FASI DELLA MATERIA

Particella

$\{m, \vec{r}, \vec{v}\}$

Corpo macroscopico

$\{m_1, \vec{r}_1, \vec{v}_1, \dots, m_N, \vec{r}_N, \vec{v}_N\} \rightsquigarrow$ variabili di stato

N numero di particelle

n quantità di sostanza

N_A numero di Avogadro

$N = n N_A$

$N \sim 10^{23}$

$n \sim 1$ mol

SI : mol (mole)

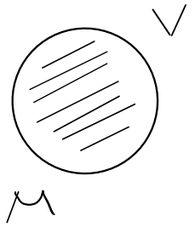
$N_A = 6.023 \times 10^{23} \ \frac{1}{\text{mol}}$

gas

liquido

cristallo

DENSITÀ



Densità $\rho = \frac{M}{V}$

Massa molare $M_A = \frac{M}{n}$

Densità di numero $\rho = \frac{N}{V}$

$M_A[C] = 12{,}01 \frac{g}{mol}$

Es.: calcola la massa molare M_A dell'acqua

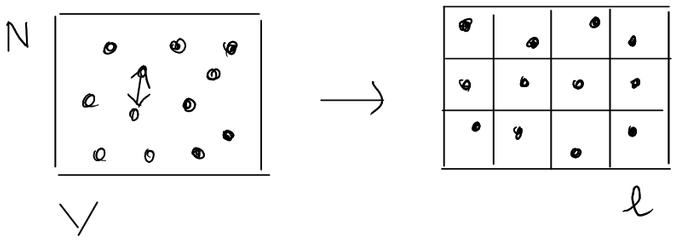
$$M_A[H_2O] = 1.01 \frac{g}{mol} \times 2 + 16 \frac{g}{mol} \times 1 = 18{,}02 \frac{g}{mol} \approx 18 \frac{g}{mol}$$

Differenza tra gas e liquido → densità!

$\rho_{liq} \approx 10^3 \frac{kg}{m^3} - 10^4 \frac{kg}{m^3}$

$\rho_{gas} \approx 1 \frac{kg}{m^3}$ condizioni ambiente

Es.: calcola l'ordine di grandezza della distanza tra atomi vicini in 1) un liquido e 2) in un gas -



$$\rho = \frac{M}{V} = \frac{n M_A}{N \ell^3} = \frac{\cancel{n} M_A}{\cancel{n} N_A \ell^3} = \frac{M_A}{N_A \ell^3}$$

$$\ell^3 = \frac{M_A}{\rho N_A} \qquad \ell = \sqrt[3]{\frac{M_A}{\rho N_A}}$$

Acqua H_2O liquida

$$\ell \approx \left(\frac{\cancel{18} \times 10^{-3}\ kg/mol}{10^3\ kg/m^3 \times \cancel{6} \times 10^{23}\ mol^{-1}} \right)^{1/3} = (3 \times 10^{-29})^{1/3}\ m \approx 3 \times 10^{-10}\ m \approx 1\ \text{Å}$$

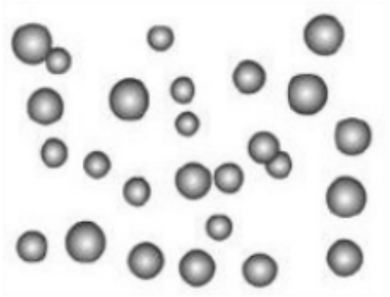
$\ell \sim \rho^{-1/3}$ $\qquad \rho' = x\rho \quad x = 10^{-3} \qquad \ell' = x^{-1/3}\ \ell$

Vapore d'acqua

$\ell \approx 10 \times 3 \times 10^{-10}\ m$

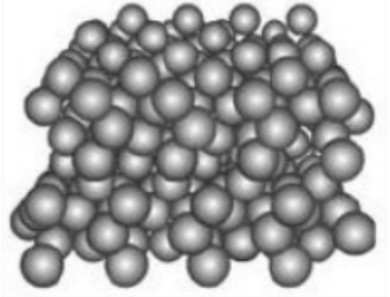
$= 3 \times 10^{-9}\ m$

$\approx 1\ nm$

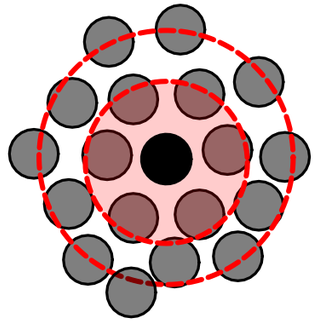


gas

fluido



liquido

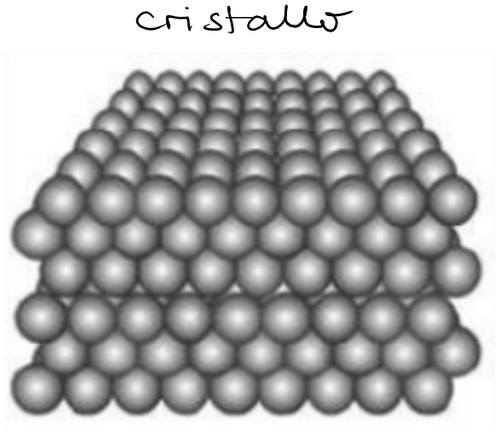


↳ ordine a corto raggio

Differenza tra liquido e solido

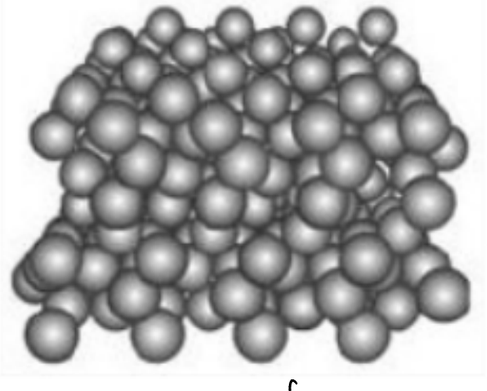
Risposta a una deformazione

- solido → rigidi
- liquidi → scorrono / fluiscono (fluidi)



Solidi cristallini → ordine a lungo raggio

es. NaCl quarzo
ghiaccio ⚠



Solidi amorfi

es: vetro finestra → SiO_2
materiali lavici

ρ [Kg/m^3]

10^3

1.

liquidi

cristalli

gas

Tipicamente

$\rho_{cryst} > \rho_{liq}$

ma eccezioni H_2O

ordine

corto raggio

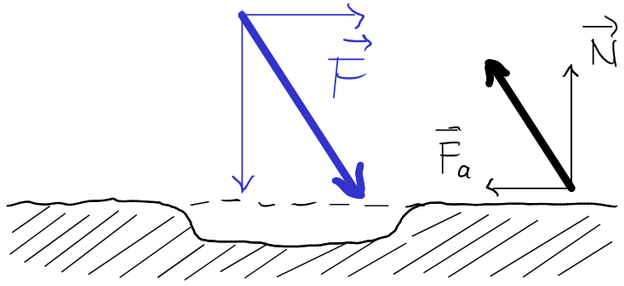
lungo raggio

PRESSIONE

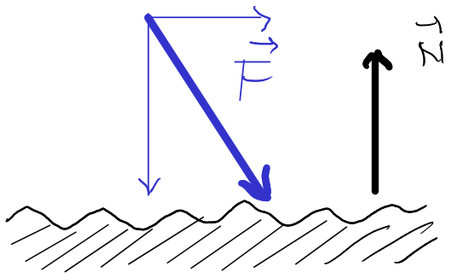
Es.: $P = 1024\ hPa = 1{,}024 \times 10^5\ Pa$

$P \approx 2{,}3\ bar$

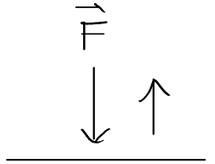
< interna
< esterna

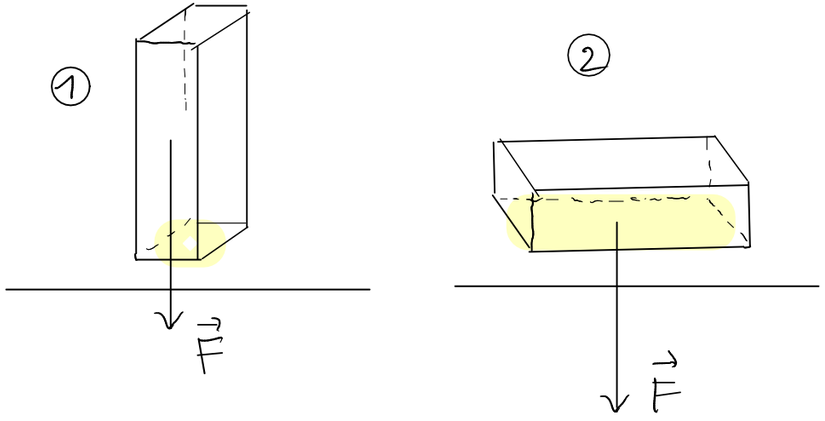


Solido
reazione normale che tangenziale



liquido
reazione normale



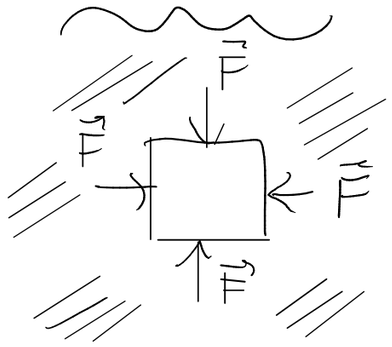


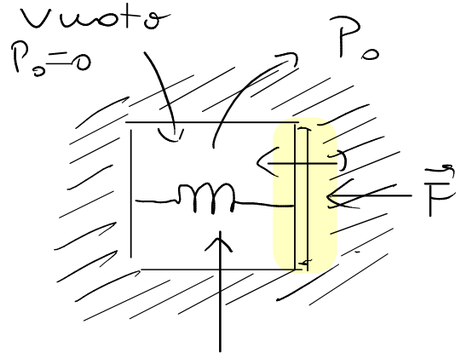
$$P = \frac{|\vec{F}|}{A}$$ pressione

SI: $\frac{N}{m^2} = Pa$ [Pascal]

↑ forza esercitata su superficie / area della superficie

Pressione nei fluidi : definizione operativa



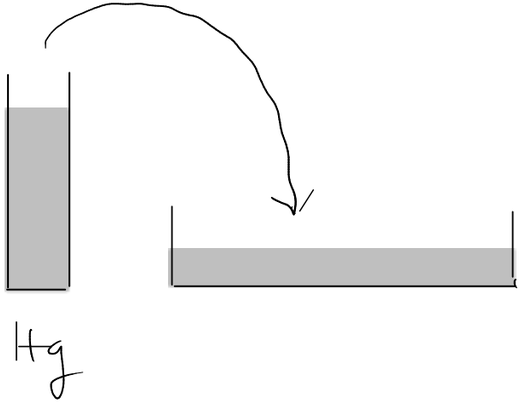


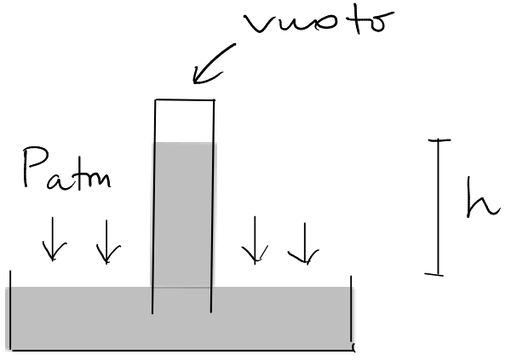
dinamometro

$$P = \frac{|F|}{A} \qquad P - P_0$$

manometro

1600 : Torricelli





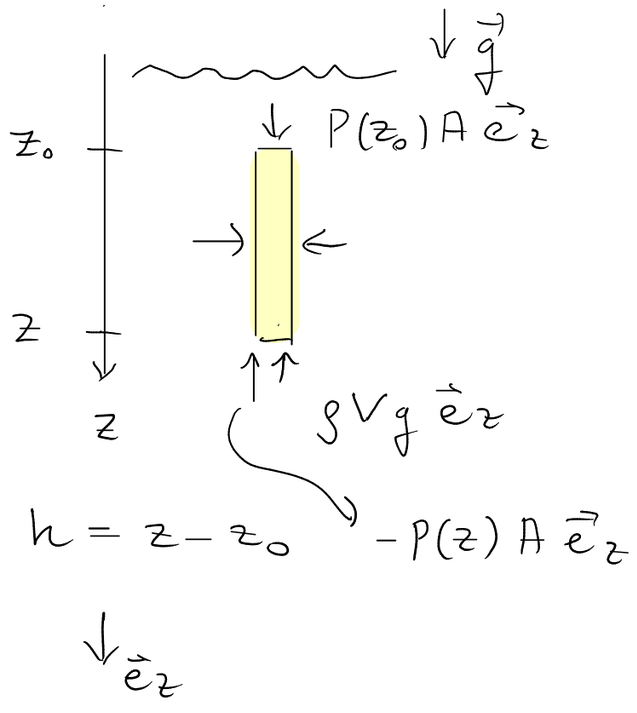
$$P_{atm} = 760 \text{ mmHg}$$

$$= 1{,}013 \times 10^5 \text{ Pa}$$

$$= 1 \text{ atm}$$

$$1 \text{ bar} = 10^5 \text{ Pa}$$

Fluido $\rho = \text{cost}$ (incomprimibile) a riposo



Eq. meccanico : $\sum \vec{F} = \vec{0}$

$$\rho V g \vec{e}_z + P(z_0) A \vec{e}_z - P(z) A \vec{e}_z = \vec{0}$$

$$P(z) = P(z_0) + \rho \frac{V}{A} g$$

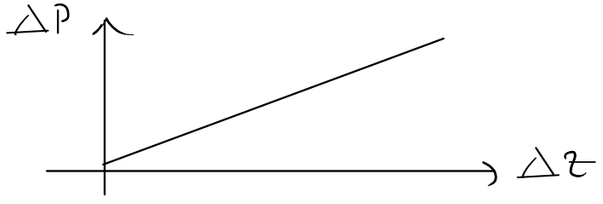
$$\boxed{P(z) = P(z_0) + \rho g h}$$

$$P(z) = P(z_0) + \rho g (z - z_0)$$

↓

legge di Stevino

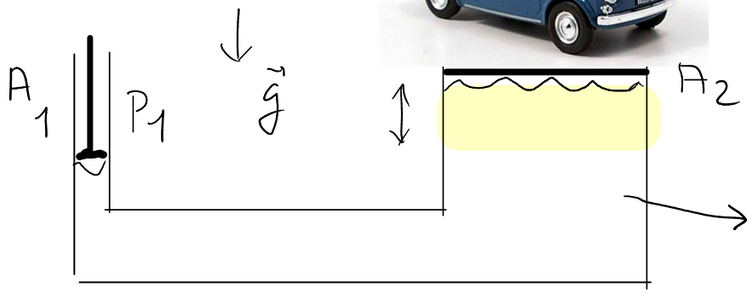
$\Rightarrow$ liquidi



$$P(z) = 0 + 13{,}6 \times 10^3 \frac{kg}{m^3} \times 9{,}8 \frac{m}{s^2} \times 0{,}760\ m$$

