

# *Solving Integrals with Excel*

LECTURE 17 – 11TH MAY 2023

Technology in Mathematics Education



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graph LR; A["Numerical method for integration"] --- B["The trapezoidal rule"]; A --- C["Simpson's rule"]
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*Numerical method for integration*

*The trapezoidal rule*

*Simpson's rule*

# *The trapezoidal rule*

The problem is to find a numerical approximation for the integral

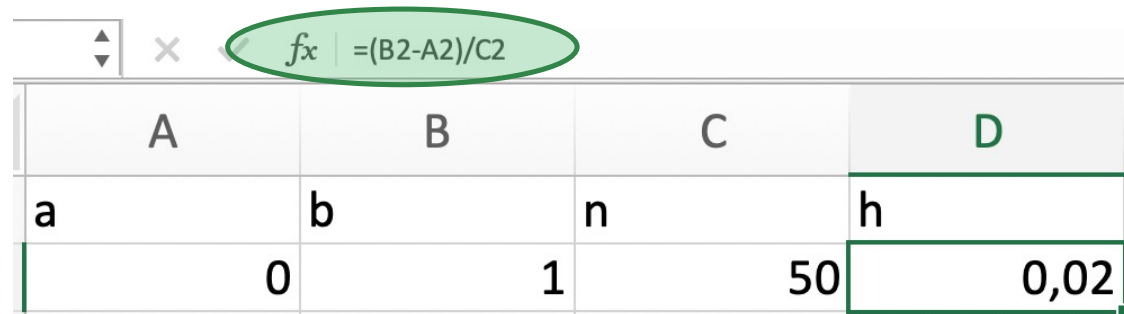
$$I = \int_a^b f(x) dx .$$

The trapezoidal rule works by approximating the function  $f(x)$  by a piecewise linear function and evaluate the integral of each piece. If the interval  $[a, b]$  is divided up into  $n$  equal subinterval, each of width  $h = \frac{b-a}{n}$ , then the approximate integral is

$$I \approx \frac{h}{2} \sum_{i=1}^n f(x_{i-1}) + f(x_i), \text{ where, } x_i = a + ih, \text{ and } i = 0, 1, \dots, n$$

**Example:**  $\int_0^1 (14x^6 + 7)dx.$

The endpoints (initial and terminal) of the interval, and the number of divisions are entered in the cells A2, B2, C2, respectively. The value of h is calculated in the cell D2 by entering the formula = (B2-A2)/C2.



	A	B	C	D
a		b	n	h
	0	1	50	0,02

**Generate**  $x_i = a + ih$ , and  $i = 0, 1, \dots, n$

The next values are generated with the formula = IF(E2>=\$B\$2; \$B\$2; E2+\$D\$2) in E3. This formula adds h to the previous value until we reach the value of  $b$ . Afterwards it keeps entering the value of  $b$ . This mechanism is used to enable changing the value of  $n$  to get more control on the accuracy of the solution

A	B	C	D	E	F
	b	n	h	x_i	f(x_i)
	0	1	50	0,02	0
				0,02	
				0,04	
				0,06	

Copy this formula to the next 100 cells or so below E4. The figure shows a part of the sheet further down, where you can see the value of  $b$  being repeated.

**Generate**  $f(x_i) = (14x^6 + 7)$

The function  $f(x_i)$  is entered in the column labeled F by entering the formula `=14*E2^6+7` in the cell F2 and copying it along the corresponding cells for the all entries.

fx					
B	C	D	E	F	
b	n	h	x_i	f(x_i)	
1	50	0,02	0	7	
			0,02	7	
			0,04	7,0000001	
			0,06	7,0000007	

# Generate

$$I \approx \frac{h}{2} \sum_{i=1}^n f(x_{i-1}) + f(x_i),$$

Then, generate each term of the trapezoidal rule, calling it A\_i in the spreadsheet. We then form the elements of the summation in the trapezoidal rule by entering the formula = (E3-E2)/2\*(F2+F3) in cell G3 and copying it along the corresponding cells for the x\_i terms.

