Predator-Prey Models

Stephanie Forrest
Dept. of Computer Science
Univ. of New Mexico
Albuquerque, NM

http://cs.unm.edu/~forrest
forrest@cs.unm.edu

Dynamical Systems vs. Computation

- Dynamics:
  - Focus on how things *change.*
  - Describe systems undergoing continual temporal change.
  - View change geometrically: Trajectories, attractors, bifurcations.
  - Stability of patterns of change as a function of system parameters.
  - Scaling limitation: Quantitative analysis only feasible for low-dimensional systems.
- Computation:
  - Focus on *internal structure.*
  - Equivalence classes*.*
  - Marr's *representation and algorithm* level of description.

Styles of Modeling I (review)
Aggregate / Differential Equations

- Describe the global behavior of a system
- Average out individual differences
- Assume infinite-sized populations
- Assume all possible genotypes always present in population.
- Easier to do theory and make quantitative predictions.
- Examples:
  - Fractals
  - Mackey-Glass systems
  - Lotka-Volterra systems

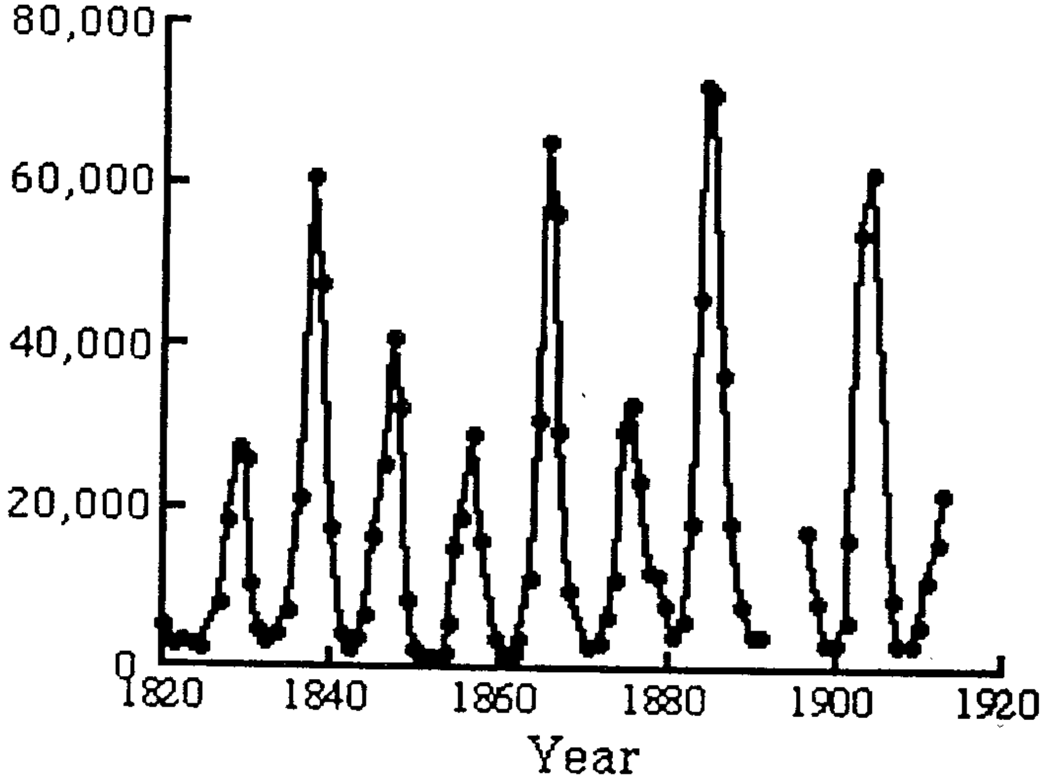
Styles of CAS Modeling II (review)

Computational / Individual-based / Agent-based

- A computational artifact that captures essential components and interactions (I.e. a computer program).
- Encodes a theory about relevant *mechanisms:*
  - Want relevant behavior to arise spontaneously as a consequence of the mechanisms. The mechanisms give rise to macro-properties without being built in from the beginning.
  - This is a very different kind of explanation than simply predicting what will happen next.
  - Example: Cooperation emerges from Iterated Prisoner's Dilemma model.
  - Simulation as a basic tool. Observe distribution of outcomes.
- Study the behavior of the artifact, using theory and simulation:
  - To understand its intrinsic properties, and wrt modeled system.

