CARDIFF UNIVERSITY
PRIFYSGOL CAERDYDD
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ENT700/EN4574

Advanced Structural Mechanics

Autumn semester a.y. 2018-2019

Prof Massimiliano Gei

November 2018

Masonry structures

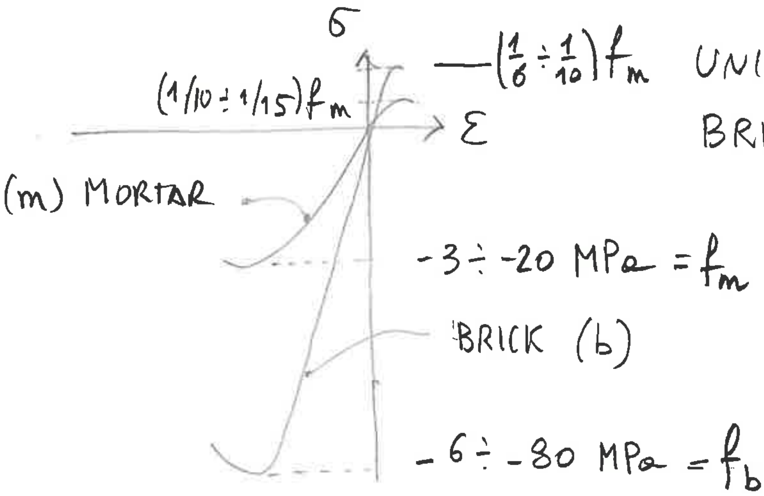
Introduction

Stability of compressed pillars

Stability of voussoir arches and domes

MASONRY STRUCTURES

MASONRY : composed of blocks (bricks, stones, ..)
+ (possibly) mortar



UNIAXIAL TESTS for BRICKWORK (BRICK + MORTAR)

$E_m = 5 \div 20000$ MPa

$E_b = 15 \div 25000$ MPa

BRICKWORK COMPRESSIVE STRENGTH : $f_K = K f_b^{0.65} f_m^{0.25}$

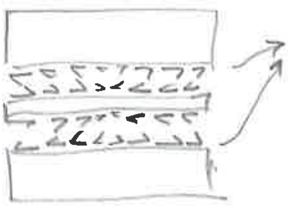
$K = 0.6 \div 0.4$ (EUROCODE)

COMPRESSION TEST ON A BRICKWORK WALL

FAILURE: axial splitting

BRICKS are under tensile stress because the mortar that tends to expand more than the bricks

The compressive strength of BRICKWORK is higher than that of the mortar.



Tensile stress (vertical cracks!)

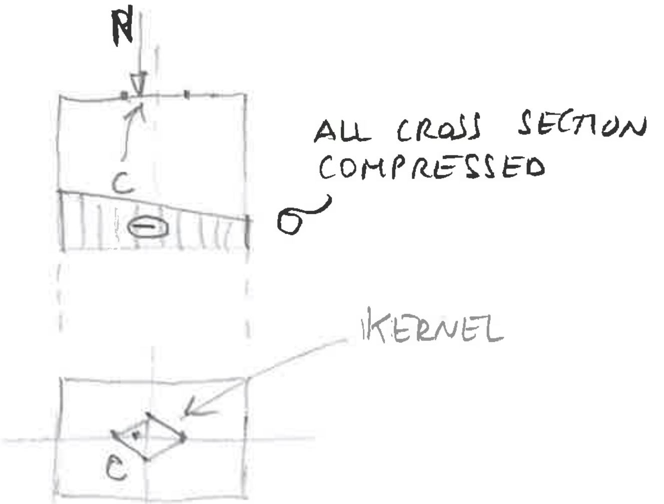
HEYMAN'S HYPOTHESES FOR MODELLING MASONRY

