Cormen Problem 12-1. Equal keys pose a problem for the implementation of binary search trees.

a. What is the asymptotic performance of TREE-INSERT when used to insert *n* items with identical keys into an initially empty binary search tree?

Cormen Problem 12-1. We propose to improve TREE-INSERT by testing before line 5 to determine whether z.key = x.key and by testing before line 11 to determine whether z.key = y.key.

TREE-INSERT(T, z)

1 y = NILx = T.root2 3 while $x \neq \text{NIL}$ 4 5 y = xif z.key < x.key6 x = x.left7 else x = x.right8 z.p = y**if** y == NIL9 10 T.root = z. 11 elseif z.key < y.keyy.left = z12 else y.right = z13

If equality holds, we implement one of the following strategies. For each strategy, find the asymptotic performance of inserting n items with identical keys into an initially empty binary search tree. (The strategies are described for line 5, in which we compare the keys of z and x. Substitute y for x to arrive at the strategies for line 11.)

b. Keep a boolean flag *x.b* at node *x*, and set *x* to either *x.left* or *x.right* based on the value of *x.b*, which alternates between FALSE and TRUE each time we visit *x* while inserting a node with the same key as *x*.

Cormen Problem 12-1. We propose to improve TREE-INSERT by testing before line 5 to determine whether z.key = x.key and by testing before line 11 to determine whether z.key = y.key.

TREE-INSERT(T, z)

- $1 \quad y = \text{NIL}$
- $2 \quad x = T.root$

3 while $x \neq \text{NIL}$

- $\begin{array}{ll} 4 & y = x \\ 5 & \text{if } z.key \end{array}$
- 5 if z.key < x.key6 x = x.left7 else x = x.right
- $8 \quad z.p = y$
- 9 **if** y == NIL
- 10
- 11 elseif z. key < y. key
- 12 y.left = z

T.root = z.

13 else y.right = z

If equality holds, we implement one of the following strategies. For each strategy, find the asymptotic performance of inserting *n* items with identical keys into an initially empty binary search tree. (The strategies are described for line 5, in which we compare the keys of *z* and *x*. Substitute *y* for *x* to arrive at the strategies for line 11.)

c. Keep a list of nodes with equal keys at *x*, and insert *z* into the list.

Cormen Problem 12-1. We propose to improve TREE-INSERT by testing before line 5 to determine whether z.key = x.key and by testing before line 11 to determine whether z.key = y.key.

TREE-INSERT(T, z)

- 1 y = NIL
- 2 x = T.root
- 3 while $x \neq \text{NIL}$
- $\begin{array}{ll} 4 & y = x \\ 5 & \text{if } z.key < x.key \end{array}$
- $\begin{array}{ll}
 6 & x = x.left \\
 7 & else \ x = x.right
 \end{array}$
- 8 z.p = y
- 9 **if** y == NIL
- 10 T.root = z
- 11 **elseif** z.key < y.key
- 12 y.left = z
- 13 else y.right = z

If equality holds, we implement one of the following strategies. For each strategy, find the asymptotic performance of inserting n items with identical keys into an initially empty binary search tree. (The strategies are described for line 5, in which we compare the keys of z and x. Substitute y for x to arrive at the strategies for line 11.)

d. Randomly set x to either *x.left* or *x.right*. (Give the worst-case performance and informally derive the expected running time.)

A preorder traversal of a tree is given by the following procedure:

- Visit (print) the root node
- Traverse the left sub-tree in pre-order
- Traverse the right sub-tree in pre-order

A postorder traversal of a tree is given by the following procedure:

- Traverse the left subtree by calling the postorder function recursively.
- Traverse the right subtree by calling the postorder function recursively.
- Visit (print) the current node.

EX. Given a BST in pre-order as {13,5,3,2,11,7,19,23}, draw this BST and determine if this BST is the same as one described in post-order as {2,3,5,7,11,23,19,13}.