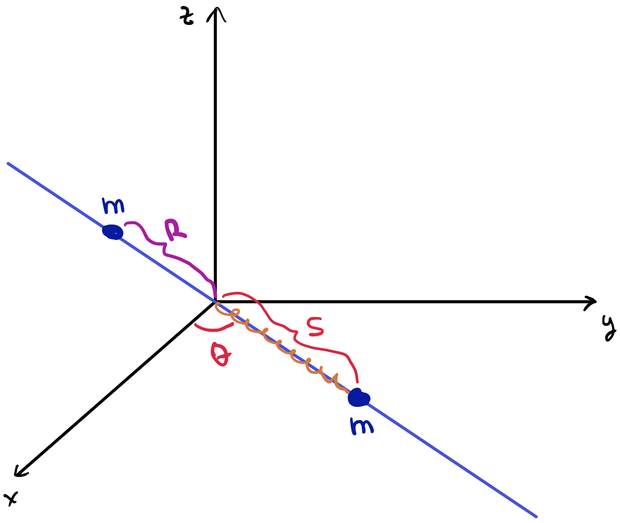


1) $T = \frac{m}{2}(R^2+s^2)\dot\theta^2 + \frac{m\dot s^2}{2}$ $\qquad V = \frac{K}{2}s^2$

$$L = \frac{m}{2}(R^2+s^2)\dot\theta^2 + \frac{m}{2}\dot s^2 - \frac{1}{2}Ks^2$$

2) $\frac{d}{dt}\frac{\partial L}{\partial \dot s} - \frac{\partial L}{\partial s} = \boxed{m\ddot s - ms\dot\theta^2 + Ks = 0}$

$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta} - \frac{\partial L}{\partial \theta} = \frac{d}{dt}\left(m(R^2+s^2)\dot\theta\right) = \boxed{m(R^2+s^2)\ddot\theta + 2ms\dot s\dot\theta = 0}$

3) θ ciclica. $\quad p_\theta = \frac{\partial L}{\partial \dot\theta} = m(R^2+s^2)\dot\theta$ è cost. del moto

(comp. lungo asse z di momento angolare)

4) $p_\theta = \ell$ cost. $\rightarrow \dot\theta = \frac{\ell}{m(R^2+s^2)}$ $\quad (*)$

Fissiamo $\ell > 0$ senza perdita di generalità

$L^* = L - \dot\theta p_\theta \Big|_{(*)} = \boxed{\frac{m}{2}\dot s^2 - \frac{K}{2}s^2 - \frac{\ell^2}{2m(R^2+s^2)}}$

5) $V_{eff} = \frac{1}{2}Ks^2 + \frac{\ell^2}{2m(R^2+s^2)}$ $\qquad [\ell] = [mR^2/T]$

$V_{eff}' = Ks - \frac{\ell^2}{\cancel{2}m(R^2+s^2)^2}\cdot\cancel{2}s = Ks\left(1 - \frac{\ell^2}{Km(R^2+s^2)^2}\right)$

$\rightarrow \boxed{s = 0}$

$$\rightarrow (s^2 + R^2) = \frac{\ell}{\sqrt{Km}} \rightsquigarrow s^2 = \frac{\ell}{\sqrt{Km}} - R^2$$

ESISTE se $\ell > R^2\sqrt{Km}$

$$s = \pm\sqrt{\frac{\ell}{\sqrt{Km}} - R^2} = \pm s_0$$

$$V_{eff}'' = K\left(1 - \frac{\ell^2}{Km(R^2+s^2)^2}\right) + Ks\left(\frac{4\ell^2 s}{Km(s^2+R^2)^3}\right)$$

$$= K\left(1 - \frac{\ell^2}{Km(s^2+R^2)^2}\right) + \frac{4\ell^2 s^2}{m(s^2+R^2)^3}$$

$$V_{eff}''(s=0) = K\left(1 - \frac{\ell^2}{KmR^4}\right)$$

STAB se $\ell^2 < KmR^4$

$$V_{eff}''(\pm s_0) = 0 + \frac{4\cancel{\ell^2}\left(\frac{\ell}{\sqrt{Km}} - R^2\right)}{m\ \cancel{\ell^3}/(Km)^{3/2}}$$