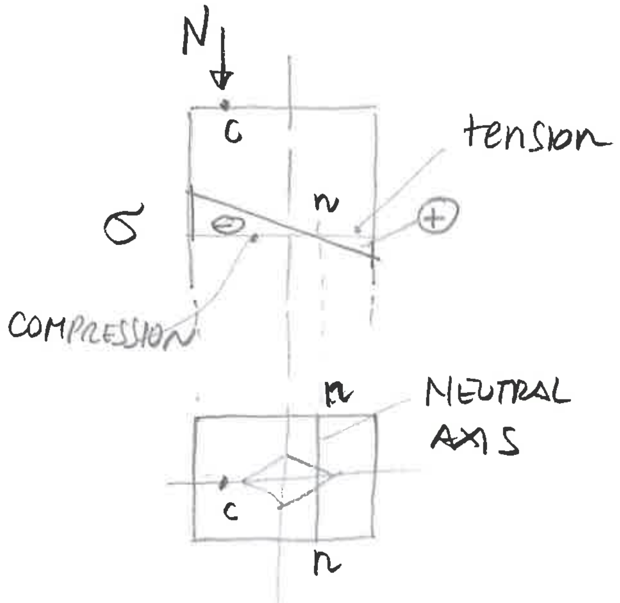
3 ASSUMPTIONS

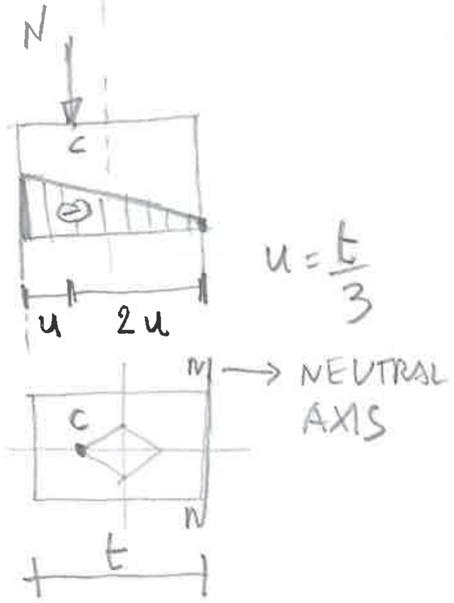
1) MASONRY HAS NO TENSILE STRENGTH.
2) " HAS UNLIMITED COMPRESSIVE STRENGTH
3) SLIDING FAILURE BETWEEN BLOCKS DOES NOT OCCUR.

THE COMPRESSED <u>ELASTIC</u> PILLAR (COMBINED COMPR + BENDING)

- LOW ECCENTRICITY LOAD; $c \in$ KERNEL OF THE CROSS SECTION
- HIGH ECCENTRICITY; $c \notin$ KERNEL OF THE CROSS SECTION



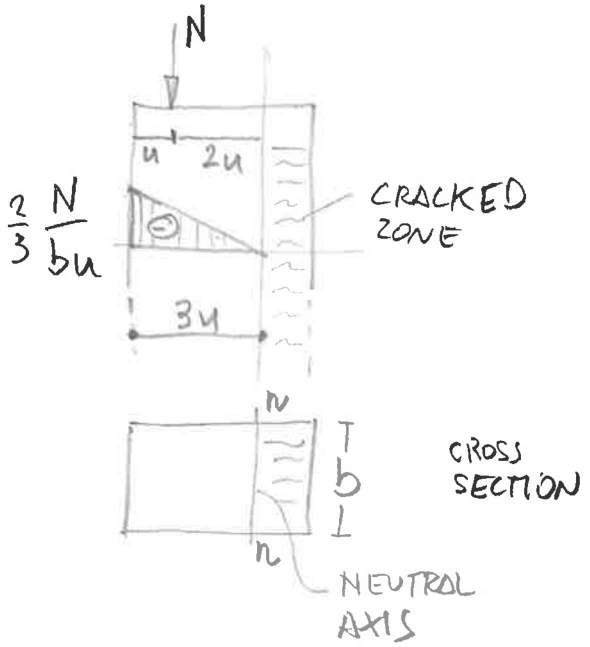




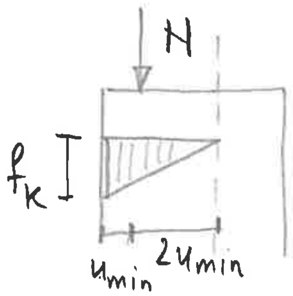
THE COMPRESSED NO-TENSION PILLAR

(NO-TENSION: MATERIAL WITH NULL TENSILE STRENGTH AND INFINITE COMPRESSIVE STRENGTH
→ HEYMAN'S ASSUMPTIONS)

- LOW ECCENTRICITY LOAD : SAME AS ELASTIC SOLUTION (see prev. page)
- HIGH ECCENTRICITY LOAD



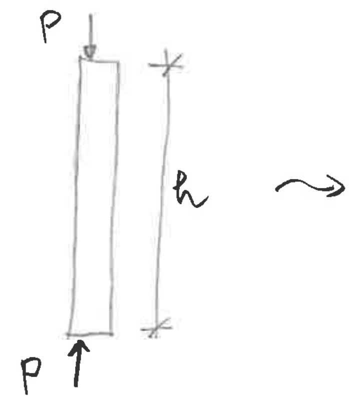
WITH A FINITE COMPRESSION STRENGTH (say f_k), which is the minimum u (u_{min})



