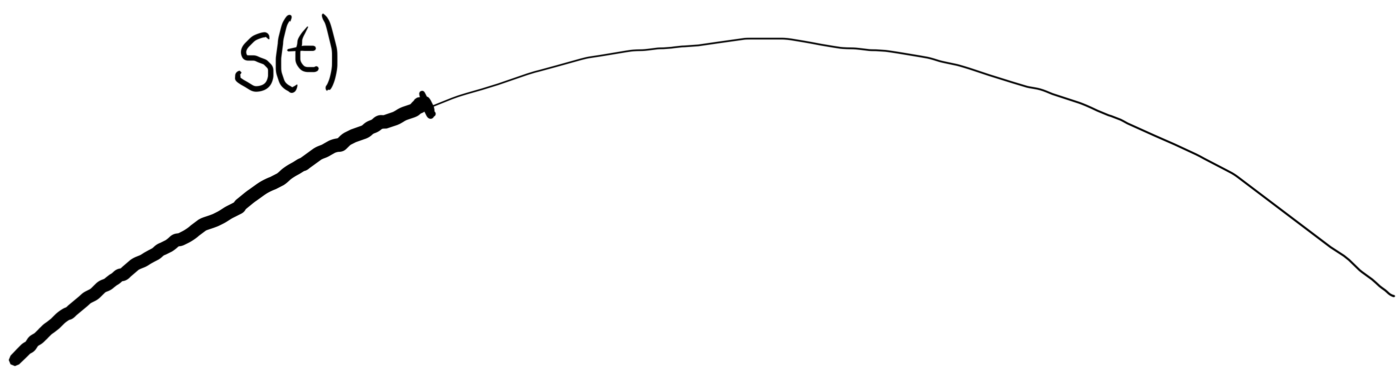
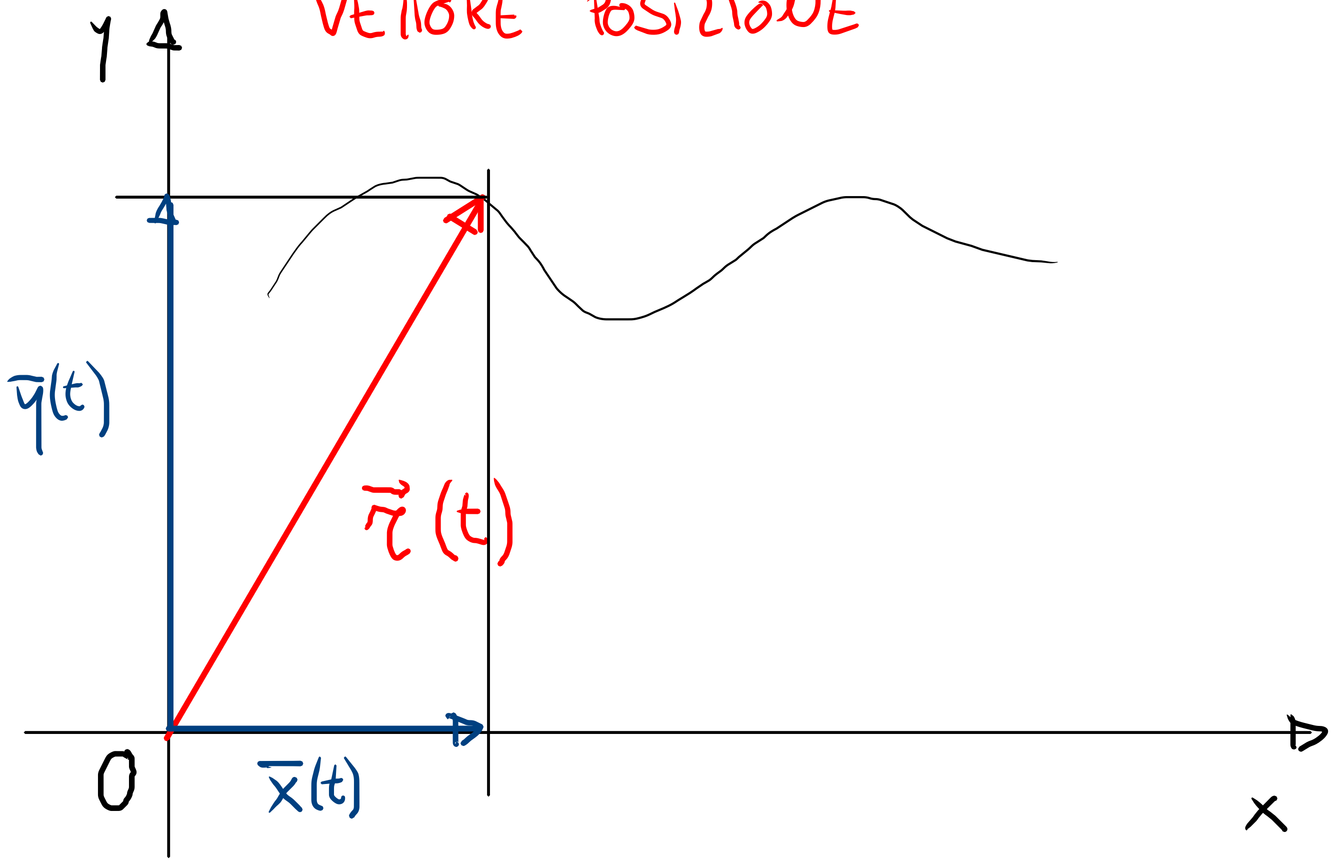
MECCANICA
- Cinematica ←
- Statica
- Dinamica

PUNTO MATERIALE punto geometrico di massa m

TRAIETTORIA luogo di punti attraversati dal P.M.



