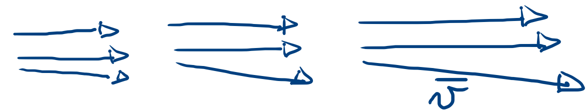
FLUIDODINAMICA

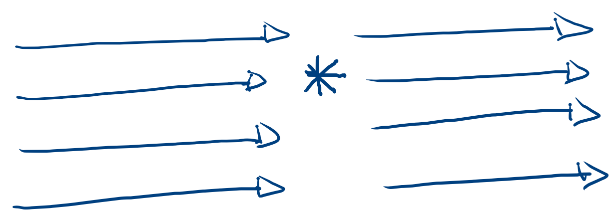
Ipotesi semplificative

fluido ideale
- incomprimibile $\rho = \text{cost.}$ (1)
- non viscoso $\eta = 0$ (2)

flusso
- stazionario (3)
  $\bar{v}$ sia costante nel tempo in ogni punto del flusso

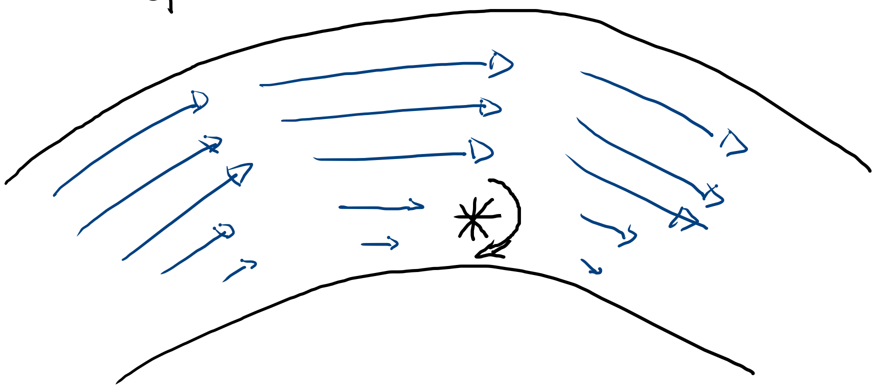


- irrotazionale



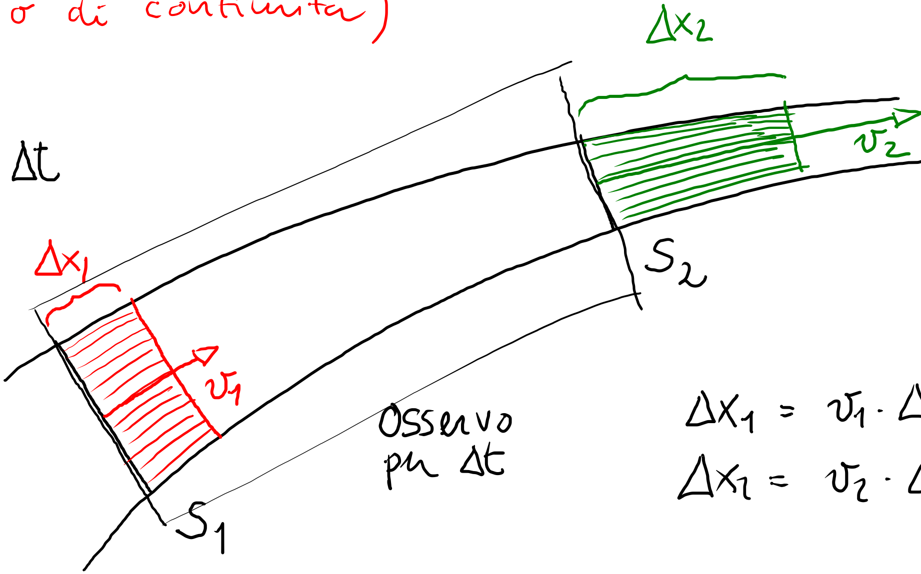
irrotazionale $\neq$ stazionario

caratteristica spaziale — caratteristica temporale



$$\vec{\nabla} \cdot \vec{v} = 0$$

EQUAZIONE DI LEONARDO
(o di continuità)



$$\Delta x_1 = v_1 \cdot \Delta t$$
$$\Delta x_2 = v_2 \cdot \Delta t$$

$$V_1 = S_1 \cdot \Delta x_1 = S_1 v_1 \Delta t$$

$$V_2 = S_2 \cdot \Delta x_2 = S_2 \cdot v_2 \Delta t$$

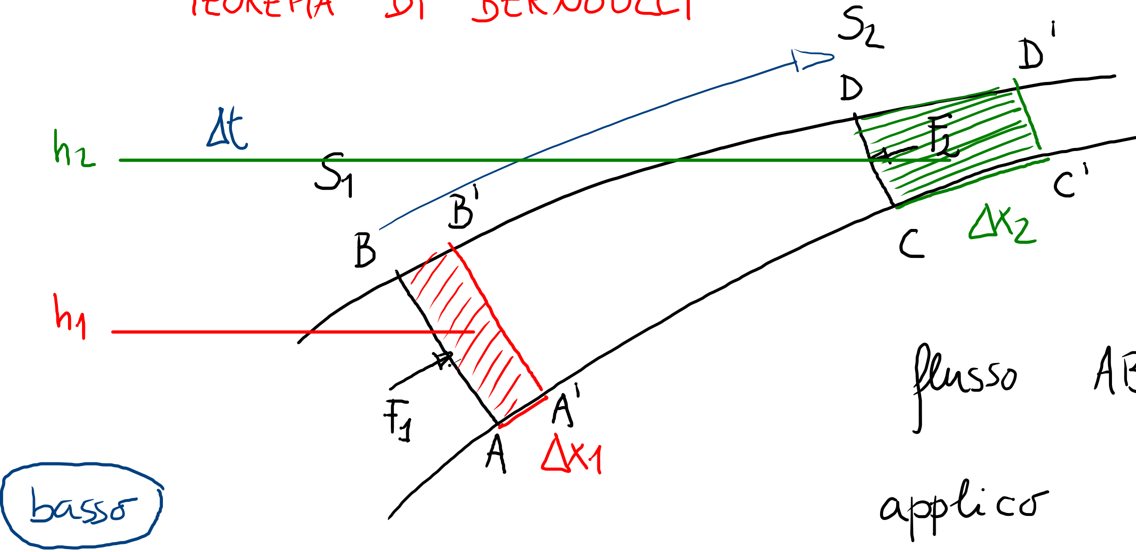
$$V_1 = V_2 = V$$
$$\cancel{\Delta t} \cdot S_1 v_1 = S_2 v_2 \cdot \cancel{\Delta t}$$
$$Q_1 = Q_2$$