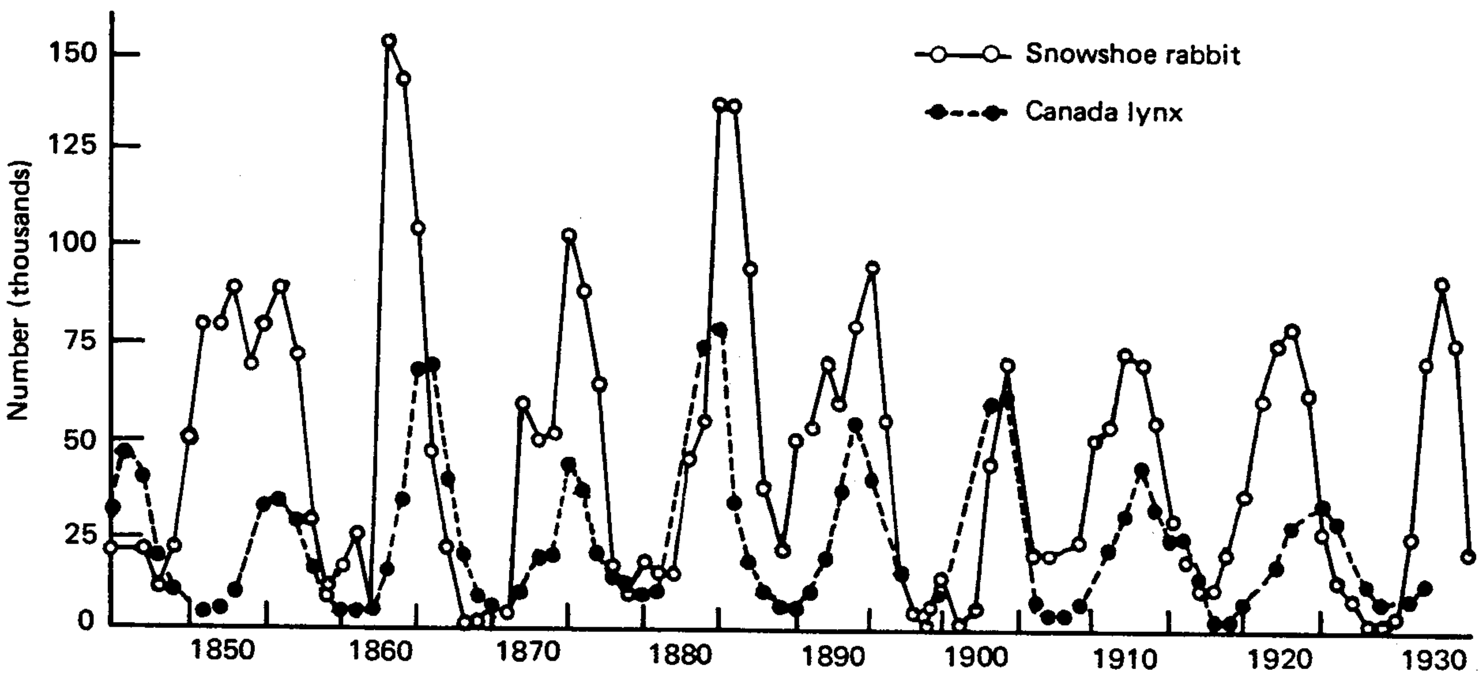
The University of New Mexico

Lynx Historical Data (Predators)