$$= 1{,}013 \times 10^5\ Pa$$

Es.: pistone idraulico



$r_1 = 5\,cm$ $r_2 = 15\,cm$

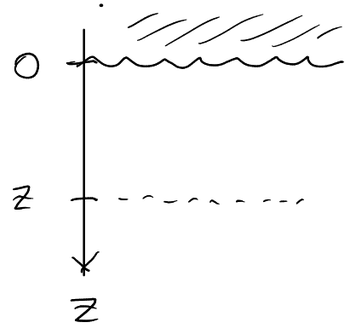
peso auto
13300 N

$P_1 = ?$ $F_1 = ?$

fluido incomprimibile

$\rho = cost$

Legge di Stevino



$P(z) = P_{atm} + \rho g z$

$P(z) = P(z_0) + \rho g (z - z_0)$

$P_1 = P_2$

$\frac{F_1}{A_1} = \frac{F_2}{A_2}$

TEMPERATURA

Calore ——

trasferimento di energia su scala microscopica

(contatto termico)

temperatura

energia cinetica immagazzinata a livello microscopico

$[W[\Sigma \vec{F}] = \Delta E_c]$

$Q \sim \Delta T$

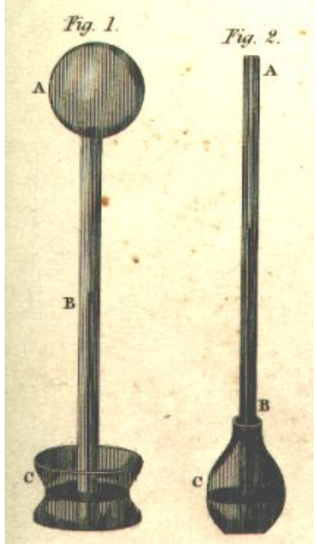
contatto - energia

velocità media

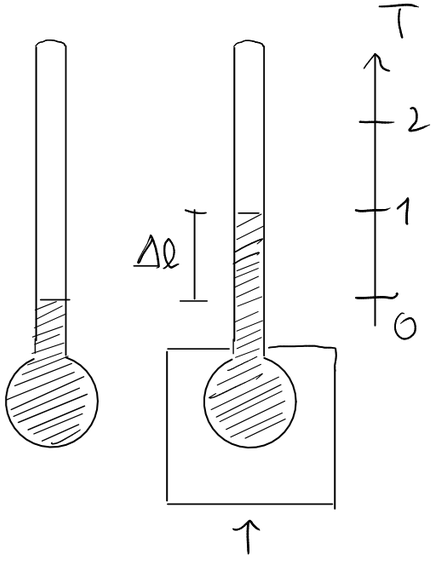
dare energia

agitazione termica

Definizione operativa



termoscopio



contatto termico

Fahrenheit 1724

dilatazione termica liquidi o gas

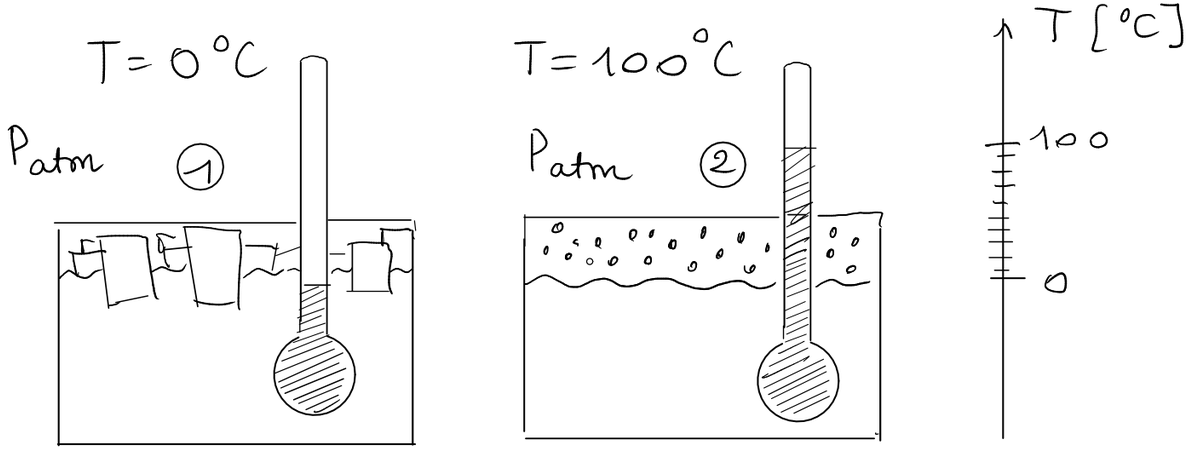
fenomeni termoelettrici

Scale di temperatura

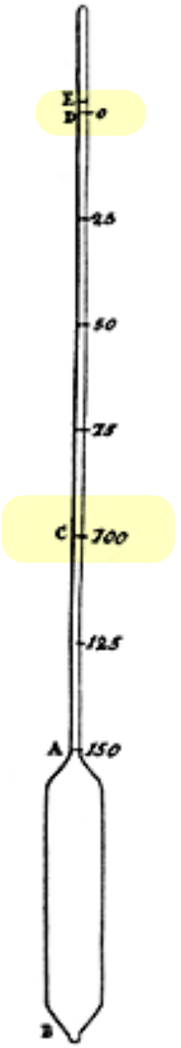
Punti fissi: coesistenza tra fasi

① H_2O liquida + ghiaccio a P_{atm}

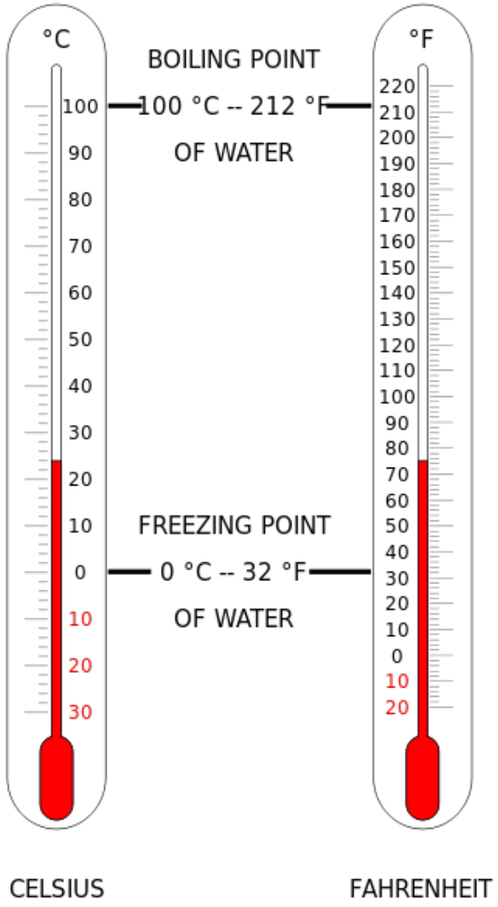
② H_2O liquida + vapore d'acqua a P_{atm}



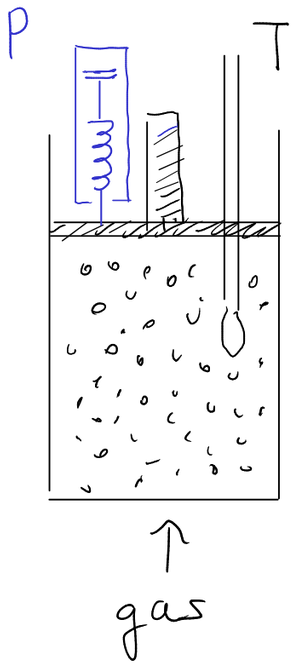
Scala Celsius della temperatura



Celsius
1742



Scala di temperatura assoluta

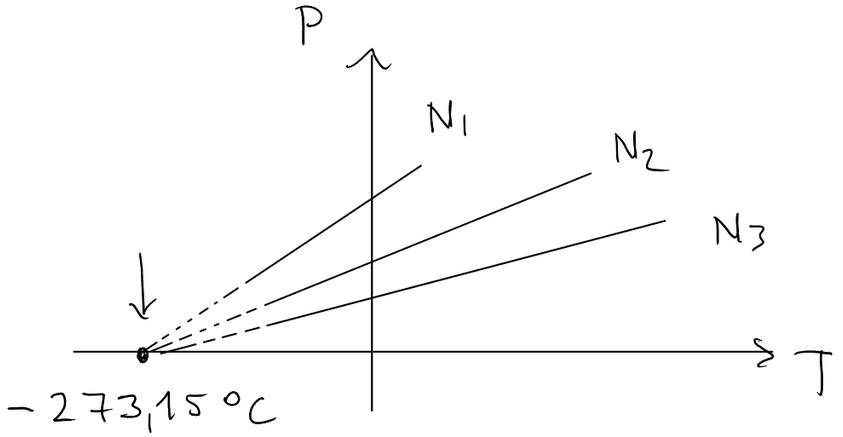


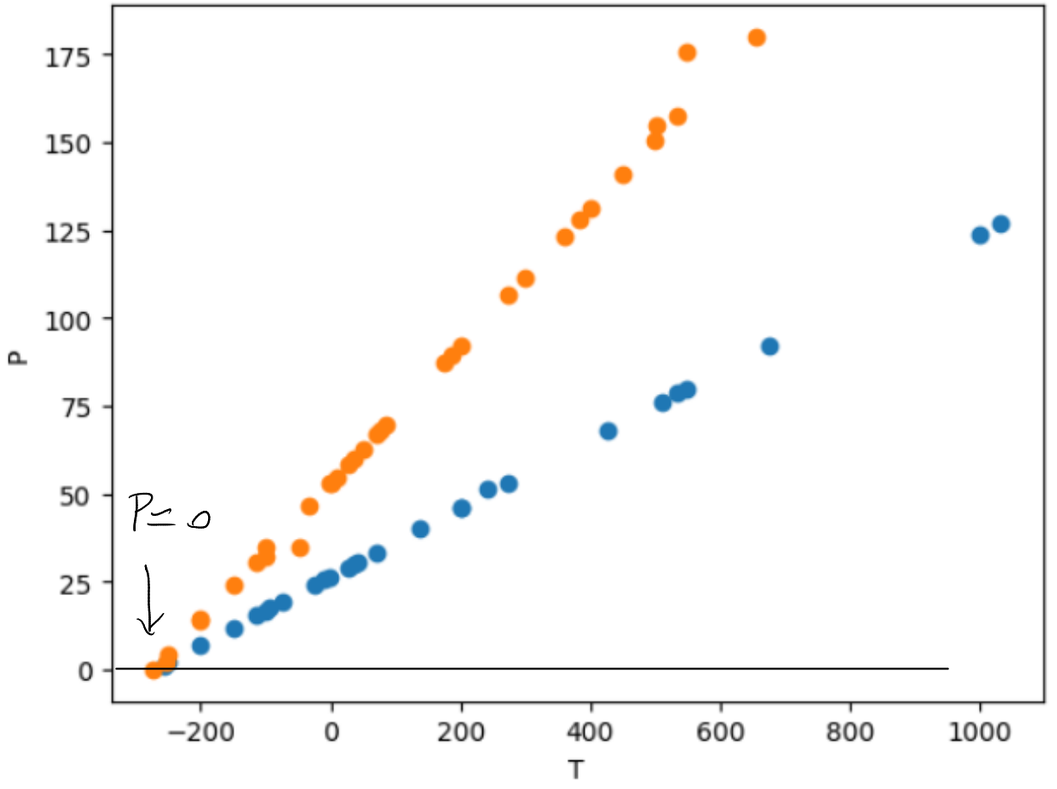
$$P = aT + b$$

$$a = a(N)$$

$$aT = -b$$

$$T = -\frac{b}{a} \approx -270\ °C$$





Scala Kelvin (1848)