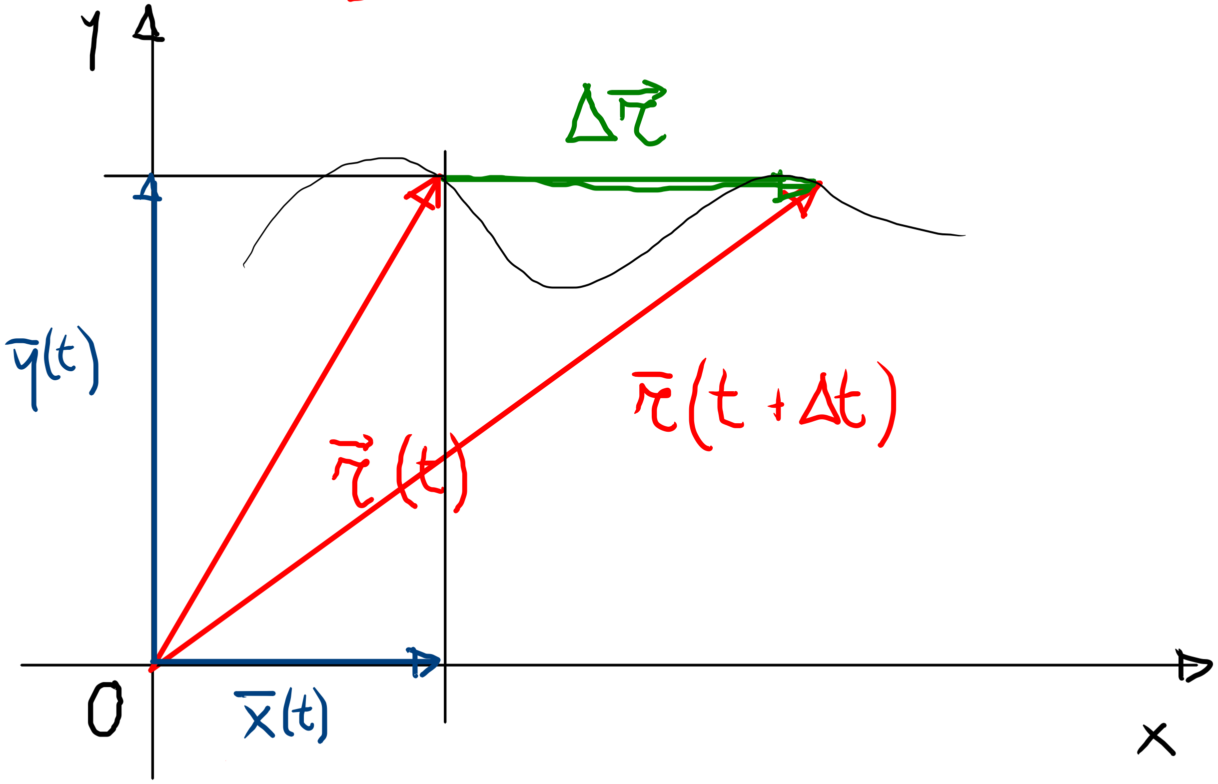
VETTORE POSIZIONE



$\vec{r}(t)$ vettore posizione

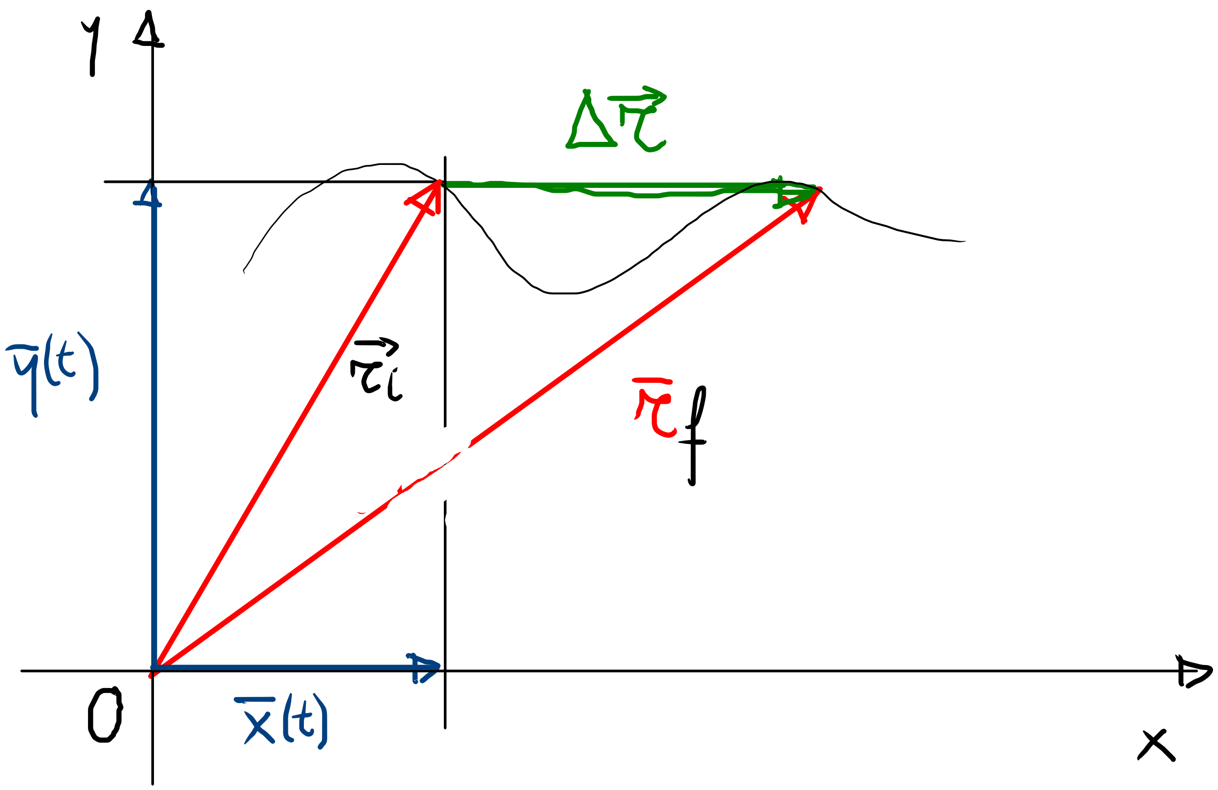
$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}$$

SPOSTAMENTO



$$\Delta t = t_f - t_i$$

$$\Delta \vec{r} = \vec{r}(t + \Delta t) - \vec{r}(t)$$



$$\begin{aligned}
\Delta \vec{r} &= \vec{r}_f - \vec{r}_i \\
&= (x_f \hat{i} + y_f \hat{j}) - (x_i \hat{i} + y_i \hat{j}) \\
&= (x_f - x_i)\hat{i} + (y_f - y_i)\hat{j} \\
&= \Delta x \hat{i} + \Delta y \hat{j}
\end{aligned}$$