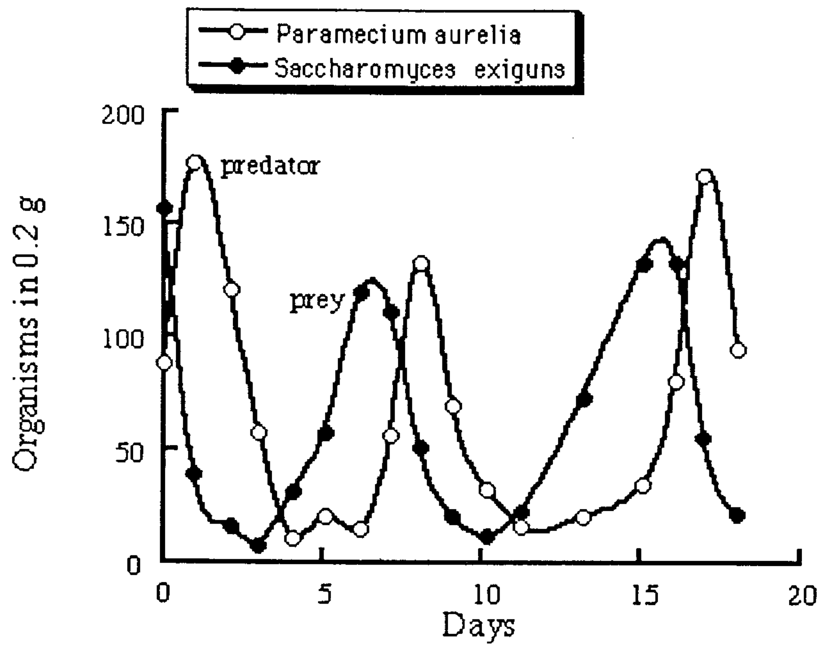
[D'Ancona, 1954]

Lynx/Rabbit Historical Data (Predators)



http://www.globalchange.umich.edu/globalchange1/current/lectures/predation/predation.html

