



Three Uses of Variable

Theoretical Tool for analysing the use and the understanding the
concept of algebraic variable

APPROACH FOR TEACHING MATHS
IN SECONDARY INSTRUCTION

REFERENCES

Warren, E., Trigueros, M., Ursini, S. (2016). *Research on the Learning and Teaching of Algebra*. In: Gutiérrez, Á., Leder, G.C., Boero, P. (eds) *The Second Handbook of Research on the Psychology of Mathematics Education*. SensePublishers, Rotterdam.

Sonia Ursini, *Il Modello 3UV: uno strumento teorico a disposizione degli insegnanti di matematica*, in: *QuaderniCIRD*, 2 (2011), pp. 59-70.

Motivational purpose

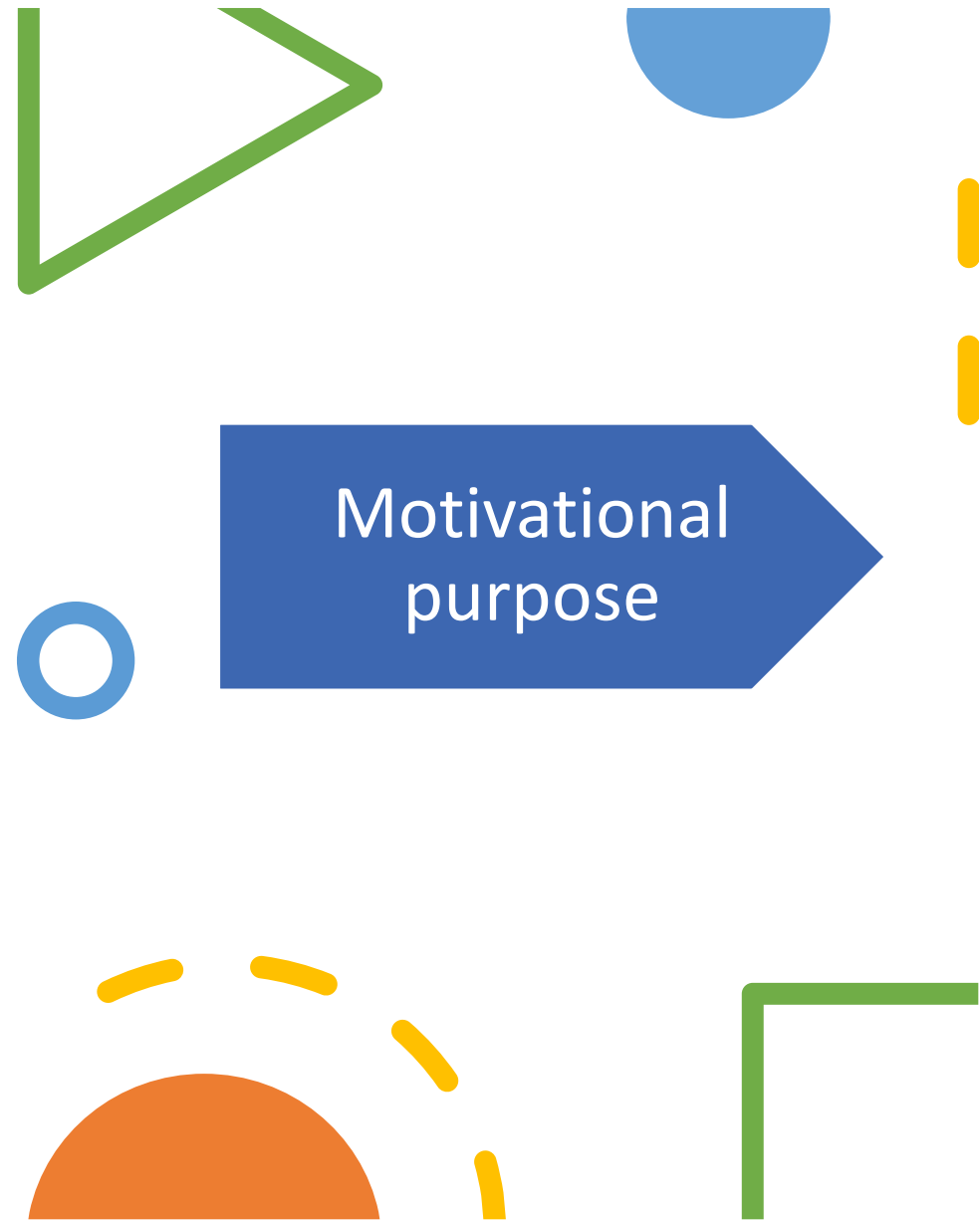
Theoretical Framework

Educational Application



Most students (junior high and high school, university) find difficulties with algebra and with problems including

- “Unknown meaning” of variable
- Generalisation and use of general expressions
- functional relations




Why it is so difficult for students to learn Algebra?