$$Q = S \cdot v$$ portata in volume

$$[Q] = m^2 \cdot \frac{m}{s} = \frac{m^3}{s}$$

TEOREMA DI BERNOULLI

alto



Δt

flusso $ABCD \rightarrow A'B'C'D'$

applico $\mathcal{L} = \Delta K$

$$\left.\begin{aligned} ① \quad F_1 &= p_1 \cdot S_1 \\ F_2 &= p_2 \cdot S_2 \end{aligned}\right\} \text{forze dovute alla pressione} \rightarrow \mathcal{L}_p$$

② $\mathcal{L}_g$ lavoro della gravità

$$\underset{①}{\mathcal{L}_p} + \underset{②}{\mathcal{L}_g} = \underset{③}{\Delta K}$$

① $\mathcal{L}_p = F_1 \cdot \Delta x_1 - F_2 \cdot \Delta x_2$

$= F_1 \, v_1 \cdot \Delta t - F_2 \cdot v_2 \cdot \Delta t$

$= p_1 \underbrace{S_1 \, v_1 \, \Delta t}_{V_1} - p_2 \underbrace{S_2 \, v_2 \cdot \Delta t}_{V_2}$ $\qquad V_1 = V_2 = V$

$= p_1 V - p_2 V = (p_1 - p_2) \cdot V$

② $\mathcal{L}_g = - \Delta U_g$

$= U_{g\,ABCD} - U_{g\,A'B'C'D'}$

$= -U_{g\,CC'D'D} + U_{g\,AA'B'B}$

$= -\rho V h_2 g + \rho V h_1 g = \rho V (-h_2 + h_1) g = \rho V (h_1 - h_2) g$

③ $\Delta K = K_{A'B'C'D'} - K_{ABCD}$

$= K_{CC'D'D} - K_{AA'B'B} = \frac{1}{2} \rho V v_2^2 - \frac{1}{2} \rho V v_1^2$

$$\mathcal{L}_p + \mathcal{L}_g = \Delta K$$

$$(p_1 - p_2)\cancel{V} + \rho \cancel{V}(h_1 - h_2) g = \frac{1}{2}\rho \cancel{V}(v_2^2 - v_1^2)$$

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$p + \rho g h + \frac{1}{2}\rho v^2$ è la stessa in ogni punto del flusso.

TEOREMA DI BERNOULLI

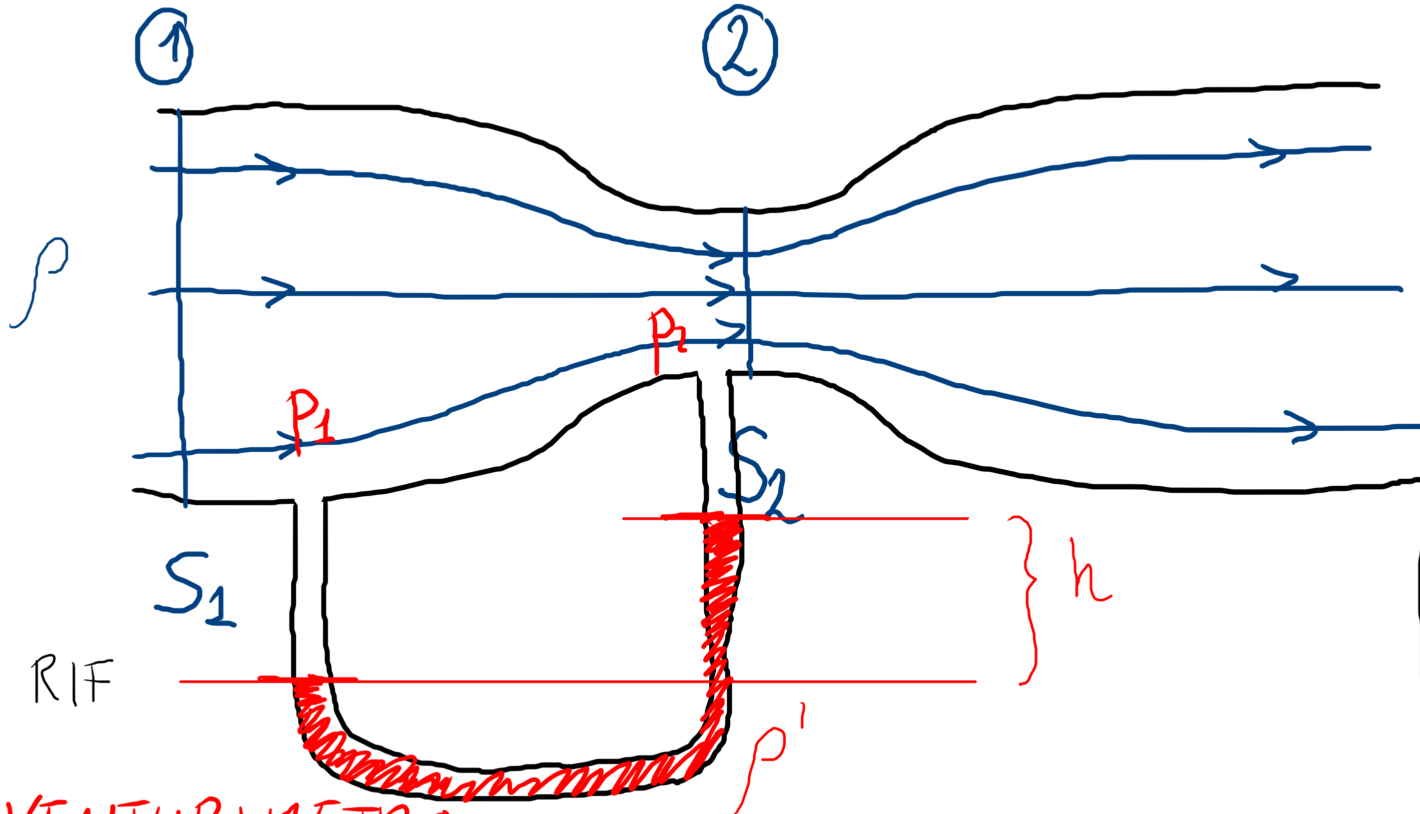
$$p_1 + \underbrace{\frac{1}{2}\rho v_1^2}_{\text{dens. en. cin.}} + \underbrace{\rho g h_1}_{\text{dens en. pot.}} = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\frac{N}{m^2}\cdot\frac{m}{m} = \frac{J}{m^3} \qquad \frac{J}{m^3} \qquad \frac{J}{m^3}$$