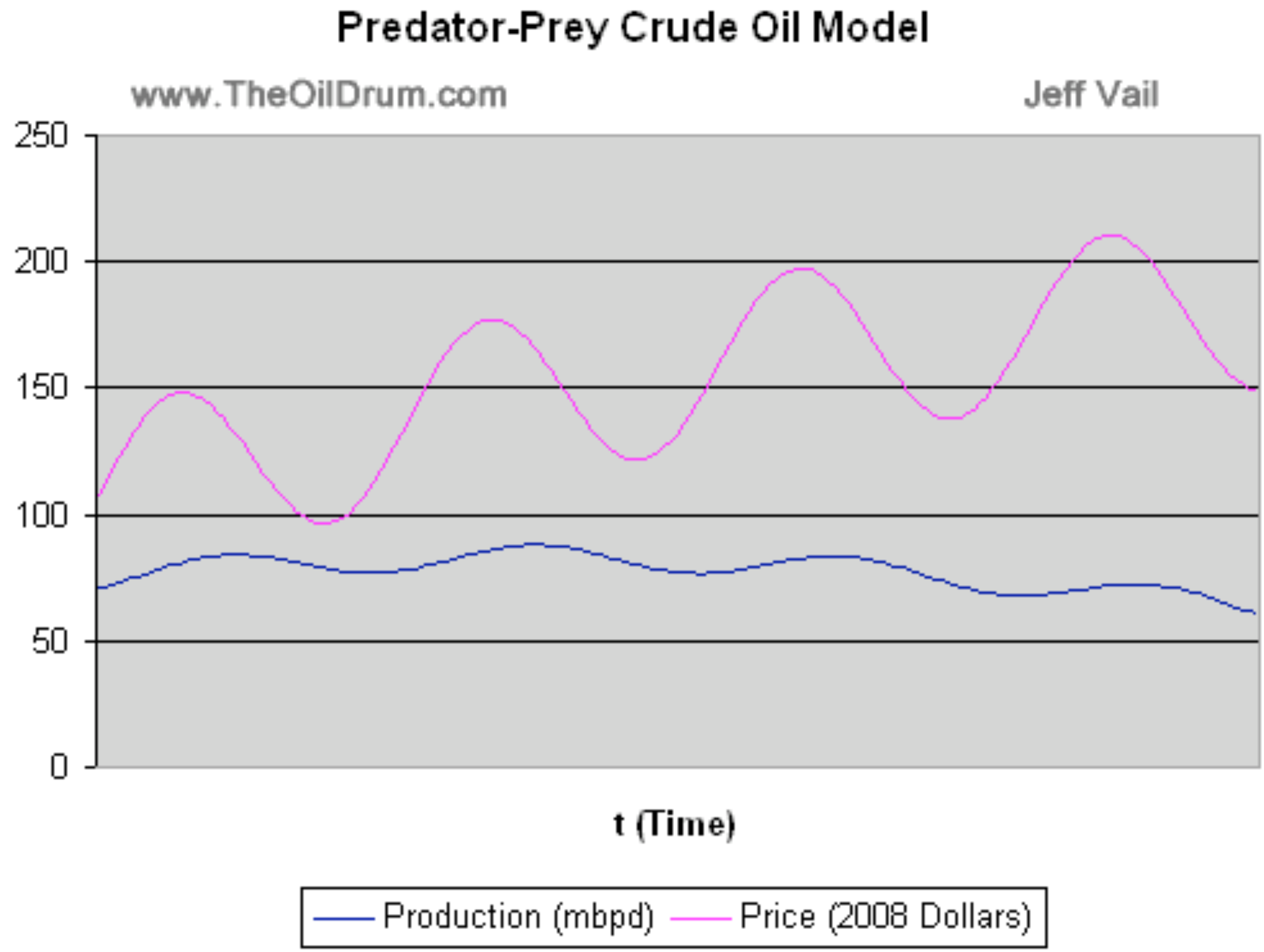
Paramecium Data



Oscillations in the populations of paramecia and yeast, from D'Ancona, 1954

Predator-Prey Dynamics in Demand Destruction and Oil Prices

The predator-prey oscillations of price increases and demand destruction/production increases superimposed on top of a geological depletion scenario--note how the volatility fo the predator-prey dynamic works to conceal the underlying geological and geopolitical trends.



Modeling Predator-Prey Interactions

- The Lotka-Volterra model is the simplest model of predator-prey interactions. It was developed independently by:
  - Alfred Lotka, an American biophysicist (1925), and
  - Vito Volterra, an Italian mathematician (1926).
- Basic idea: Population change of one species depends on:
  - Its current population.
  - Its reproduction rate.
  - Its interactions with other species (predation or prey).
- Model expressed as coupled differential equations:

$$\frac{dx}{dt} = Ax - Bxy$$

$$\frac{dy}{dt} = -Cy + Dxy$$

The Lotka-Volterra Model cont.

- Example: Rabbits and Lynxes (bobcats):
  - Rabbits reproduce at a rate proportional to their population. Let x denote the number (density) of rabbits.
  - Lynxes eat rabbits, and die at a constant rate. Let y denote the density of lynxes.
  - See population plots from real experiments (previous slides).
- The model:
  - Population x (the rabbits) increases at rate $dx = Axdt$.
  - Population x (the rabbits) decreases at rate $dx = -Bxydt$.
  - Population y (the lynxes) increases at rate $dy = Dxydt$.
  - Population y (the lynxes) decreases at rate $dy = -Cydt$.
- The parameters:
  - A = natural reproduction rate of rabbits in the absence of predation.
  - B = death rate per encounter of rabbits due to predation.
  - C = natural death rate of lynxes in the absence of food (rabbits).
  - D = the efficiency of turning predated rabbits into new lynxes.

The University of New Mexico

Lotka-Volterra Model
Rabbit and Lynx Population

- The change in the rabbit population is equal to how many rabbits are born minus the number eaten by lynxes:

$$\frac{dx}{dt} = Ax - Bxy$$

- The change in lynx population is equal to how fast they reproduce (depends on how many rabbits are available to eat) minus their death rate:

$$\frac{dy}{dt} = -Cy + Dxy$$

- Note: In some versions B=D.

Can we predict how the model will behave?

- Are the populations ever stable?
- That is,

$$\frac{dx}{dt} = \frac{dy}{dt} = 0$$

$$A - By = 0 \qquad \mathrm{y} = \frac{\mathrm{A}}{\mathrm{B}}$$

$$-C + Dx = 0 \qquad \mathrm{x} = \frac{\mathrm{C}}{\mathrm{D}}$$

- Stationary point at (x,y) = (C/D,A/B).
  - Fig. 12.1 fixed point (F,S): (1.167,0.74)
- Typical behavior is one of oscillating populations.