$[T] = \Theta$ fondamentale

$$T = T_c + 273{,}15\ °C$$

SI: K [Kelvin]

$$\Delta T_c = \Delta T$$

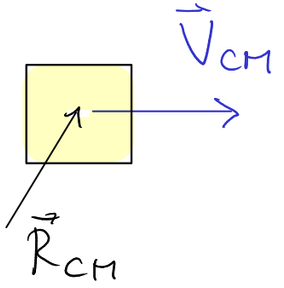
EQUAZIONI DI STATO

Variabili di stato :

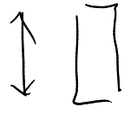
N, M, V

$\rho = \frac{M}{V}$, $\rho_N = \frac{N}{V}$

P, T



Equilibrio termodinamico : tutte le variabili di stato del sistema hanno valori "ben definiti" e indipendenti dal tempo

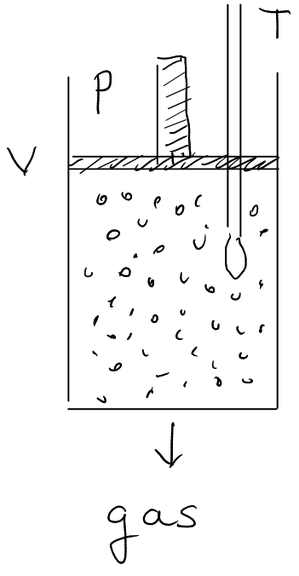


$\Delta P = \rho g \Delta z \approx 10^3 \frac{kg}{m^3} \times 10 \frac{m}{s^2} \times 0.1\,m \approx 10^3\,Pa \ll P_{atm}$

Equazioni di stato

$$f(N, V, P, T, \dots) = 0$$

1. Leggi empiriche dei gas diluiti



11700 scala assoluta ⚠

↓

$V = \text{cost} : P \sim T$ legge di GAY-LOUSSAC

$P = \text{cost} : V \sim T$ legge di CHARLES

$T = \text{cost} : PV \sim \text{cost}$ legge di BOYLE-MARIOTTE

$$PV = a\,T \qquad a(N)$$

Equazione di stato dei gas perfetti

$$PV = K_B N T$$

↑ costante di Boltzmann

$K_B = 1{,}38 \times 10^{-23}\ J/K$

$$n = \frac{N}{N_A} \rightarrow N = n N_A$$

$$PV = K_B N_A n T = R n T$$

$K_B N_A$: costante universale dei gas

$R = 8{,}314\ J/K/mol$

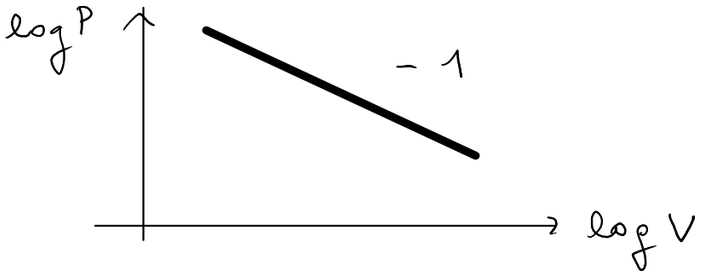
Limiti di validità dell'equazione di stato dei gas perfetti

$PV = nRT$ $\quad R = N_A K_B$ $\quad PV = NK_BT$ $\quad f(N, V, P, T, \dots) = 0$

$PV - NK_BT = 0$

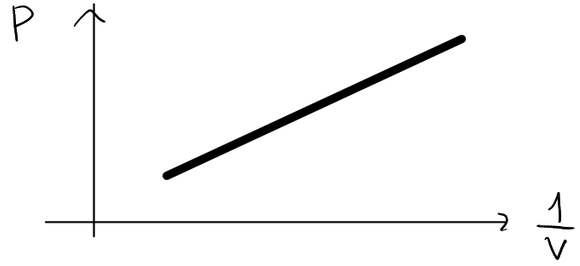
$T = cost \quad n = 1\ mol$

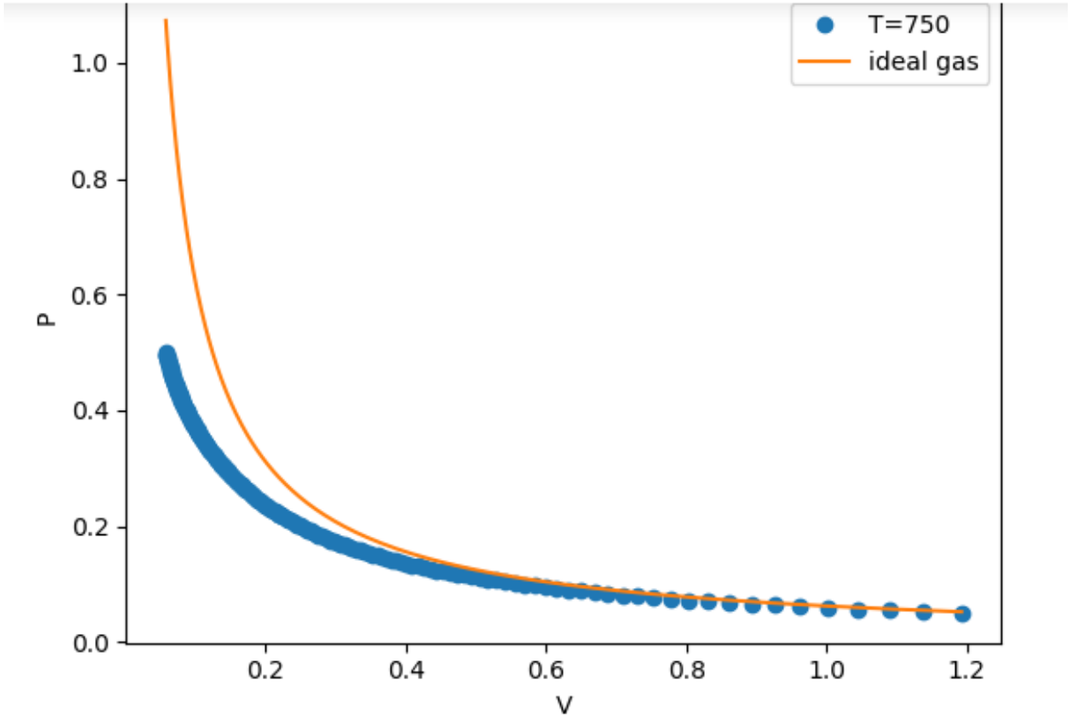
$P \sim \frac{1}{V} \quad H_2O$



$P = aV^{-1}$

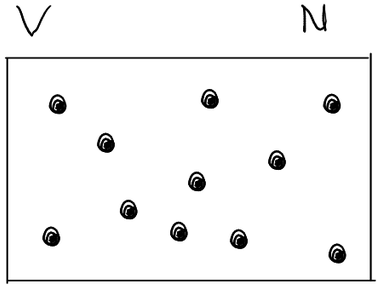
$\log P = \log a - \log V$





Interpretazione microscopica di pressione e temperatura

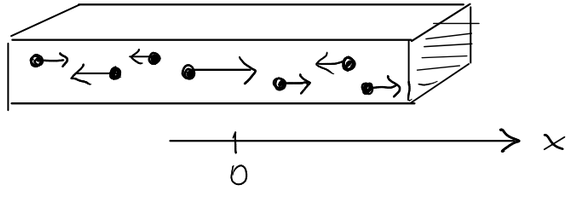
Modello di gas perfetto



① molecole = particelle (massa m)

② particelle non-interagenti
urti elastici con pareti

Sistema 1d



$v_{xi} \neq 0 \quad , \quad v_{yi} = 0 \quad , \quad v_{zi} = 0 \qquad i = 1, \dots, N$

③ non ci sono posizioni privilegiate (OMOGENEO)

④ non ci sono direzioni / sensi privilegiati (ISOTROPO)

Posizione media di una particella

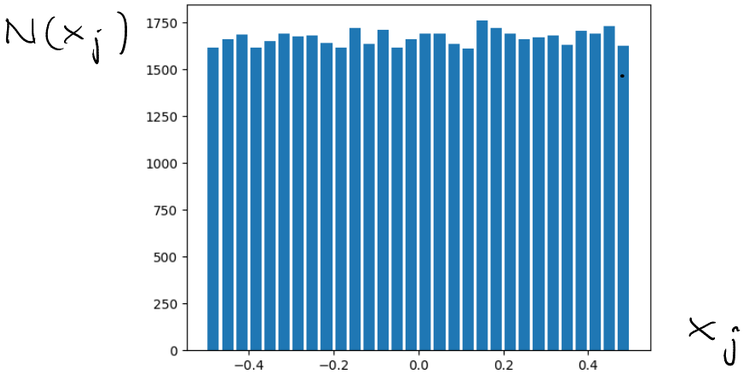
$$\langle x \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$\vec{R}_{CM} = \frac{1}{M} \sum_{i=1}^{N} m_i \vec{r}_i \qquad m_i = m$$

Raggruppo le posizioni entro n intervalli di ampiezza Δx → istogramma

$$\langle x \rangle \approx \frac{1}{N} \sum_{j=1}^{n} N(x_j)\, x_j = \sum_{j=1}^{n} \frac{N(x_j)}{N} \frac{\Delta x}{\Delta x} x_j$$

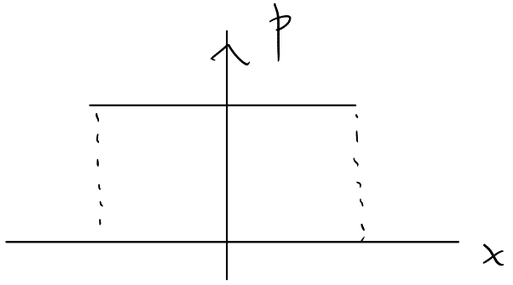
distribuzione di probabilità

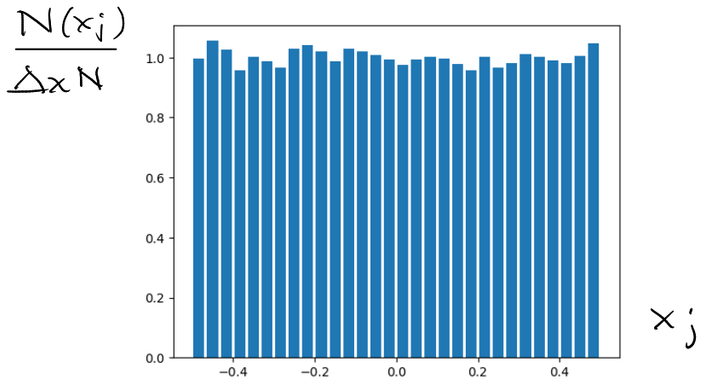


$$p(x_j) = \frac{1}{\Delta x} \frac{N(x_j)}{N} \rightarrow \text{densità di probabilità}$$

Limite $\Delta x \to 0$, $N \to \infty$

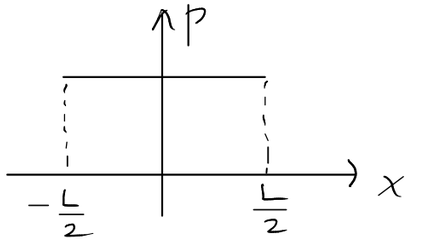
$$\langle x \rangle = \int_{-\infty}^{\infty} p(x)\, x\, dx$$

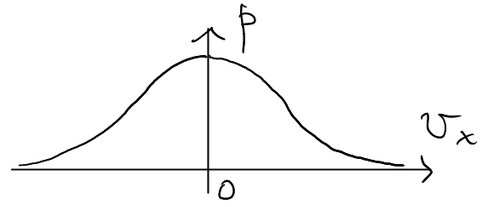




$$\langle x \rangle = \int_{-\infty}^{\infty} p(x)\, x\, dx = 0 \qquad \langle v_x^2 \rangle \neq 0$$

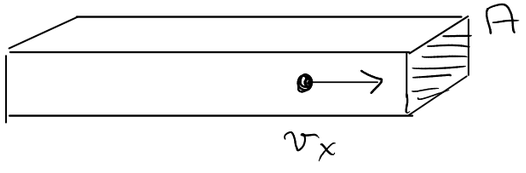
$$\langle v_x \rangle = \int_{-\infty}^{\infty} p(v_x)\, v_x\, dx = 0 \qquad \left\langle \frac{1}{2} m v_x^2 \right\rangle \to \langle E_c \rangle$$





Maxwell - Boltzmann

1 particella, intervallo di tempo dt

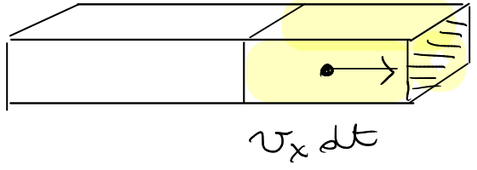


$$\frac{d\vec{p}}{dt} = \vec{F} \longrightarrow dp_x = F_x\, dt \to m v_{xf} - m v_{xi} = F_x\, dt$$

$\vec{F}$ sulla particella

$$-2 m v_x = F_x\, dt$$

Fisso $v_x > 0$



$v_x\, dt\, A$ (volume) $\cdot \frac{N}{V}$ (densità di numero) → rimbalzano contro parete in dt

Forza esercitata da queste particelle sulla parete — II Newton

$$F = \frac{2 m v_x}{dt} \cdot v_x\, dt\, A \cdot \frac{N}{V} = 2 m v_x^2 A \frac{N}{V}$$

Media sulle velocità delle particelle

$$\langle F \rangle = \frac{1}{2} \cdot 2 m \langle v_x^2 \rangle A \frac{N}{V} \qquad v_x > 0 \Rightarrow \frac{1}{2}$$

Forza media per unità di superficie

$$\frac{\langle F \rangle}{A} = m \langle v_x^2 \rangle \frac{N}{V}$$

$$\begin{cases} \frac{\langle F \rangle}{A} \cdot V = N m \langle v_x^2 \rangle \\ P \cdot V = N K_B T \end{cases} \rightarrow \begin{cases} P = \frac{\langle F \rangle}{A} \\ T = \frac{m}{K_B} \langle v_x^2 \rangle = \frac{2}{K_B} \langle \frac{1}{2} m v_x^2 \rangle \end{cases}$$

Pressione: urti delle molecole sulle pareti

Temperatura: $\sim$ energia cinetica media di una particella

3d: $T = \frac{2}{3K_B} \langle \frac{1}{2} m v^2 \rangle$ $\quad \vec{V}_{CM} = \vec{0}$ $\quad T$ non dipende da $\vec{V}_{CM}$!