THE MAIN CHARACTERISTIC OF ALGEBRA IS THE USE OF VARIABLES, I.E. LETTERS INSTEAD OF NUMBERS.



FOR STUDENTS, WORKING WITH LETTERS AND UNDERSTANDING THEIR MEANING POSES CONSIDERABLE DIFFICULTIES.



Students fail to appropriate the essence of this concept, finding it difficult to switch flexibly between the distinct uses of the variable

(Matz, 1982; Usiskin, 1988; Trigueros and Ursini, 1999).

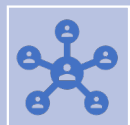
What are these distinct uses of the variable?



Unknown (variabile come INCOGNITA)



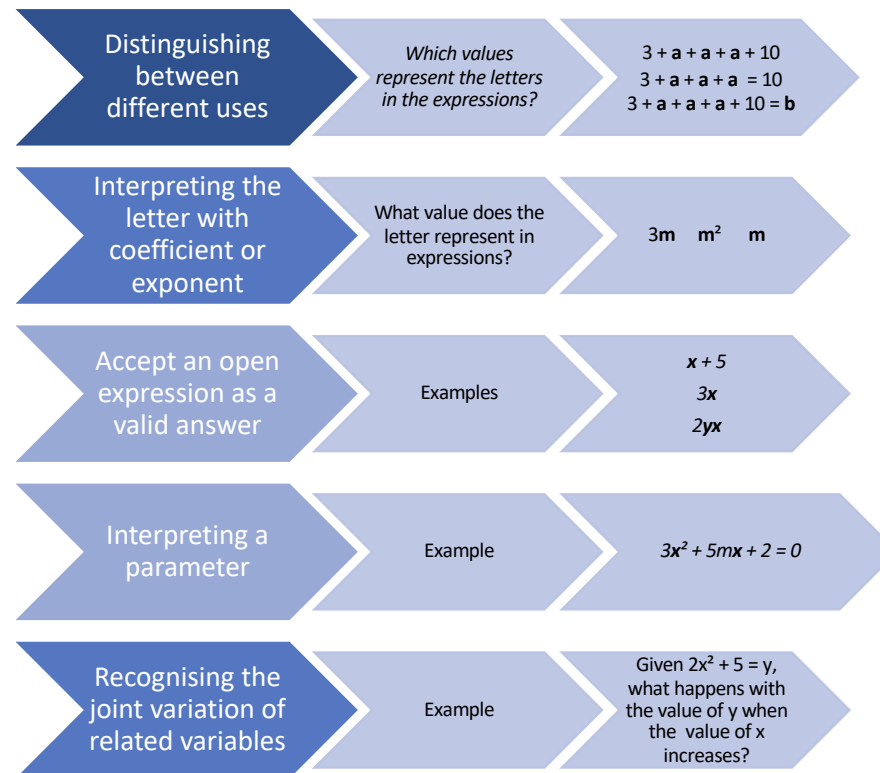
General numbers (variabile con NUMERI GENERICO)



relazioni funzionali (variabile in RELAZIONE FUNZIONALE)

Common errors and difficulties

For each use there are specific epistemological and didactic obstacles



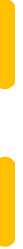
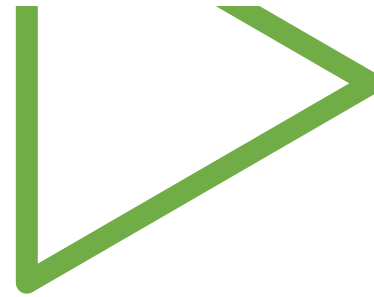


What a good understanding of the concept of VARIABLE implies?

