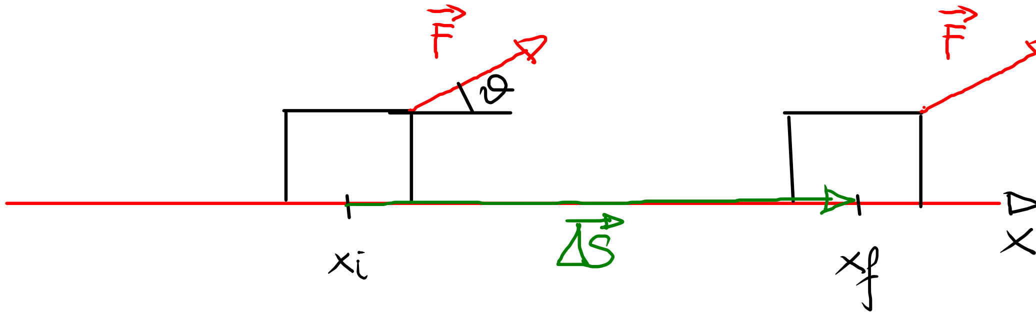
LAVORO di $\vec{F}$ su uno $\vec{\Delta S}$



livello base: $\vec{\Delta S}$ è rettilineo
$\vec{F}$ è costante
ϑ è l'angolo tra $\vec{\Delta S}$ e $\vec{F}$

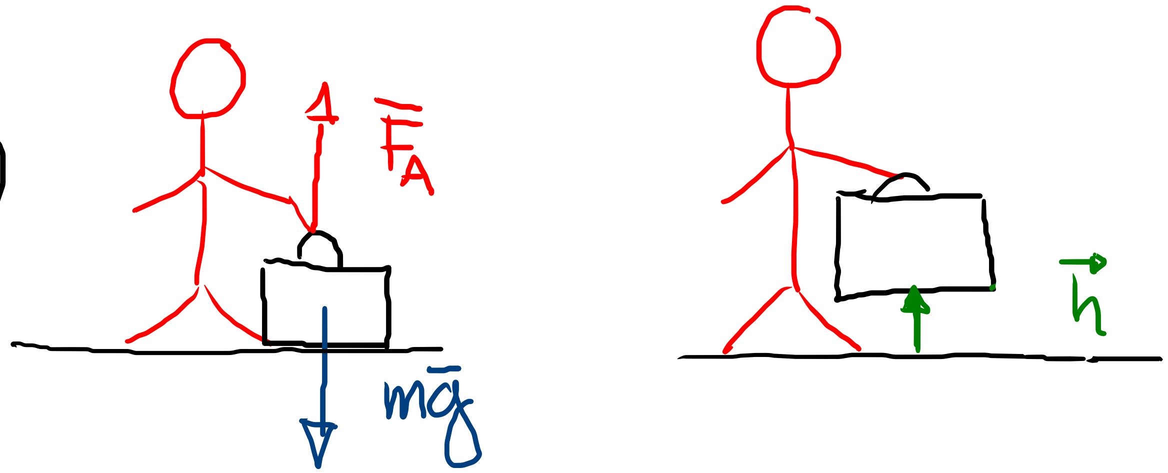
Definisco $\mathcal{L} = \vec{F} \cdot \vec{\Delta S} = |\vec{F}| \cdot |\vec{\Delta S}| \cdot \cos\vartheta$ $\begin{cases} + & 0 \le \vartheta < \frac{\pi}{2} \\ 0 & \text{per } \vartheta = \frac{\pi}{2} \\ - & \frac{\pi}{2} < \vartheta \le \pi \end{cases}$

$$[\mathcal{L}] = [\vec{F}] \cdot [\vec{\Delta S}] = N \cdot m = J \quad (\text{Joule})$$

L'UOMO CON LA VALIGIA

$$\vec h = h\hat{j}$$

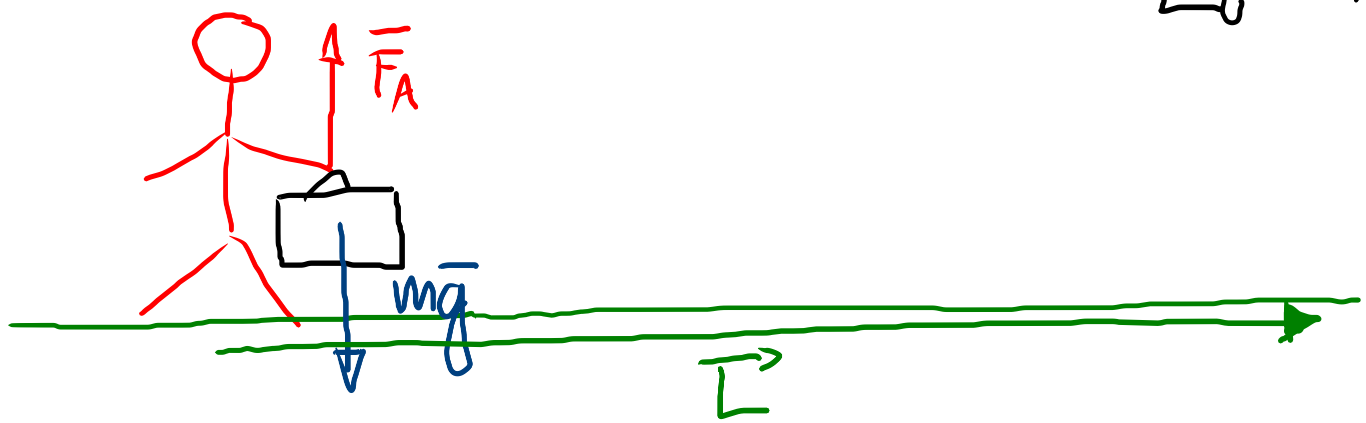
(1)