densità energia

$$\rho = \frac{m}{V}$$

TUBO DI VENTURI



VENTURIMETRO

$S_1 > S_2$

$S_1 v_1 = S_2 v_2 \Rightarrow v_2 > v_1$

$h_1 = h_2$

$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$

$\Rightarrow \boxed{p_2 < p_1}$

$$p_1 - p_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{1}{2}\rho\left(v_1^2\frac{S_1^2}{S_2^2} - v_1^2\right) = \frac{1}{2}\rho v_1^2 \underbrace{\left(\frac{S_1^2}{S_2^2} - 1\right)}_{>0}$$

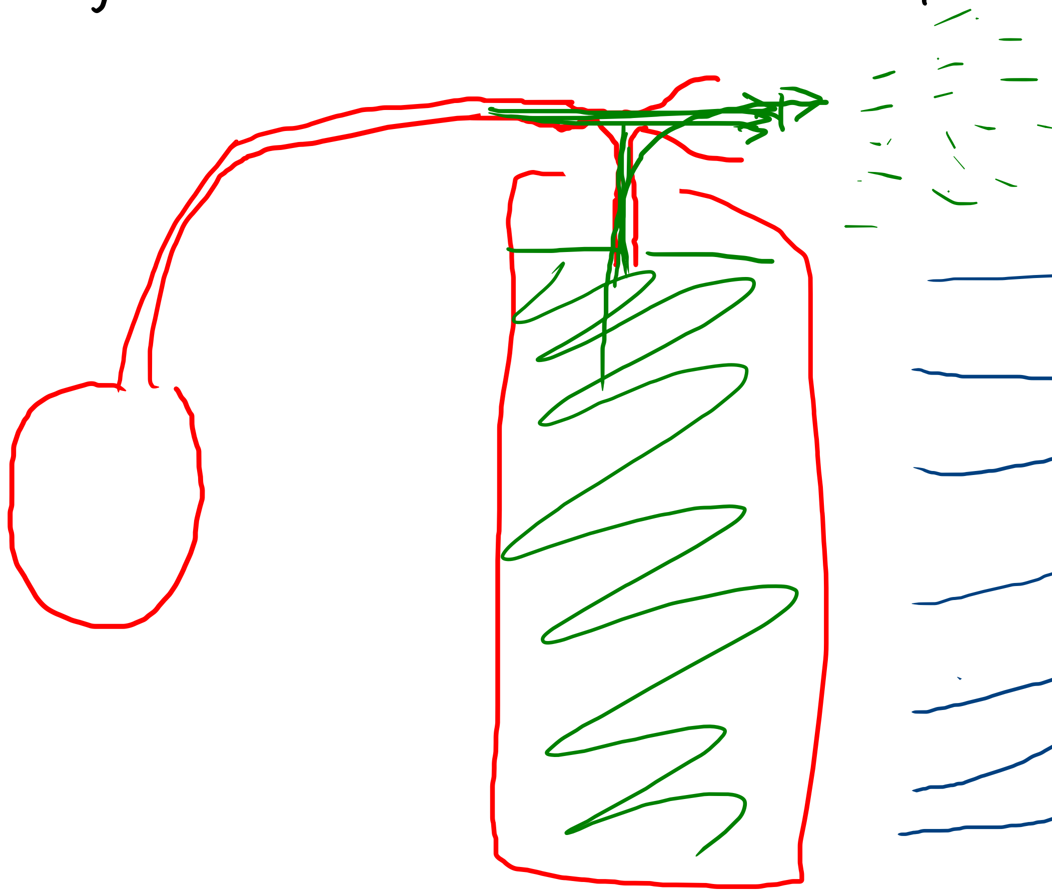
In RIF: SX = dx

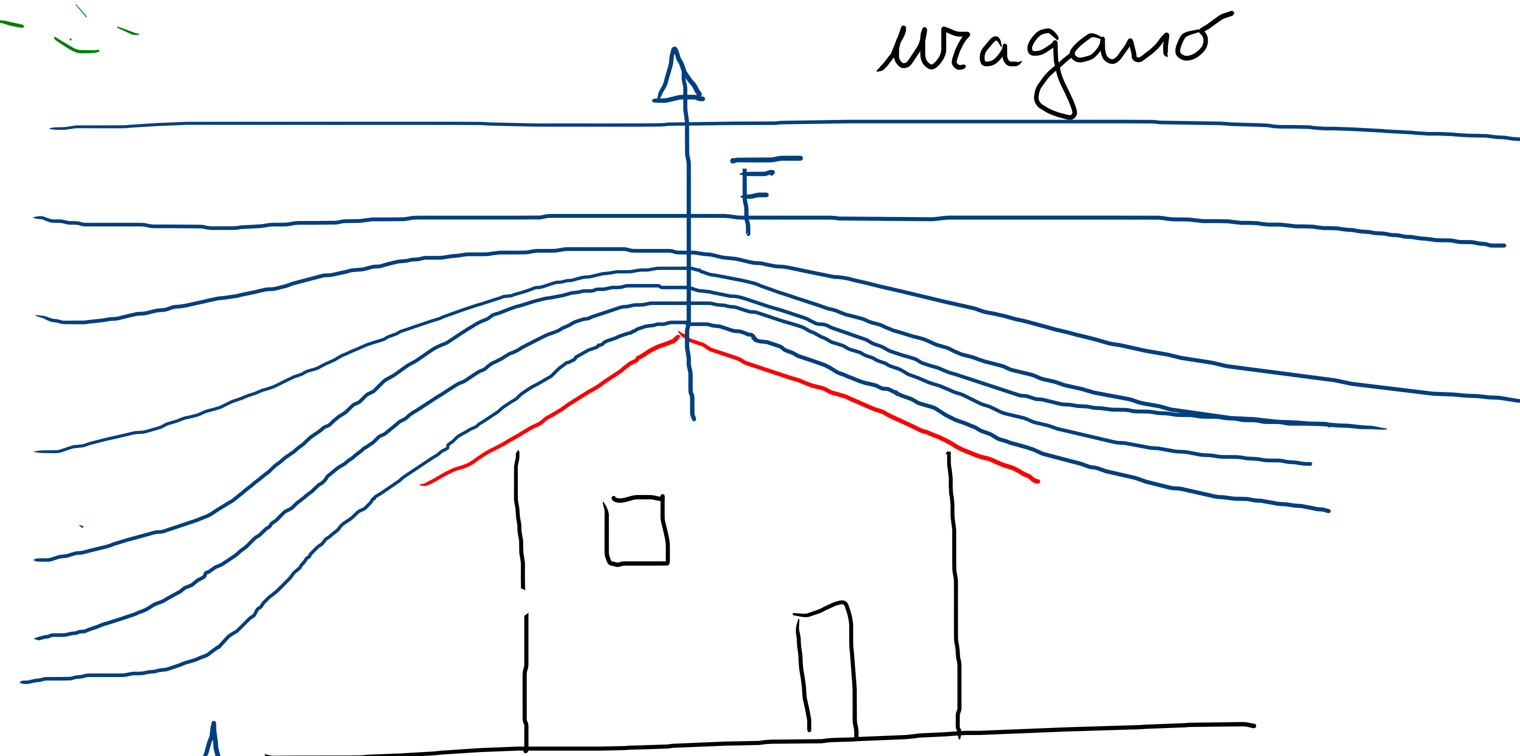
$p_1 = p_2 + \rho' g h$

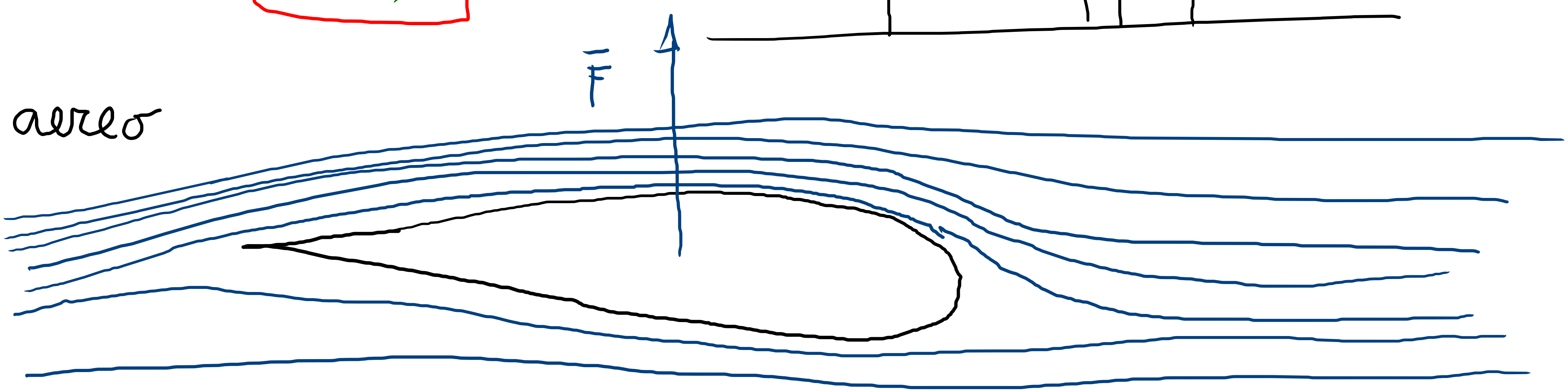
$$h = \frac{p_1 - p_2}{\rho' g} = \frac{\rho\left(\frac{S_1^2}{S_2^2} - 1\right)}{2\rho' g} v_1^2$$

$h = C v_1^2 \Rightarrow v_1 = \sqrt{\frac{h}{C}}$

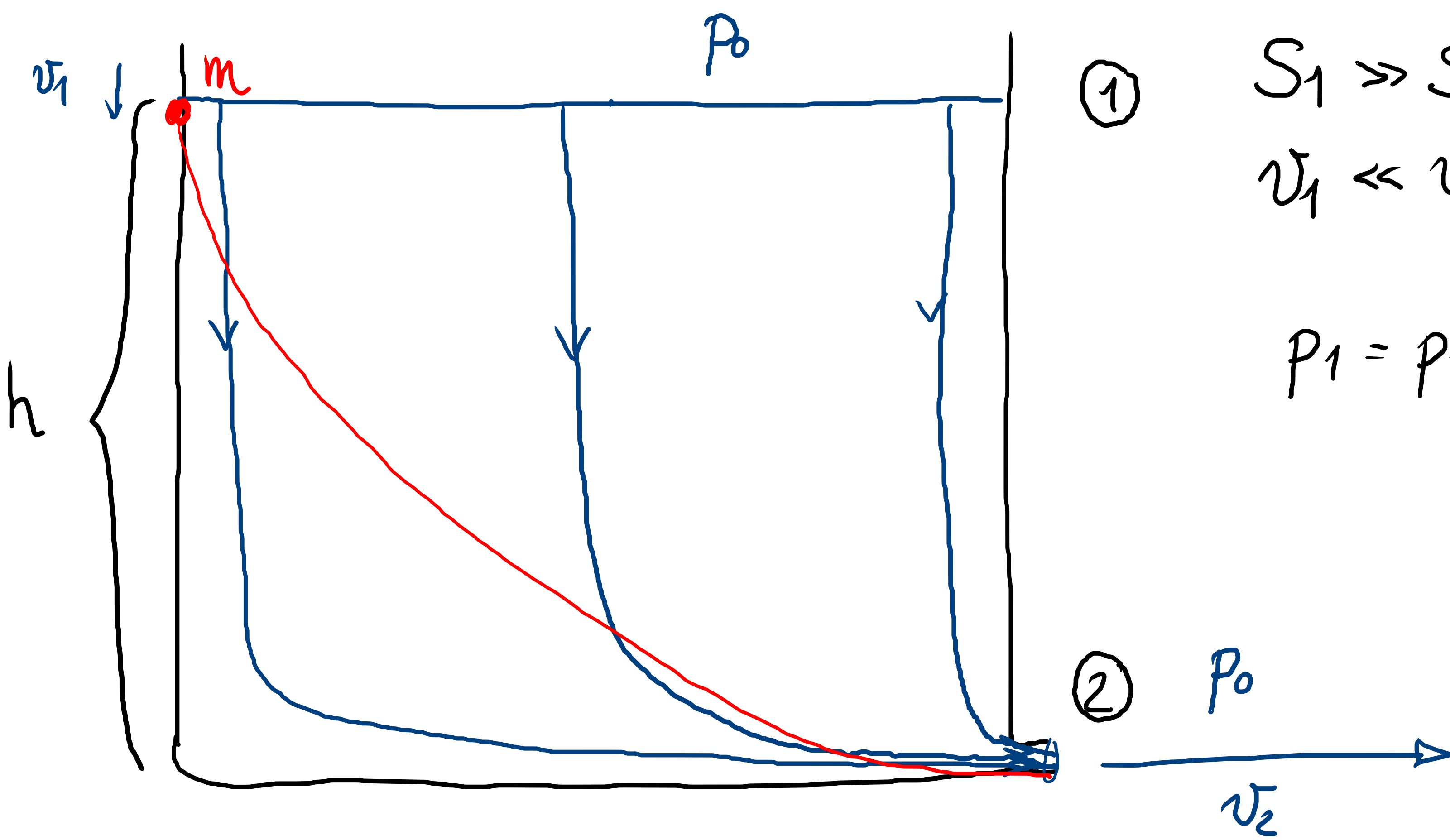
Profumi, carburatori, aspiratori del dentista etc.