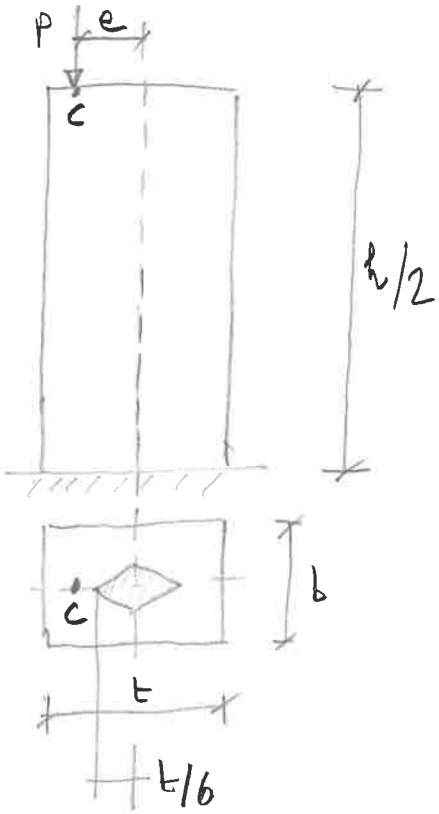
$$f_k = \frac{2}{3}\frac{N}{b\,u_{min}} \Rightarrow \boxed{u_{min} = \frac{2}{3}\frac{N}{b\,f_k}}$$

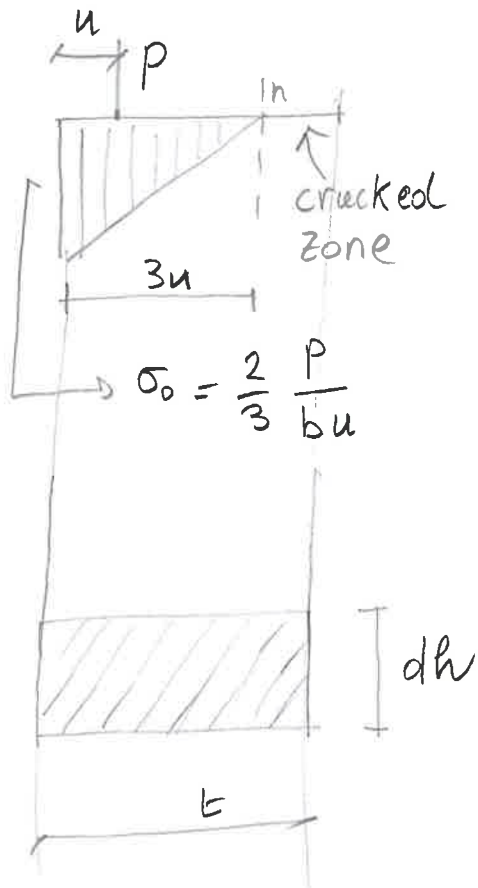
RECTANGULAR NO-TENSION PILLAR WITH A HIGH-ECCENTRICITY LOAD

(YOKEL, 1971)