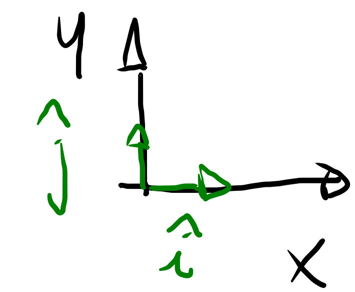
$$\sum \vec F = \vec F_A + m\vec g = 0$$

$$L_U = \vec F_A \cdot \vec h = m\vec g \cdot \vec h = mgh$$

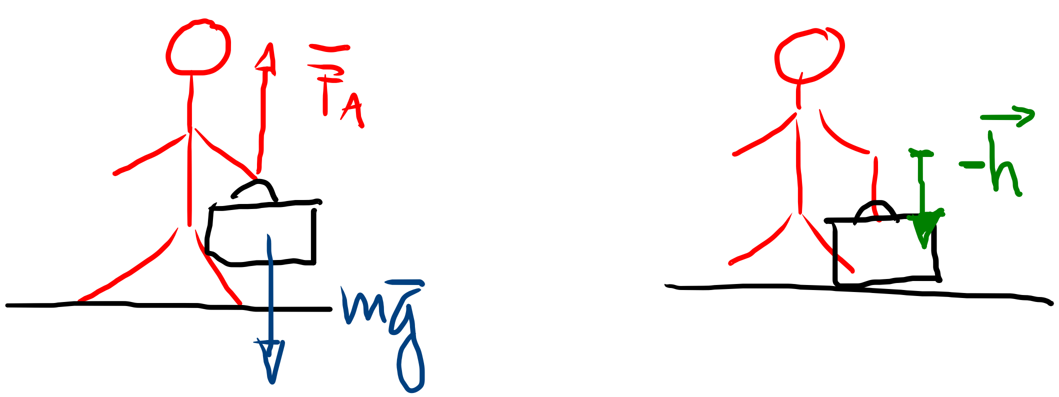
(2)



$$L_U = \vec F_A \cdot \vec L = 0$$



(3)



$$L_G = m\vec g \cdot (-\vec h) = -m\vec g \cdot \vec h$$

$$= -m|\vec g||\vec h| \cancel{\sin} \pi = mgh$$

COS π

LAVORO DI UNA F VARIABILE
(caso 1D)