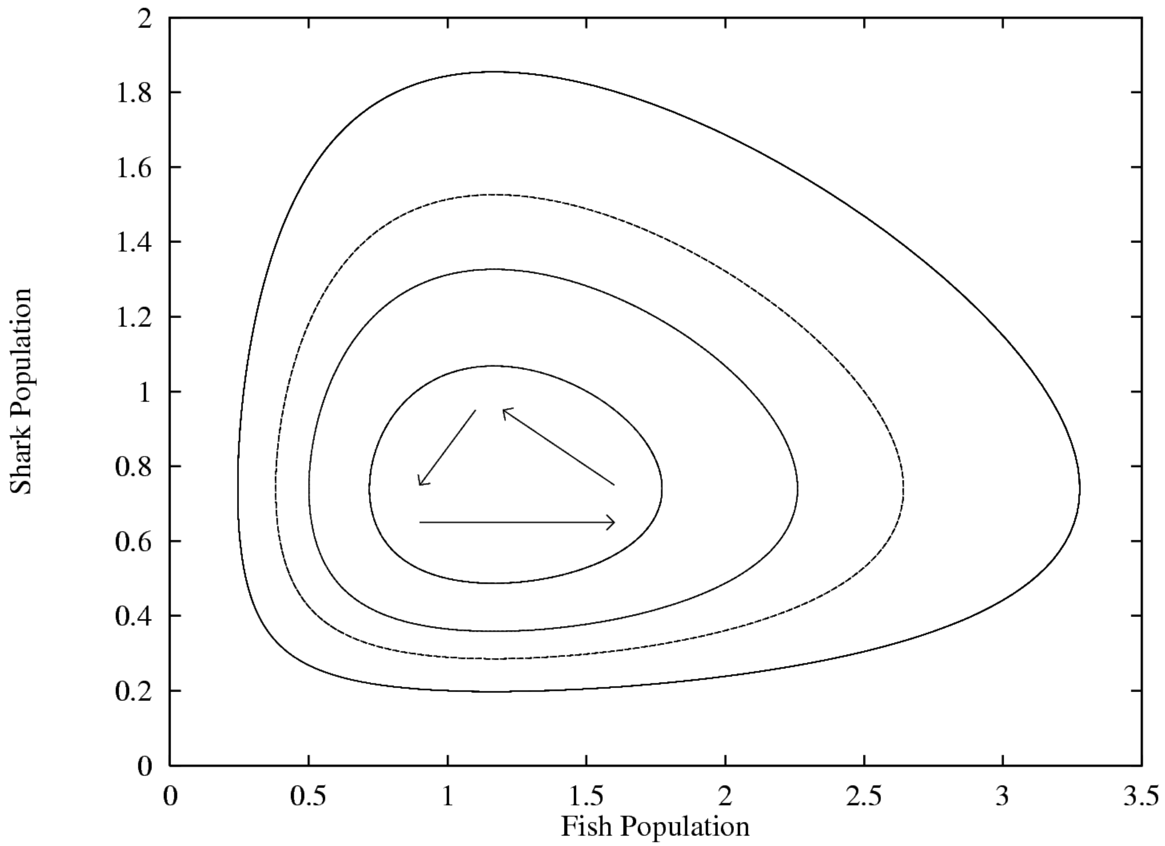


Figure 12.1 A simple Lotka-Volterra attractor which shows four (out of an infinite number of possible) limit cycles. The value of the four parameters are equal to 3.029850, 4.094132, 1.967217, and 2.295942, which yields a fixed point at 1.1671, 0.740047.

Stability of Fixed Point

- What happens if we perturb the system?
  - Perturb initial conditions
  - Perturb system coefficents
  - Stability of system
  - Can analyze mathematically (compute the Jacobian and study eigenvalues)
- Experiment with small perturbations
  - What happens if you increase A slightly (rabbit reproduction rate)?
  - What happens if you increase C slightly (lynx death rate)?

Comments on the Lotka-Volterra Model

- Doesn't consider competition among prey or predators:
  - Prey population may grow infinitely without any resource limits (the rabbits never run out of food).
  - Carrying capacity
  - Predators have no saturation: Their consumption rate is unlimited (the lynxes never get full).
  - Only considers two interacting species.
- Nanofoxes?
- Many extensions and refinements exist, including models of three interacting species model. The 3-species model can have chaotic dynamics.
- Extending Lotka-Volterra to 3 species:

$$\frac{dx_i}{dt} = x_i \sum_{j=1}^{n} A_{ij}(1 - x_j)$$

- X_i represents the i-th species.
- A_{ij} represents the effect that species *j* has on species *i.*
  - Represent A as a matrix.