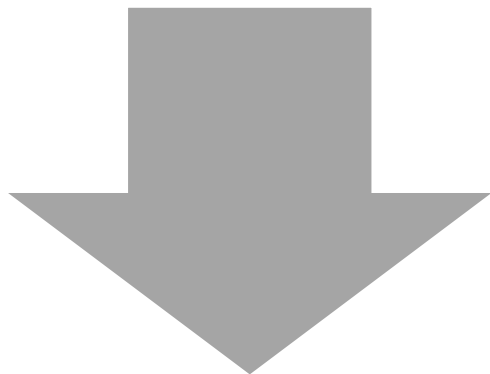
The ability of:

Calculate and operate	calculate and operate with letters
Understand	understand why it is necessary to do these operations
Realise	realise the importance of variables in modelling situations
Distinguish	distinguish between different uses
Switch	switch flexibly between one use and another
Integrate	integrate the distinct uses by considering them as distinct sides of the same mathematical object that become visible depending on the specific situation.

Theoretical framework



detailed analysis of what is involved in solving typical school problems in algebra (emphasis on the use of the variable)



from the results of the first stage, identify the aspects characterising the distinct uses of the variable



Examples of problematic situation in algebra analysed:

Unknown

Paolo ha 12 caramelle e vuole dividerle tra i suoi 3 amici, quante caramelle potrà dare a ciascun amico?

Generic number

Scrivi la proprietà commutativa della somma per due numeri naturali

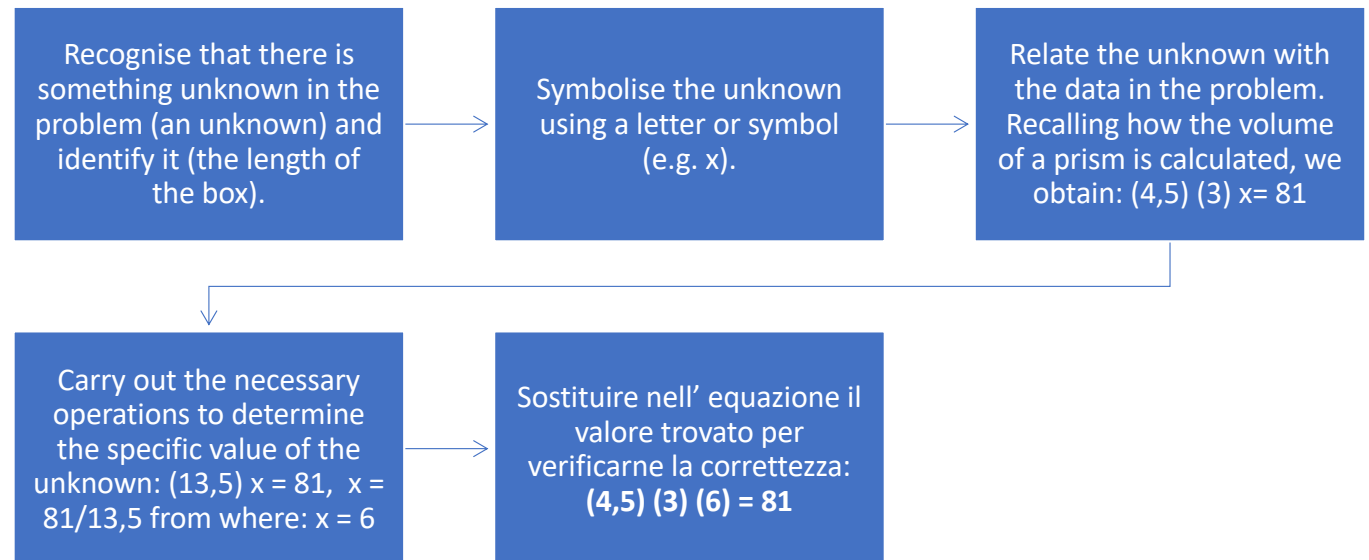
Functional relation

Se percorro 3km in 30 min, quanti km percorro in 60 min?