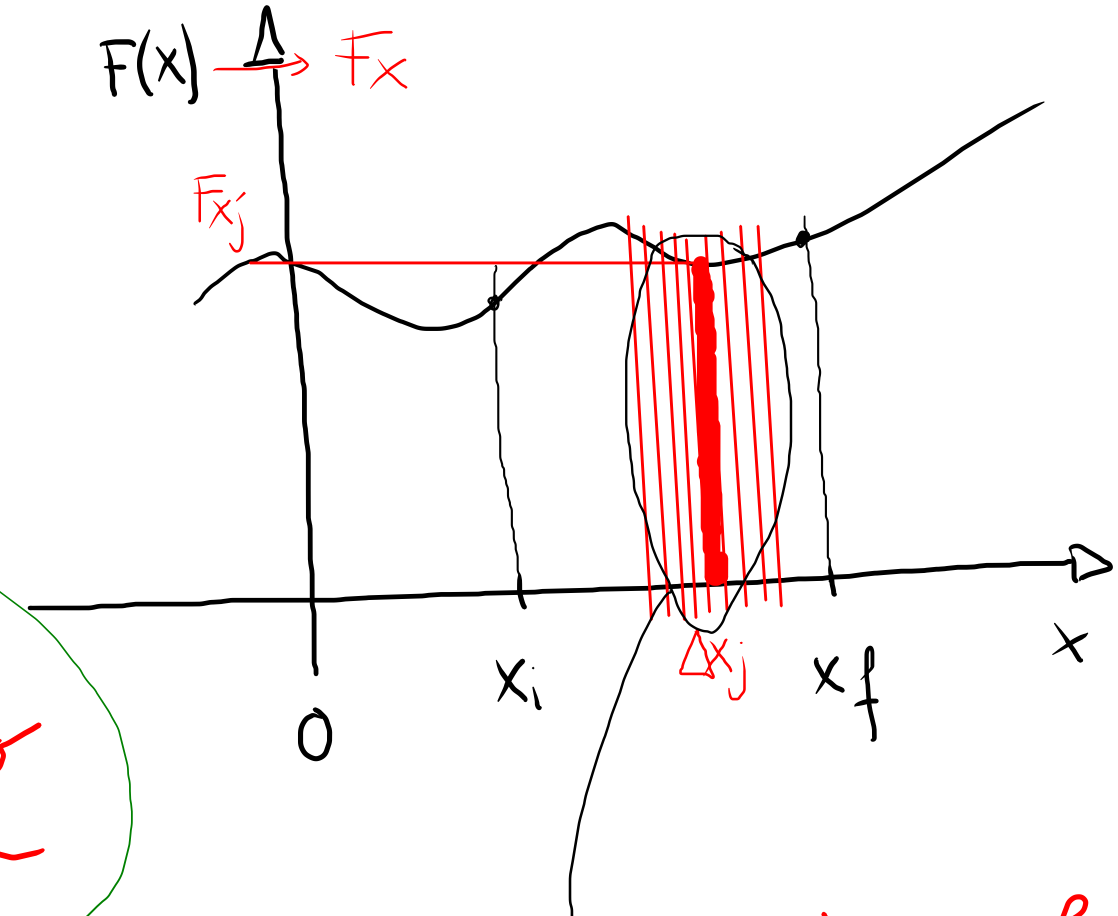
1D lungo asse x : $x_i \to x_f$

F è variabile: $F(x) \to F_x$

$\mathcal{L}_j = F_{x_j} \cdot \Delta x_j$

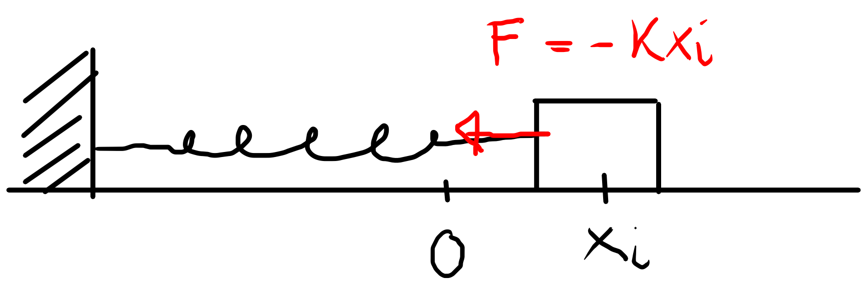
$$\mathcal{L} = \sum_j \mathcal{L}_j = \sum_j \underbrace{F_{x_j} \cdot \Delta x_j}_{A_j} = \sum_j A_j = \text{area sotto la curva}$$

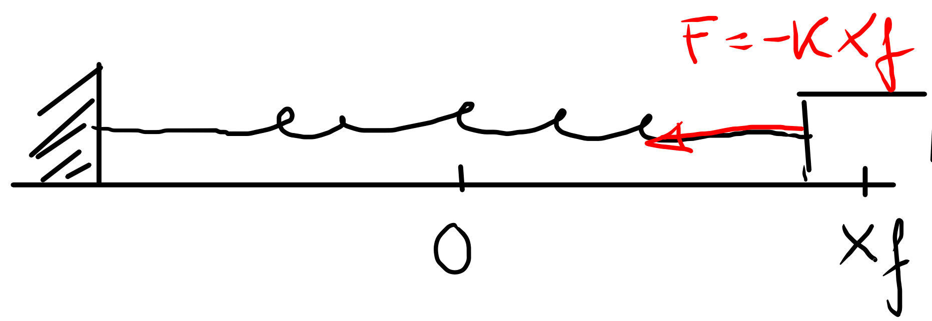
$$\mathcal{L} = \lim_{\Delta x_j \to 0} \sum_j \mathcal{L}_j = \int_{x_i}^{x_f} F(x)\, dx = \text{area sotto la curva}$$

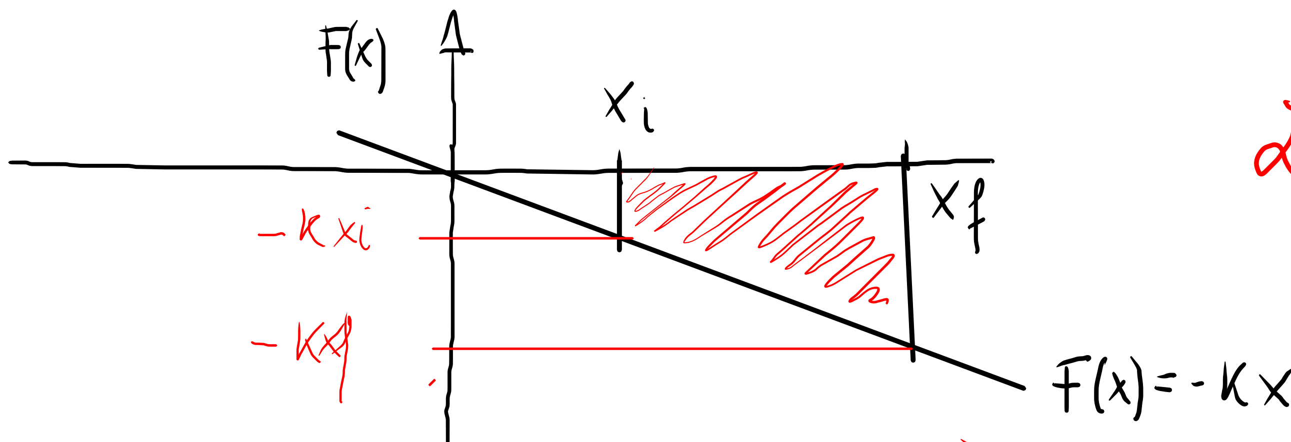


$A_j = F_{x_j} \cdot \Delta x_j = \mathcal{L}_j$

ESEMPIO: $\vec{F} = -K\vec{x}$ (forza elastica)







$$\mathcal{L} = \int_{x_i}^{x_f} F(x)\,dx = \int_{x_i}^{x_f} -Kx\,dx$$

$$= -K \int_{x_i}^{x_f} x\,dx = -K \, \frac{1}{2} x^2 \Big|_{x_i}^{x_f}$$

$$= -\frac{1}{2} K \left(x_f^2 - x_i^2\right) = \frac{1}{2} K \left(x_i^2 - x_f^2\right)$$