VELOCITÀ MEDIA E ISTANTANEA

$$\vec{v}_m = \frac{\Delta \vec{r}}{\Delta t} = \frac{1}{\Delta t} \cdot (\Delta \vec{r})$$

$$|\vec{v}_m| = \frac{|\Delta \vec{r}|}{\Delta t}$$, direzione e verso di $\Delta \vec{r}$.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}$$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$|\vec{v}| = \left|\frac{d\vec{r}}{dt}\right|$$, direzione tangente alla traiettoria

verso del moto

1D (asse x)

$$v_m = \frac{\Delta x}{\Delta t}$$

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

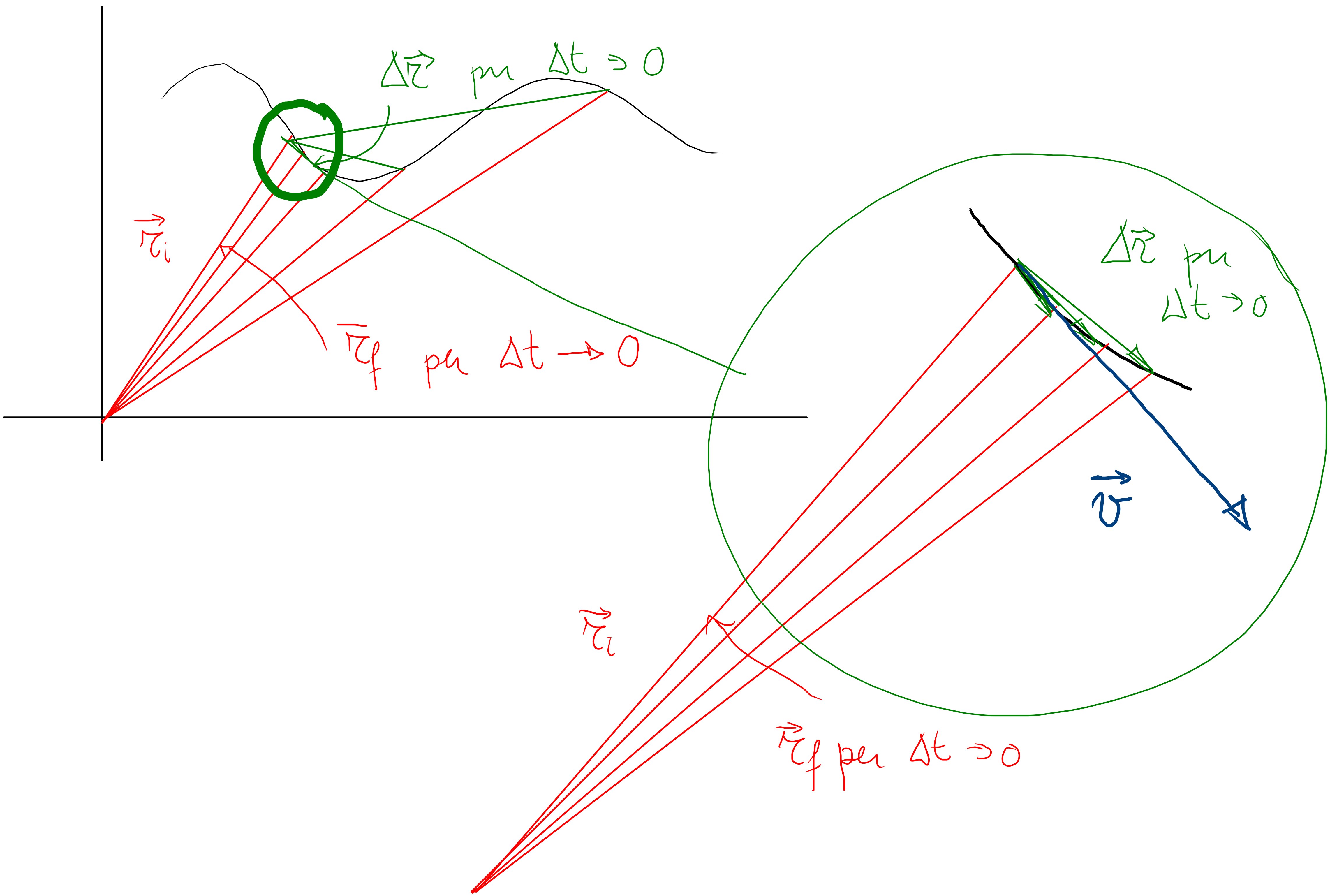
unità di misura

SI : $\frac{m}{s}$

cgs : $\frac{cm}{s}$

$$\frac{Km}{h} = \frac{1000\ m}{3600\ s} = \frac{1}{3{,}6} \frac{m}{s}$$

$$1 \frac{m}{s} = 3{,}6 \ Km/h$$



$\Delta\vec{r}$ pu $\Delta t \Rightarrow 0$
$\vec{r}_i$
$\vec{r}_f$ pu $\Delta t \rightarrow 0$
$\Delta\vec{r}$ pu $\Delta t \rightarrow 0$
$\vec{v}$
$\vec{r}_i$
$\vec{r}_f$ pu $\Delta t \Rightarrow 0$

ACCELERAZIONE MEDIA E ISTANTANEA

Δt

$t_i \; : \; \vec{v}_i$

$t_f \; : \; \vec{v}_f$

$$\vec{a}_m = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

1D

$$a_m = \frac{\Delta v}{\Delta t}$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2 \vec{r}}{dt^2}$$

unità di misura

SI $\frac{m}{s^2}$

cgs $\frac{cm}{s^2}$

nelle puntate precedenti....

(previously on Fisica @ STB)

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

$\vec{a}$ è un vettore!

