Fondamenti di Automatica

Esercizi su
Z-Trasformata
e Trasformata di
Laplace

(•)

$$y(k+2) = y(k+1) + y(k)$$

$$\begin{cases} y(0) = 0 \\ y(1) = +1 \end{cases}$$

ec. diferente
ordine 2

$$\{y(k)\}\quad k \geq 0$$

$$\mathcal{Z}\{y(k)\} = Y(z)$$

$$\mathcal{Z}\{y(k+1)\} = z\left[Y(z) - y(0)\right] = zY(z)$$

$$\mathcal{Z}\{y(k+2)\} = z^2\left[Y(z) - \cancel{\overset{0}{y(0)}} - \underbrace{y(1)}_{1} z^{-1}\right]$$

$$= z^2 Y(z) - z$$

$$\mathcal{Z}\{y(k+2)\} = \mathcal{Z}\{y(k+1)\} + \mathcal{Z}\{y(k)\}$$

$$z^2 Y(z) - z = z Y(z) + Y(z)$$

$$(z^2 - z - 1) Y(z) = +z$$

$$Y(z) = \frac{z}{z^2 - z - 1}$$

$m = 1$

$n = 2$

$n - m$ valori nulli

$$y(0) = 0 \qquad y(1) = \frac{b_m}{a_n} = 1$$

$\lim_{k \to \infty} y(k) \to ?$

poli

NB

$$P_1 = + \frac{1+\sqrt{5}}{2} \simeq +2{,}12$$

$$P_2 = + \frac{1-\sqrt{5}}{2} \simeq -0{,}12$$

$$Y(z) = \frac{z}{\left(z - \frac{1-\sqrt{5}}{2}\right)\left(z - \frac{1+\sqrt{5}}{2}\right)} \quad \Big/ \cdot \frac{1}{z} \qquad 1$$

$$\left[\frac{Y(z)}{z}\right] = \frac{1}{\left(z - \frac{1-\sqrt{5}}{2}\right)\left(z - \frac{1+\sqrt{5}}{2}\right)} =$$

$$= \frac{C_1}{z - \left(\frac{1-\sqrt{5}}{2}\right)} + \frac{C_2}{\left(z - \frac{1+\sqrt{5}}{2}\right)} \qquad 3$$

$$C_1 = \lim_{z \to \frac{1-\sqrt{5}}{2}} \left[\frac{Y(z)}{z}\right]\left(z - \frac{1-\sqrt{5}}{2}\right) =$$

$$= \lim_{z \to \frac{1-\sqrt{5}}{2}} \frac{1}{\cancelto{1}{\left(z - \frac{1-\sqrt{5}}{2}\right)}\left(z - \frac{1+\sqrt{5}}{2}\right)} \cancelto{1}{\left(z - \frac{1-\sqrt{5}}{2}\right)} =$$

$$C_1 = \lim_{z \to \frac{1-\sqrt{5}}{2}} \frac{1}{z - \frac{1+\sqrt{5}}{2}} = \frac{1}{\frac{1-\sqrt{5}}{2} - \frac{1+\sqrt{5}}{2}}$$

$$= \frac{1}{\frac{1}{2} - \frac{\sqrt{5}}{2} - \frac{1}{2} - \frac{\sqrt{5}}{2}} = \frac{1}{-\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$C_2 = \lim_{z \to \frac{1+\sqrt{5}}{2}} \left[\frac{Y(t)}{z}\right]\left(z - \frac{1+\sqrt{5}}{2}\right) =$$

$$= \lim_{z \to \frac{1+\sqrt{5}}{2}} \frac{\cancel{\left(z - \frac{1+\sqrt{5}}{2}\right)}^{1}}{\left(z - \frac{1-\sqrt{5}}{2}\right)\cancel{\left(t - \frac{1+\sqrt{5}}{2}\right)}_{1}} =$$

$$= \frac{1}{\frac{1}{2} + \frac{\sqrt{5}}{2} - \frac{1}{2} + \frac{\sqrt{5}}{2}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

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$$\left[\frac{Y(t)}{t}\right] = \left(-\frac{\sqrt{5}}{5}\right)\frac{1}{\left[z - \left(\frac{1-\sqrt{5}}{2}\right)\right]} + \frac{\sqrt{5}}{5}\,\frac{1}{\left[z - \left(\frac{1+\sqrt{5}}{2}\right)\right]}$$

$$\left[\frac{Y(z)}{z}\right] = \left(-\frac{\sqrt{5}}{5}\right)\frac{1}{\left[z-\left(\frac{1-\sqrt{5}}{2}\right)\right]} + \frac{\sqrt{5}}{5}\frac{1}{\left[z-\left(\frac{1+\sqrt{5}}{2}\right)\right]} \cdot z$$