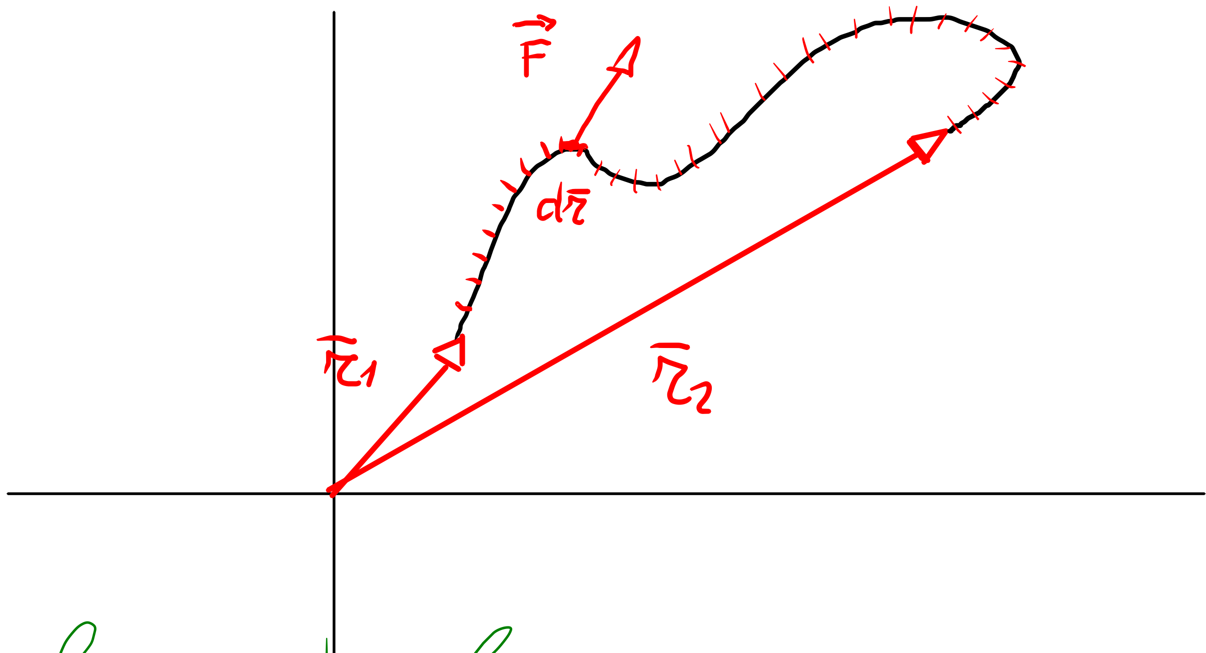
$$\mathcal{L} = -\frac{1}{2} K (x_i + x_f)(x_f - x_i)$$

$$= \frac{1}{2} K \left(x_i^2 - x_f^2\right)$$

DEFINIZIONE GENERALE di $\mathcal{L}$

$$\mathcal{L} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}) \cdot d\vec{r}$$

prodotto scalare!



integrale di linea!

ovvero va calcolato lungo la particolare traiettoria del moto

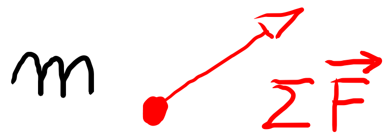
Se ho molte Forze, posso valutare $\Sigma\vec{F}$ e

$$\mathcal{L} = \int_{\vec{r}_1}^{\vec{r}_2} \Sigma\vec{F} \cdot d\vec{r}$$

ENERGIA

Energia cinetica: m, $\vec{v}$ $\quad K = \frac{1}{2} m v^2$

TEOREMA LAVORO - ENERGIA



$$\mathcal{L} = \Delta K = K_f - K_i$$

$\mathcal{L}$ di $\Sigma\vec{F}$: $\mathcal{L} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} m (v_f^2 - v_i^2)$

($\mathcal{L}$ è anche il lavoro totale, ovvero la somma dei lavori delle singole forze prese singolarmente)

Dimostrazione di $\mathcal{L} = \Delta K$

II principio dinamica

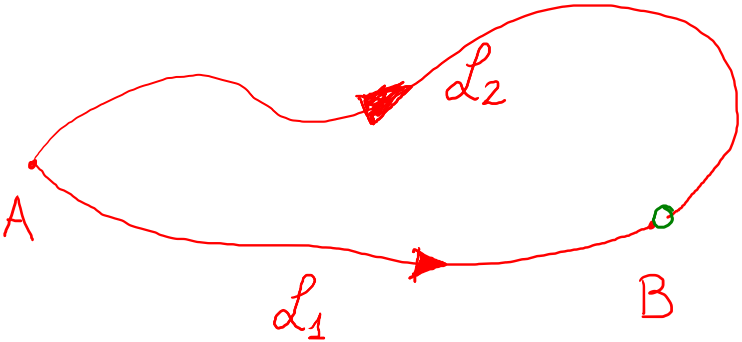
$$\mathcal{L} = \int_{x_i}^{x_f} \underbrace{\sum F(x)}_{ma}\, dx = \int_{x_i}^{x_f} ma\, dx = \int_{x_i}^{x_f} m \frac{dv}{dt}\, dx$$

$$= \int_i^f m\, dv \underbrace{\frac{dx}{dt}}_{v} = \int_{v_i}^{v_f} m v\, dv = m \int_{v_i}^{v_f} v\, dv =$$

$$= m\, \frac{1}{2} v^2 \Big|_{v_i}^{v_f} = \frac{1}{2} m \left(v_f^2 - v_i^2 \right) = K_f - K_i = \Delta K$$

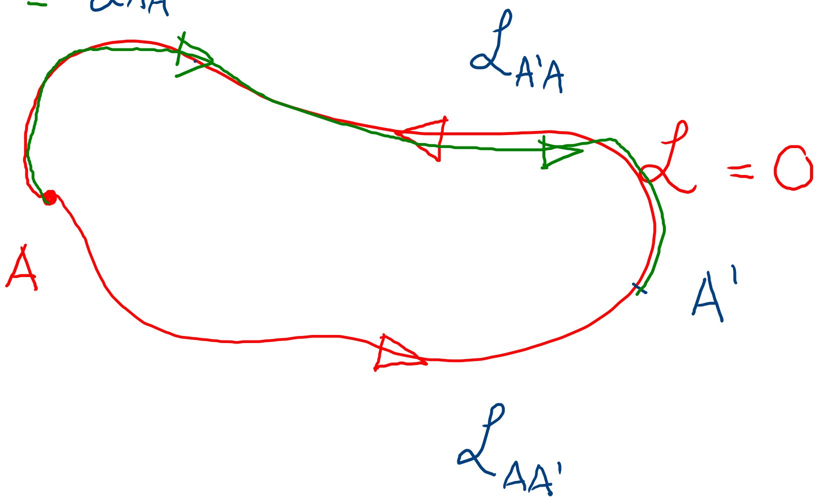
$$\boxed{\vec{F} = m\vec{a} \iff \mathcal{L} = \Delta K}$$