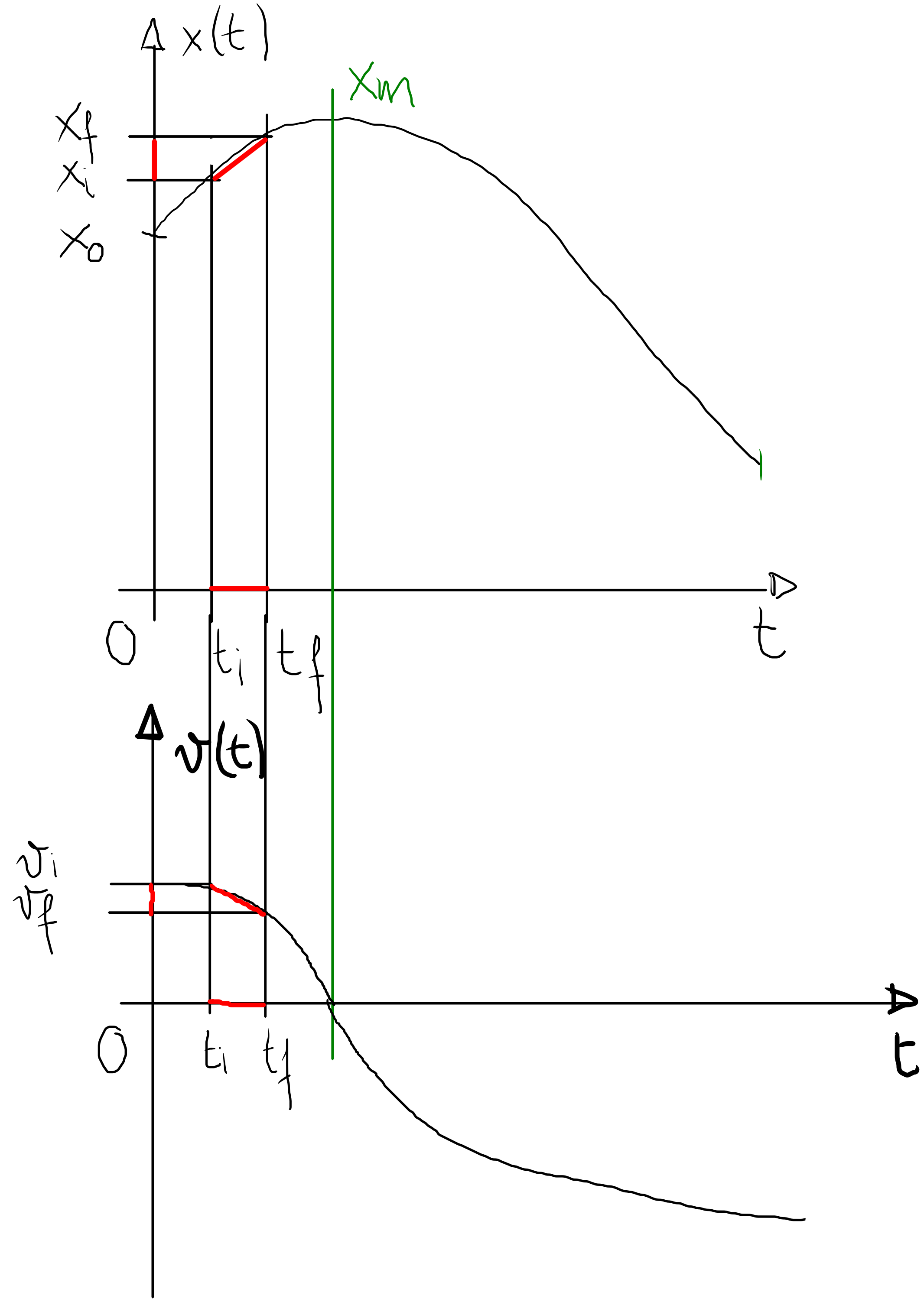
Quali sono direzione e verso di $\vec{a}$?

– TO BE CONTINUED –

LEGGE ORARIA DI UN MOTO (1D)

$x = x(t)$

(esprime la posizione in funzione di t)



$$\Delta t = t_f - t_i$$

$$\Delta x = x_f - x_i$$

$$v_m = \frac{\Delta x}{\Delta t}$$

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

} pendenza della curva $x(t)$

$$a_m = \frac{\Delta v}{\Delta t} \quad (<0)$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

} pendenza della curva $v(t)$

MOTO RETTILINEO UNIFORME

RETTILINEO → 1D

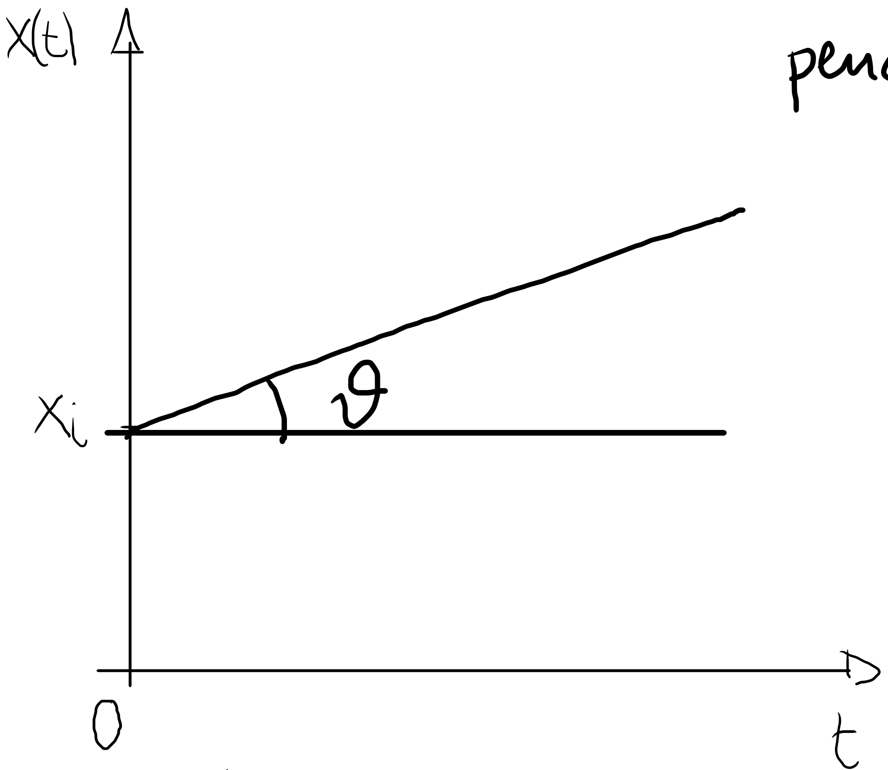
UNIFORME → $\vec{v}$ costante nel tempo $= \vec{v}_i$