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$$Y(z) = \left(-\frac{\sqrt{5}}{5}\right)\frac{z}{\left[z-\left(\frac{1-\sqrt{5}}{2}\right)\right]} + \left(\frac{\sqrt{5}}{5}\right)\frac{z}{\left[z-\left(\frac{1+\sqrt{5}}{2}\right)\right]}$$

$$\frac{z}{z-a}$$

$$\left(\frac{1-\sqrt{5}}{2}\right)^{k} \cdot 1(k) \qquad \left(\frac{1+\sqrt{5}}{2}\right)^{k} \cdot 1(k)$$

$$y(k) = \left(-\frac{\sqrt{5}}{5}\right)\left(\frac{1-\sqrt{5}}{2}\right)^{k} \cdot 1(k) + \left(\frac{\sqrt{5}}{5}\right)\left(\frac{1+\sqrt{5}}{2}\right)^{k} \cdot 1(k)$$

(•) Risolvere l'equazione alle differenze

$$y(k+2) = -\sqrt{5}\, y(k+1) - y(k)$$

con condizioni iniziali $y(0) = 0$

$y(1) = +1$

$$\{y(k)\}_{k \geq 0} \qquad k \in \mathbb{Z}$$

$$\mathcal{Z}\{y(k)\} = Y(z)$$

$$\mathcal{Z}\{y(k+1)\} = z\left[Y(z) - \overset{0}{\cancel{y(0)}}\right] = zY(z)$$

$$\mathcal{Z}\{y(k+2)\} = z^2\left[Y(z) - \overset{0}{\cancel{y(0)}} - \underset{+1}{y(1)}\, z^{-1}\right]$$

$$= z^2 Y(z) - z$$

$$\mathcal{Z}\{y(k+2)\} = -\sqrt{5}\,\mathcal{Z}\{y(k+1)\} - \mathcal{Z}\{y(k)\}$$

$$z^2 Y(z) - z = -\sqrt{5} z Y(z) - Y(z)$$

$$(z^2 + \sqrt{5} z + 1) Y(z) = z$$

$$Y(z) = \frac{z}{z^2 + \sqrt{5} z + 1} =$$

$$= \frac{z}{\left(z - \frac{1-\sqrt{5}}{2}\right)\left(z + \frac{1+\sqrt{5}}{2}\right)}$$

$$Y(z) = \frac{z}{\underline{\left(z - \frac{1-\sqrt{5}}{2}\right)}\underline{\left(z + \frac{1+\sqrt{5}}{2}\right)}} \Big/ \cdot \frac{1}{z} \quad ①$$

$$\left[\frac{Y(z)}{z}\right] = \frac{1}{\cancel{z}} \frac{\cancel{z}}{\left(z - \frac{1-\sqrt{5}}{2}\right)\left(z + \frac{1+\sqrt{5}}{2}\right)} \quad ②$$

③ $$\left[\frac{Y(z)}{z}\right] = \frac{C_1}{z - \frac{1-\sqrt{5}}{2}} + \frac{C_2}{z + \frac{1+\sqrt{5}}{2}}$$

④ $C_1 = ?$ $\qquad C_2 = ?$

$$C_1 = \lim_{z \to \frac{1-\sqrt{5}}{2}} \left[\frac{Y(z)}{z}\right]\left(z - \frac{1-\sqrt{5}}{2}\right) =$$

$$C_1 = \lim_{z \to \frac{1-\sqrt{5}}{2}} \frac{1}{\cancel{\left(z - \frac{1-\sqrt{5}}{2}\right)}\left(z + \frac{1+\sqrt{5}}{2}\right)} \cancel{\left(z - \frac{1-\sqrt{5}}{2}\right)}$$

$$= \lim_{z \to \frac{1-\sqrt{5}}{2}} \frac{1}{z + \frac{1+\sqrt{5}}{2}} = \frac{1}{\frac{1-\sqrt{5}}{2} + \frac{1+\sqrt{5}}{2}}$$

$$= +1$$

$$C_2 = \lim_{z \to -\left(\frac{1+\sqrt{5}}{2}\right)} \left[\frac{Y(z)}{z}\right]\left(z + \frac{1+\sqrt{5}}{2}\right) =$$

$$= \lim_{z \to (\dots)} \frac{1}{\left(z - \frac{1-\sqrt{5}}{2}\right)\cancel{\left(z + \frac{1+\sqrt{5}}{2}\right)}} \cancel{\left(z + \frac{1+\sqrt{5}}{2}\right)}$$

$$= \lim_{z \to (\dots)} \frac{1}{z - \frac{1-\sqrt{5}}{2}} = \frac{1}{-\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2}}$$

$$C_2 = \frac{1}{\frac{-1-\cancel{\sqrt{5}}-1+\cancel{\sqrt{5}}}{2}} = -1$$

$$\left[\frac{Y(z)}{z}\right] = \frac{1}{z-\frac{1-\sqrt{5}}{2}} + \frac{(-1)}{z+\frac{1+\sqrt{5}}{2}} \Bigg/ \cdot z$$