Es.: velocità tipica delle molecole d'aria a temperatura ambiente

aria = gas perfetto $\quad T = 300\ K$

$\langle \vec{v} \rangle = \vec{0}$ $\quad$ velocità tipica : $\sqrt{\langle v^2 \rangle}$

$\langle |\vec{v}|^2 \rangle \neq 0$

$$\frac{3}{2} k_B T = \left\langle \frac{1}{2} m v^2 \right\rangle \qquad k_B N_A = R \qquad m N_A = M_A$$

$$\frac{3 k_B T}{m} = \langle v^2 \rangle \rightarrow \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3 k_B T}{m}} = \sqrt{\frac{3RT}{M_A}} \qquad M_A [N_2] \approx 28 \frac{g}{mol}$$

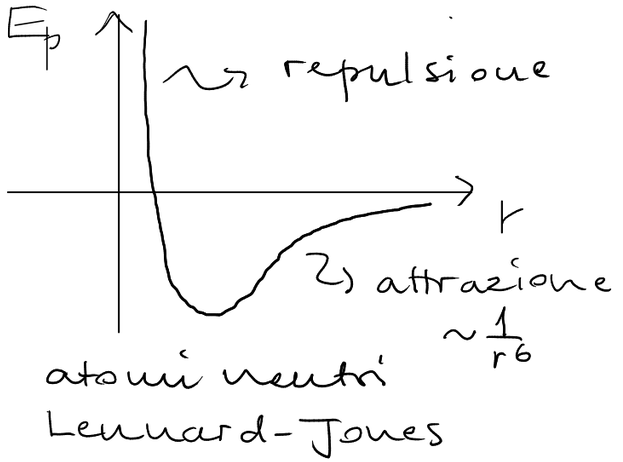
$$\sqrt{\langle v^2 \rangle} = \sqrt{\frac{3 \times 8\ J/K/mol \times 300\ K}{28 \times 10^{-3}\ kg/mol}} \approx 500 \frac{m}{s} \sim c = 330 \frac{m}{s}$$

□

2. Equazione di stato di van der Waals

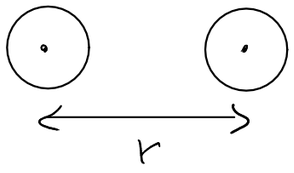
↗ repulsione
↘ attrazione a "corto" raggio

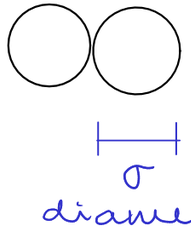
} approccio fenomenologico

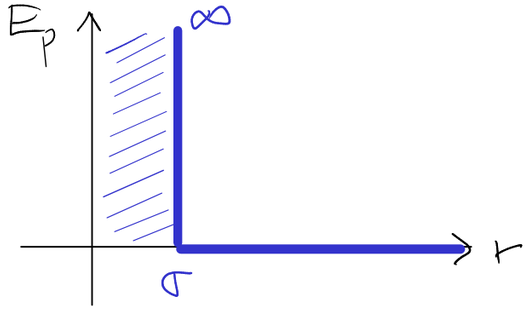


atomi neutri
Lennard-Jones

a) repulsione intermolecolare



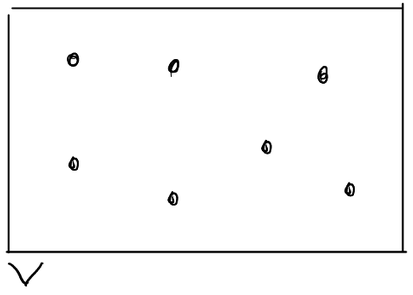




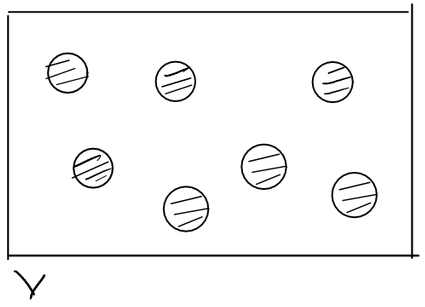
modello di sfera dura

$$E_p = \begin{cases} 0 & r > \sigma \\ \infty & 0 < r \leq \sigma \end{cases}$$

$PV = nRT$



$V \longrightarrow V - V_0$



Volume escluso

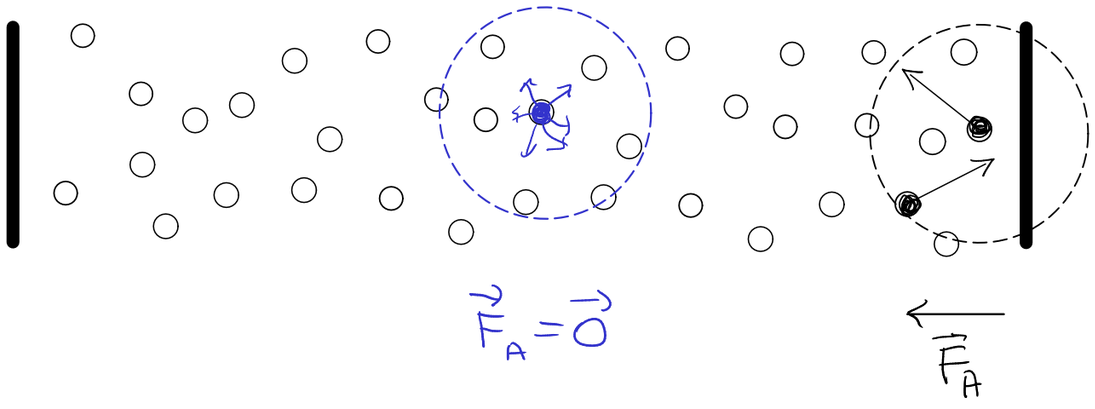
$V_0 \sim n$ ← covolume

$V_0 = bn$

$PV = nRT \rightarrow P(V - bn) = nRT$

(es.) : traccia P in funzione di V $\qquad P = P(V)$

b) attrazione intermolecolare a corto raggio



$$P = \frac{nRT}{V-nb} - P_A \qquad (P_A > 0)$$

$$P_A \sim \left(\frac{n}{V}\right)^2 \sim \left(\frac{N}{V}\right)^2 \Rightarrow P_A = a\left(\frac{n}{V}\right)^2$$

Eq. di stato di van der Waals

$$P = \frac{nRT}{V-nb} - a\left(\frac{n}{V}\right)^2$$

$$P + a\left(\frac{n}{V}\right)^2 = \frac{nRT}{V-nb}$$

$$\left(P + a\left(\frac{n}{V}\right)^2\right)(V-nb) = nRT$$

P V nRT

< attrazione repulsione

a, b dipendono dal materiale

Dimensioni di a e b

$$[P] = [a]\frac{[n]^2}{[V]^2} \qquad [a] = \frac{[Pa][V]^2}{[n]^2} \qquad SI: \ \frac{Pa \cdot m^6}{mol^2}$$

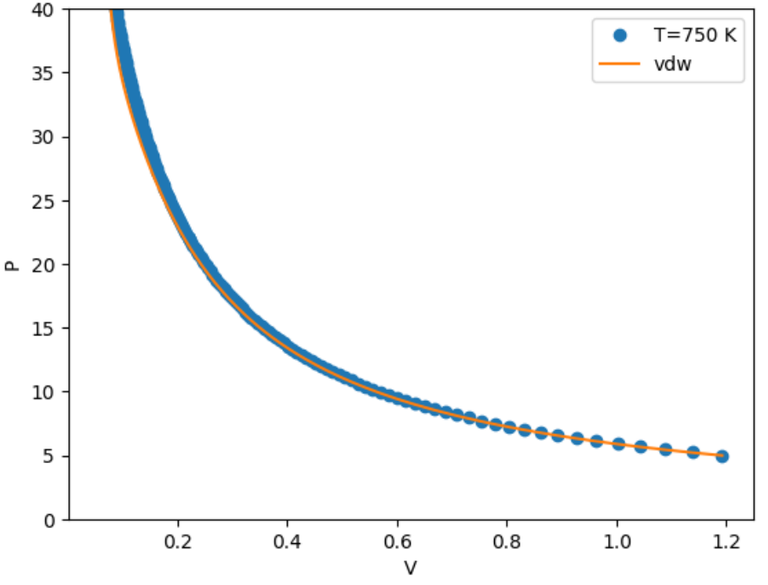
$$[V] = [n][b] \qquad [b] = \frac{[V]}{[n]} \qquad SI: \ \frac{m^3}{mol}$$

$$\begin{cases} \dfrac{l}{mol} = x \ \dfrac{m^3}{mol} & (1) \\ \dfrac{bar \cdot l^2}{mol^2} = y \ \dfrac{Pa \cdot m^6}{mol^2} \end{cases}$$

(1) $10^{-3}\, m^3 = x\, m^3 \Rightarrow x = 10^{-3}$

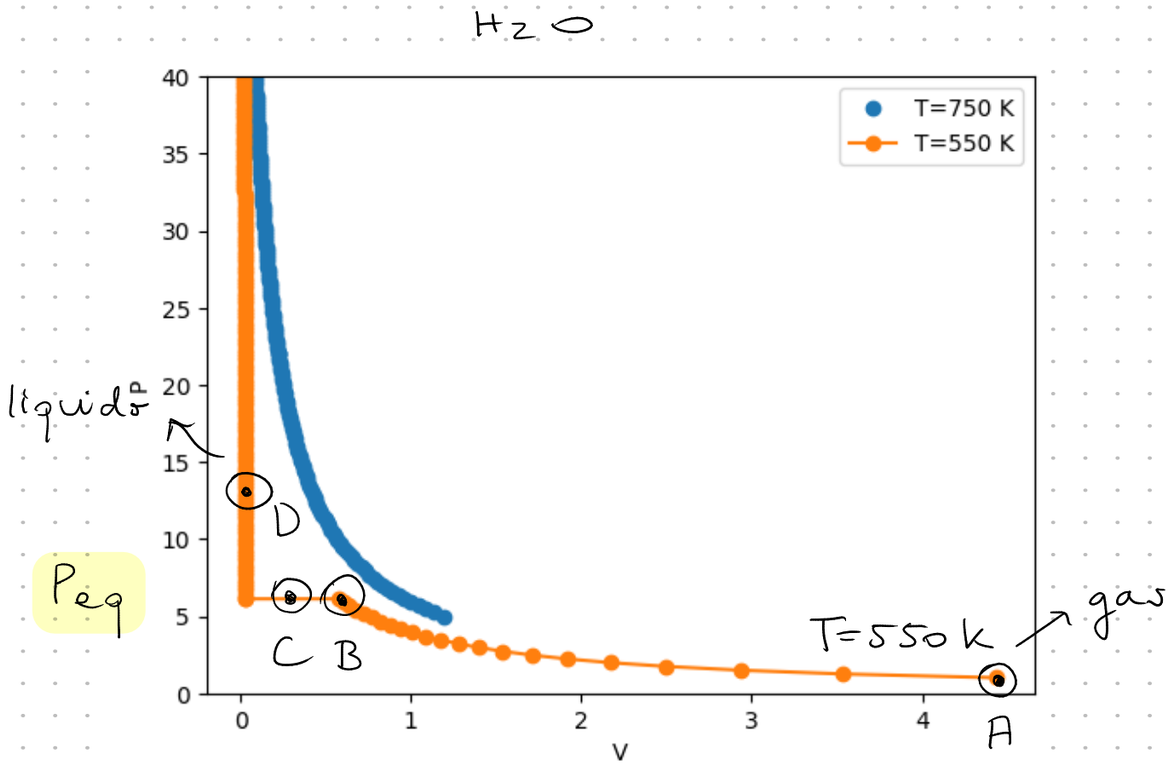
(2) $10^5\, Pa \times (10^{-3}\, m^3)^2 = y \times Pa \times m^6$