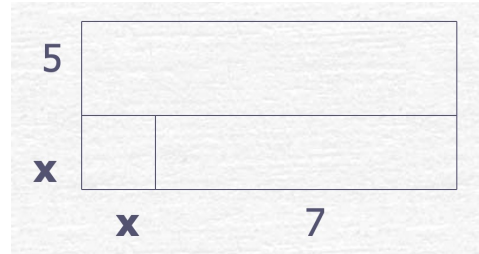
Una scatola a forma di prisma è larga 4,5cm, alta 3cm ed il suo volume è di 81cm³. Quanto è lunga?

To resolve this kind of problem means:

Example for
the variable
with
“Unknown
meaning”



Scrivi l'espressione che rappresenta l' area della seguente figura:



To resolve this kind of problem means:

Example for
the variable
with “Generic
number”

Interpret the letter x as
the representation of a
general number.

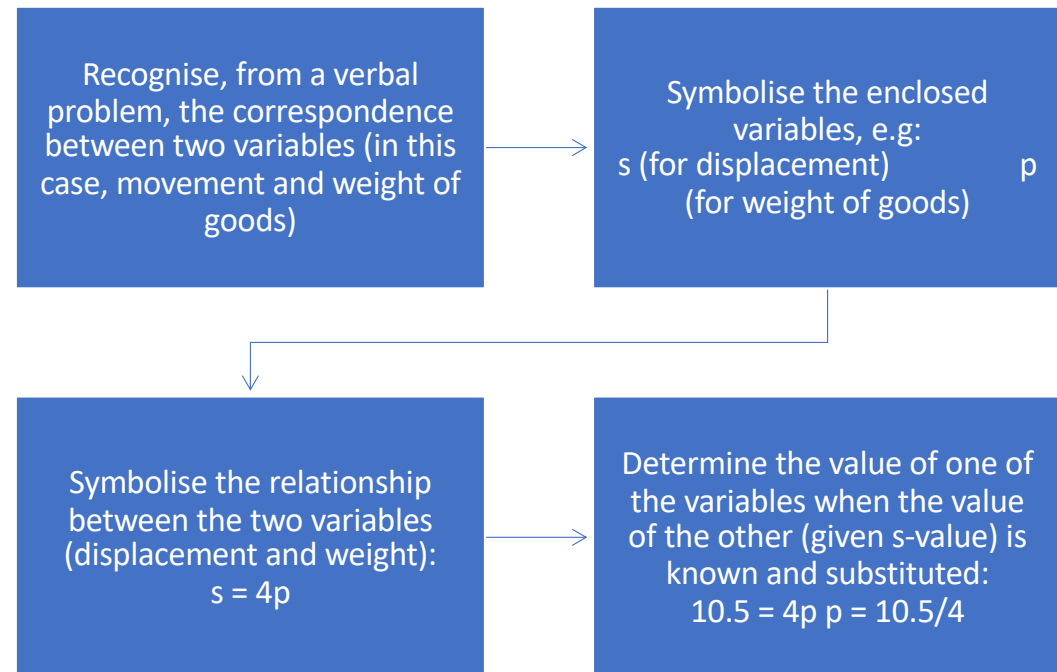
Use the letter to write
expressions, i.e. work
with the letter (base and
height of the rectangle):
 $x + 7$ and $x + 5$

Use the expressions
produced to obtain
another, i.e. operate with
expressions (area of the
figure): $(x + 7)(x + 5)$

Per ogni kilogrammo di peso il piatto d'una bilancia si sposta di 4 cm. Scrivi la relazione che c'è tra il peso della merce e lo spostamento del piatto della bilancia. Se il piatto della bilancia si sposta di 10,5cm, quanti kilogrammi pesa la merce?

To resolve this kind of problem means:

Example for
the variable
with
“Functional
relationship”



Characteristic aspects of the variable as an “unknown”