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$$Y(z) = \frac{z}{z-\frac{1-\sqrt{5}}{2}} - \frac{z}{z+\frac{1+\sqrt{5}}{2}}$$

$$\frac{z}{z-p} \longleftrightarrow p^k \cdot 1(k)$$

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$$y(k) = \left(\frac{1-\sqrt{5}}{2}\right)^k \cdot 1(k) - \left[-\frac{1+\sqrt{5}}{2}\right]^k \cdot 1(k)$$

$|\cdot| < 1$ $\qquad$ $|\cdot| > 1$

$$\left[\frac{Y(z)}{z}\right] = \frac{C_1}{z - \frac{1-\sqrt{5}}{2}} + \frac{C_2}{z + \frac{1+\sqrt{5}}{2}}$$

$$= \frac{1}{\left(z - \frac{1-\sqrt{5}}{2}\right)\left(z + \frac{1+\sqrt{5}}{2}\right)}$$

Utilizzo il principio di identità dei polinomi per determinare C_1 e C_2

$$C_1\left(z + \frac{1+\sqrt{5}}{2}\right) + C_2\left(z - \frac{1-\sqrt{5}}{2}\right) = 1$$

$$(C_1 + C_2)\, z + \frac{1}{2}(C_1 - C_2) +$$

$$+ \frac{\sqrt{5}}{2}(C_1 + C_2) = +1$$

$$\begin{cases} C_1 + C_2 = 0 \\ \frac{1}{2}(C_1 - C_2) + \frac{\sqrt{5}}{2}\cancel{(C_1 + C_2)} = +1 \end{cases}$$

0

$$\begin{cases} C_1 = -C_2 \\ C_1 - C_2 = 2 \end{cases} \qquad \begin{cases} C_1 = -C_2 \\ -2C_2 = 2 \end{cases} \qquad \begin{matrix} C_1 = +1 \\ C_2 = -1 \end{matrix}$$

⊙ $$Y(z) = \frac{2z-1}{z^4 - \frac{13}{5}z^3 + \frac{11}{5}z^2 - \frac{3}{5}z}$$

$y(k) = ?$

$m = 1$

$n = 4$

$n - m = 3 \quad y(0) = y(1) = y(2) = 0$

$y(3) = 2$

(1) $N(z) = 2z - 1 = 2\left(z - \frac{1}{2}\right)$

$$D(z) = z(z-1)^2\left(z - \frac{3}{5}\right)$$

$$Y(z) = \frac{2\left(z - \frac{1}{2}\right)}{z(z-1)^2\left(z - \frac{3}{5}\right)} \Bigg/ \cdot \frac{1}{z}$$

2b

$$\left[\frac{Y(z)}{z}\right] = \frac{2z-1}{z^2(z-1)^2(z-3/5)}$$

(3) $$\left[\frac{Y(z)}{z}\right] = \frac{C_{1,1}}{z} + \frac{C_{1,2}}{z^2} + \frac{C_{2,1}}{z-1} + \frac{C_{2,2}}{(z-1)^2} + \frac{C_3}{z - 3/5}$$

$$C_3 = \lim_{z \to \frac{3}{5}} \left[\frac{Y(z)}{z}\right]\left(z - \frac{3}{5}\right) = \lim_{z \to \frac{3}{5}} \frac{(2z-1)\cancel{(z-3/5)}}{z^2(z-1)^2\cancel{\left(z-\frac{3}{5}\right)}} = 125/36$$

$$C_{1,2} = \lim_{z \to 0} \left[\frac{Y(z)}{z}\right] \cdot z^2 =$$

$$= \lim_{z \to 0} \frac{(2z-1)\cdot \cancel{z^2}}{\cancel{z^2}(z-1)^2\left(z-\frac{3}{5}\right)} = +5/3$$

$$C_{1,1} = \lim_{z \to 0} \frac{d}{dz}\left(\left[\frac{Y(z)}{z}\right]\cdot z^2\right) =$$

$$= \lim_{z \to 0} \frac{d}{dz}\left[\frac{(2z-1)}{(z-1)^2\left(z-3/5\right)}\right] =$$

$$= \lim_{z \to 0} \frac{2\left(z-3/5\right)(z-1)^2 - (2z-1)\left[(z-1)^2 + 2(z-1)\left(z-3/5\right)\right]}{(z-1)^4\left(z-3/5\right)^2}$$

$$= +\frac{25}{9}$$

$$C_{2,2} = \lim_{z \to +1} \left[\frac{Y(z)}{z}\right](z-1)^2 =$$

$$= \lim_{z \to +1} \frac{(2z-1)\cdot \cancel{(z-1)^2}}{z^2 \cancel{(z-1)^2}\left(z-\frac{3}{5}\right)} =$$