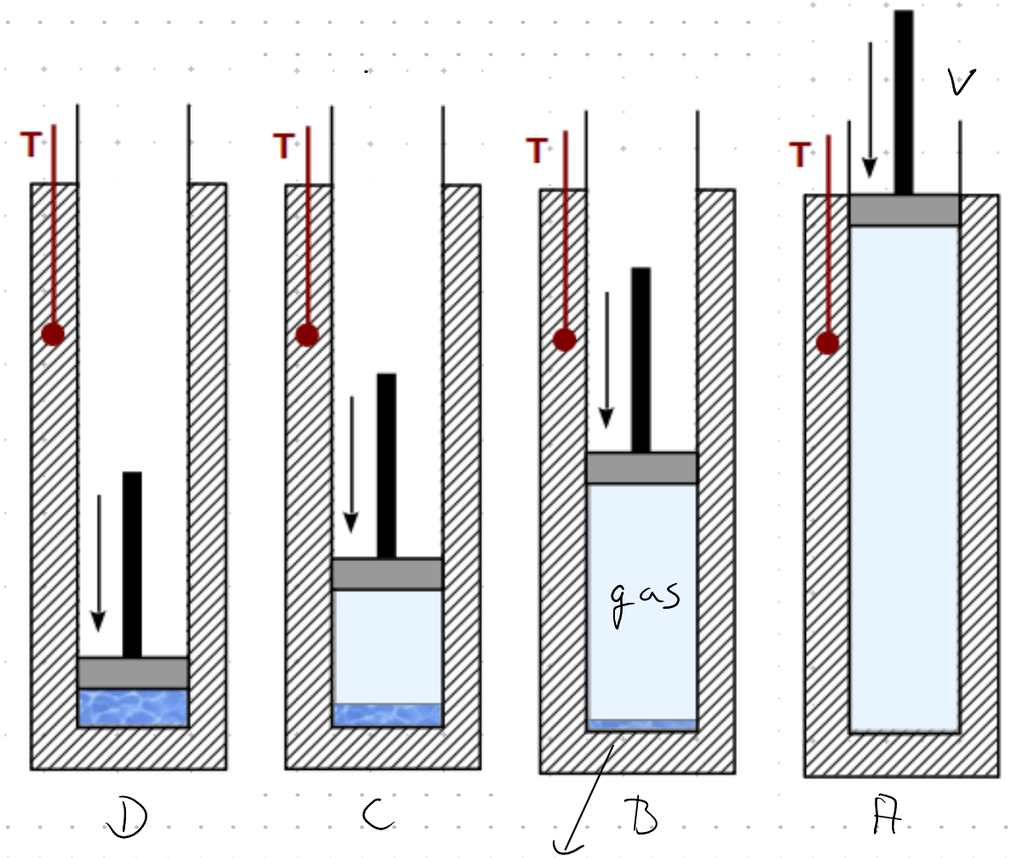
$10^5 \times 10^{-6}\, m^6 = y\, m^6 \Rightarrow y = 10^{-1}$



Coesistenza di fase

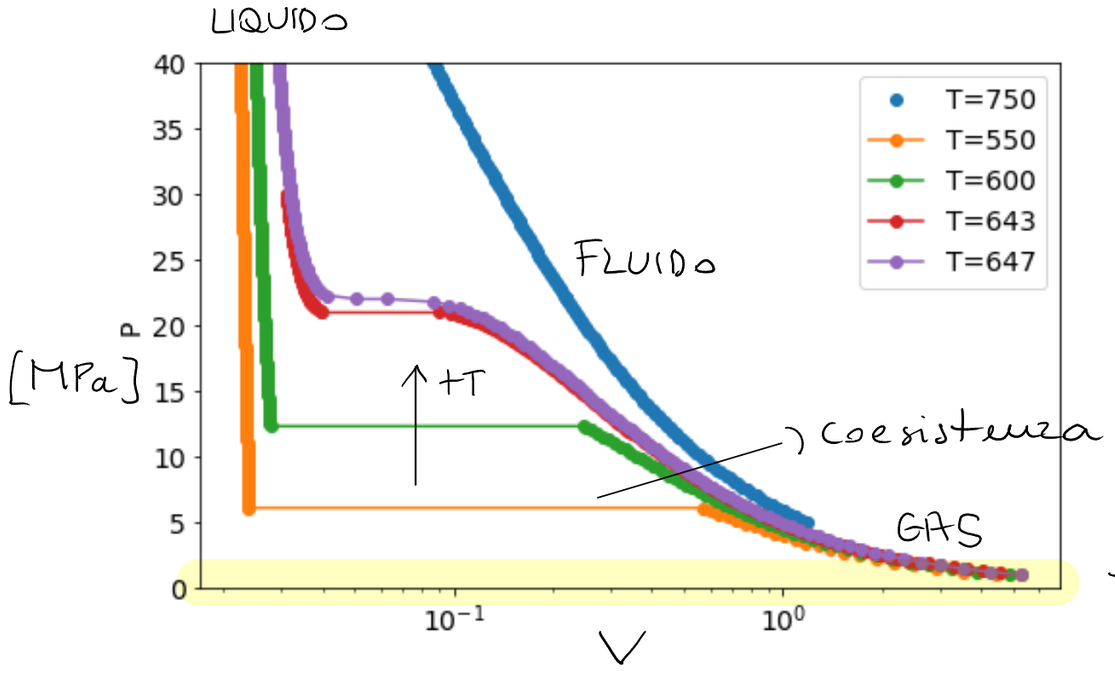
Fase: stato del sistema caratterizzato da proprietà fisiche omogenee



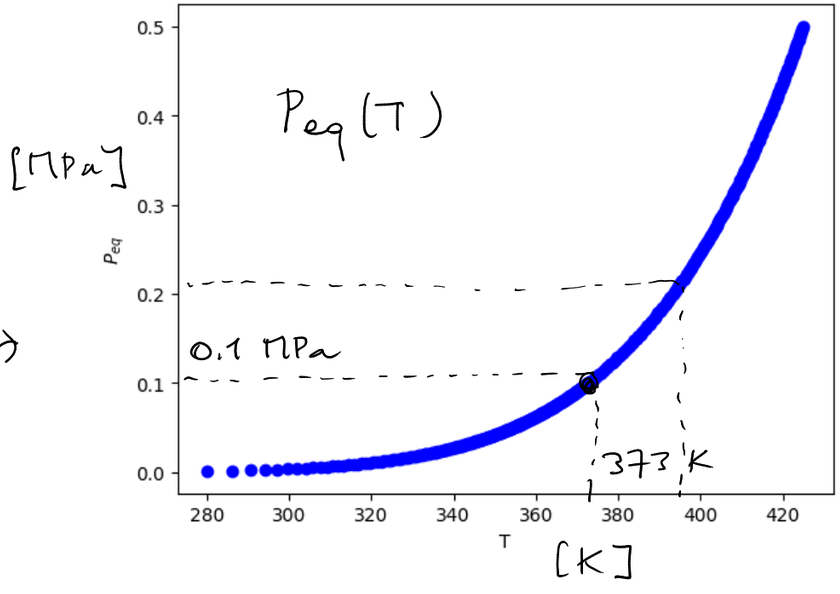


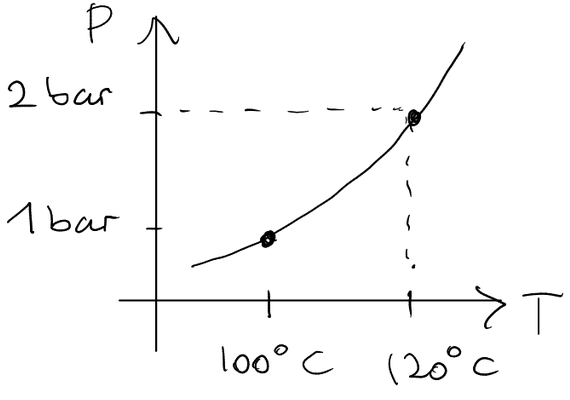
Coesistenza di fase: all interno del sistema sono presenti due o più fasi

Fasi di equilibrio → stato di equilibrio

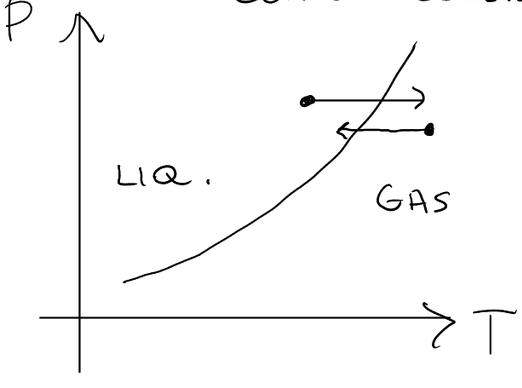


H_2O $T_c = 647\ K$ temperatura critica





curva coesistenza



$$\log P \sim \frac{1}{T}$$

$$\log P = \frac{A}{T} + B$$

$$P = \exp(B)\exp\left(\frac{A}{T}\right) = P_0 \exp\left(\frac{A}{T}\right) = P_0 \exp\left(-\frac{E}{T}\right)$$

E → calore latente

→ si può ottenere da un modello fisico!