FORZE CONSERVATIVE



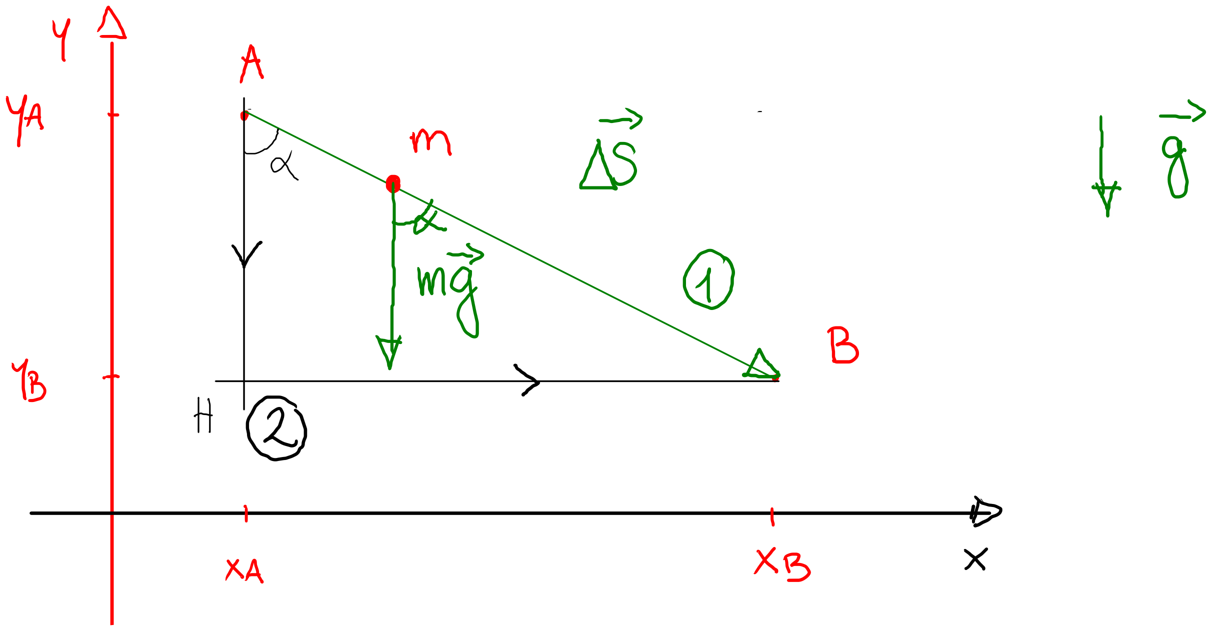
$$\mathcal{L}_1 = \mathcal{L}_2 = \mathcal{L}_{AB}$$

$$\mathcal{L}'_{AA'} = -\mathcal{L}_{A'A}$$



$$\mathcal{L} = \mathcal{L}_{AA'} + \mathcal{L}_{A'A} = \mathcal{L}_{AA'} - \mathcal{L}'_{AA'} = 0$$

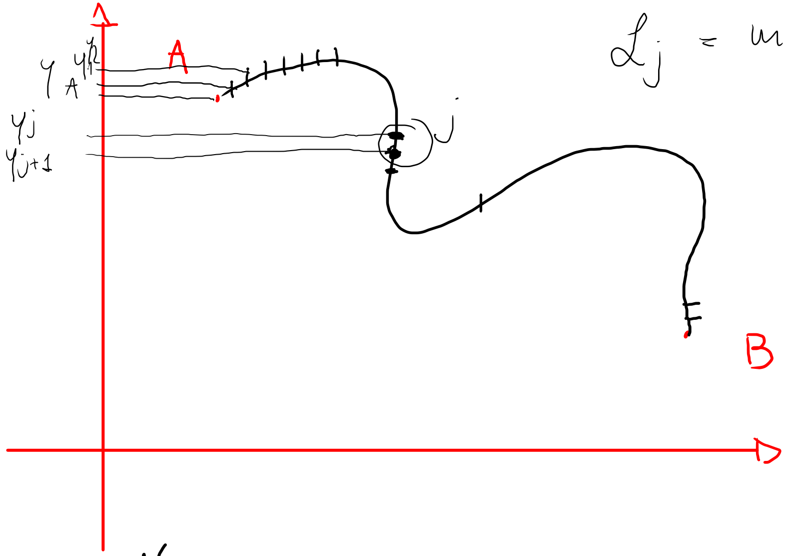
ESEMPIO : FORZA PESO



1 $$\mathcal{L} = \vec{F} \cdot \vec{\Delta S} = |m\vec{g}| \cdot |\Delta \vec{S}| \cos\alpha = mg\Delta s \cos\alpha$$
$$= mg\,(y_A - y_B)$$

2 $$\mathcal{L} = \mathcal{L}_{AH} + \mathcal{L}_{HB}$$
$$= mg\,(y_A - y_B) + 0 = mg\,(y_A - y_B)$$