$$= \lim_{z \to +1} \frac{2z-1}{z^2\left(z-3/5\right)} = 5/2$$

$$C_{2,1} = \lim_{z \to +1} \frac{d}{dz}\left\{\left[\frac{Y(z)}{z}\right](z-1)^2\right\} =$$

$$= \lim_{z \to +1} \frac{d}{dz}\left[\frac{(2z-1)}{z^2\left(z-3/5\right)}\right] =$$

$$= \lim_{z \to +1} \frac{2z^2\left(z-3/5\right)-(2z-1)\left[2z\left(z-3/5\right)+z^2\right]}{z^4\left(z-3/5\right)^2}$$

$$C_{2,1} = -\frac{25}{4}$$

$$\left[\frac{Y(z)}{z}\right] = \frac{5/3}{z^2} + \frac{25/9}{z} + \frac{5/2}{(z-1)^2} + \frac{\left(-\frac{25}{4}\right)}{(z-1)} + \frac{175/36}{z-\frac{3}{5}} \quad / \cdot z$$

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$$Y(z) = 1 \cdot \frac{25}{9} + \frac{5}{3} \cdot \frac{1}{z} + \boxed{\frac{5}{2} \frac{z}{(z-1)^2}} - \frac{25}{4} \frac{z}{z-1} + \frac{175}{36} \frac{z}{z-3/5}$$

$1 \leftrightarrow \delta(k)$

$\frac{1}{z} \leftrightarrow \delta(k-1)$? $z^{-1} \cdot 1$

$\frac{z}{(z-1)^2} \leftrightarrow k \cdot 1(k)$

$\frac{z}{z-1} \leftrightarrow 1(k)$

$\frac{z}{z-3/5} \leftrightarrow \left(\frac{3}{5}\right)^k \cdot 1(k)$

$$y(k) = \frac{5}{3}\cdot\delta(k-1) + \frac{25}{9}\delta(k) +$$

$$+\frac{5}{2}k\cdot 1(k) - \frac{25}{4}\cdot 1(k) +$$

$$+\frac{175}{36}\left(\frac{3}{5}\right)^k\cdot 1(k) \qquad k\in\mathbb{Z}$$

$$y(0) = 0 + \frac{25}{9} + 0 - \frac{25}{4} + \frac{175}{36} = 0$$

$$y(1) = \frac{5}{3} + 0 + \frac{5}{2}\cdot 1 - \frac{25}{4} + \frac{175}{36}\cdot\frac{3}{5} = 0$$

$$y(2) = 0$$

$$y(3) = 2$$

$$Y(s) = \frac{(s+1)}{s(s^2+2s-3)}$$

(1) $y(0) = ?$ $\dot{y}(0) = ?$

$\lim_{t \to +\infty} y(t) = ?$

(2) $y(t) =$

$y(0) = ?$

Teorema val. iniziale ?

$Y(s)$ strett. propria

$m = 1$

$n = 3$

$$y(0) = \lim_{t\to 0} y(t) = \lim_{s\to\infty} sY(s)$$

$$= \lim_{s\to\infty} \frac{\overset{1}{\cancel{s}}\,(s+1)}{1\,\cancel{s}\,(s^2+2s-3)} = 0$$

$$\mathcal{L}\{\dot{y}(t)\} = sY(s) - y(0) = sY(s)$$

$m = 2$

$n = 3$

$$\dot{y}(0) = \lim_{s\to\infty} s\left[\mathcal{L}(\dot{y})\right] = \lim_{s\to\infty} \frac{\cancel{s^2}^{\,1}(s+1)}{\cancel{s}(s^2+2s-3)}$$

$$= 1$$

$$\ddot{y}(0) = ?$$

$$\mathcal{L}\{\ddot{y}\} = s^2Y(s) - \cancel{sy(0)}^{\,0} - \dot{y}(0)$$

$+1$

$$= s^2Y(s) - 1$$

$$\mathcal{L}\{\ddot{y}\} = \frac{\cancel{s^2}^{\,1}(s+1)}{\cancel{s}(s^2+2s-3)} - 1 = \frac{s^2+s-(s^2+2s-3)}{s^2+2s-3}$$

$$\mathcal{L}\{\ddot{y}\} = \frac{\cancel{s^2}+s-\cancel{s^2}-2s+3}{s^2+2s-3} = \frac{-s+3}{s^2+2s-3} \qquad \begin{matrix} m=1 \\ n=2 \end{matrix}$$