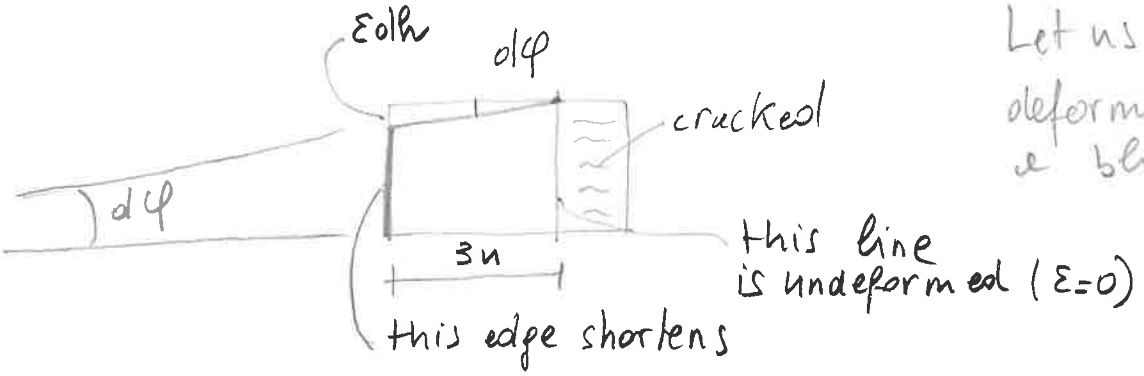


$$\boxed{e \geq \frac{t}{6}}$$



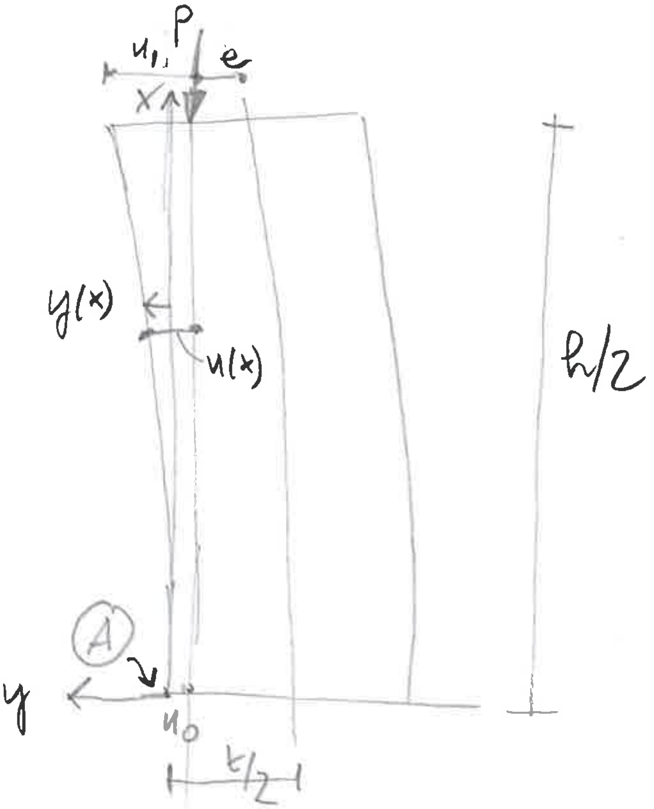


Let us study the deformation of such a block



$$\varepsilon = \frac{\sigma_0}{E} = \frac{2}{3}\frac{P}{Ebu} \quad ; \qquad d\varphi = \frac{\varepsilon\, dh}{3u} = \frac{2P}{9Ebu^2}\, dh$$

the induced CURVATURE is $\dfrac{d\varphi}{dh} = \dfrac{2P}{9Ebu^2}$



In this reference system y, x the curvature is $\frac{d^2y}{dx^2}$. Therefore

$$\frac{d^2y}{dx^2} = \frac{2}{9}\frac{P}{Ebu^2}$$

But, as $u(x) = y(x) + u_0 \Rightarrow$

$$\boxed{\frac{d^2y}{dx^2} = \frac{2}{9}\frac{P}{Eb(y+u_0)^2}}$$

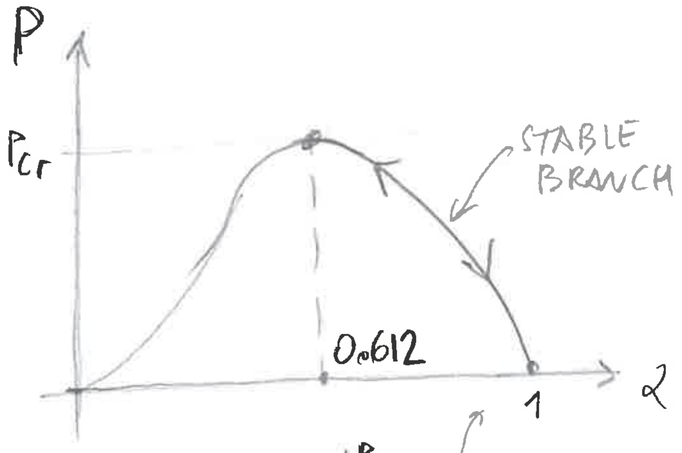
The unknown y(x) appears

The solution in close form is available but quite complicated. The boundary conditions are

$$\boxed{y(0)=0 \quad ; \quad \frac{dy}{dx}(0)=0}$$

(both at point A)

$$\alpha = \frac{u_0}{u_1}$$



STRAIGHT PILLAR (P=0)

$$P_{cr} = 0.285\ P_{eq}$$

where $P_{eq} = \frac{\pi^2}{h^2} EI_{eq}$

with

$$I_{eq} = \frac{b(3u_1)^3}{12}$$

second moment of area of the "reactive" cross section

The CRITICAL LOAD P_{cr} can be also expressed in terms of EULERIAN CRITICAL load of the piller

$$P_E = \frac{\pi^2}{h^2} EI \quad ; \text{with} \quad I = \frac{bt^3}{12}$$

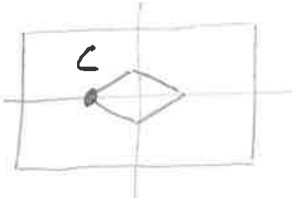
$$P_{cr} = 0.285 \frac{I_{eq}}{I} P_E = 0.285 P_E \left[\frac{(3u_1)^3}{t^3} \right]$$

but $u_1 = \frac{t}{2} - e$, so that

$$P_{cr} = 0.285 P_E \left[\left(\frac{3}{2} t - 3e \right) \frac{1}{t} \right]^3 = 0.285 P_E \left[\frac{3}{2} - \frac{3e}{t} \right]^3$$