↓

pendenza costante

↓

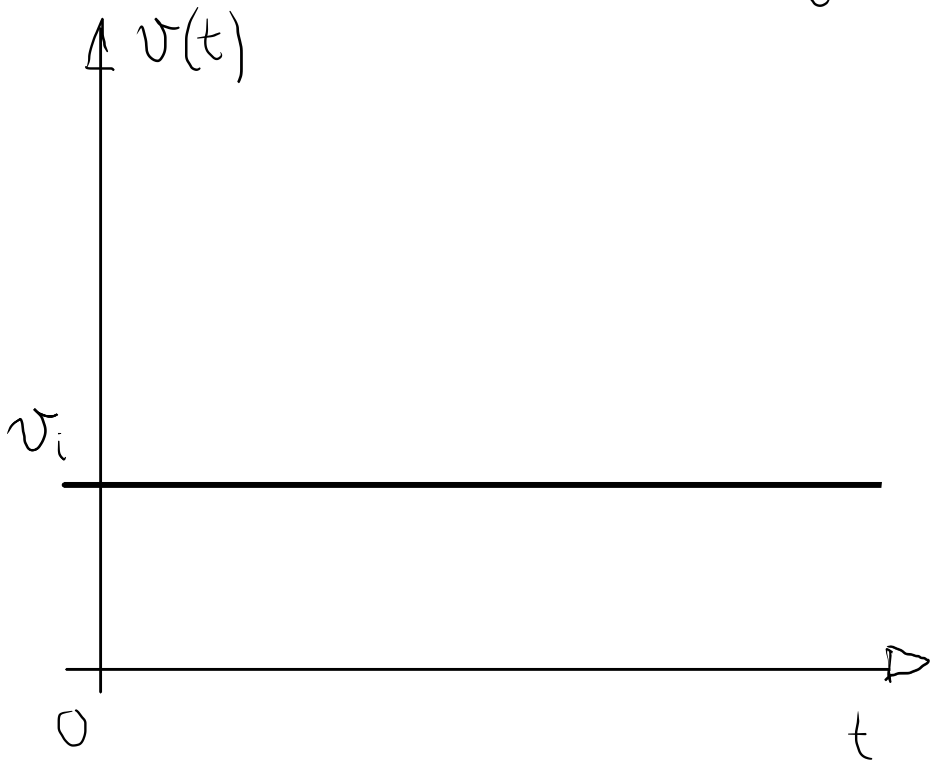
retta



$$x(t) = v_i \cdot t + x_i \qquad (v_i = tg\,\vartheta)$$

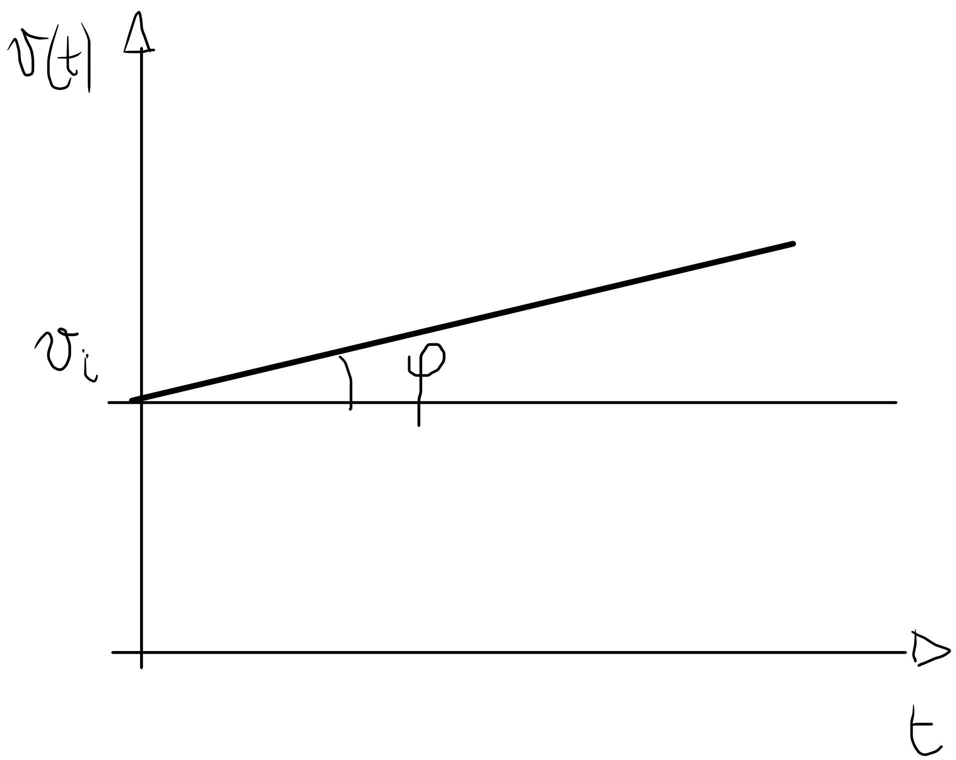
$$\boxed{x(t) = x_i + v_i \cdot t} \quad \text{in 1D}$$