$$\ddot{y}(0) = \lim_{s\to\infty} -\frac{s(s-3)}{s^2+2s-3} = -1$$

$$\lim_{t\to\infty} y(t) = ? \qquad Y(s) = \frac{(s+1)}{s(s-1)(s+3)}$$

Sviluppo in fratti semplici

$$Y(s) = \frac{C_0}{s} + \frac{C_1}{s-1} + \frac{C_2}{s+3} \to C_2 e^{-3t} \cdot 1(t)$$

$$C_0 \cdot 1(t) \qquad C_1 e^{+t} \cdot 1(t)$$

$$\frac{1}{s-a} \Rightarrow e^{at} \cdot 1(t)$$

$$C_0 = \lim_{s \to 0} Y(s) \cdot s = \lim_{s \to 0} \cancel{s} \cdot \frac{(s+1)}{\cancel{s}(s-1)(s+3)} =$$

$$= \lim_{s \to 0} \frac{s+1}{(s-1)(s+3)} = \frac{1}{(-1)(+3)} = -\frac{1}{3}$$

$$C_1 = \lim_{s \to +1} (s-1) Y(s) = \lim_{s \to 1} \frac{\cancel{(s-1)}(s+1)}{s\cancel{(s-1)}(s+3)} =$$

$$= \lim_{s \to +1} \frac{s+1}{s(s+3)} = \frac{2}{1 \cdot 4} = +\frac{1}{2}$$

$$C_2 = \lim_{s \to -3} (s+3) Y(s) = \lim_{s \to -3} \frac{\cancel{(s+3)}(s+1)}{s(s-1)\cancel{(s+3)}}$$

$$= \lim_{s \to -3} \frac{s+1}{s(s-1)} = \frac{(-2)}{(-3)(-4)} = -\frac{1}{6}$$

$$Y(s) = \left(-\frac{1}{3}\right) \cdot \frac{1}{s} + \frac{1}{2} \cdot \frac{1}{s-1} - \frac{1}{6} \frac{1}{s+3}$$

$$1(t) \qquad e^{+t} \cdot 1(t) \qquad e^{-3t} 1(t)$$

$$y(t) = -\frac{1}{3} \cdot 1(t) + \frac{1}{2} e^{t} \cdot 1(t) - \frac{1}{6} e^{-3t} \cdot 1(t)$$

(·) $X(s) = \dfrac{1}{s^2+2s+10}$

$s^2+2s+10=0$

$p_{1,2} = -1 \pm \sqrt{1-10}$

$= -1 \pm j3$

$p_1 \triangleq -1-j3$

$p_2 \triangleq -1+j3$

(1) sviluppo in fratti semplici

$X(s) = \dfrac{C_1}{s-p_1} + \dfrac{C_2}{s-p_2} \leftarrow p_1 = p_2^*$

ha coeff. reali

$\downarrow$

$C_1 = C_2^*$

$\downarrow$

(a) formula dei residui

(b) principio identità polinomi

(c) completamento quadrato

$$s^2+2s+10=(s-p_1)(s-p_2)=$$

$$p_1=-1-j3$$

$$p_2=-1+j3$$

$$=(s+1+j3)(s+1-j3)$$

$$=\left[(s+1)+j3\right]\left[(s+1)-j3\right]=$$

$$a+b \qquad a-b$$

$$=(s+1)^2-(j3)^2=(s+1)^2-(-1)\cdot 3^2$$

$$=(s+1)^2+3^2$$

$$(s-\sigma)^2+\omega^2$$

$$p_{1,2}=\sigma\pm j\omega$$

$$-1\pm j3$$

$$X(s)=\frac{1}{(s+1)^2+3^2}$$

$$\mathcal{L}\left[\frac{\omega}{(s-\sigma)^2+\omega^2}\right] \qquad \frac{s-\sigma}{(s-\sigma)^2+\omega^2}$$

$$X(s)=\frac{3}{\underbrace{(s+1)}_{s-\sigma}{}^2+\underbrace{3^2}_{\omega^2}}\cdot\frac{1}{3}$$

$$x(t)=\frac{1}{3}\cdot e^{-t}\,\mathrm{sen}(3t)\,1(t)$$

$$X(s)=\frac{1}{s^2+2s+10}$$

$$p_1=-1-j3$$

$$p_2=-1+j3$$

$$X(s)=\frac{C_1}{s-p_1}+\frac{C_2}{s-p_2} \qquad C_2=C_1^*$$

$$C_1 = \lim_{s \to p_1} X(s)(s-p_1) =$$

$$= \lim_{s \to p_1} \frac{1}{\cancel{(s-p_1)}(s-p_2)} \cancel{(s-p_1)} =$$

$$= \lim_{s \to -1-j3} \frac{1}{s+1-j3} =$$

$$= \frac{1}{\cancel{-1}-j3 \cancel{+1}-j3} = \frac{1}{-2j3} \cdot \frac{j}{j}$$

$$= \frac{j}{(-2)j^2\, 3} = +\frac{j}{2 \cdot 3} = +\frac{j}{6}$$

$$+ \quad -1$$

$$C_2 = C_1^* = -\frac{j}{6}$$

$$X(s) = \frac{j}{6}\,\frac{1}{s+1+j3} - \frac{j}{6}\,\frac{1}{s+1-j3}$$

$$\frac{1}{s-a} \xrightarrow{\mathcal{L}^{-1}} e^{at}\cdot 1(t)$$

$$x(t) = \frac{j}{6}\, e^{-(1+j3)t}\cdot 1(t) - \frac{j}{6}\, e^{-(1-j3)t}\cdot 1(t)$$