Diagramma di fase di materiali "normali" es. gas rari

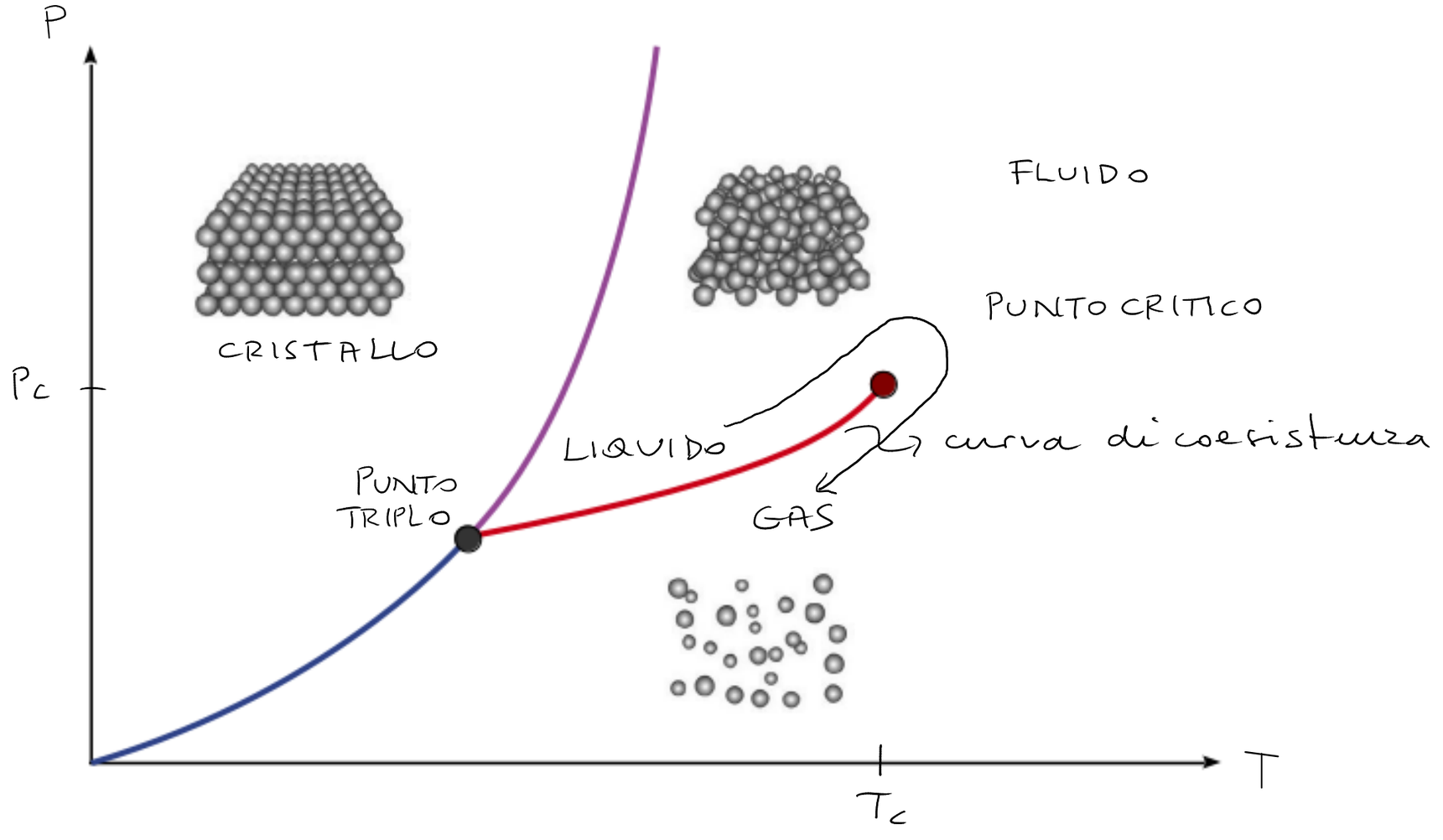
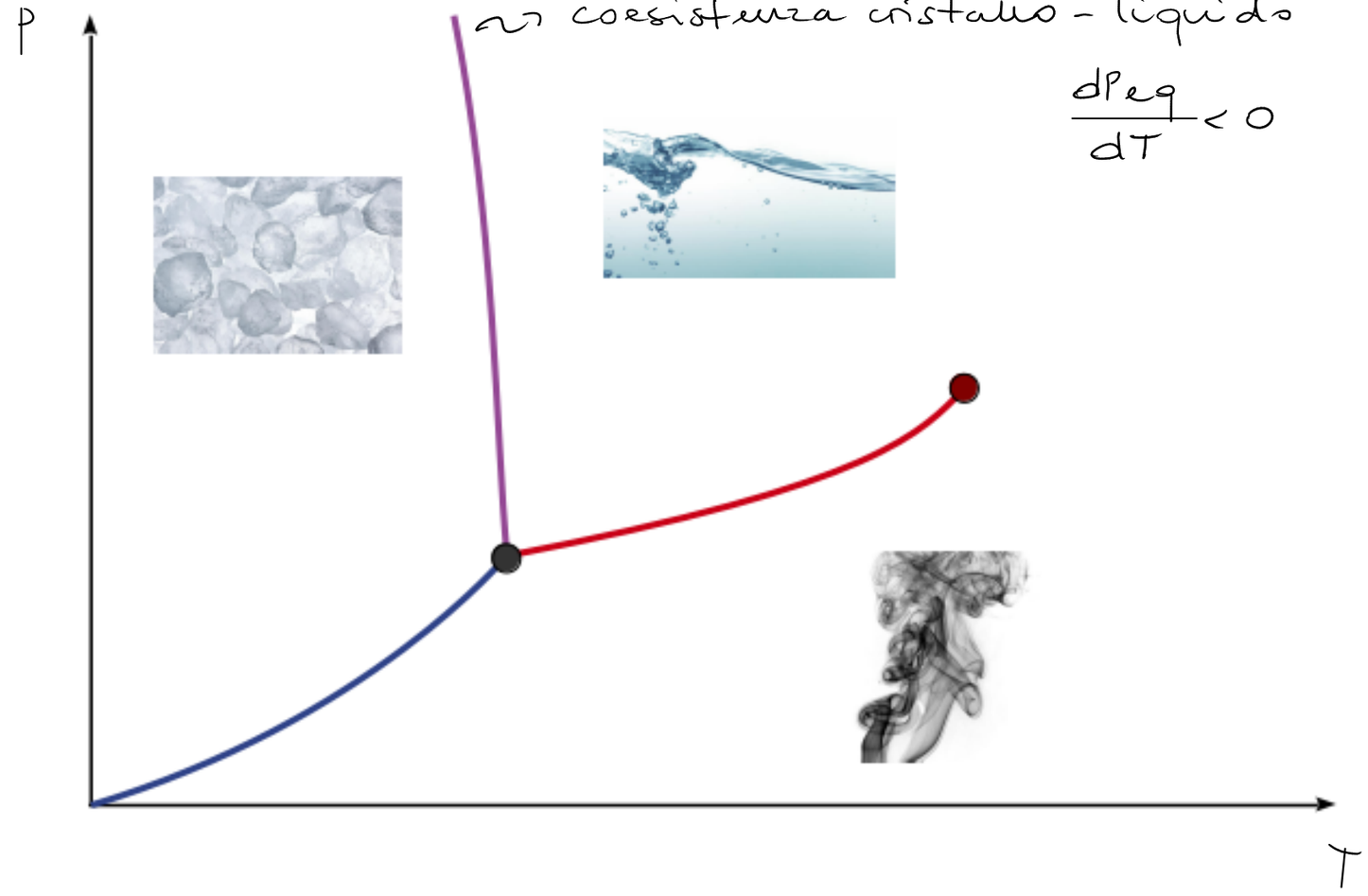
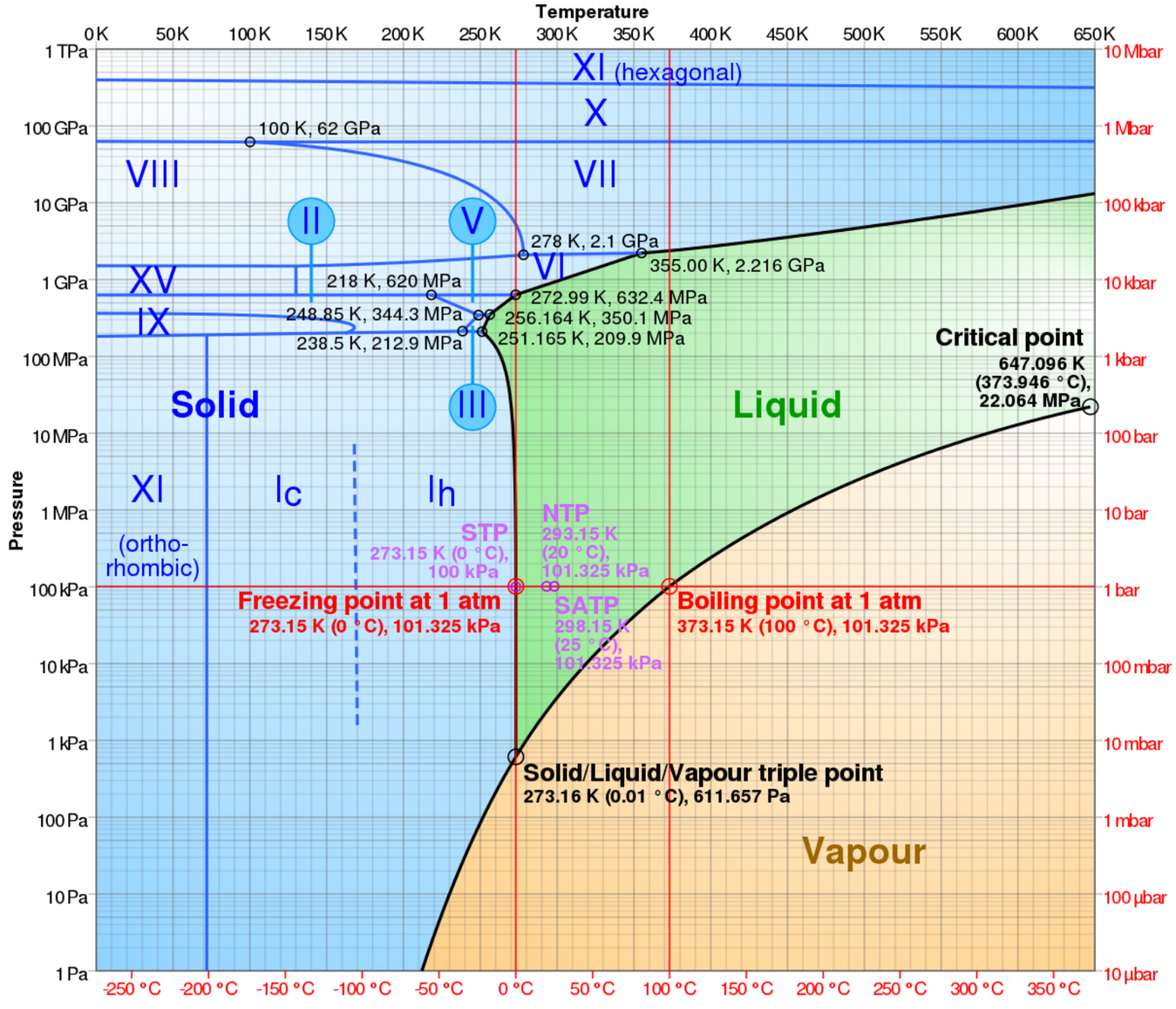


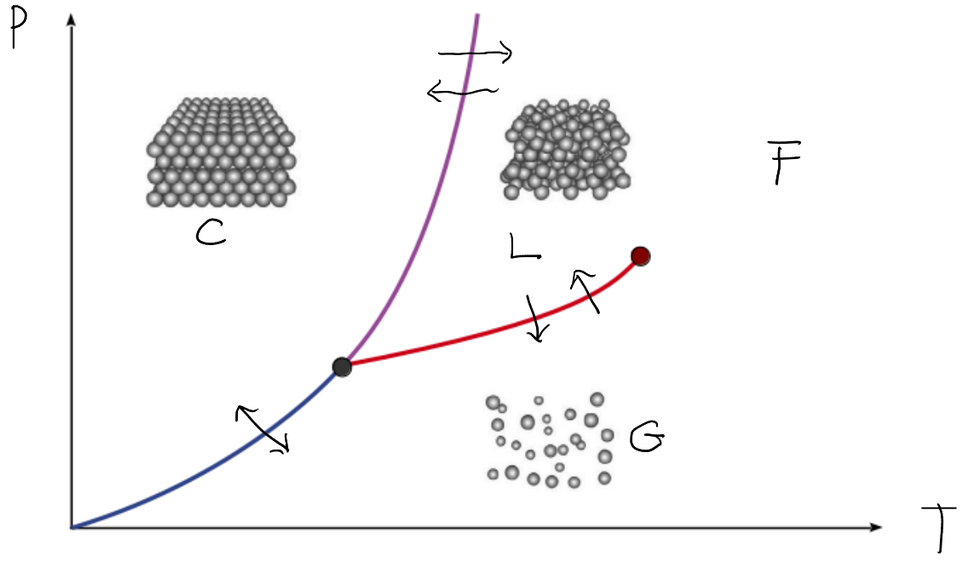
Diagramma di fase dell'acqua

$\rho_{liq} > \rho_{crist.}$




Temperature
0 K
50 K
100 K
150 K
200 K
250 K
300 K
350 K
400 K
450 K
500 K
550 K
600 K
650 K
Pressure
1 TPa
100 GPa
10 GPa
1 GPa
100 MPa
10 MPa
1 MPa
100 kPa
10 kPa
1 kPa
100 Pa
10 Pa
1 Pa
10 Mbar
1 Mbar
100 kbar
10 kbar
1 kbar
100 bar
10 bar
1 bar
100 mbar
10 mbar
1 mbar
100 µbar
10 µbar
-250 °C
-200 °C
-150 °C
-100 °C
-50 °C
0 °C
50 °C
100 °C
150 °C
200 °C
250 °C
300 °C
350 °C
XI (hexagonal)
X
VIII
VII
II
V
VI
XV
IX
III
100 K, 62 GPa
278 K, 2.1 GPa
355.00 K, 2.216 GPa
218 K, 620 MPa
272.99 K, 632.4 MPa
248.85 K, 344.3 MPa
256.164 K, 350.1 MPa
238.5 K, 212.9 MPa
251.165 K, 209.9 MPa
Solid
Liquid
Vapour
XI
(ortho-
rhombic)
Ic
Ih
Critical point
647.096 K
(373.946 °C),
22.064 MPa
STP
273.15 K (0 °C),
100 kPa
NTP
293.15 K
(20 °C),
101.325 kPa
SATP
298.15 K
(25 °C),
101.325 kPa
Freezing point at 1 atm
273.15 K (0 °C), 101.325 kPa
Boiling point at 1 atm
373.15 K (100 °C), 101.325 kPa
Solid/Liquid/Vapour triple point
273.16 K (0.01 °C), 611.657 Pa

MECCANICA DEI FLUIDI



F

Liquido

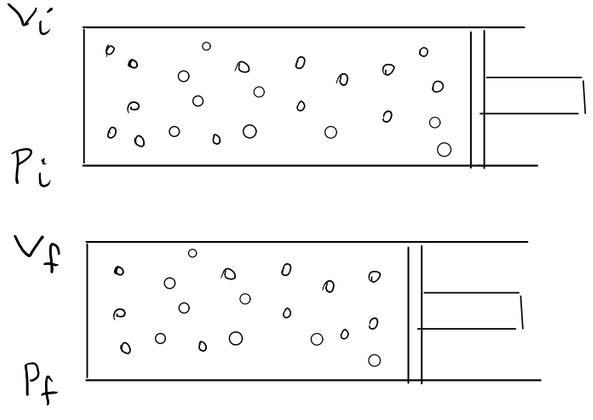
molto denso

poco comprimibile

Gas

poco denso

molto comprimibile



$P_i \rightarrow P_f \qquad \Delta P = P_f - P_i$

$V_i \rightarrow V_f \qquad \Delta V = V_f - V_i$

$\Delta V \sim \Delta P$

$\Delta V = - k \Delta P \qquad k > 0$

$$\frac{\Delta V}{V} = -\chi_T \Delta P \qquad T = \text{cost} \qquad N = \text{cost}$$

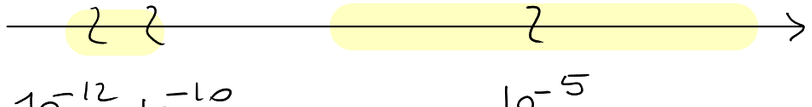
$$\frac{1}{V}\frac{\Delta V}{\Delta P} = -\chi_T$$

$$\chi_T = -\frac{1}{V}\frac{\Delta V}{\Delta P}$$

Comprimibilità isoterma:

$$\chi_T = -\frac{1}{V}\frac{dV}{dP} \qquad T = \text{cost}$$

$$\chi_T = -\frac{1}{V}\frac{\partial V}{\partial P}\bigg|_T \qquad \text{SI: } Pa^{-1}$$