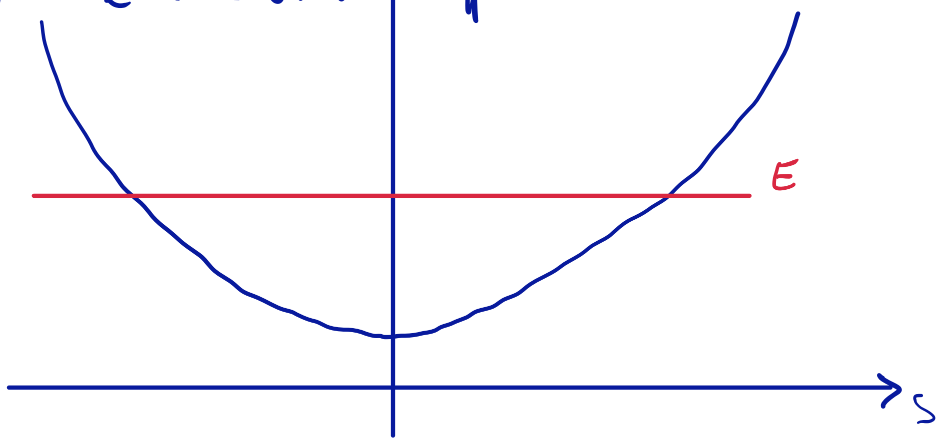
STAB se $\ell > R^2\sqrt{Km}$ cioè qndo esiste

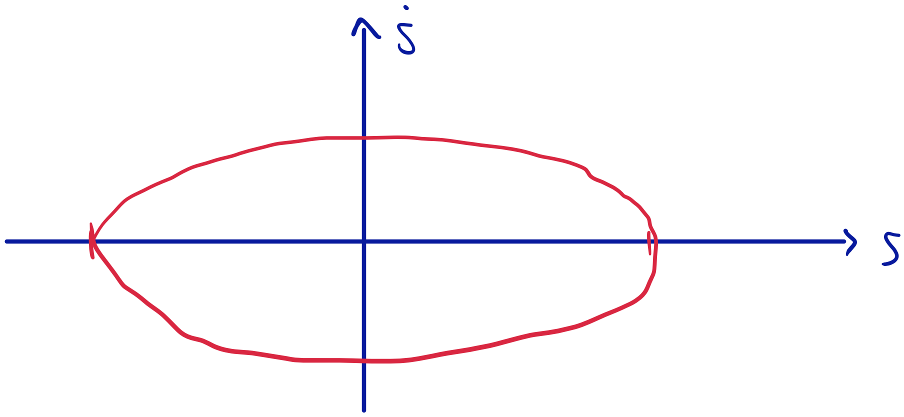
6) $L^* = \frac{1}{2}m\dot{s}^2 - V_{eff}(s) \rightarrow L^*_{lin} = \frac{1}{2}m\dot{\delta s}^2 - \frac{1}{2}V_{eff}''(MIN)\,\delta s^2$

$$\ell < R^2\sqrt{Km} \rightsquigarrow \omega^2 = \frac{V_{eff}''(0)}{m} = \frac{K}{m}\left(1 - \frac{\ell^2}{KmR^4}\right)$$

$$\ell > R^2\sqrt{Km} \rightsquigarrow \omega^2 = \frac{V_{eff}''(\pm s_0)}{m} = \frac{4R^2\sqrt{Km}}{\ell}\frac{K}{m}\left(\frac{\ell}{R^2\sqrt{Km}} - 1\right)$$

7) $\ell < R^2\sqrt{Km}$





$\ell > R^2\sqrt{Km}$