$$x(t) = \frac{j}{6}\left[e^{-t}e^{-j3t} - e^{-t}e^{+j3t}\right]\cdot 1(t) =$$

$$= \frac{j}{6}\left[e^{-j3t} - e^{+j3t}\right]\cdot e^{-t}\cdot 1(t)$$

$$\operatorname{sen}(\omega t) = \frac{e^{+j\omega t} - e^{-j\omega t}}{2j} \qquad \cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$x(t) = \frac{j}{\cancel{6}_{\,3}}\,(-1)\,\frac{\cancel{-j}^{\,1}}{2j}\underbrace{\left[e^{j3t}-e^{-j3t}\right]}_{\text{sen}(3t)} e^{-t}\cdot 1(t) \qquad (+1)$$

$$x(t) = +\frac{1}{3}\left[\text{sen}(3t)\right]\cdot e^{-t}\cdot 1(t)$$

$$= +\frac{1}{3}e^{-t}\,\text{sen}(3t)\; 1(t)$$

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principio di identità dei polinomi

$$X(s) = \frac{1}{s^2+2s+10} = \frac{c_1}{s-p_1} + \frac{c_1^*}{s-p_2}$$

$$p_1 = -1-j3$$

$$p_2 = -1+j3$$

$$c_1 = A+jB$$

$$c_1^* = A-jB$$

$$\frac{A+jB}{s+1+j3} + \frac{A-jB}{s+1-j3} = \frac{1}{s^2+2s+10}$$

$$(A+jB)(s+1-j3)+(A-jB)(s+1+j3)=1$$

$$As+A-\cancel{3Aj}+\cancel{sBj}+\cancel{Bj}-j^2 3B+$$

$(-1)(-1)$

$$+As+A+\cancel{3Aj}-\cancel{sBj}-\cancel{Bj}-j^2 3B=1$$

$(-1)(-1)$

$$2As+2A+6B=1$$

$$\begin{cases} 2A=0 \\ 2A+6B=1 \end{cases} \qquad \begin{cases} A=0 \\ B=\frac{1}{6} \end{cases}$$

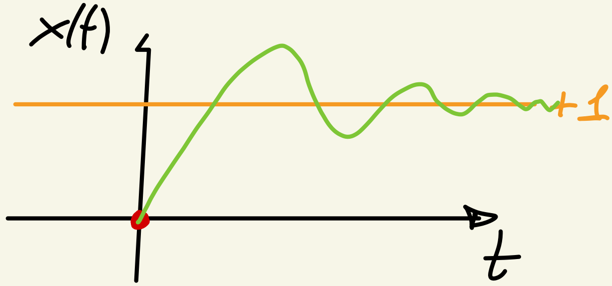
$$C_1 = A+jB \longrightarrow C = j\frac{1}{6}$$

$$C_2^* = -j\frac{1}{6}$$

(•) $$X(s) = \frac{10}{s(s^2+2s+10)}$$

$$x(t) \xrightarrow[t \to +\infty]{} 1$$

$$x(0) = 0$$



$$X(s) = \frac{C_0}{s} + \frac{C_1}{s+1+j3} + \frac{C_2}{s+1-j3}$$

$$= \frac{C_0}{s} + \frac{As+B}{s^2+2s+10}$$

$$C_0 = \lim_{s \to 0} X(s) \cdot s = \lim_{s \to 0} \frac{10}{\cancel{s}(s^2+2s+10)} \cdot \cancel{s}$$

$$= 1$$

$$X(s) = \boxed{\frac{1}{s} + \frac{As+B}{s^2+2s+10}} = \frac{10}{s(s^2+2s+10)}$$

$$(s^2+2s+10)\cdot 1 + s(As+B) = 10$$

$$s^2 + 2s + 10 +$$
$$As^2 + Bs \qquad = 10$$

$$(A+1)s^2 + (2+B)s + 10 = 10$$

$$\begin{cases} A+1=0 \\ 2+B=0 \end{cases} \qquad \begin{cases} A=-1 \\ B=-2 \end{cases}$$