fx = (E3-E2)/2*(F2+F3)						
Insert Function						
B	C	D	E	F	G	
b	n	h	x_i	f(x_i)	A_i	
	1	50	0,02	0	7	
				0,02	7	0,14
				0,04	7,0000001	0,14
				0,06	7,0000007	0,14

## ***Note:***

Observe that, instead of using the value of  $h$  generated in cell D2, we used the equivalent difference E3-E2. This has two advantages:

1. The formula produces zeros when we go past the right endpoint  $b$ . In this way, the final sum of these numbers is not affected by the repetition of  $b$ .
2. It allows the use of the trapezoidal rule with non-uniform divisions of the interval  $[a, b]$ .



# Calculating the Integral

The last step is to add the terms in column G to get the approximation of the integral. Select the range of cells that contains the summation terms and then click the sum button ( $\Sigma$ ) on the toolbar.

	C	D	E	F	G	H
	n	h	$x_i$	$f(x_i)$	$A_i$	I
1	50	0,02	0	7		9,0027996
			0,02	7	0,14	
			0,04	7,0000001	0,14	

## *More...*

- (a) If you now change the number of divisions  $n$  to 100, the new, more accurate approximation will appear in the same cell (G103).
- (b) To change the interval of integration all you need to do is to change the values  $a, b$  in cells A2, B2.
- (c) To change the integrated function enter the new formula in cell F2 and copy it to cell F103.

	B	C	D	E	F	G	H
	b	n	h	$x_i$	$f(x_i)$	$A_i$	I
0	1	100	0,01	0	7		9,0007
				0,01	7	0,07	
				n n2	7	n n7	

## *Simpson's rule*

Simpson's rule finds an approximation of the value of the integral I by approximating the integrand with a piecewise polynomial of degree 2 and then evaluate the integral over each piece.

Simpson's formula is:

$$\int_a^b f(x) dx \approx \frac{b-a}{6} \left[ f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \quad \text{with } n=2$$

$$\int_a^b f(x) dx \approx \frac{1}{3}h \sum_{i=1}^{n/2} [f(x_{2i-2}) + 4f(x_{2i-1}) + f(x_{2i})] \quad \text{with } n \text{ an even number}$$

## *Simpson's rule*

Simpson's rule finds an approximation of the value of the integral  $I$  by approximating the integrand with a piecewise polynomial of degree 2 and then evaluate the integral over each piece.

Simpson's formula is:

$$I \approx \frac{h}{3} \sum_{i=1}^{n-1} [f(x_{i-1}) + 4f(x_i) + f(x_{i+1})] \quad \text{with } n \text{ an even number}$$

Where the interval  $[a, b]$  is divided into  $n$  intervals, each of length  $h = \frac{b-a}{n}$

# Implementation

The Excel implementation of Simpson's rule is very much similar to that of the trapezoidal rule, except for some details. The following figure shows the upper part of the worksheet implementation.

fx   =(E4-E2)/6*(F2+4*F3+F4)							
	B	C	D	E	F	G	H
	b	n	h	x <sub>i</sub>	f(x <sub>i</sub> )	S <sub>i</sub>	I
0	1	40	0,03	0	7		9,0000036
				0,03	7	0,35	
				0,05	7,0000002		
				0,08	7,0000025	0,3500002	

Since Simpson's formula spans two subintervals for each entry in the summation, the copying of the formula is done as follows. Select the range of two cells G2, G3 (note that G2 is actually empty). Using the formula copying technique, drag the two cells down to cell G103. The result is that the formula is copied to every other cell. One more thing to notice here is that, in the above formula, the value of  $h$  is replaced by the difference over 3 cells divided by 2.

	$x_i$	$f(x_i)$	$S_i$
3	0		7
	0,03		7
	0,05	7,0000000	

This way the same skipping is achieved and no problem arises as a result of repeating the values of  $b$ . When you select the range G2:G103 and click the sum button, you will see the result 9.000003644, which is more accurate than the result of the trapezoidal rule as the theory predicts.