ESERCITAZIONE 9

#9.1

NO VISCOSITÀ $\rightarrow$ FLUIDO IDEALE

(a)

EQUAZIONE DI CONTINUITÀ: $v_1 A_1 = v_{2a} A_2$

$$\Rightarrow v_{2a} = \frac{v_1 A_1}{A_2} = v_1 \frac{\cancel{\pi} r_1^2}{\cancel{\pi} r_2^2} = v_1 \frac{(d_1/\cancel{2})^2}{(d_2/\cancel{2})^2} = v_1 \frac{d_1^2}{d_2^2} \Rightarrow \boxed{v_2 = v_1 \left(\frac{d_1}{d_2}\right)^2}$$

$\simeq 19{,}2\ m/s$

(b) EQ. DI BERNOULLI

(qui $h_1 = h_2$)

$$p_1 + \cancel{\rho g h_1} + \frac{1}{2}\rho v_1^2 = \underbrace{p_2}_{= p_0 \text{ PRESSIONE ATM (apertura)}} + \cancel{\rho g h_2} + \frac{1}{2}\rho v_{2a}^2$$

(*)

$$\Rightarrow \boxed{p_1 = p_0 + \frac{1}{2}\rho (v_2^2 - v_1^2)} = 2{,}84 \cdot 10^5\ Pa$$

(c)

EQ. BERNOULLI di nuovo, ma stavolta $v_1 \rightarrow v_{1c}$
$v_2 \rightarrow v_{2c}$

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_{1c}^2 = p_0 + \rho g h_2 + \frac{1}{2}\rho v_{2c}^2$$

$$\Rightarrow p_1 - p_0 = \rho g \overbrace{(h_2 - h_1)}^{h} + \frac{1}{2}\rho (v_{2c}^2 - v_{1c}^2) \quad (**)$$

Ma $p_1 - p_0$ è la stessa del caso precedente, quindi da ⊛:

$p_1 - p_0 = \frac{1}{2}\rho(v_{2a}^2 - v_1^2)$ uguaglio con ⊛⊛:

$\frac{1}{2}\cancel{\rho}(v_{2a}^2 - v_1^2) = \cancel{\rho} g h + \frac{1}{2}\cancel{\rho}(v_{2c}^2 - v_{1c}^2)$ uso eq. continuità:

$v_{1c} S_1 = v_{2c} S_2 \Rightarrow v_{1c} = \frac{v_{2c} S_2}{S_1} = v_{2c}\frac{d_2^2}{d_1^2}$ e $v_1 S_1 = v_{2a} S_2 \Rightarrow v_1 = v_{2a}\frac{d_2^2}{d_1^2}$

$\Rightarrow v_{2a}^2\left(1 - \frac{d_2^4}{d_1^4}\right) = 2gh + v_{2c}^2\left(1 - \frac{d_2^4}{d_1^4}\right) \Rightarrow \boxed{v_{2c} = \sqrt{v_{2a}^2 - \frac{2gh}{1 - \left(\frac{d_2}{d_1}\right)^4}}}$

$\simeq 17{,}6 \text{ m/s}$

3.2

(a) PORTATA: $Q = Av$

$\Rightarrow Q_A = A_A \cdot v_A$ PORTATA NELL'AORTA

$\Rightarrow v_A = \frac{Q_A}{A_A} \Rightarrow \boxed{v_A = \frac{Q}{\pi R_A^2}} \simeq 27 \text{ cm/s}$

portata non dipende da dove ci si trova

NB: $5{,}0 \text{ l/min} = \frac{5.0 \cdot 10^{-3} \text{ m}^3}{60 \text{ s}}$

(b) $Q_c = Q_A = Q$ portata nei capillari sarà uguale, però stavolta la Q è divisa tra tutti i capillari

$\Rightarrow Q = N_c \cdot S_c \cdot v_c \Rightarrow \boxed{v_c = \frac{Q}{N_c \pi R_c^2}} \simeq 0{,}33 \text{ mm/s}$

(c) LEGGE DI POISEUILLE

$$Q = \frac{\pi R_A^4}{8\eta}\frac{\Delta P_A}{l_A} \Rightarrow \boxed{\eta = \frac{\pi R_A^4}{8Q}\frac{\Delta P_A}{l_A}} \simeq 4{,}3\cdot 10^{-3}\ Pa\cdot s$$

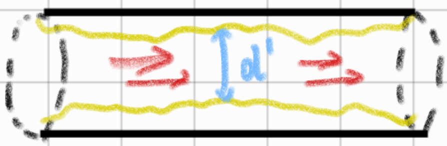
(d) ANCORA POISEUILLE, MA LA PORTATA IN UN CAPILLARE É: $\frac{Q}{N_c}$

$$\Rightarrow \frac{Q}{N_c} = \frac{\pi R_c^4}{8\eta}\frac{\Delta P_c}{l_c} \Rightarrow \boxed{\Delta P_c = \frac{Q}{N_c}\frac{8 l_c}{\pi R_c^4}\eta} \simeq 780\ Pa$$