$$\vec{r}(t) = \vec{r}_i + \vec{v}_i \cdot t \quad \text{in 2+ D}$$



MOTO (RETTILINEO) UNIFORMEMENTE ACCELERATO

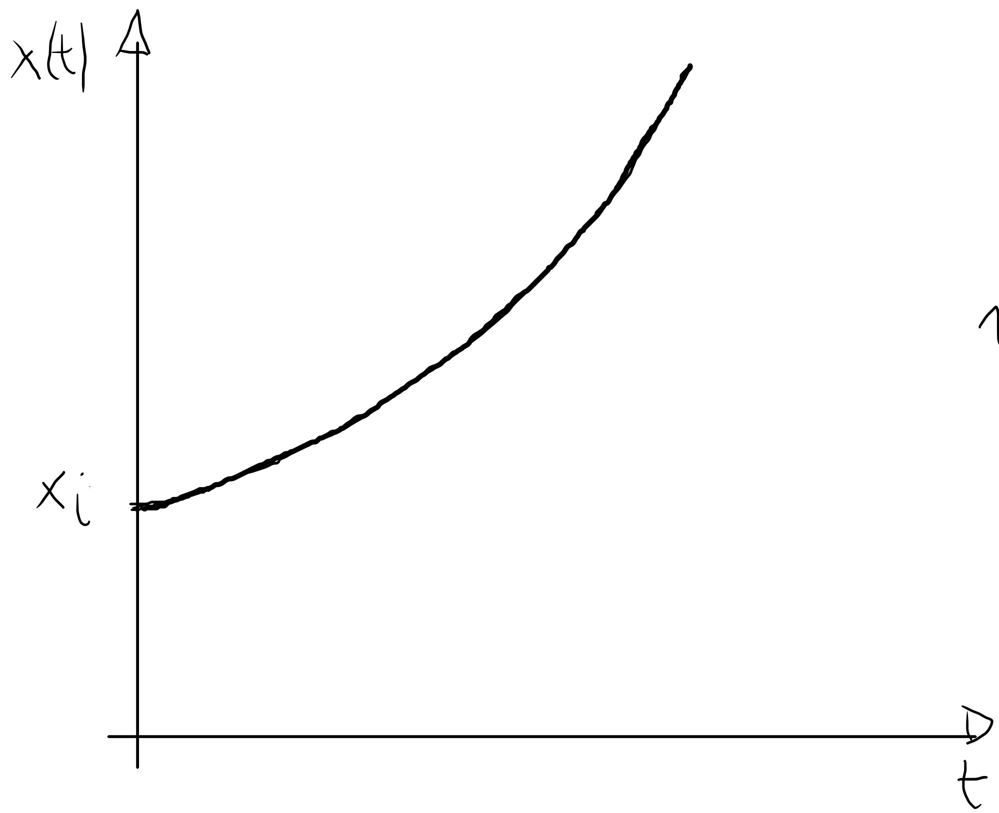
$$\vec{a} = \text{cost} = \vec{a}_i$$



I $\boxed{v(t) = v_i + a_i t}$ $\quad (a_i = \text{tg}\,\varphi)$

$$\vec{v}(t) = \vec{v}_i + \vec{a}_i t$$

$$a(t) = \frac{dv(t)}{dt} = a_i$$



II $\boxed{x(t) = x_i + v_i t + \frac{1}{2} a t^2}$

$$v(t) = \frac{dx(t)}{dt} = v_i + \frac{1}{2} a\, 2t = v_i + at$$

$$\vec{r}(t) = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a}\, t^2$$

I) $\boxed{v(t) = v_i + at}$

II) $\boxed{x(t) = x_i + v_i t + \frac{1}{2}at^2}$

$$\begin{cases} v = v_i + at \\ x = x_i + v_i t + \frac{1}{2}at^2 \end{cases}$$

$$\begin{cases} t = \dfrac{v - v_i}{a} \\ x = x_i + v_i\left(\dfrac{v - v_i}{a}\right) + \dfrac{1}{2}a\left(\dfrac{v - v_i}{a}\right)^2 \end{cases}$$

$$x - x_i = \frac{v v_i}{a} - \frac{v_i^2}{a} + \frac{1}{2}\cancel{a}\left(\frac{v^2 - 2v v_i + v_i^2}{a^{\cancel{2}}}\right)$$

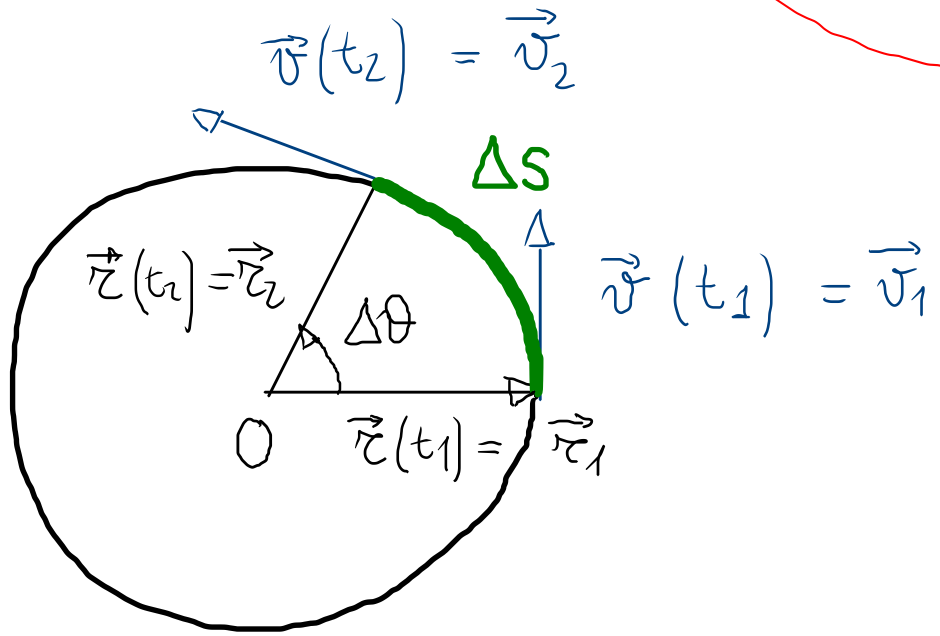
$$x - x_i = \cancel{\frac{v v_i}{a}} - \frac{v_i^2}{a} + \frac{v^2}{2a} - \cancel{\frac{v v_i}{a}} + \frac{v_i^2}{2a}$$

$$x - x_i = \frac{v^2}{2a} - \frac{v_i^2}{2a} = \frac{v^2 - v_i^2}{2a}$$

$$2a(x - x_i) = v^2 - v_i^2$$

III) $\boxed{v^2 = v_i^2 + 2a(x - x_i)}$

MOTO CIRCOLARE UNIFORME



$$|\vec{r}_1| = |\vec{r}_2| = R$$

$$|\vec{v}_1| = |\vec{v}_2| = v \qquad \text{nota: } \vec{v}_1 \neq \vec{v}_2 \,!$$

$$\Delta t = t_2 - t_1$$

$$v = \frac{\Delta s}{\Delta t}$$

- velocità angolare: $\omega_m = \dfrac{\Delta\vartheta}{\Delta t}$ media

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\vartheta}{\Delta t} = \frac{d\vartheta}{dt} \quad \text{istantanea}$$

$$\Delta\vartheta = \frac{\Delta s}{R}$$

$$\omega = \frac{\Delta\vartheta}{\Delta t} = \frac{1}{\Delta t} \cdot \frac{\Delta s}{R} = \frac{1}{R}\frac{\Delta s}{\Delta t} = \frac{1}{R} v = \frac{v}{R}$$

$$\boxed{\omega = \frac{v}{R}} \quad \text{o} \quad \boxed{v = R\omega} \quad \Rightarrow \omega \text{ è costante !}$$

- accelerazione angolare: $\alpha_m = \frac{\Delta \omega}{\Delta t}$ media

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\vartheta}{dt^2}$$ istantanea

nel moto circolare uniforme ω cost.

$$\Delta \omega = 0$$

$$\alpha_m = \alpha = 0$$

Non c'è accelerazione angolare ma c'è ....

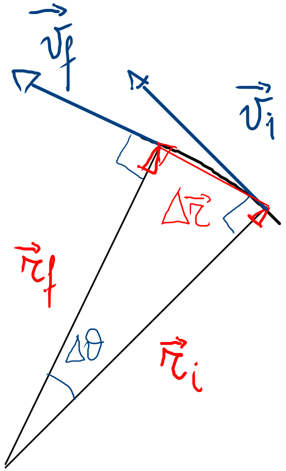
- accelerazione $\vec{a}_m = \frac{\Delta \vec{v}}{\Delta t}$

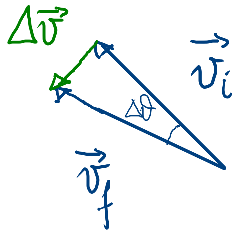
$\Delta \vec{v} \neq 0$ $\Rightarrow$ c'è acc.