Es.: χ_T per un gas perfetto

$$PV = nRT \rightarrow V = \frac{nRT}{P}$$

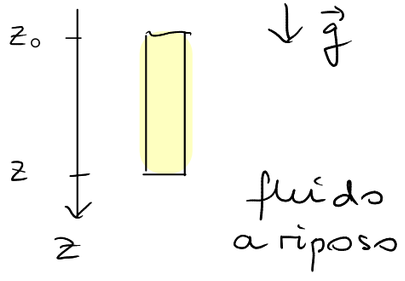
$$\chi_T = -\frac{1}{V}\frac{\partial V}{\partial P}\bigg|_T = -\frac{P}{nRT}\cdot\frac{\partial}{\partial P}\left(\frac{nRT}{P}\right)\bigg|_T$$

$$= -P\frac{d}{dP}\left(\frac{1}{P}\right) = \frac{1}{P} \qquad \square$$

$P = 10^5\ Pa \Rightarrow \chi_T = 10^{-5}\ Pa^{-1}$

FLUIDOSTATICA

$\rho = \text{cost} \Rightarrow$ legge di Stevino



$$P(z) = P(z_0) + \rho g (z - z_0)$$

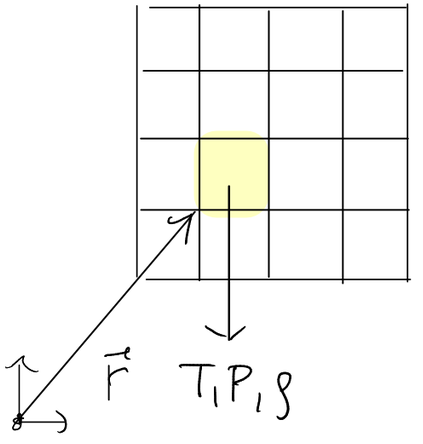
$$\Delta P = \rho g \Delta z$$

$T(\vec{r})$

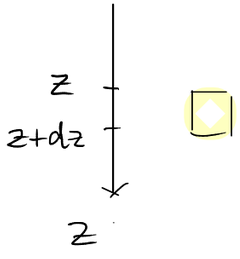
$P(\vec{r})$

$\rho(\vec{r})$

campi



Fluido a riposo $\Rightarrow$ principio fondamentale della fluidostatica



$$P(z+dz) = P(z) + \rho g dz$$

$$dP = \rho g dz$$

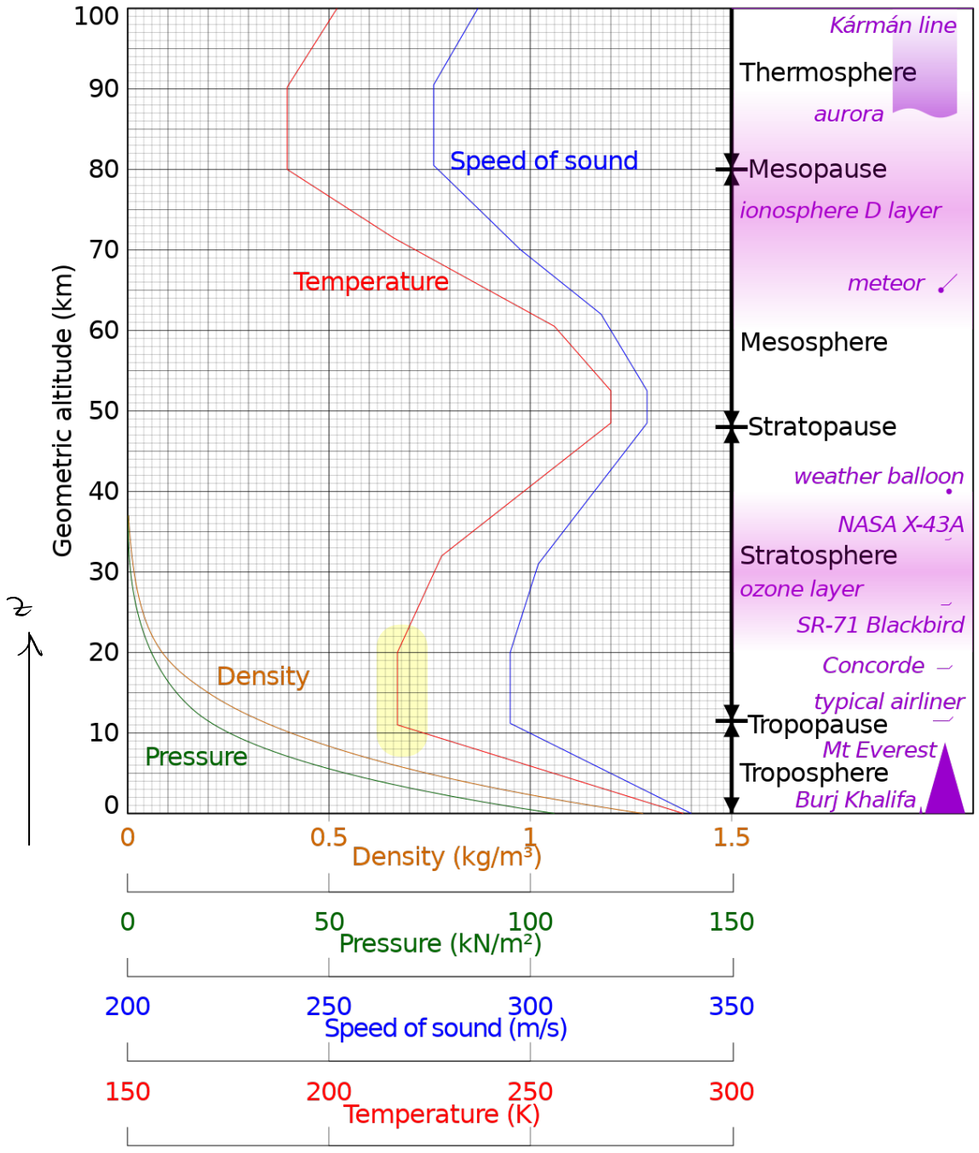
$$\frac{dP}{dz} = \rho g$$

$\uparrow$

$\rho = \rho(z)$ ⚠

$$dP = -\rho g dz$$

$$\frac{dP}{dz} = -\rho g$$



By Cmglee - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=17311330

Modellizzazione dell'atmosfera terrestre

→ tropopausa

① $T = \text{cost}$

② atmosfera a riposo

③ gas perfetto

④ $M_A = 28 \frac{g}{mol}$

$$\begin{cases} \frac{dP}{dz} = -\rho g & \rho = \rho(z) \quad ① \\ PV = nRT & ② \end{cases}$$

$$\frac{n}{V} = \frac{P}{RT}$$

$$\rho = \frac{M}{V} = \frac{n M_A}{V} = \frac{M_A P}{RT} \Rightarrow (1)$$

(2)

$$\frac{dP}{dz} = -\underbrace{\frac{M_A g}{RT}}_{=\frac{1}{\ell}} P \qquad \text{eq. differenziale per } P(z)$$

$$\frac{dP}{dz} = -\frac{1}{\ell} P \qquad \ell = \frac{RT}{M_A g} \qquad \left[\frac{dv}{dt} = -kv \right]$$

Separazione delle variabili

$$\frac{dP}{P} = -\frac{1}{\ell} dz$$

$$\int_{P_i}^{P_f} \frac{dP}{P} = -\frac{1}{\ell} \int_{z_i}^{z_f} dz \rightarrow \log P_f - \log P_i = -\frac{1}{\ell}(z_f - z_i)$$

$$\log\left(\frac{P_f}{P_i}\right) = -\frac{z_f - z_i}{\ell}$$

$P_f \to P \qquad P_i \to P_0 \qquad z_i = z_0 \qquad z_f = z$

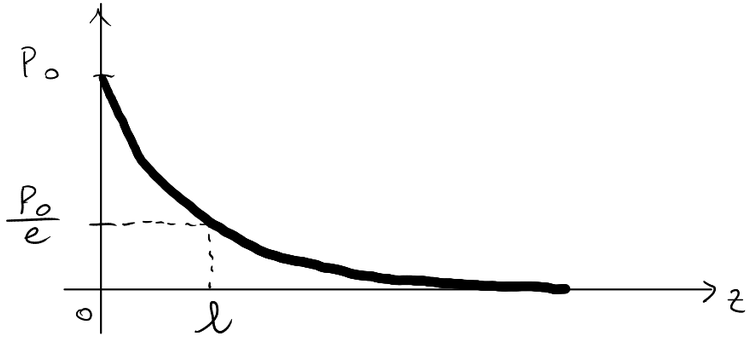
$$P(z) = P_0 \exp\left(-\frac{z - z_0}{\ell}\right)$$

$$\ell = \frac{RT}{M_A g}$$

$z_0 = 0$ (livello del suolo)

$$P(z) = P_0 \exp\left(-\frac{z}{\ell}\right)$$

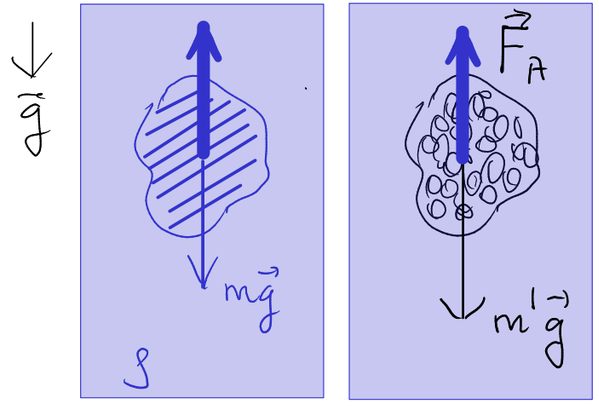
$$P(z=\ell) = P_0 \exp(-1) = \frac{P_0}{e}$$



$$\ell \approx \frac{8\ \text{J/K/mol} \times \cancel{300}\ \text{K}}{\cancel{28} \times 10^{-3}\ \text{kg/mol} \times \cancel{10}\ \frac{\text{m}}{\text{s}^2}}$$

$$\approx 8 \times 10^3\ \text{m} \approx 10\ \text{km} \qquad \square$$

Principio di Archimede



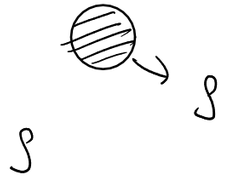
Un corpo immerso in un fluido subisce una forza uguale in modulo e direzione al peso del fluido spostato dal corpo e di verso opposto a tale peso.

$$\vec{F}_A = -m\vec{g} = -\rho V \vec{g}$$

(ρ: del fluido spostato; V: del fluido spostato)

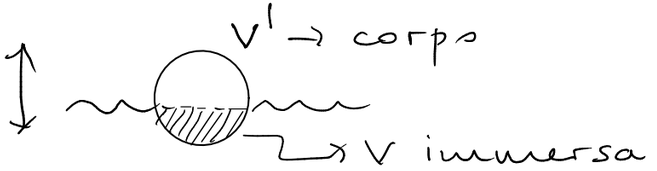
Casi particolari:

a) completamente immerso $\quad \Sigma\vec{F} = \vec{P} + \vec{F}_A = V\rho'\vec{g} - V\rho\vec{g} = (\rho' - \rho)V\vec{g}$



- $\rho' > \rho$: $\Sigma\vec{F} = (\underbrace{\rho' - \rho}_{>0})V\vec{g} \sim \vec{g}$ ⇓
- $\rho' < \rho$: $\Sigma\vec{F} = (\underbrace{\rho' - \rho}_{<0})V\vec{g} \sim -\vec{g}$ ⇑
- $\rho' = \rho$: $\Sigma\vec{F} = \vec{0}$ equilibrio meccanico → indifferente

b) parzialmente immerso $\rho' < \rho$



$$\Sigma \vec{F} = V'\rho'\vec{g} - V\rho\vec{g} = (V'\rho' - V\rho)\vec{g}$$

Equilibrio meccanico $\Sigma \vec{F} = \vec{0}$

$$V'\rho' = V\rho \Rightarrow V = \frac{\rho'}{\rho}V' \Rightarrow \frac{V}{V'} = \frac{\rho'}{\rho} < 1$$

↑ frazione di volume immerso

Es: H_2O

$\rho_{gh} < \rho_{liq}$

$\rho_{gh} = 0.9\,\rho_{liq} \Rightarrow \frac{V}{V} = 0.9 = 90\%$

Es. 1) di quanto varia il livello dell'acqua una volta fuso il ghiaccio?

2) caso b) equilibrio è stabile / instabile / indifferente?

DINAMICA DEI FLUIDI

Fluidodinamica

→ fluido $\rho \neq cost$

Idrodinamica

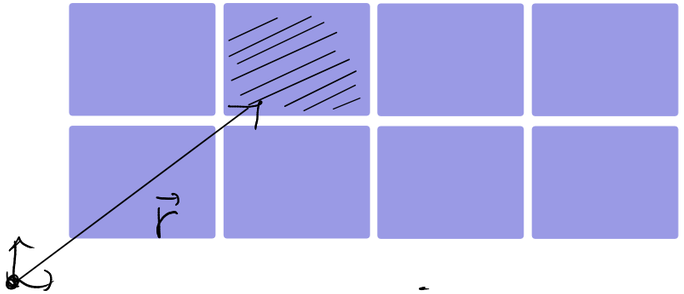
fluido $\rho = cost$ → liquidi

→ bassa velocità

→ alta velocità

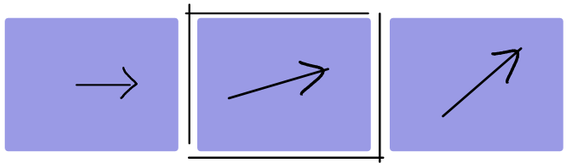
✓ 3 + 4 ✓

× 3 × 4



campo di velocità

v_i singole particelle



$\vec{v}(\vec{r},t)$

$\vec{v} = \vec{v}(x,y,z,t)$

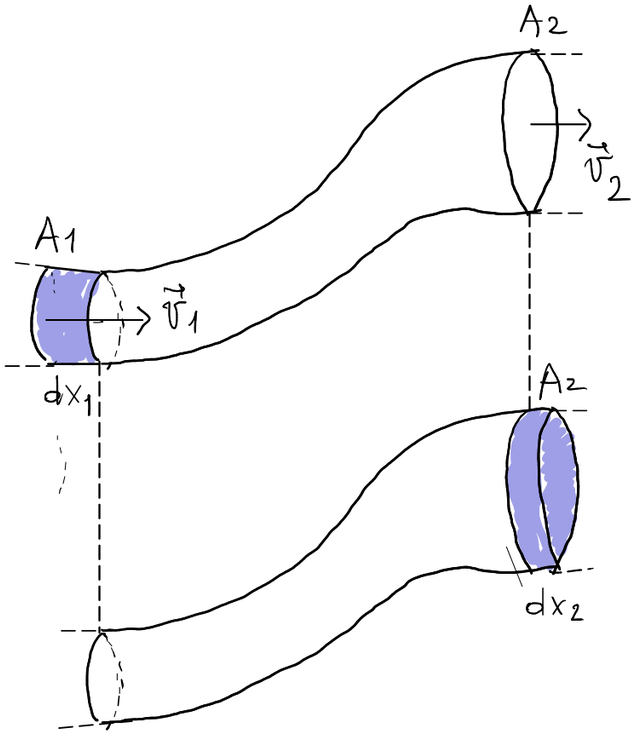
1. Incomprimibile $\rho = cost$
2. non viscoso