Three-Species Lotka-Volterra

Example of Chaotic Behavior

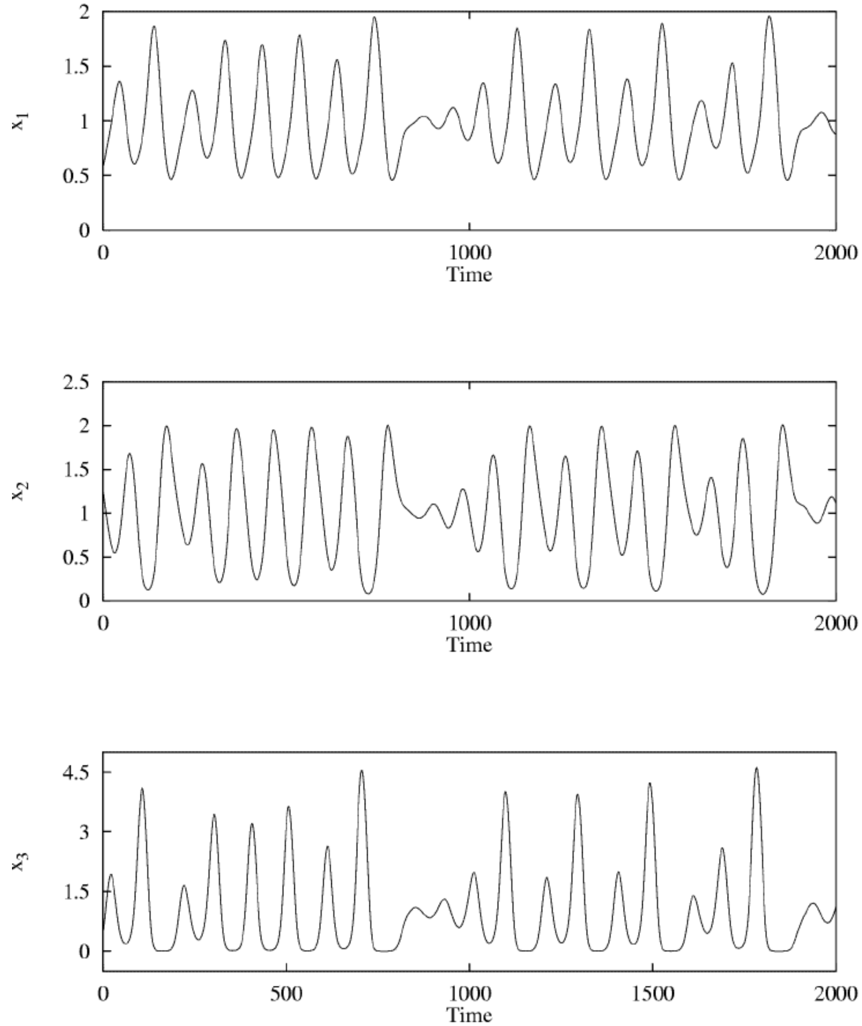


Figure 12.3 Population levels for the three-species Lotka-Volterra system

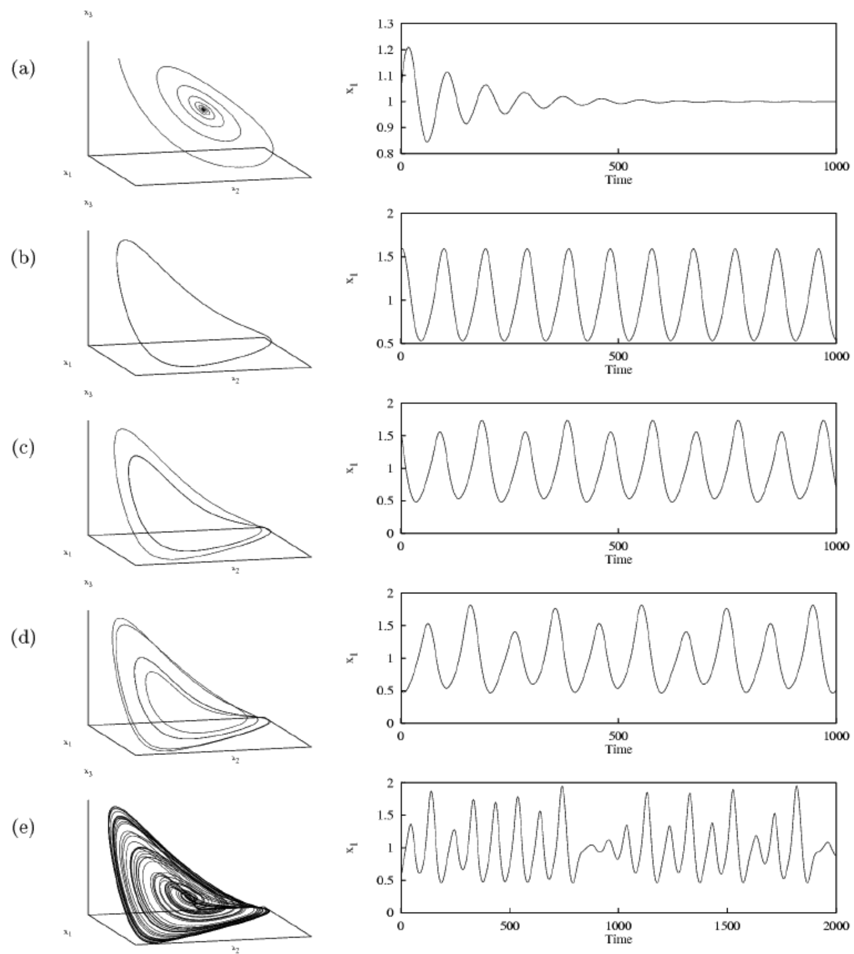
The Interaction Terms

- A_{ij} represents the effect that species *j* has on species *I:*

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0.1 \\ -0.5 & -0.1 & 0.1 \\ \alpha & 0.1 & 0.1 \end{bmatrix}$$

- This choice of A was discovered to have chaotic dynamics:
  - Tune α to see full range of dynamical behaviors.
  - See Figure 12.4.

Three-Species Lotka-Volterra cont.



Stable Fixed Point

Simple Periodic Behavior

Period Doubling

Period 4 Attractor

Chaotic

Figure 12.4 Period doublings in a three-species Lotka-Volterra system: phase space is on the left and x_1 is plotted on the right. (a) spiral fixed point, (b) simple periodic orbit, (c) period-2 orbit, (d) period-4 orbit, (e) chaos

An Alternative Approach
(Agent-based modeling)