$$X(s) = \frac{1}{s} - \frac{s+2}{s^2+2s+10}$$

$1(t)$

$$\frac{s+2}{s^2+2s+10} = \frac{s+2}{(s+1)^2+3^2}$$

$$(s-\sigma)^2+\omega^2$$

$$(s+2) = K_1(s+1) + K_2 3$$

$$\frac{(s-\sigma)}{(s-\sigma)^2+\omega^2} \qquad \frac{\omega}{(s-\sigma)^2+\omega^2}$$

$$e^{\sigma t}\cos\omega t \cdot 1(t) \qquad e^{\sigma t}\operatorname{sen}\omega t \cdot 1(t)$$

$$s+2 = K_1(s+1) + K_2 \cdot 3$$

$$= K_1 s + (K_1 + 3K_2)$$

$$\begin{cases} K_1 = 1 \\ K_1 + 3K_2 = 2 \end{cases} \qquad \begin{cases} K_1 = 1 \\ K_2 = \frac{1}{3} \end{cases}$$

$$X(s) = \frac{1}{s} - \frac{s+2}{s^2+2s+10} = \frac{1}{s} - \frac{k_1(s+1)+k_2 \cdot 3}{(s+1)^2+3^2}$$

$$= \frac{1}{s} - \left[1 \cdot \frac{s+1}{(s+1)^2+3^2} + \frac{1}{3} \cdot \frac{3}{(s+1)^2+3^2} \right]$$

$$= \frac{1}{s} - \frac{s+1}{(s+1)^2+3^2} - \frac{1}{3} \frac{3}{(s+1)^2+3^2}$$

$1(t)$

$$\frac{(s-\sigma)}{(s-\sigma)^2+\omega^2} \qquad \frac{\omega}{(s-\sigma)^2+\omega^2}$$

$$x(t) = 1(t) - e^{-t}\cos(3t) \cdot 1(t) - \frac{1}{3} e^{-t} \operatorname{sen}(3t) 1(t)$$

$$= \left\{ 1 - e^{-t} \left[\cos(3t) + \frac{1}{3} \operatorname{sen}(3t) \right] \right\} \cdot 1(t)$$

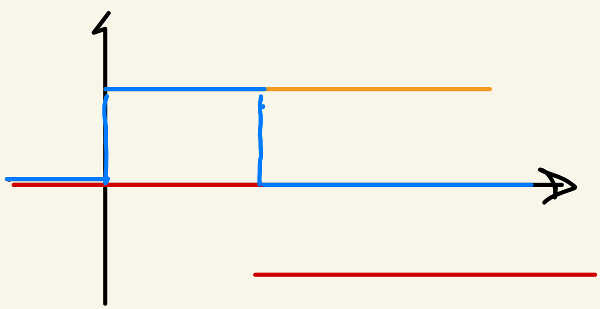
$\tan \alpha$

•

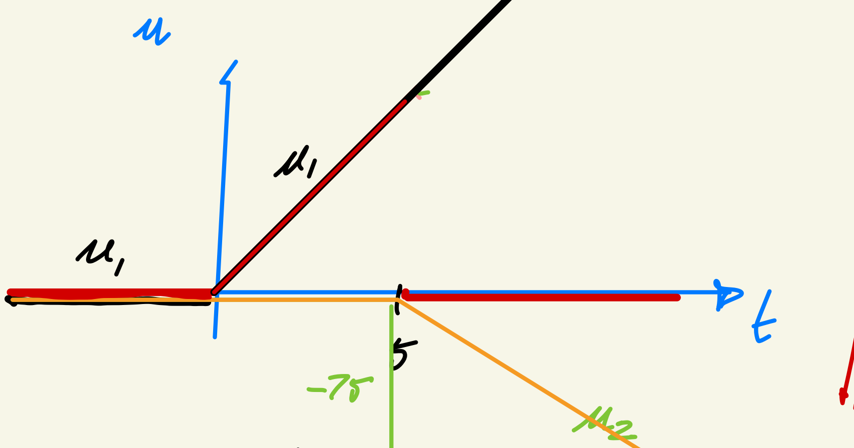
$$U(s) = 10\,\frac{1-e^{-5s}}{s} \qquad u(t) = ?$$

$$U(s) = \frac{10}{s} - \frac{10}{s}e^{-5s} = 10 \cdot 1(t) - 10 \cdot 1(t-5)$$

$$10 \cdot \frac{1}{s} \qquad \frac{10}{s}\, e^{-5s}$$



$$u(t) = \begin{cases} 0 & t < 0 \\ 5t & 0 \le t < 5 \\ 0 & t \geq 5 \end{cases}$$