Percorso qualsiasi... (approfondimento, non in programma)



$$\mathcal{L}_j = mg\left(y_j - y_{j+1}\right)$$

$$\mathcal{L} = \sum_{1}^{N} \mathcal{L}_j = mg\left(y_A - \cancel{y_1}\right) + mg\left(\cancel{y_1} - \cancel{y_2}\right) + \cancel{\ldots}$$

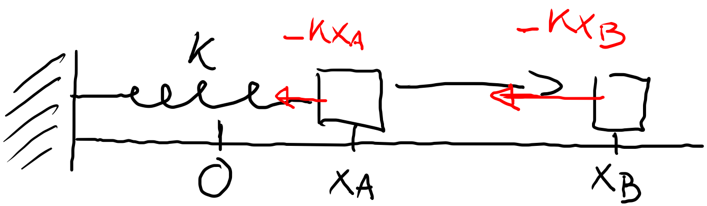
$$+ \ldots\, mg\left(\cancel{y_j} - \cancel{y_{j+1}}\right) + mg\left(\cancel{y_{j+1}} - \cancel{y_{j+2}}\right)$$

$$+ mg\left(\cancel{y_N} - y_B\right) = mg\left(y_A - y_B\right)$$

ULTERIORE ESEMPIO: Forza elastica

$$\vec{F} = -k\vec{x}$$

Ieri abbiamo calcolato il lavoro della forza elastica per andare da x_A a x_B:



$$\mathcal{L} = \frac{1}{2} k\left(x_A^2 - x_B^2\right)$$

Il lavoro dipende solo da x_A e x_B

$\Rightarrow$ La forza elastica è conservativa!

ENERGIA POTENZIALE

Per ogni forza CONSERVATIVA posso definire

$U(\vec{r})$ tale che $\mathcal{L}_{AB} = U(\vec{r}_A) - U(\vec{r}_B) \overset{!}{=} U_A - U_B = -\Delta U$

Esempi precedenti

Forza peso : $\mathcal{L}_{AB} = mg(y_A - y_B)$

$\boxed{U_g = mgy}$

gravitazionale

$\mathcal{L}_{AB} = U_A - U_B \overset{!}{=} mgy_A - mgy_B$

Forza elastica $\mathcal{L}_{AB} = \frac{1}{2}k(x_A^2 - x_B^2)$

$\boxed{U_e = \frac{1}{2}kx^2}$

elastica

$\mathcal{L}_{AB} = U_A - U_B \overset{!}{=} \frac{1}{2}kx_A^2 - \frac{1}{2}kx_B^2$

ENERGIA MECCANICA

$$\boxed{E_{mecc} = K + U}$$

Sistema conservativo: il $\mathcal{L}$ è fatto da forze conservative

① $\mathcal{L} = \Delta K = K_B - K_A$ (vale sempre)

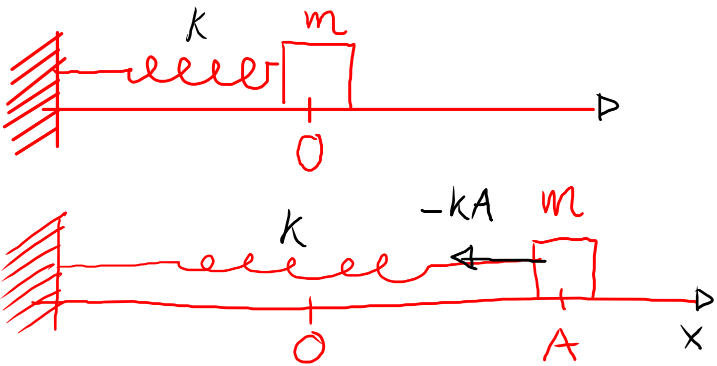
② $\mathcal{L} = -\Delta U = U_A - U_B$ (solo per sistemi conservativi)

①② $\underbrace{0 = \Delta K + \Delta U}$ $=$ $\underbrace{K_B + U_B - (K_A + U_A)}$

$$\Delta K + \Delta U = 0$$
$$\Delta(K + U) = 0$$
$$\boxed{\Delta E_{mecc} = 0}$$

$$K_B + U_B - (K_A + U_A) = 0$$
$$E_{mecc\,B} - E_{mecc\,A} = 0$$
$$\boxed{\Delta E_{mecc} = 0}$$

L'Emecc si conserva.



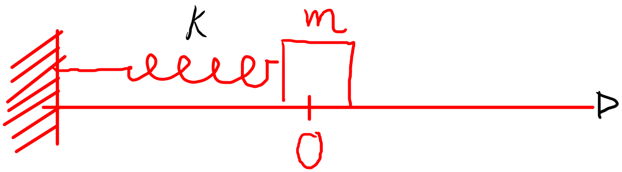
cinematica

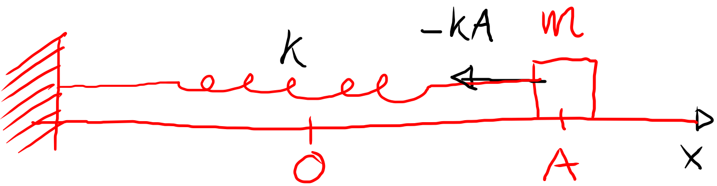
$$
\begin{cases}
x(t) = A\cos(\omega t) & (*) \\
v(t) = -A\omega\,\mathrm{sen}(\omega t) & (**) \\
a(t) = -A\omega^2\cos(\omega t) = -\omega^2 x(t)
\end{cases}
\qquad \omega = \frac{2\pi}{T}
$$