TEOREMA DI TORRICELLI



$$S_1 \gg S_2$$

$$v_1 \ll v_2$$

$$p_1 = p_2 = p_0$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v_2^2$$

$$v_2 = \sqrt{2gh}$$

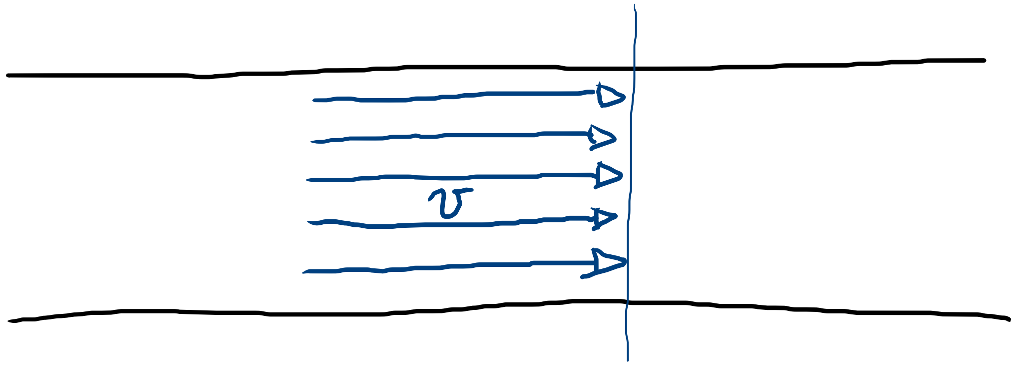
$$\cancel{p_1} + \frac{1}{2}\rho v_1^2 + \rho g h_1 = \cancel{p_2} + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\cancel{\rho} g \underbrace{(h_1 - h_2)} = \frac{1}{2}\cancel{\rho}(v_2^2 - \cancel{v_1^2}) \rightarrow \text{perché } v_1^2 \ll v_2^2$$

$$gh \cong \frac{1}{2} v_2^2$$

$$v_2 \cong \sqrt{2gh}$$

FLUIDI "REALI" Vs FLUIDI "IDEALI"