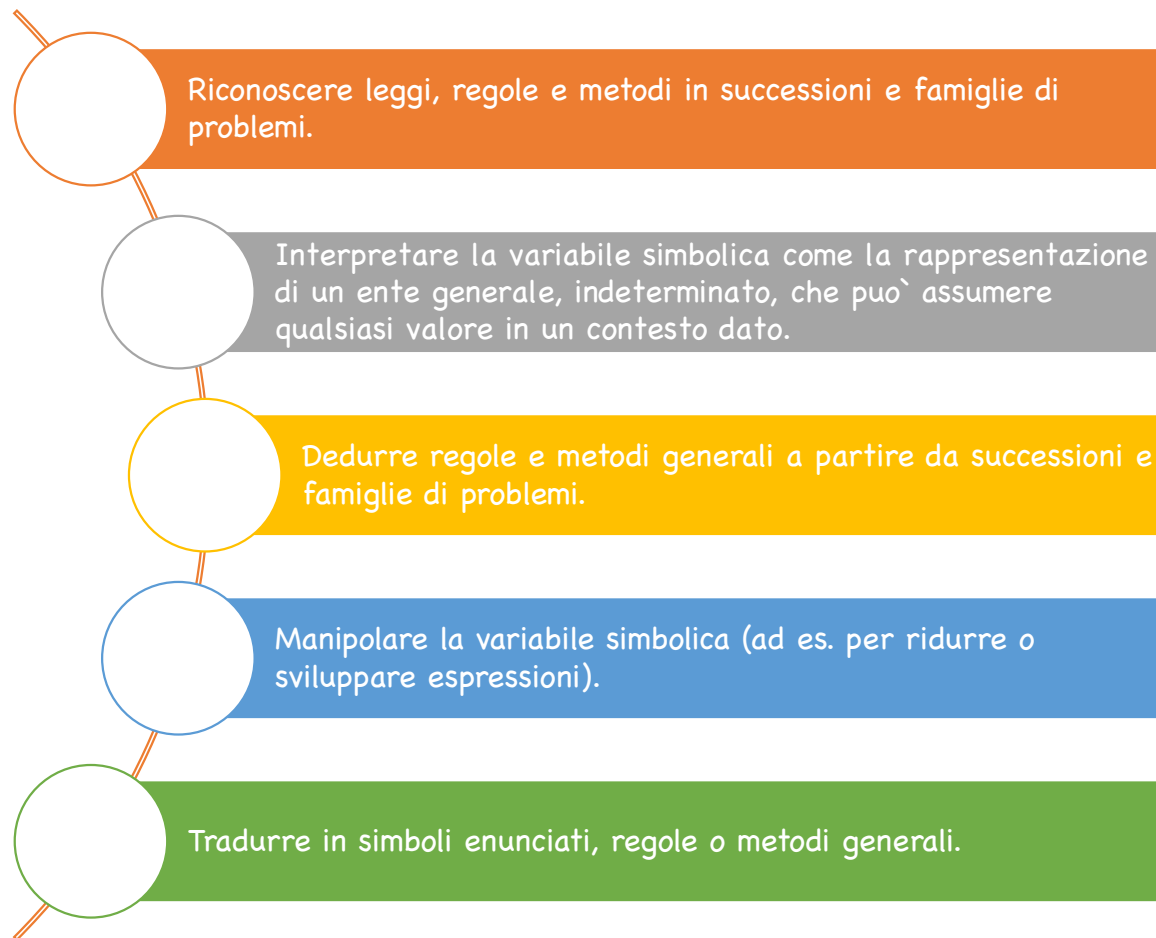
Riconoscere e identificare l'incognita in una situazione problematica

Interpretare la variabile simbolica che appare in un'equazione come rappresentazione di valori specifici

Sostituire alla variabile i valori che rendono corretta un'espressione

Determinare il valore sconosciuto che appare in una equazione

Esprimere con simboli i valori sconosciuti identificati in una data situazione e usare tali simboli per scrivere le equazioni



Characteristic aspects of the variable as a generic number

Characteristic aspects of the variable as a functional relationship



Riconoscere la corrispondenza tra le variabili in relazione, indipendentemente dalla rappresentazione (tabella, grafico, problemi verbali, espressioni analitiche).



Determinare i valori della variabile dipendente, dati i valori di quella indipendente (e viceversa).



Riconoscere la variazione congiunta delle variabili in relazione funzionale indipendentemente dalla rappresentazione (tabella, grafico, problemi verbali, espressioni analitiche).



Determinare l'intervallo di variazione di una delle variabili, dato l'intervallo dell'altra.