dinamica: $\omega = \sqrt{\frac{k}{m}}$ $\quad \omega^2 = \frac{k}{m}$

energia potenziale elastica: $U_e = \frac{1}{2}kx^2$

$$
\begin{aligned}
E_{mecc} = K + U_e &= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \\
&= \frac{1}{2}m\left[\underset{**}{-A\omega\,\mathrm{sen}(\omega t)}\right]^2 + \frac{1}{2}k\left[\underset{*}{A\cos(\omega t)}\right]^2 \\
&= \frac{1}{2}\underline{m}\,A^2\underline{\omega^2}\,\mathrm{sen}^2(\omega t) + \frac{1}{2}kA^2\cos^2(\omega t) \qquad *** \\
&= \frac{1}{2}kA^2\,\mathrm{sen}^2(\omega t) + \frac{1}{2}kA^2\cos^2(\omega t) \\
&= \frac{1}{2}kA^2\underbrace{\left[\mathrm{sen}^2(\omega t) + \cos^2(\omega t)\right]}_{1} = \frac{1}{2}kA^2
\end{aligned}
$$





$$E_{mecc} = K + U_e = \frac{1}{2} k A^2$$

cinematica

$$\begin{cases} x(t) = A\cos(\omega t) & * \\ v(t) = -A\omega\, \mathrm{sen}(\omega t) & ** \\ a(t) = -A\omega^2 \cos(\omega t) = -\omega^2 x(t) \end{cases} \qquad \omega = \frac{2\pi}{T}$$

dinamica : $\omega = \sqrt{\frac{k}{m}}$ $\quad \omega^2 = \frac{k}{m}$

energia potenziale elastica : $U_e = \frac{1}{2} k x^2$

$x = 0 \Rightarrow U_e = 0$

$$K = \frac{1}{2} k A^2$$

$$\cancel{\frac{1}{2}} m v^2 = \cancel{\frac{1}{2}} k A^2$$

$$v^2 = \frac{k}{m} A^2 = \omega^2 A^2$$

$$v = \pm \omega A$$

$x \neq 0$

$$\cancel{\frac{1}{2}} k A^2 = \cancel{\frac{1}{2}} m v^2 + \cancel{\frac{1}{2}} k x^2$$

$$\omega^2 A^2 = v^2 + \omega^2 x^2$$

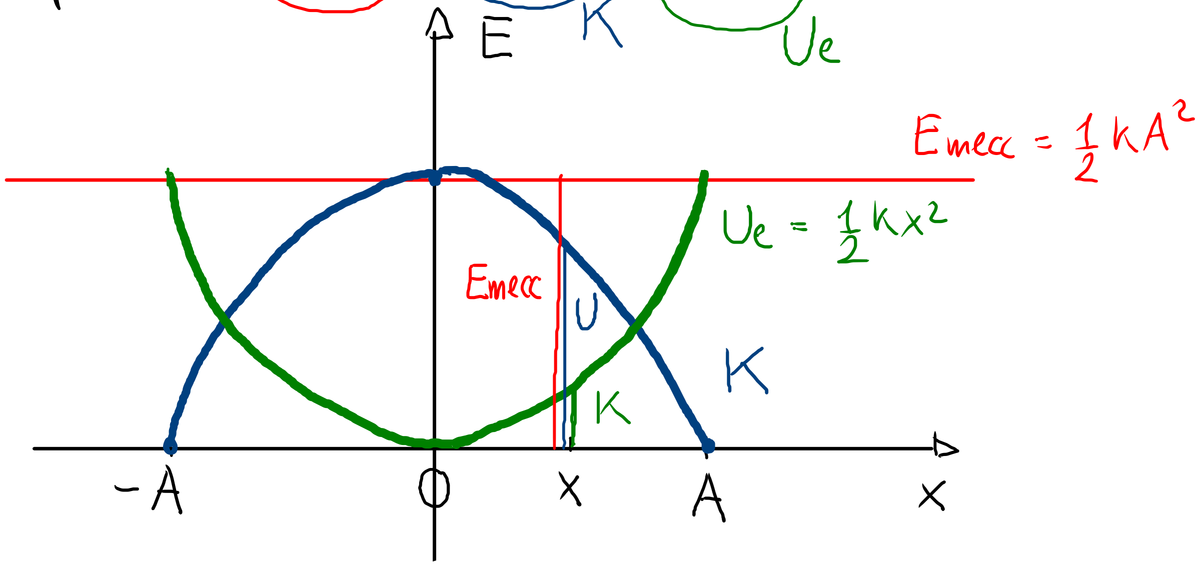
$$v^2 = \omega^2 (A^2 - x^2)$$

$$v = \pm \omega \sqrt{A^2 - x^2}$$

In forma grafica:

$$E_{mecc} = \underset{K}{\frac{1}{2} m v^2} + \underset{U_e}{\frac{1}{2} k x^2} = \frac{1}{2} k A^2$$

$K = E_{mecc} - U_e$



- $\mathcal{L} = \Delta K$ — Vale SEMPRE

$\mathcal{L} = -\Delta U$ — vale solo x sistemi conservativi

In un sistema conservativo valgono entrambe

$$\begin{cases} \mathcal{L} = \Delta K \\ \mathcal{L} = -\Delta U \end{cases} \Rightarrow \Delta K = -\Delta U \qquad \Delta K + \Delta U = 0$$