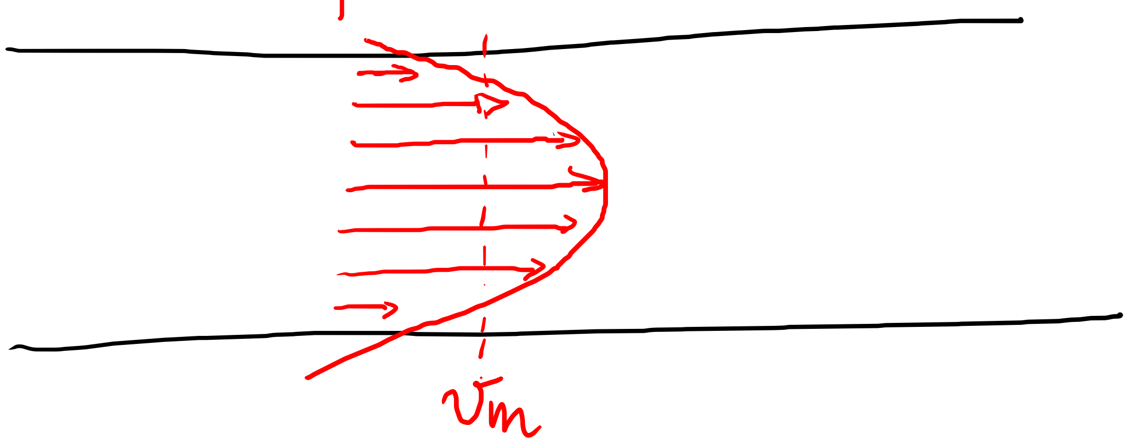
no attrito

v è la stessa su tutta la sezione

moto stazionario

moto laminare

parabola



$\eta \neq 0$ c'è attrito

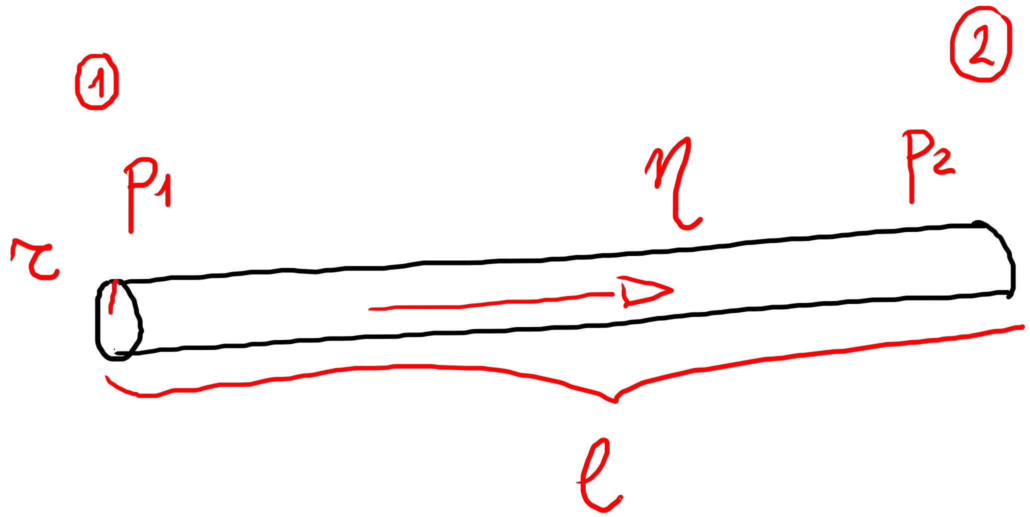
v cambia ma possiamo considerare la velocità media v_m

moto stazionario

Sì eq. Leonardo ma con v_m: $S_1 v_{m1} = S_2 v_{m2}$

No Bernoulli

LEGGE DI POISEUILLE
(liquidi Newtoniani)



$p_1 > p_2$

fluido "reale", moto stazionario, laminare e non turbolento,

$$\boxed{Q = \frac{\pi}{8}\,\frac{r^4}{\eta}\cdot\frac{(p_1-p_2)}{l}}$$

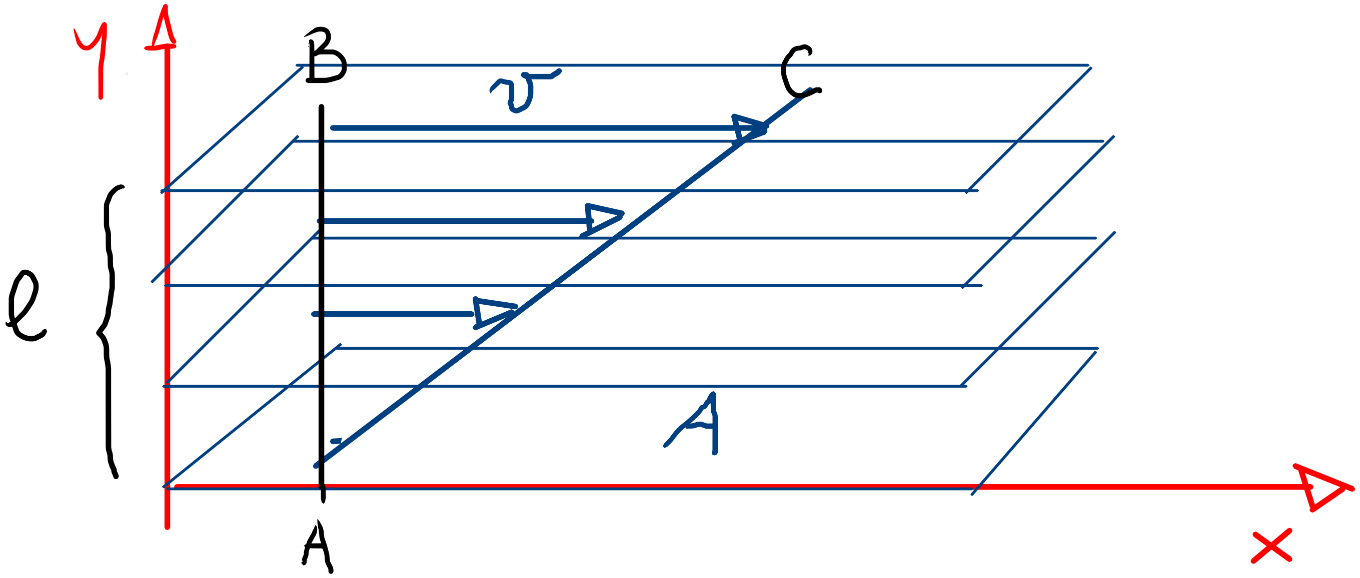
$$= \frac{\pi}{8}\,\frac{r^4}{\eta}\,\frac{\Delta p}{l} \rightarrow \frac{dp}{dl}$$ gradiente di pressione