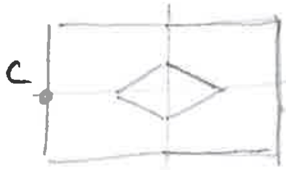
$\frac{t}{2} \geq e \geq t/6$

- For $e = \frac{t}{6}$



$P_{cr} = 0.285 P_E$

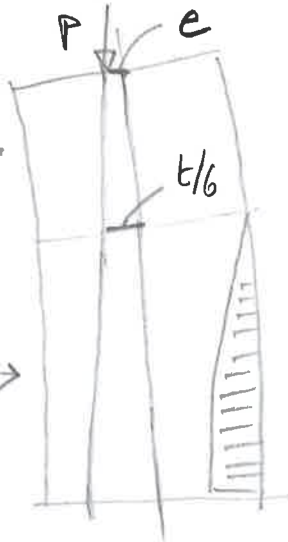
- For $e = \frac{t}{2}$



$P_{cr} = 0$

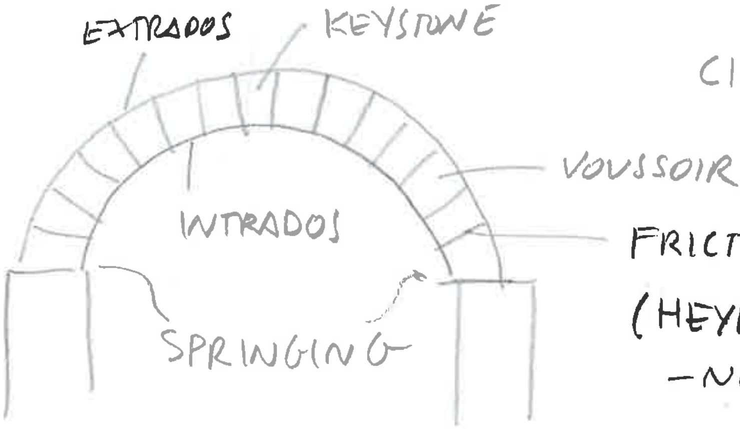
LOAD WITH LOW ECCENTRICITY (SAHLIN, 1971)

The problem is much more complicated as there is a part of the piller cracked and another undamaged.



See Sahlin, 1971; Frish-Fey, 1975.

EQUILIBRUM OF VOUSSOIR ARCHES



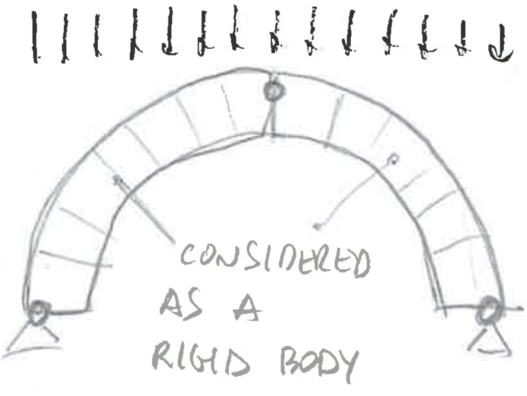
CIRCULAR ARCH

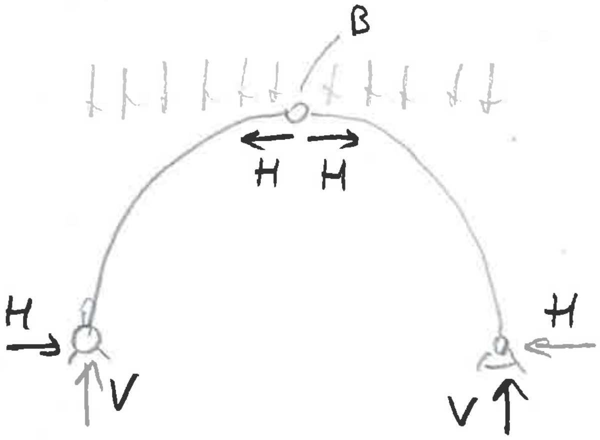
FRICTION AT THE INTERFACE
(HEYMAN'S HYPOTHESES APPLY)
– NO TENSION MATERIAL

LINE OF THRUST : LINE OF ACTION OF RESULTANT OF LOADS

HOW TO COMPUTE THE LINE OF THRUST ?