- Taken from *The Computational Beauty of Nature*.
- Represent each individual in the population explicitly.
  - Rules for encounters between individuals.
- Write down interaction rules between individuals.
- Represent physical space explicitly:
  - 2-d grid.
  - Each grid site is either empty or contains a single individual.
- Three types of individuals:
  - Plants.
  - Herbivores (rabbits).
  - Carnivores (lynxes).

Individual-based Models

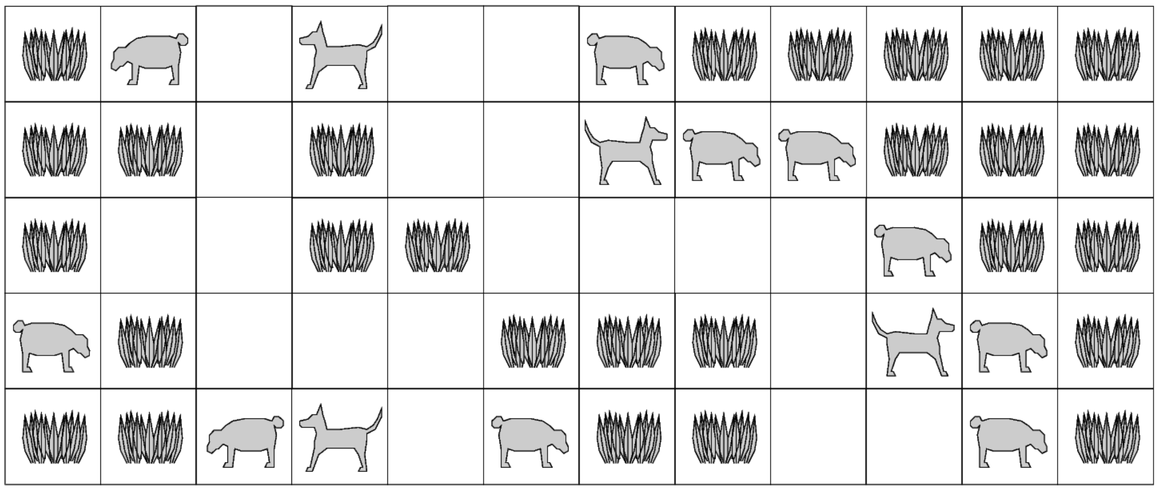


Figure 12.5 An individual-based three-species ecosystem

Agent Rules

- Plants can:
  - Spread into contiguous empty space.
  - Be eaten by herbivores.
- Herbivores can:
  - Die (by starving to death or being eaten by carnivores).
  - Move into contiguous locations.
  - Eat plants.
  - Have babies (if they have stored enough energy).
- Carnivores can:
  - Die (by starving to death).
  - Move to a contiguous location.
  - Eat herbivores.
  - Have babies (if they have stored enough energy).