$p_1 - p_2 = \Delta p > 0$

vale per $v_m < v_c = N_R \frac{\eta}{\rho r}$ con N_R numero di Reynolds

qui ~ 1000

DEFINIZIONE DI VISCOSITÀ



1) $\dfrac{dv}{dy} = \text{cost} = \dfrac{v}{l}$

2) $F \propto \dfrac{dv}{dy} \cdot A$

$\Rightarrow$ si definisce η come

$$F = \eta \cdot \frac{dv}{dy} \cdot A$$

Definiamo anche il carico di scorrimento

$$\frac{F}{A} = \eta \cdot \frac{dv}{dy}$$

CIRCOLAZIONE SANGUIGNA

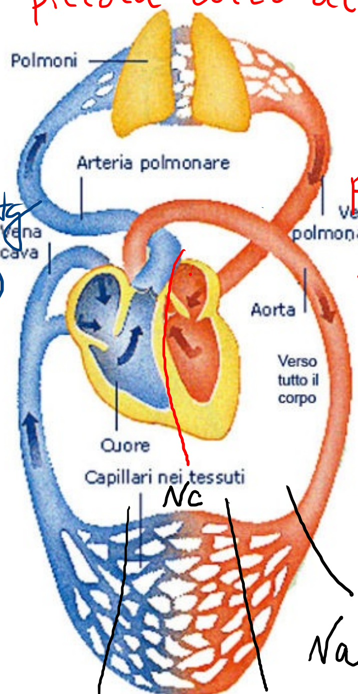
polmonare
piccola circolazione

$p' \sim 20$ mmHg

$p \sim 0$

$p \sim 0$

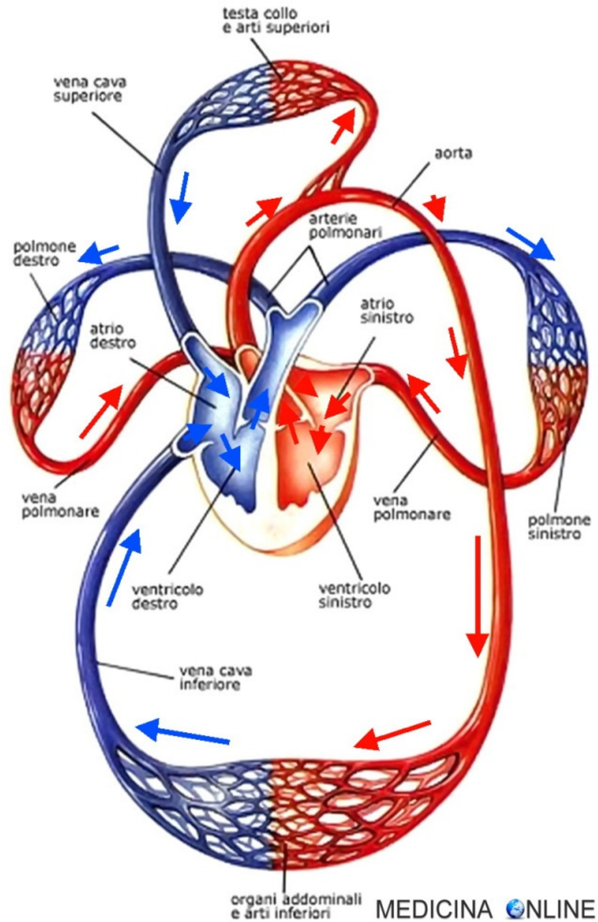
$p \sim 100$ mmHg



Nc

Na

grande circolazione
sistemica



Na arteriole in parallelo
Nc capillari in parallelo

Legge di Poiseuille $\longleftrightarrow$ Legge di Ohm

$$Q = \frac{\pi}{8} \frac{r^4}{\eta} \frac{\Delta p}{\ell}$$

resistenza

$$\Delta p = \frac{8 \eta \ell}{\pi r^4} \cdot Q \qquad \qquad \Delta V = R\, I$$

R

differenza di pressione ai capi del vaso sanguigno

portata

differenza di potenziale ai capi di R

intensità di corrente elettrica

Resistenze in serie $R_1, R_2 \dots R_N$

$$R_{eq} = R_1 + R_2 + \dots + R_N$$

Resistenze in parallelo $R_1, R_2, \dots R_N$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

N_a arteriole

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_{Na}}$$

$$R_1 = R_2 = \dots = R_{Na} \equiv R_a$$

$$= \underbrace{\frac{1}{R_a} + \frac{1}{R_a} + \dots + \frac{1}{R_a}}_{N_a} = N_a \cdot \frac{1}{R_a} = \frac{N_a}{R_a}$$

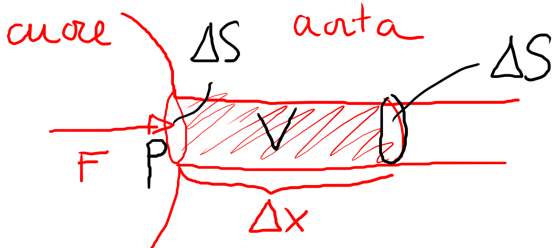
$$R_{eq} = \frac{R_a}{N_a}$$

N_c capillari, ciascuno con resistenza R_c

$$\frac{1}{R_{eq}} = \frac{N_c}{R_c} \qquad R_{eq} = \frac{R_c}{N_c} \qquad N_c \cong 2 \cdot 10^9$$

LAVORO CARDIACO

sx



1 contrazione $\rightarrow \Delta V$

$$\mathcal{L} = F \cdot \Delta x$$

$$F = p \cdot \Delta S$$

$$= p \cdot \Delta S \cdot \Delta x = p \Delta V$$

dx $\mathcal{L}' = p' \Delta V$

$$\mathcal{L}_{TOT} = \mathcal{L} + \mathcal{L}' = p\Delta V + p'\Delta V = (\underbrace{p+p'}_{\sim 120\ mmHg}) \underbrace{\Delta V}_{60\ cm^3}$$

$$= \frac{120\ \cancel{mmHg}}{760\ \cancel{mmHg}} \cdot 10^5\ Pa \cdot 60\ (10^{-2} m)^3$$

$$= 0{,}16 \cdot \cancel{10^5} \cdot \cancel{60} \cdot \cancel{10^{-6}}\ Pa\,m^3 \cong 1\ J$$

$$\mathcal{P} = \frac{\mathcal{L}}{\Delta t} = \frac{1\,J}{1\,s} = 1\ W \quad (\text{assumo } 1\ \text{puls/s})$$

VISCOSITÀ DEL SANGUE

→ aumenta con ematocrito

→ η dipende dal raggio r del vaso sanguigno

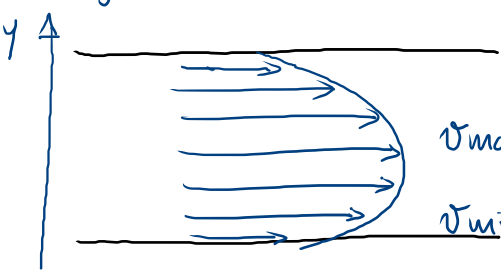
$$\eta(r) = \frac{\eta_\infty}{\left(1 + \frac{d}{r}\right)^2}$$