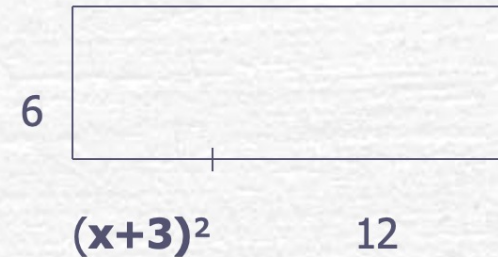


Tradurre in simboli una relazione funzionale basandosi sull'analisi dei dati di un problema.

EXAMPLES:

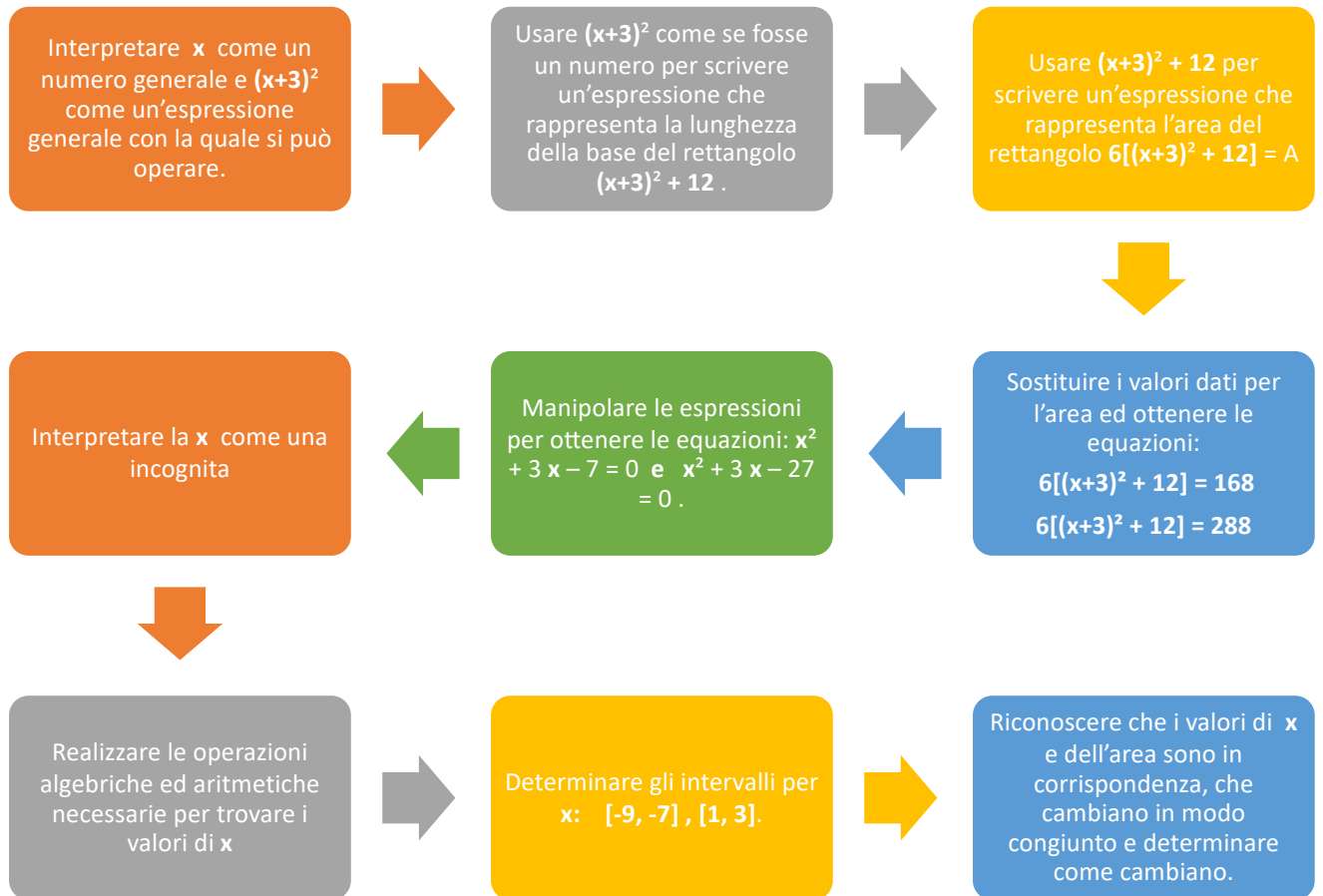
*Per quali valori di x l'area della seguente figura è tra 168
y 288?*

