$y = y(x) \quad \frac{dy}{dx} dx = dx$$

$$\int f(y(x)) \frac{dy}{dx} dx = \int f(y) dy$$

↓

$$\int \frac{dv'}{v'}$$

$$\int_{x_i}^{x_f} f(y(x)) \frac{dy}{dx} dx = \int_{y_i}^{y_f} f(y) dy$$

$$y_i = y(x_i) \quad y_f = y(x_f)$$

$$-\frac{m}{\xi} \int_{g - \frac{\xi}{m} v_i}^{g - \frac{\xi}{m} v_f} \frac{dv}{v} = \int_0^{t_f} dt \quad \leftarrow \text{specifico estremi di integrazione}$$

$$-\frac{m}{\xi} \left[\ln\left(g - \frac{\xi}{m} v_f\right) - \ln\left(g - \frac{\xi}{m} v_i\right) \right] = t_f$$

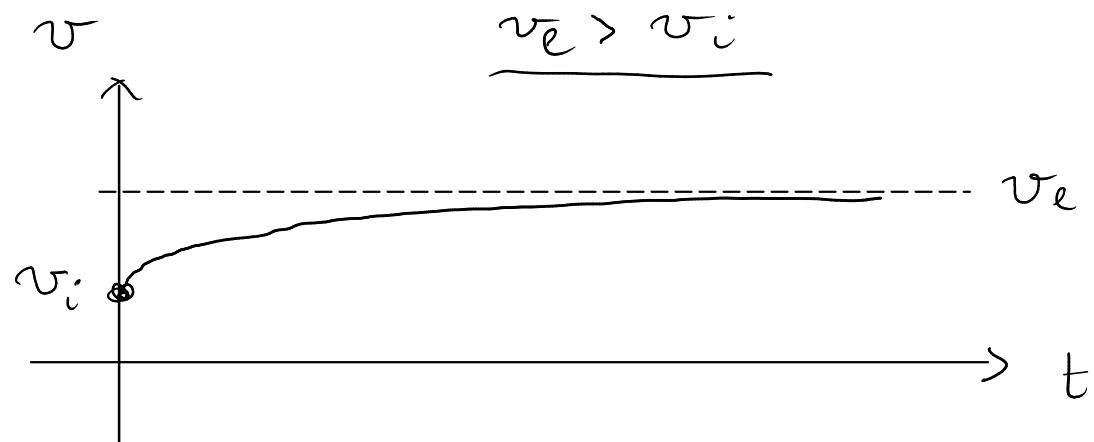
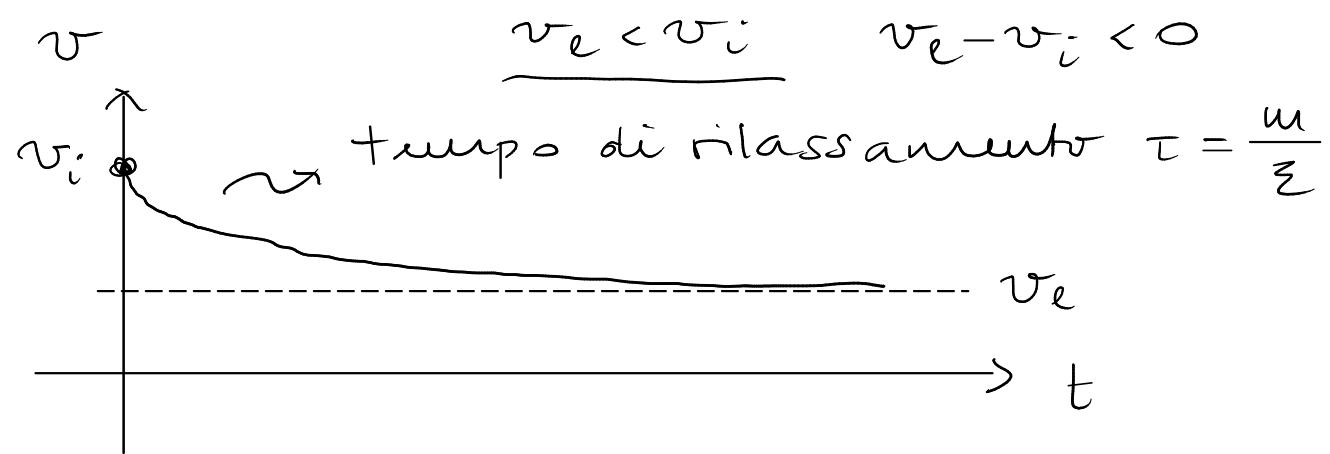
$$\ln\left(\frac{g - \frac{\xi}{m} v_f}{g - \frac{\xi}{m} v_i}\right) = -\frac{\xi}{m} t_f$$

$$\frac{g - \frac{\xi}{m} v_f}{g - \frac{\xi}{m} v_i} = \exp\left(-\frac{\xi}{m} t_f\right) \quad \text{Rinominando: } t_f = t, \quad v_f = v$$

$$g - \frac{\xi}{m} v = \left(g - \frac{\xi}{m} v_i\right) \exp\left(-\frac{\xi}{m} t\right)$$

$$v = \frac{m}{\xi} g - \frac{m}{\xi} \left(g - \frac{\xi}{m} v_i\right) \exp\left(-\frac{\xi}{m} t\right) = \frac{mg}{\xi} - \left(\frac{mg}{\xi} - v_i\right) \exp\left(-\frac{\xi}{m} t\right)$$

$$v = v_e - (v_e - v_i) \exp\left(-\frac{\xi}{m} t\right)$$



Casi limite:

$$\begin{cases} \lim_{t \rightarrow \infty} v = \frac{mg}{\xi} = v_e \\ v(t=0) = v_e - (v_e - v_i) = v_i \end{cases}$$