η_∞ ← viscosità x vasi suff. grandi

d ← diametro eritrociti 6-9 μm

→ η non è costante in tutta la sezione

fluido newtoniano

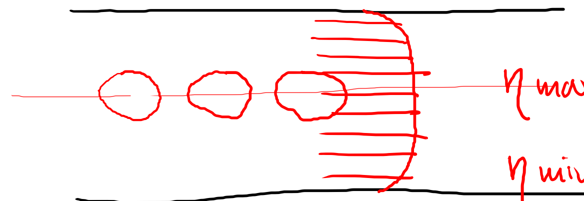


η cost

v_{max} $\quad \frac{dv}{dy} \sim 0$

v_{min} $\quad \frac{dv}{dy} \sim max$

sangue



v cost

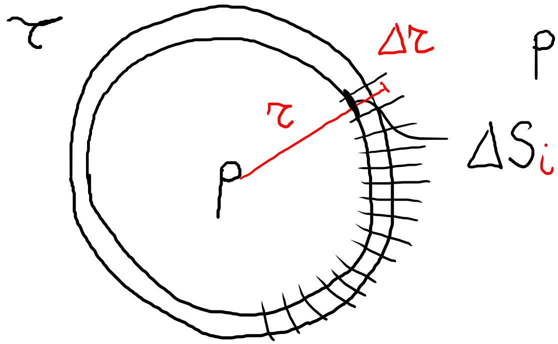
η_{max} $\quad \frac{dv}{dy} \sim 0$

η_{min} $\quad \frac{dv}{dy} \sim max$

$$\frac{F}{A} = \eta \cdot \left(\frac{dv}{dy}\right)$$

← minimizza il carico di scorrimento

LEGGE DI LAPLACE



$$p = \frac{4\tau}{r}$$ (sovrapressione rispetto a p_0)

$$\sum_i \Delta S_i = S = 4\pi r^2$$

$r \to r + \Delta r$ ← piccolo!

$$\tau = \frac{\mathcal{L}}{\Delta S}$$ dove $\mathcal{L}$ è lavoro contro τ

$$\mathcal{L} = \tau \cdot \Delta S = \tau \left[4\pi (r+\Delta r)^2 - 4\pi r^2 \right] \cdot 2$$
$$= \tau \left[\cancel{4\pi r^2} + 8\pi r \Delta r + \cancel{4\pi \Delta r^2} - \cancel{4\pi r^2} \right] \cdot 2$$
$$= 16\pi r \Delta r \cdot \tau$$

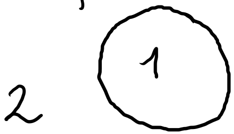
$$\mathcal{L}_p^i = F \cdot \Delta r = p \Delta S_i \cdot \Delta r$$
$$\mathcal{L}_p = \sum_i \mathcal{L}_p^i = \sum_i p \cdot \Delta S_i \cdot \Delta r = p \Delta r \underbrace{\sum_i \Delta S_i}_{4\pi r^2} = 4\pi r^2 \Delta r \cdot p$$

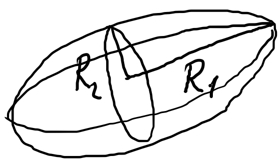
$$\mathcal{L} = \mathcal{L}_p$$

$$4\;\cancel{16}\,\cancel{\pi}\,\cancel{r}\cdot\cancel{\Delta r}\cdot\tau = \cancel{4}\cancel{\pi}\cancel{r^2}\,\cancel{\Delta r}\cdot p \qquad p = \frac{4\,\tau}{r}$$

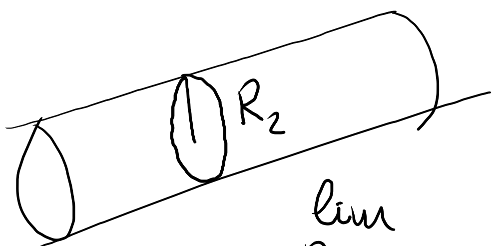
Per una goccia 1 in 2 :
sferica

$$p = \frac{2\,\tau}{r} \overset{!}{=} \frac{\tau}{r} + \frac{\tau}{r}$$





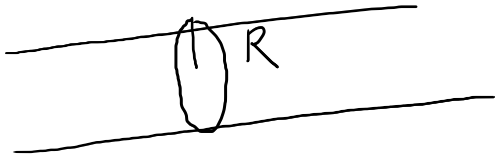
$$p = \frac{\tau}{R_1} + \frac{\tau}{R_2}$$



$\lim\limits_{R_1 \to \infty}$

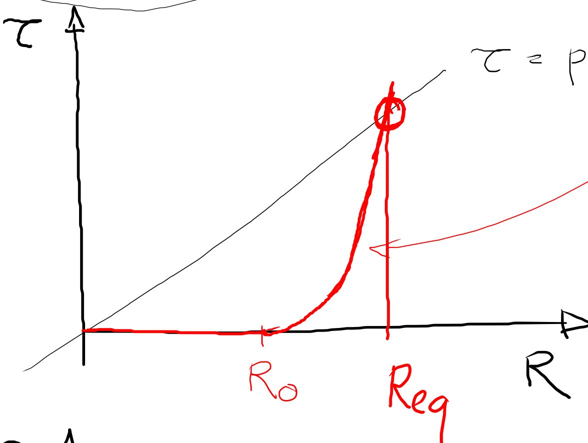
$$p = \frac{\tau}{R_2}$$

Applicazione della legge di Laplace ai vasi sanguigni



$$p = \frac{\tau}{R} \quad \rightarrow \quad \tau = \tau(R)$$

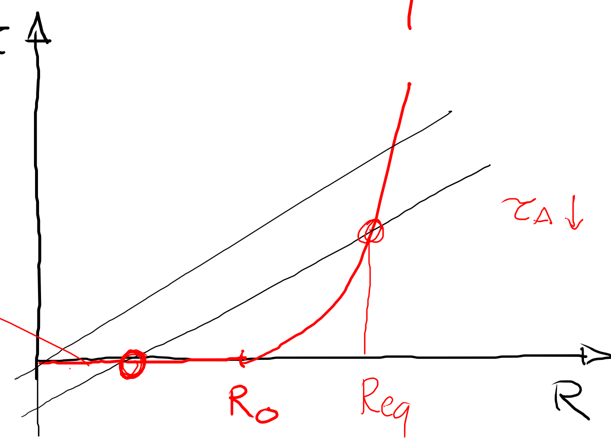
$$\tau = pR$$



tensione attiva dovuta alla muscolatura

$$\tau + \tau_A = pR$$

$$\tau = pR - \tau_A$$



$\tau_A \downarrow \Rightarrow R \uparrow$

$\downarrow \tau_A \Rightarrow R \downarrow$

rischio chiusura vaso