*Se il valore di x cresce o decresce, cosa succede con il
valore dell'area?*

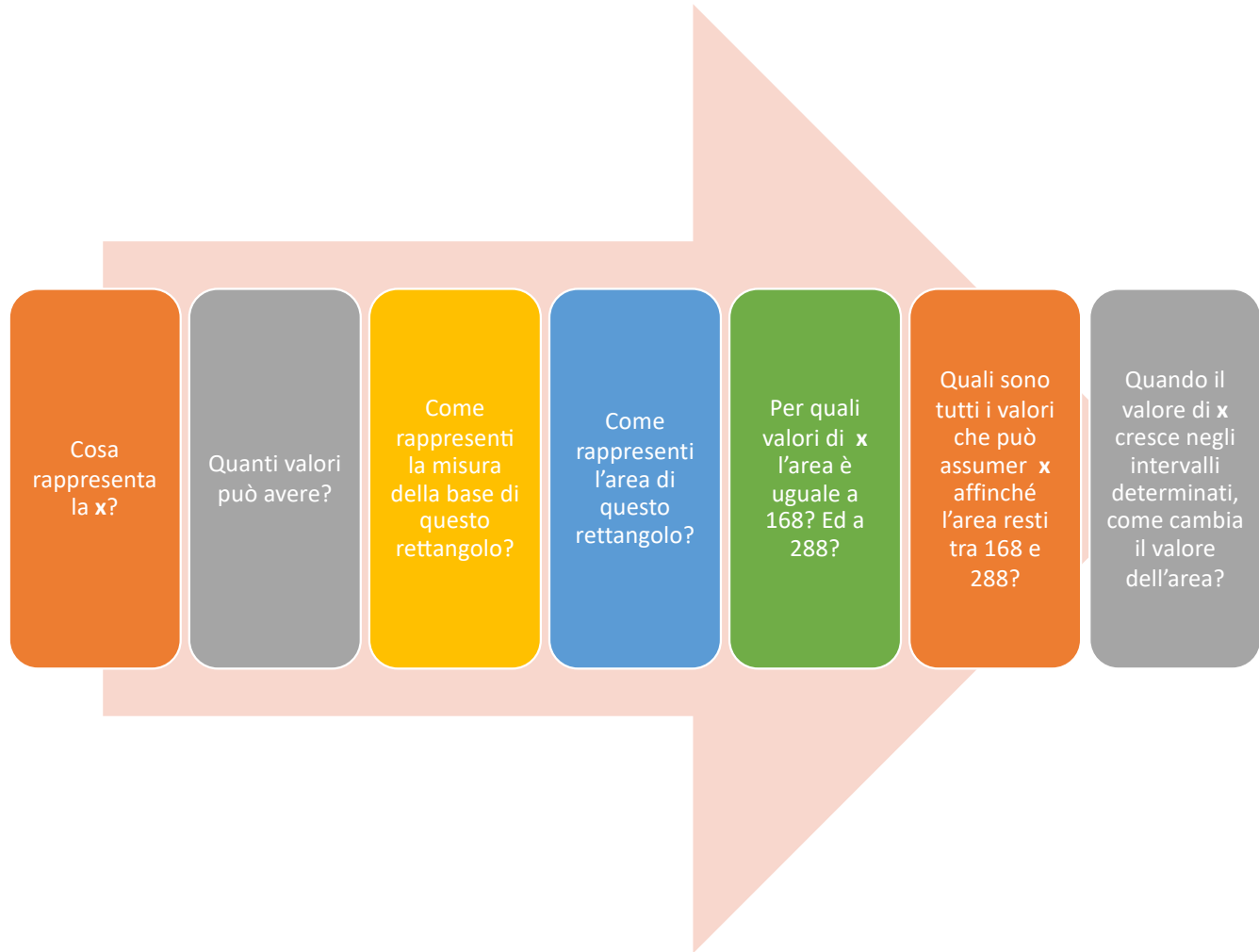


- ☞ Cosa implica risolvere questo problema?
- ☞ Quali sono le domande che potremmo fare agli studenti?

Resolving this problem means:



Guiding questions for this problem:





LAB ACTIVITY

GAME: IO HO CHI HA