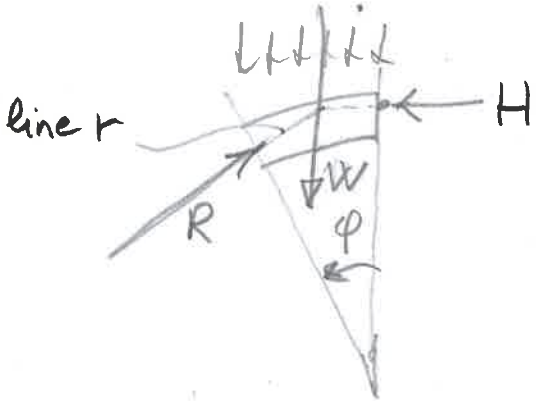
FOR SYMM. LOADINGS, A REASONABLE ASSUMPTION IS TO SELECT 3 HINGES AND STUDY GLOBAL EQUILIBRIUM (WITH 3 HINGES THE PROBLEM IS STATICALLY DETERMINED) :

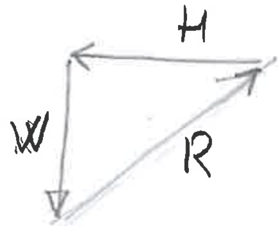




FOR SYMMETRY, REACTION IN B IS ONLY HORIZONT.

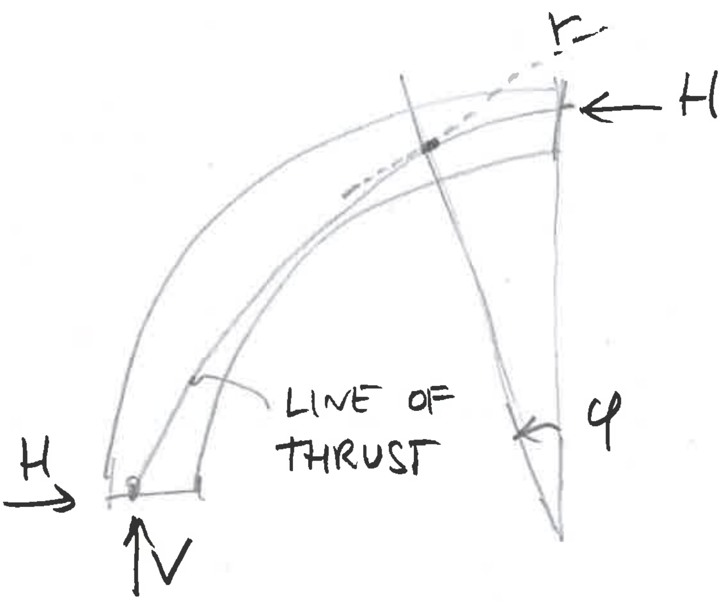
HOW TO OBTAIN THE LINE OF TRUST AT A GENERIC ANGLE ?





H, W, R in equilibrium

the line r represents the LINE OF THRUST at angle φ



For symmetry, the line of thrust in the right-hand side is readily obtained.

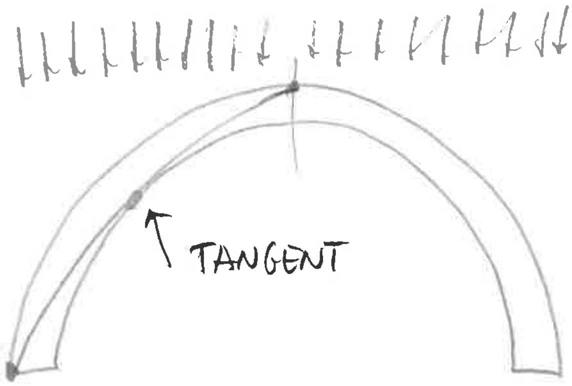
In an arch, we can choose several positions for the hinges, therefore H and V can change slightly. then, in principle, an infinite no of LINE OF THRUST exists.

HEYMAN'S MASTER "SAFE" THEOREM

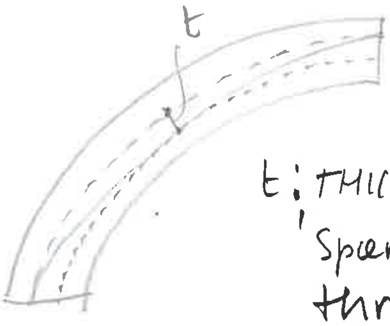
To avoid any tensile stress in the arch, the line of thrust must lie within the boundaries of the arch. therefore:

THE ARCH IS IN EQUILIBRIUM IF AT LEAST ONE LINE OF THRUST IN EQUILIBRIUM WITH THE APPLIED LOADS CAN BE FOUND WHICH LIE WITHIN ITS BOUNDARIES.