$\vec{v}_1 \neq \vec{v}_2$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

ACCELERAZIONE NEL MOTO CIRCOLARE UNIFORME





$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

$$|\vec{r}_i| = |\vec{r}_f| = R \qquad |\vec{v}_i| = |\vec{v}_f| = v$$

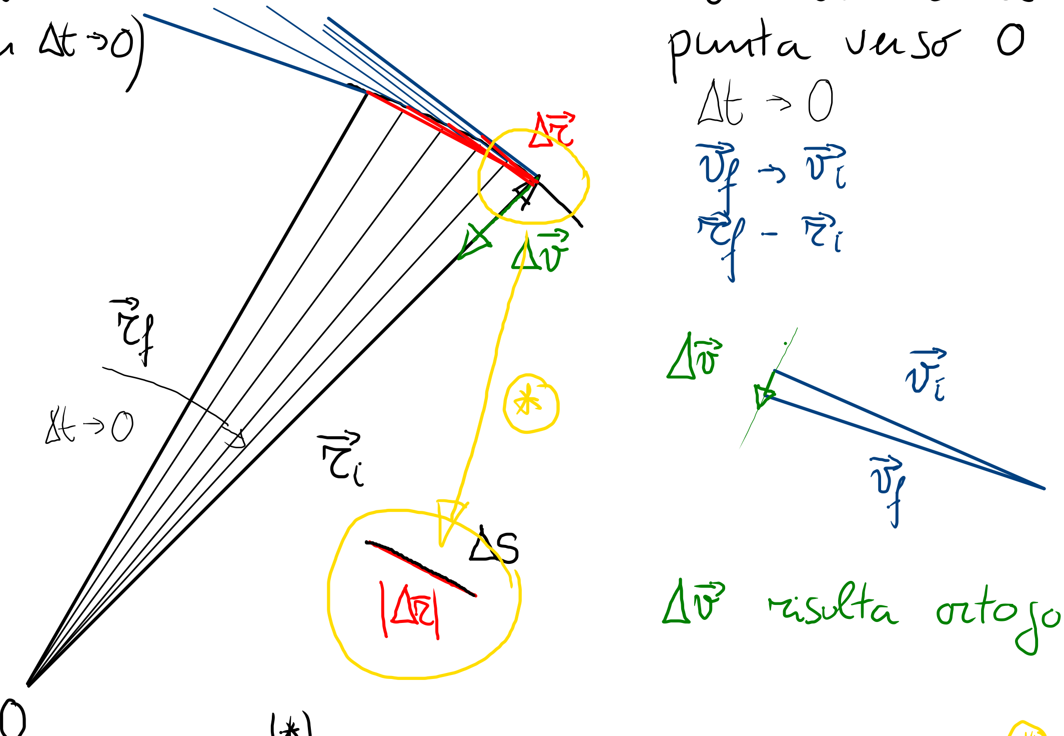
Sono triangoli simili: $\dfrac{|\Delta\vec{r}|}{R} = \dfrac{|\Delta\vec{v}|}{v}$

$$\vec{a}_m = \frac{\Delta\vec{v}}{\Delta t}$$

$$|\vec{a}_m| = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v}{\Delta t}\frac{|\Delta\vec{r}|}{R} = \frac{v}{R}\frac{|\Delta\vec{r}|}{\Delta t} \qquad (*)$$

$\vec{a}_m \to \vec{a}$
$(\lim \Delta t \to 0)$

$\vec{a}$ ha direzione radiale, punta verso O } é CENTRIPETA



$\Delta \vec{v}$ risulta ortogonale a $\vec{v}$

$$|\vec{a}| = \lim_{\Delta t \to 0} |\vec{a}_m| \overset{(*)}{=} \lim_{\Delta t \to 0} \frac{v}{R} \frac{|\Delta \vec{r}|}{\Delta t} = \frac{v}{R} \lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta t} = \frac{v}{R} \underbrace{\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}}_{v}$$