#9.3

CASO NON PATOLOGICO

CASO PATOLOGICO



$$Q = \frac{\pi}{8}\frac{r^4}{\eta}\frac{\Delta p}{l} \qquad Q' = \frac{\pi}{8}\frac{r'^4}{\eta'}\frac{\Delta p'}{l'}$$

con $d' = d - \frac{25}{100}d = \frac{75}{100}d = \frac{3}{4}d \Rightarrow \frac{d'}{d} = \frac{3}{4} \Rightarrow \frac{r'}{r} = \frac{3}{4}$

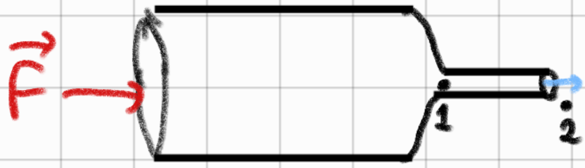
(a) $Q = Q'$ e $\eta = \eta'$

$$\Rightarrow \cancel{\frac{\pi}{8}}\frac{r^4}{\cancel{\eta}}\frac{\Delta p}{\cancel{l}} = \cancel{\frac{\pi}{8}}\frac{r'^4}{\cancel{\eta}}\frac{\Delta p'}{\cancel{l}} \Rightarrow \boxed{\frac{\Delta p'}{\Delta p} = \left(\frac{r}{r'}\right)^4} = \left(\frac{4}{3}\right)^4 = 3{,}16.$$

(b) $\Delta p = \Delta p'$ e $\eta = \eta'$

$$\Rightarrow \frac{\cancel{8} Q \cancel{l \eta}}{\cancel{\pi} r^4} = \frac{\cancel{8} Q' \cancel{l \eta}}{\cancel{\pi} r'^4} \Rightarrow \boxed{\frac{Q'}{Q} = \left(\frac{r'}{r}\right)^4} = \left(\frac{3}{4}\right)^4 = 0{,}316$$

#9.4



(a) pressione dovuta a $\vec{F}$: $P = \frac{F}{A} = \frac{F}{\pi\left(\frac{d}{2}\right)^2} \Rightarrow \boxed{P = \frac{4F}{\pi d^2}} \simeq 3{,}98 \cdot 10^4$ Pa

$$\Rightarrow P_{TOT} = P_{atm} + P = 1{,}4 \cdot 10^5 \text{ Pa}$$

(b) eq. Bernoulli tra 1 e 2: (NB: $h_1 = h_2$)

$$P_1 + \cancel{\rho g h_1} + \frac{1}{2}\cancel{\rho v_1^2} = P_2 + \cancel{\rho g h_2} + \frac{1}{2}\rho v_2^2$$

P_{TOT} dentro la siringa

$v_1 = 0$

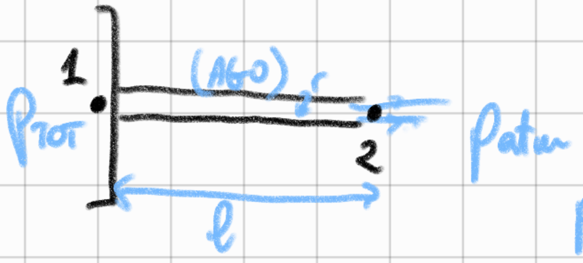
velocità dentro siringa trascurabile

$= P_{atm}$ (foro aperto)

$$\Rightarrow v_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho}} = \sqrt{\frac{2(\cancel{P_{atm}} + P - \cancel{P_{atm}})}{\rho}}$$

$\Rightarrow \boxed{v_2 = \sqrt{\frac{2P}{\rho}}} = 8{,}9$ m/s con P pressione dovuta a $\vec{F}$.

(c) $Q = \frac{\pi r^4}{8\eta}\frac{\Delta p}{\ell}$



con Δp differenza di pressione tra ext e interno: $|\overset{P_{atm}}{p_2} - \overset{P_{TOT}}{p_1}| =$

$= |\cancel{P_{atm}} - p + \cancel{p_{atm}}| = p$

(NB: 1 poise = 0,1 Pa·s)

$\Rightarrow \boxed{Q = \frac{\pi r^4 p}{8\eta\ell}} \simeq 0.031 \ cm^3/s$

$\Rightarrow v = \frac{Q}{S} \Rightarrow \boxed{v = \frac{Q}{\pi r^2}} \simeq 1 \ m/s$

(d) VELOCITÀ CRITICA $v_c = \frac{N_R \cdot \eta}{r \cdot \rho} = 10 \ m/s$

poiché $v < v_c \Rightarrow$ regime laminare.