Definiamo ora $K + U = E_{mecc}$

$$\Delta K + \Delta U = 0$$

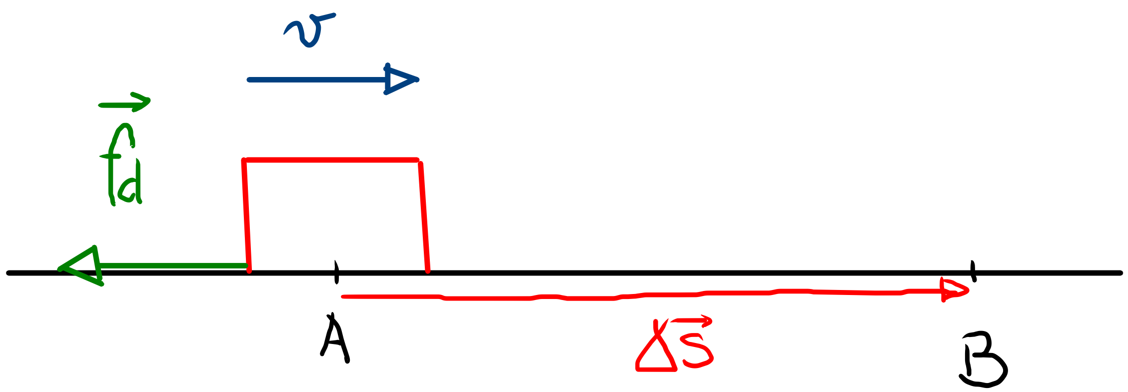
$$\Delta(K + U) = 0$$

$$\Delta E_{mecc} = 0$$

Emecc si conserva!

FORZE DISSIPATIVE (ovvero non conservative)

Es: attrito dinamico



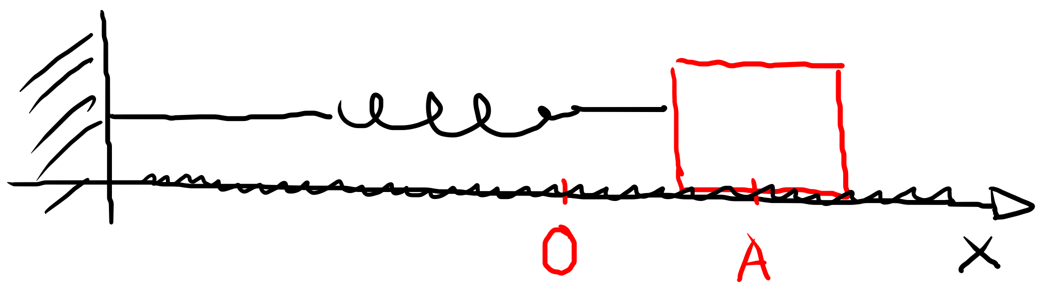
$$\mathcal{L}_d = \vec{f_d} \cdot \vec{\Delta s} = f_d \, \Delta s \cdot \cos\pi \overset{!}{=} -f_d \, \Delta s$$

negativo!

dipende dal percorso!

SISTEMA CON FORZE CONSERVATIVE E DISSIPATIVE

Es.



piano liscio

$\Rightarrow$ conservativo

$$U = \frac{1}{2} K x^2$$

piano ruvido $\Rightarrow$ c'è attrito

non conservativo

$$\mathcal{L} = \Delta K$$

↑ somma dei lavori fatti da ciascuna forza

assumiamo: una forza conservativa $\mathcal{L}_C$

una forza dissipativa $\mathcal{L}_D$

$$\mathcal{L} = \mathcal{L}_C + \mathcal{L}_D$$

$$\mathcal{L}_C + \mathcal{L}_D = \Delta K$$

$$\mathcal{L}_C = -\Delta U$$

$$-\Delta U + \mathcal{L}_D = \Delta K$$

$$\mathcal{L}_D = \boxed{\Delta K + \Delta U = \Delta E_{mecc}} \quad *$$

$$\Delta E_{mecc} = \mathcal{L}_D < 0$$

Emecc NON si conserva ma diminuisce ($\Delta E_{mecc} < 0$)

SISTEMI ISOLATI

Non scambia né materia né energia con l'ambiente che lo circonda

Definisco E_{int}, energia interna del sistema.

E_{TOT}, energia totale del sistema. ← si conserva!

$$E_{TOT} = E_{mecc} + E_{int}$$

$$\Delta E_{TOT} = \underbrace{0 = \Delta E_{mecc} + \Delta E_{int}}$$

$$\Delta E_{int} = -\underbrace{\Delta E_{mecc}}_{<0} > 0$$

Quindi da *

$$\boxed{\Delta K + \Delta U = \Delta E_{mecc}}$$

$$\Delta K + \Delta U - \Delta E_{mecc} = 0$$

$$\Delta K + \Delta U + \Delta E_{int} = 0$$

$$\Delta E_{TOT} = 0$$ → si conserva!

POTENZA

$$P_m = \frac{\mathcal{L}}{\Delta t} \qquad \frac{J}{s} = \text{Watt} \qquad \text{(potenza media)}$$

$$P = \lim_{\Delta t \to 0} \frac{\mathcal{L}}{\Delta t} \qquad \text{(potenza istantanea)}$$

$$\mathcal{L} = \vec{F} \cdot \Delta\vec{x}$$

$$= \lim_{\Delta t \to 0} \frac{\vec{F} \cdot \Delta\vec{x}}{\Delta t} = \lim_{\Delta t \to 0} \vec{F} \cdot \frac{\Delta\vec{x}}{\Delta t} = \vec{F} \cdot \vec{v} \qquad \boxed{P = \vec{F} \cdot \vec{v}}$$

---

nota: $\quad 1 \text{ kWh} = \underbrace{1 \text{ kW}}_{\frac{kJ}{\cancel{s}}} \cdot \underbrace{1h}_{3600\,\cancel{s}} = 3600 \text{ kJ}$

RENDIMENTO

$$\eta = \frac{\mathcal{L}}{E}$$

$\mathcal{L}$ ← lavoro compiuto dalla macchina

E ← energia necessaria a far funzionare

in percentuale

$$\eta = \frac{\mathcal{L}}{E} \cdot 100 \qquad (\%)$$

(fine programma <u>obbligatorio</u> meccanica)

<u>seguono approfondimenti</u>