Limit condition (on the verge on INSTABILITY)



Here only one line of thrust exists!
Equilibrium is assured (THEORETICALLY), but it is very close to a critical condition.



h : thickness of the arch

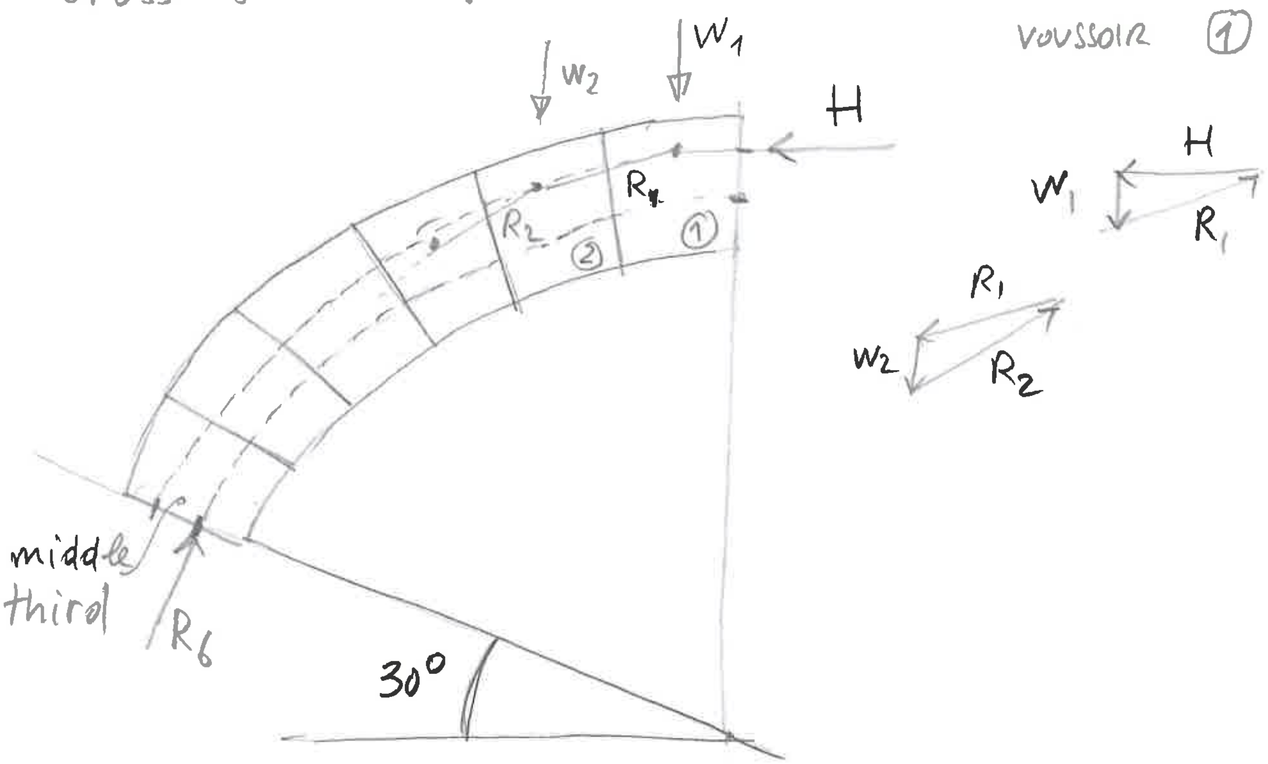
t : THICKNESS of the arch 'spanned' by the line of thrust.

ideally

$$\frac{h}{t} \geq 2$$ for a real arch.

MERY'S METHOD (1840)

This method is quite popular to assess the stability of a voussoir arch under a symmetric loading. It is based on the principle for which the line of thrust should lie in the MIDDLE THIRD OF THE arch cross section:



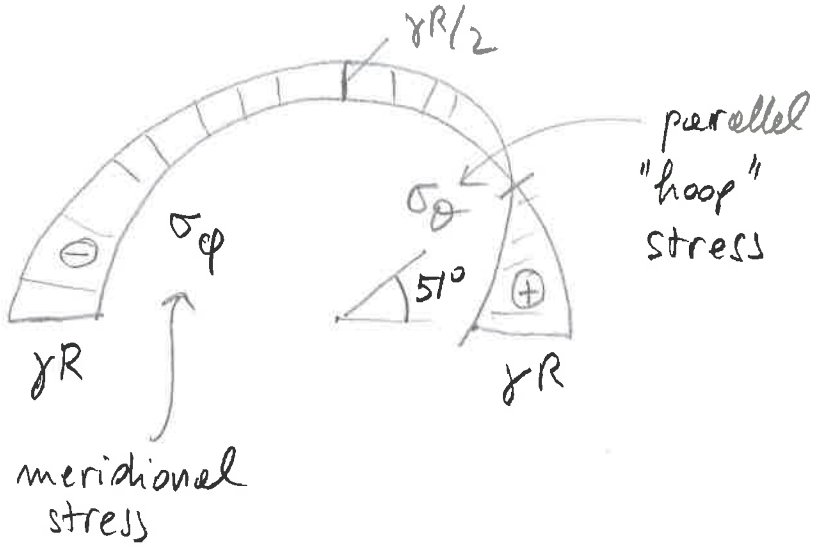
The discrete line of thrust is composed of the collection of segments $R_1, R_2, \dots R_6$.
The arch is safe according to Mery's method if the LINE OF THRUST lies within the middle third.