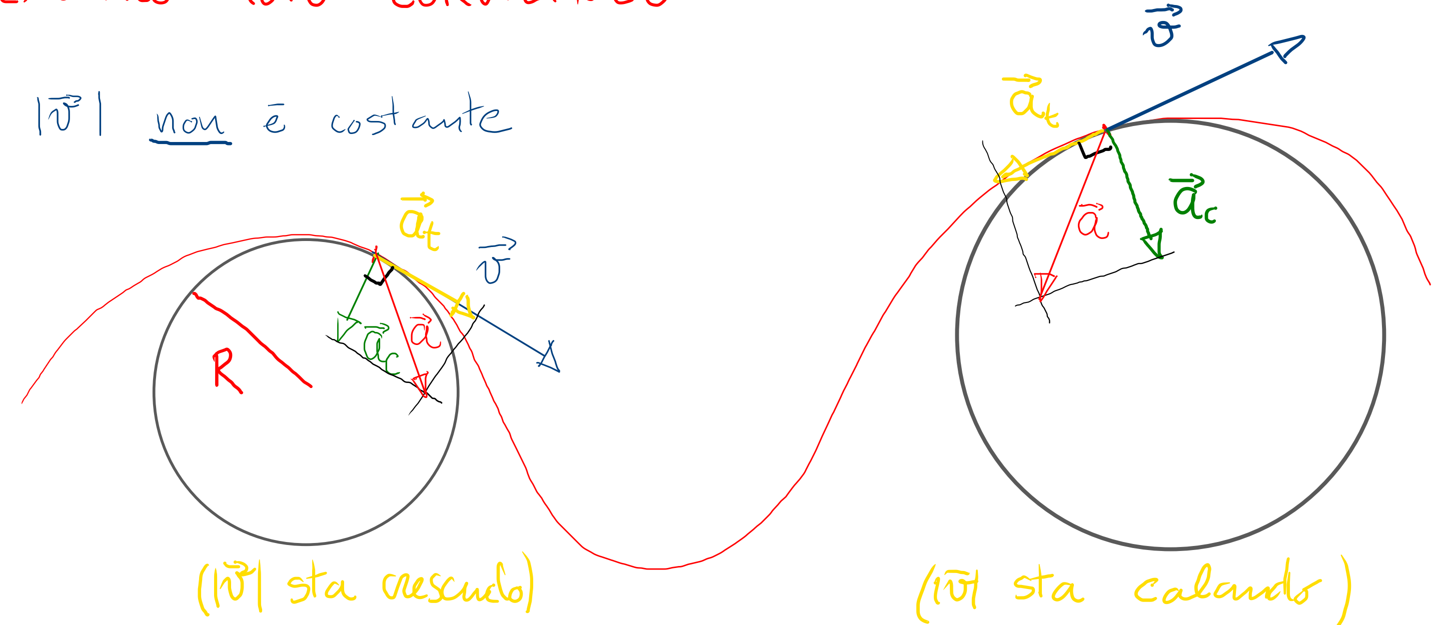
$$|\vec{a}| = \frac{v^2}{R}$$

$$v = \omega R$$

$$|\vec{a}| = \frac{\omega^2 R^2}{R} = \omega^2 R$$

GENERICO MOTO CURVILINEO

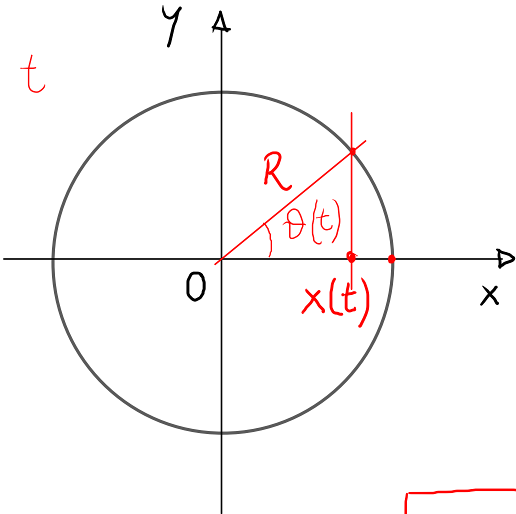
$|\vec{v}|$ non $\bar{e}$ costante



$$|\vec{a}_t| = \frac{d|\vec{v}|}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d}{dt}\left(\frac{R\,d\vartheta}{dt}\right) = R\,\frac{d}{dt}\underbrace{\left(\frac{d\vartheta}{dt}\right)}_{\omega} = R\,\frac{d\omega}{dt} = R\alpha$$

$$|\vec{a}_c| = \omega^2 R = \frac{v^2}{R}$$

MOTO ARMONICO (1D)



$t=0 \Rightarrow \vartheta = 0$

$$\vartheta(t) = \omega \cdot t \qquad \left(\omega = \frac{\Delta\vartheta}{\Delta t} = \frac{\vartheta}{t}\right)$$

$$x(t) = R\cos(\vartheta(t))$$

$$\left[\, y(t) = R\,\text{sen}(\vartheta(t)) \,\right]$$

$$x(t) = R\cos(\vartheta(t)) = R\cos(\omega t)$$

(esempio di moto armonico)

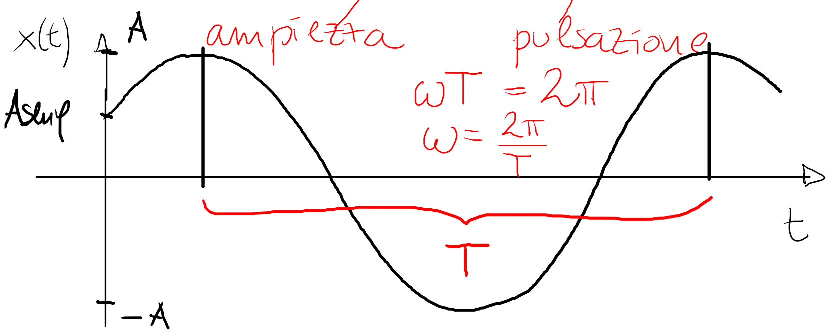
In generale:

$$x(t) = A\,\text{sen}(\omega t + \varphi)$$

ampiezza (A), pulsazione (ω), fase (φ) (condizioni iniziali)

$\omega T = 2\pi$

$\omega = \frac{2\pi}{T}$



Frequenza $\nu = \dfrac{1}{T}$ si misura in $s^{-1} = Hz$

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

$$T = \frac{2\pi}{\omega} = \frac{1}{\nu}$$

$$x(t) = A \,\text{sen}(\omega t + \varphi)$$

$$v(t) = \frac{dx(t)}{dt}$$

$$v(t) = A\omega \cos(\omega t + \varphi)$$

$$a(t) = \frac{dv(t)}{dt}$$

$$a(t) = -A\omega^2 \,\text{sen}(\omega t + \varphi)$$

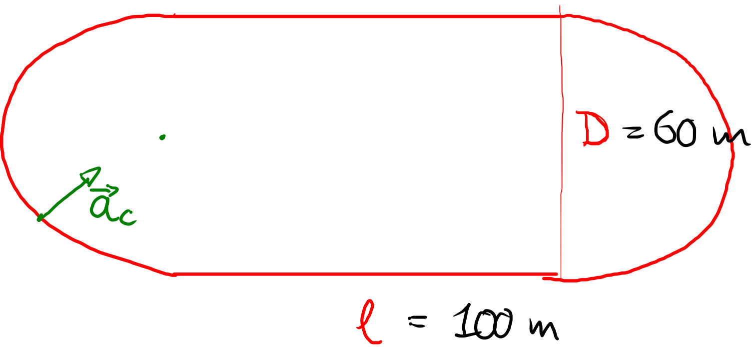
Esempio: $\varphi = \frac{\pi}{2}$

$$x(t) = A\,\text{sen}\left(\omega t + \frac{\pi}{2}\right) = A\left(\text{sen}(\omega t)\cdot\cos\frac{\pi}{2} + \cos(\omega t)\,\text{sen}\frac{\pi}{2}\right) = A\cos(\omega t)$$

$$a(t) = -\omega^2 x(t)$$

La pattinatrice

(esercizio 2,39 a pag. 57 del Ragozzino, Elementi di Fisica)



$$v = 45\ \frac{km}{h} = 45\ \frac{\overset{1}{\cancel{1000}}\ m}{\underset{3,6}{\cancel{3600}}\ s} = 12{,}5\ \frac{m}{s}$$

a) $$T = \frac{2 \cdot \ell + \pi D}{v} = \frac{2 \cdot 100\ m + \pi \cdot 60\ m}{12{,}5\ \frac{m}{s}} = 31{,}1\ s$$

b) $$|\vec{a}_c| = \frac{v^2}{R} = \frac{2v^2}{D} = \frac{2\left(12{,}5\ \frac{m}{s}\right)^2}{60\ m} = 5{,}21\ \frac{m}{s^2}$$