} fluido ideale

3. Stazionario $\vec{v}(\vec{r})$
4. irrotazionale

Equazione di continuità per fluidi incomprimibili

Fluido ideale in un tubo



iniziale

dt

finale

Conservazione della massa (materia)

$$A_1\, dx_1 = A_2\, dx_2$$

$$A_1 v_1 \cancel{dt} = A_2 v_2 \cancel{dt}$$

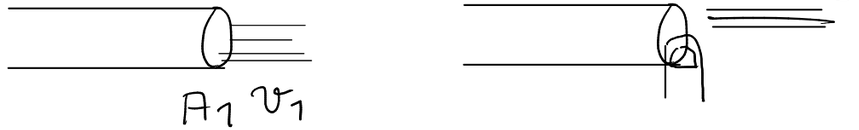
$$A_1 v_1 = A_2 v_2$$

$$A\,v = \text{cost}$$ eq. di continuità

$$[A\,v] = \frac{L^2 L}{T} = \frac{L^3}{T}$$

Av : corrente di volume

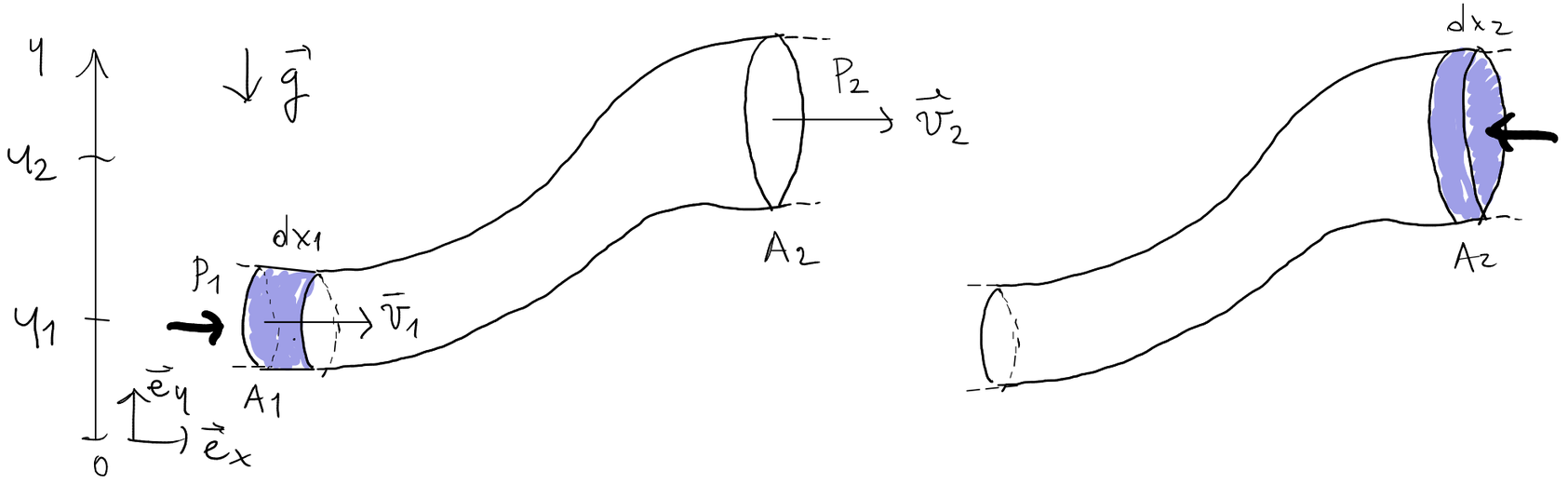
Es.: $A_1 v_1 = A_2 v_2$



$$v_2 = \left(\frac{A_1}{A_2}\right) v_1 \qquad \frac{A_1}{A_2} > 1$$

Teorema di Bernoulli

Fluido ideale + forze : pressione + gravità



iniziale $\longrightarrow$ finale

dt

Bernoulli 1738

- $F_1 = P_1 A_1 \qquad F_2 = P_2 A_2$

  $\vec{F}_1 = P_1 A_1 \vec{e}_x \qquad \vec{F}_2 = - P_2 A_2 \vec{e}_x$

- $E_p = mgy$

Dim.: $\Delta E_c = W(\Sigma \vec{F}) = W(\Sigma \vec{F}_{est})$ — teor. energia cinetica

ΔE_c ↑ ③ ; $\Sigma \vec{F}_{est}$ ← ① p , $\vec{g}$ ②

① $W_1 = P_1 A_1 \vec{e}_x \cdot \vec{v}_1 dt = P_1 A_1 dx_1 = P_1 V_1$

$W_2 = -P_2 A_2 \vec{e}_x \cdot \vec{v}_2 dt = -P_2 A_2 dx_2 = -P_2 V_2$

$\Rightarrow \quad W_1 + W_2 = (P_1 - P_2) V$

② $W = -\Delta E_p$

$\Delta E_p = (mgy_2 + E_p^\circ) - (mgy_1 + E_p^\circ) = mg(y_2 - y_1)$

③ $\Delta E_c = \left(\frac{1}{2} m v_2^2 + E_c^\circ\right) - \left(\frac{1}{2} m v_1^2 + E_c^\circ\right) = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

↖ [3] stazionario

$\Delta E_c + \Delta E_p = W_1 + W_2$

$\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + mgy_2 - mgy_1 = P_1 V - P_2 V$

$\frac{1}{2} \rho v_2^2 + \rho g y_2 + P_2 = \frac{1}{2} \rho v_1^2 + \rho g y_1 + P_1 \quad \Rightarrow \quad \frac{1}{2} \rho v^2 + \rho g y + P = \text{cost}$ □

Teorema di Bernoulli

$$\frac{1}{2}\rho v^2 + \rho g y + P = \text{cost}$$

$$\frac{1}{2}\rho v_1^2 + \rho g y_1 + P_1 = \frac{1}{2}\rho v_2^2 + \rho g y_2 + P_2$$

Stevino

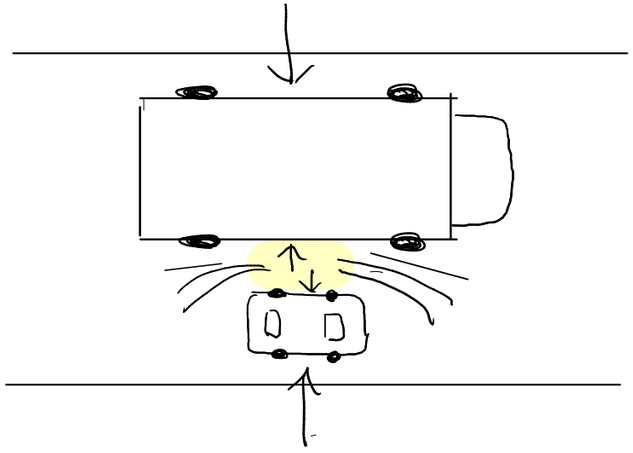
$$\Delta P = -\rho g \Delta y$$

Casi particolari:

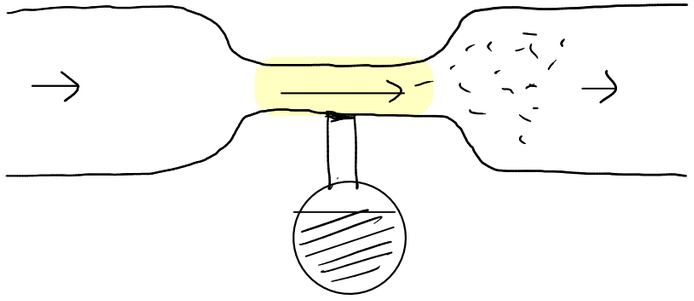
1) $\vec{v} = \vec{0} \Rightarrow P_1 + \rho g y_1 = P_2 + \rho g y_2 \rightarrow P_2 - P_1 = \rho g (y_1 - y_2) = -\rho g (y_2 - y_1)$

2) $\vec{v} = \text{cost}$ idem!

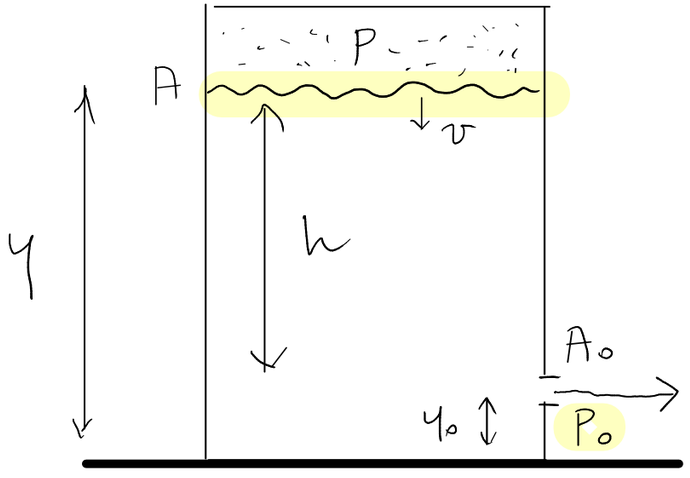
3) $y = \text{cost} \Rightarrow \frac{1}{2}\rho v^2 + P = \text{cost}$, $A v = \text{cost}$ $\quad A \downarrow \; v \uparrow \; P \downarrow$



Effetto Venturi



Es: velocità di uscita di un fluido da un contenitore



Fluido ideale $\rho = cost$

$A_0 << A \Rightarrow v << v_0 \Rightarrow v \approx 0$

Teor. Bernoulli $\Rightarrow$ $v_0 = ?$

$$\frac{1}{2}\rho v^2 + \rho g y + P = cost$$

$$\frac{1}{2}\rho v^2 + \rho g y + P = \frac{1}{2}\rho v_0^2 + \rho g y_0 + P_0$$

($\frac{1}{2}\rho v^2 \approx 0$)

$$\frac{1}{2}\rho v_0^2 = P - P_0 + \rho g (y - y_0) = P - P_0 + \rho g h$$

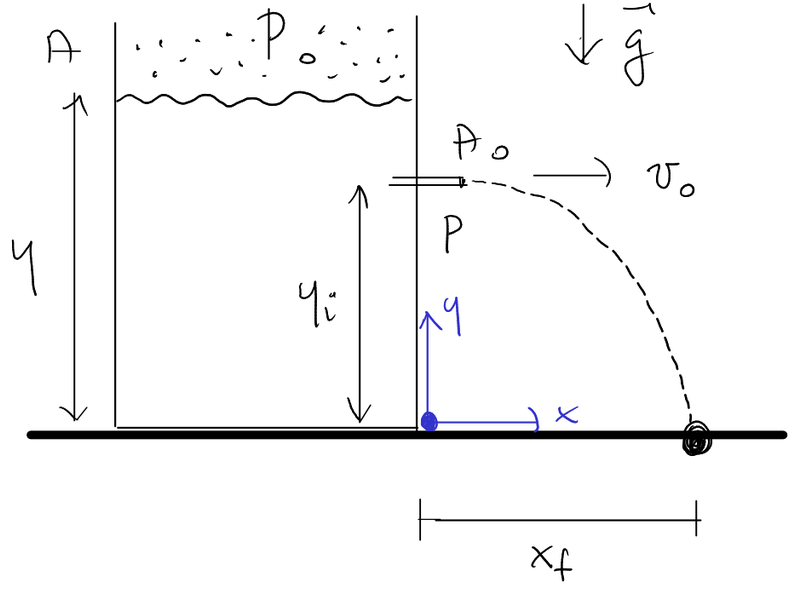
$$v_0 = \sqrt{2\frac{P - P_0}{\rho} + 2gh}$$

casi limite :

1) $P >> P_0$: $v_0 \approx \sqrt{\frac{2P}{\rho}}$

2) $P = P_0$: $v_0 = \sqrt{2gh}$ → legge di Torricelli

Variante: contenitore aperto, gittata massima?



fluido ideale

$A_0 \ll A$

$P = P_0$

gittata x_f = ?

quale y_i determina la gittata massima?

$$v_0 = \sqrt{2g(y - y_i)}$$

Moto uniformemente accelerato in 2d

$$\begin{cases} x = x_i + v_{xi} t \\ y = y_i + v_{yi} t - \frac{1}{2} g t^2 \end{cases}$$

Condizioni iniziali:

$x_i = 0$, $y_i = y_i$

$v_{xi} = \sqrt{2g(y - y_i)}$ $\quad v_{yi} = 0$

Gittata x_f : $\quad y(t_f) = 0 \quad \Rightarrow \quad x_f = x(t_f)$

$$\begin{cases} x_f = \sqrt{2g(y - y_i)}\; t_f \\ 0 = y_i - \frac{1}{2} g t_f^2 \end{cases}$$

$$t_f = \sqrt{\frac{2y_i}{g}}$$

$$x_f = \sqrt{2g(y - y_i)} \cdot \sqrt{\frac{2y_i}{g}} = \sqrt{4\, y_i\, (y - y_i)} \qquad \text{gittata} \qquad \sqrt{y\, y_i - y_i^2} \;\uparrow\; \sqrt{y_i (y - y_i)}$$

Massimo di x_f in funzione di y_i

$$\frac{dx_f}{dy_i} = 2\, \frac{y - 2y_i}{2\sqrt{y_i(y - y_i)}}$$

$$\frac{dx_f}{dy_i}(y_i = y_i^*) = 0 \quad \Rightarrow \quad y = 2y_i^*$$

$$\Rightarrow y_i^* = \frac{1}{2} y \qquad \square$$