MASONRY DOME MODELLED WITH A NO-TENSION MATERIAL

E11

We consider self-weight only.

We have already come across the solution of a semi-spherical dome when tension/compression strengths are equal:



Estimate of $\sigma_\varphi = -\sigma_\theta$ at the springing for $\gamma = 20 \frac{kN}{m^3}$

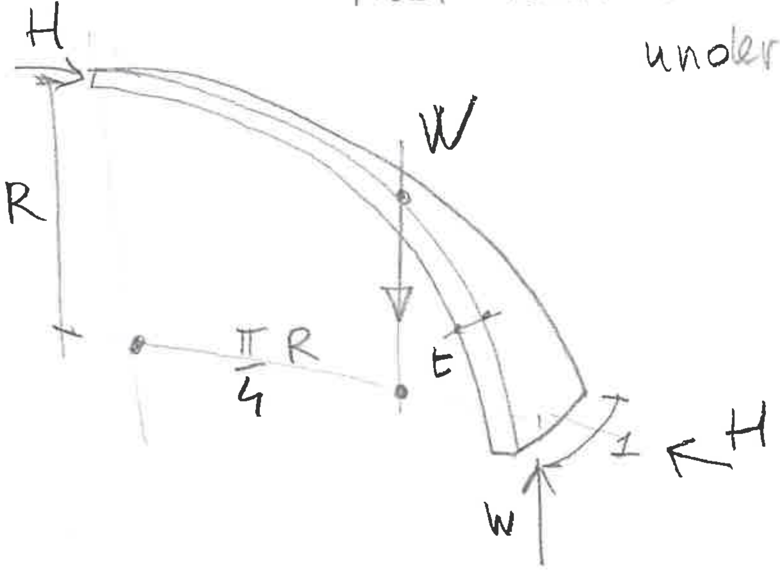
$$\sigma_\varphi = \gamma R = 20\,000 \cdot 10 = 200{,}000 \ \frac{N}{m^2} = 0.2 \text{ MPa}$$

$R = 10$ m

For a NO-TENSION material, the restraint action of "parallels" will no longer apply, then CRACKS occur well beyond the 51° angle indicated above. The dome is subdivided in "slices" connected to each other on top of the dome.

CRACKS extend almost to 65° starting from the springing.

Each slice applies a horizontal thrust per unit length H at the base of the dome that must be equilibrated by the underlying drum.



w: weight + unit length of springing circumference.

$$W = \gamma \frac{2\pi R^2 t}{2\pi R}$$ → volume of the dome → length of the circumf.

$$W = \gamma R t$$

Equilibrium of moment:

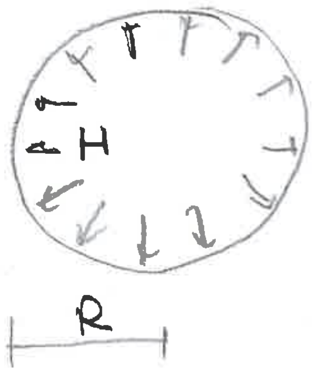
$$W R \left(1 - \frac{\pi}{4}\right) = H R \rightarrow \boxed{H \simeq W \cdot 0.215}$$

EXAMPLE: $R = 10\,m$, $t = 20\,cm$, $\gamma = 20\ KN/m^3$

$$W = \gamma R t = 20000 \cdot 10 \cdot 0.2 = 40\,000\ \frac{N}{m} = 40\ \frac{KN}{m}$$

$$H \simeq W \cdot 0.215 = 40 \cdot 0.215 = 8.6\ KN/m$$

the horizontal thrust can be absorbed by a circular tie-rod (or a steel ring) according to the scheme:



[Recall that the axial force in the ring is $N = HR$]

We can assume a design strength f_{yd}, then the cross-section area (A) of the tie-rod is:

$$N = HR \rightsquigarrow \sigma = \frac{N}{A} \rightarrow A = \frac{N}{f_{yd}} = \frac{HR}{f_{yd}}$$

EXAMPLE

Consider the example in the previous page, where $R = 10\ m$, $H = 8.6\ KN/m$ and assume $f_{yd} = 1200\ N/mm^2$ [steel tie-rod].

then: $$A = \frac{HR}{f_{yd}} = \frac{8600 \cdot 10}{1200 \cdot 10^6} = 7.17 \cdot 10^{-5}\ m^2$$

$$= 71.7\ mm^2$$