$$u_1(t) = 5t \cdot 1(t)$$

$$u_2(t) = -75 \cdot 1(t-5)$$

$$u_3(t) = -5(t-5) \cdot 1(t-5)$$

$$\int_0^t 5\tau \cdot 1(\tau-5) e^{5\tau} d\tau$$

$$\cancel{\mathcal{L}\{5t \cdot 1(t-5)\} = e^{-5s} Y(s)}$$

$$U_1(s) + U_2(s) + U_3(s) =$$

$$\frac{5}{s^2} - \frac{75}{s} e^{-5s} - \frac{5}{s^2} e^{-5s} = U(s)$$

- $\dot{y} = -2y + u$ $\quad y(t)$ causale

$Y(s)$

$y(t) = ?$

$y(0) = 0$

$U(s) = \frac{5}{s^2}\left(1 - e^{-5s}\right) + -\frac{25}{s}e^{-5s}$

$$\mathcal{L}\{\dot{y}\} = \mathcal{L}\{-2y + u\}$$

$$sY(s) - \cancel{y(0)}^{\,0} = -2Y(s) + U(s)$$

$$(s+2)Y(s) = U(s)$$

$$Y = \frac{1}{s+2} \cdot U(s) = \frac{1}{s+2}\left[\frac{5}{s^2}\left(1 - e^{-5s}\right) - \frac{25}{s}e^{-5s}\right]$$

$$Y = \boxed{\frac{5}{s^2(s+2)}} - \frac{5}{s^2(s+2)}e^{-5s} - \boxed{\frac{25}{s(s+2)}e^{-5s}}$$

$$\frac{5}{s^2(s+2)} = \frac{C_{0,1}}{s} + \frac{C_{0,2}}{s^2} + \frac{C_1}{s+2}$$

$$C_1 = \lim_{s \to -2} \left[\frac{5}{s^2 \cancel{(s+2)}} \right] \cancel{(s+2)} = \lim_{s \to -2} \frac{5}{s^2} = \frac{5}{4}$$

$$C_{0,2} = \lim_{s \to 0} \left(\frac{5}{\cancel{s^2}(s+2)} \right) \cancel{s^2} = \lim_{s \to 0} \frac{5}{s+2} = 5/2$$

$$C_{0,1} = \lim_{s \to 0} \frac{d}{ds} \left[\frac{5}{\cancel{s^2}(s+2)} \cdot \cancel{s^2} \right] = \lim_{s \to 0} \left[\frac{d}{ds} \left(\frac{5}{s+2} \right) \right] =$$

$$= \lim_{s \to 0} \left[-\frac{5}{(s+2)^2} \right] = -\frac{5}{4}$$

$t \cdot 1(t)$

$$\frac{5}{s^2(s+2)} = -\frac{5}{4} \cdot \frac{1}{s} + \frac{5}{2} \frac{1}{s^2} + \frac{5}{4} \cdot \frac{1}{s+2}$$

$1(t)$ $\qquad$ $e^{-2t} \cdot 1(t)$

$$\mathcal{L}^{-1}\left[\frac{5}{s^2(s+2)}\right] = \left(-\frac{5}{4} + \frac{5}{2}t + \frac{5}{4}e^{-2t}\right)\cdot 1(t)$$

$$\mathcal{L}^{-1}\left[-\frac{5}{s^2(s+2)}e^{-5s}\right] = -\boxed{\mathcal{L}^{-1}\left[\frac{5}{s^2(s+2)}e^{-5s}\right]}$$

$$= -\left[-\frac{5}{4} + \frac{5}{2}(t-5) + \frac{5}{4}e^{-2(t-5)}\right]\cdot 1(t-5)$$

$$\mathcal{L}^{-1}\left[-\frac{25}{s(s+2)}e^{-5s}\right] \Rightarrow \mathcal{L}^{-1}\left[-\frac{25}{s(s+2)}\right]$$

$$-\frac{25}{s(s+2)} = \frac{B_0}{s} + \frac{B_1}{s+2}$$

$$B_0 = \lim_{s \to 0} \left[-\frac{75}{\cancel{s}(s+2)} \right] \cdot \cancel{s} = -\frac{75}{2}$$

$$B_1 = \lim_{s \to -2} \left[-\frac{75}{s\cancel{(s+2)}} \right] \cdot \cancel{(s+2)} = -\frac{75}{-2} = \frac{75}{2}$$

$$-\frac{75}{s(s+2)} = -\frac{75}{2} \cdot \frac{1}{s} + \frac{75}{2} \cdot \frac{1}{s+2}$$

$1(t)$ $\quad$ $e^{-2t} \cdot 1(t)$

$$\mathcal{L}^{-1}\left[-\frac{75}{s(s+2)} \right] = -\frac{75}{2} \cdot 1(t) + \frac{75}{2} e^{-2t} \cdot 1(t)$$

$$\mathcal{L}^{-1}\left[-\frac{75}{s(s+2)} e^{-5s} \right] = -\frac{75}{2} \cdot 1(t-5) + \frac{75}{2} e^{-2(t-5)} \cdot 1(t-5)$$