Population Dynamics (Time Series) in an Individual-based Model

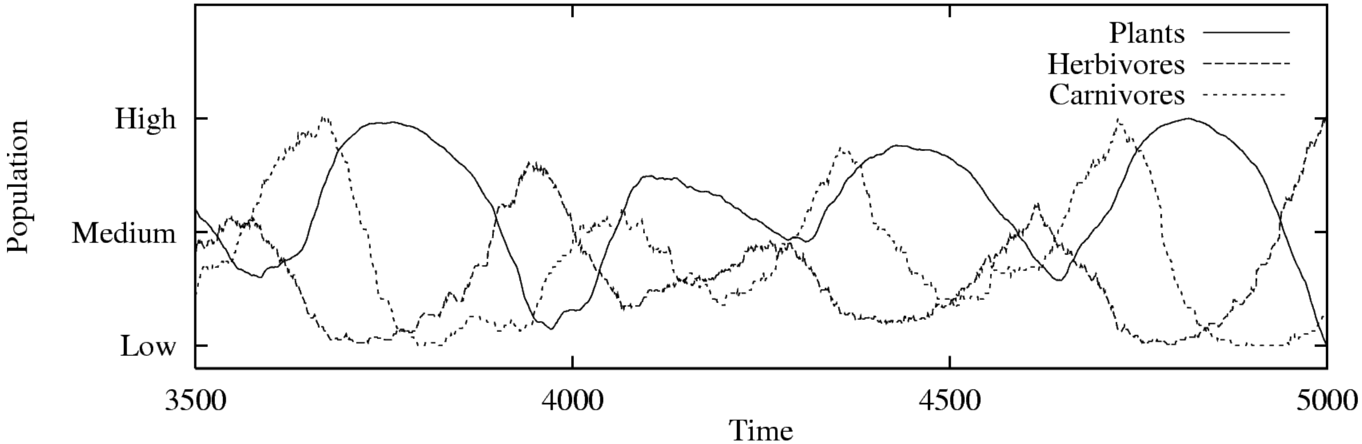


Figure 12.7 Population levels for all creatures, normalized for comparison

The University of New Mexico

Comparison with Lotka-Volterra Model

- How is this model different from Lotka-Volterra?
- How is this model the same as Lotka-Volterra?
- Do we expect fixed points?

The University of New Mexico

Comparison with Lotka-Volterra Model

- How is this model different from Lotka-Volterra?
  - Potential for non-uniform mixing (because space is represented explicitly)
  - Non-deterministic movement into adjacent spaces.
  - Discrete time.
  - Discrete population values.
  - Discrete threshhold for reproduction.
- How is this model the same as Lotka-Volterra?
  - Similar interaction rules.
  - Similar dynamics.
- Do we expect fixed points?

How Should We Study/Analyze the Behavior of this Agent-based Model?

- Look at one run carefully?
- Look at many runs and “note” the analogy to Lotka-Volterra models?
- Look at many runs and compute statistics?
- Look at very large runs?
- Can we make the analogy more precise?
  - Suppose we ignore space and simply record the relative population values for the entire grid, over time.
  - What is the state space?
  - Record how population changes at each point in state space.
  - Compare with phase portrait for Lotka-Volterra.

The University of New Mexico

Conclusions

- Species eye view of the world vs. individual eye:
  - Lotka-Volterra ignores variations among individuals.
  - Lotka-Volterra assumes infinite-size populations and perfect mixing.
- The agent-based alternative is not exactly a cellular automaton.
- These models form the basis of many more complicated models.
- Examples:
  - Echo (Holland, 1994)
    - One type of agent. Agents can: mate, fight, trade.
    - Agent rules and preferences can evolve over time.
  - Maley's models of Barrett's Esophagus, CancerSim.
  - Disease modeling