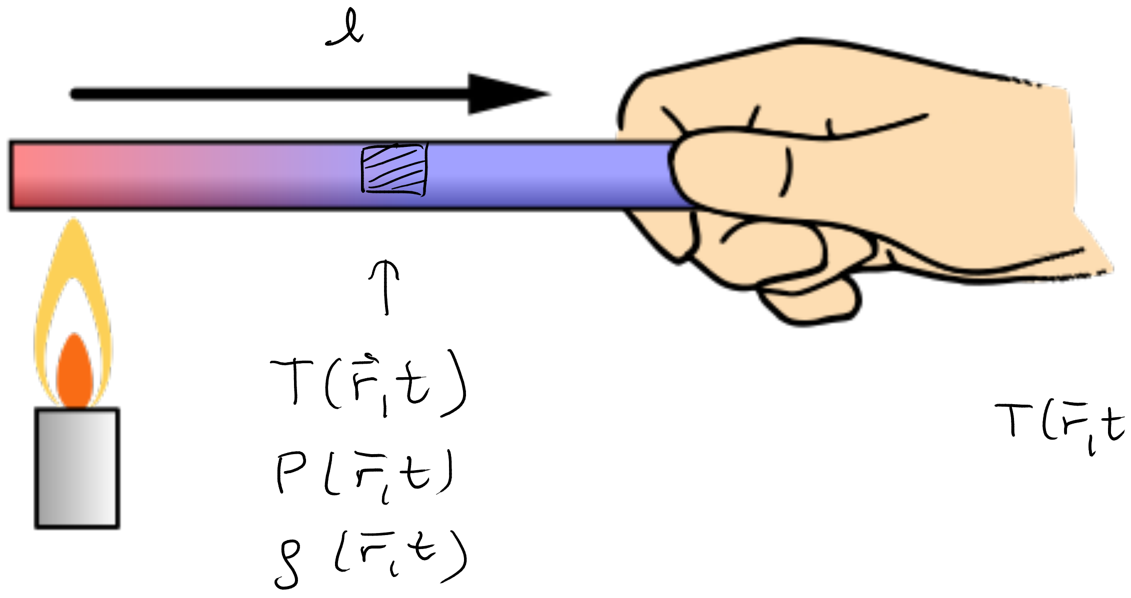
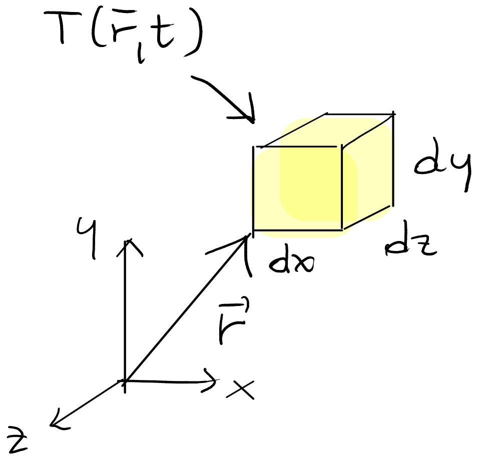
CONDUZIONE TERMICA

$$dS = dS_1 + \dots + dS_M \geq 0$$

Evoluzione temporale delle variabili di stato → termodinamica fuori equilibrio



Equilibrio locale: tra t e $t+dt$, in ogni punto $\vec{r}$ esiste un sottosistema (macro) in equilibrio termodinamico



$$l_0 \ll dx, dy, dz \ll l$$

micro macro

10^{-9} m

$$\tau_0 \ll dt \ll \tau$$

micro macro

$$c\tau_0 = l_0$$

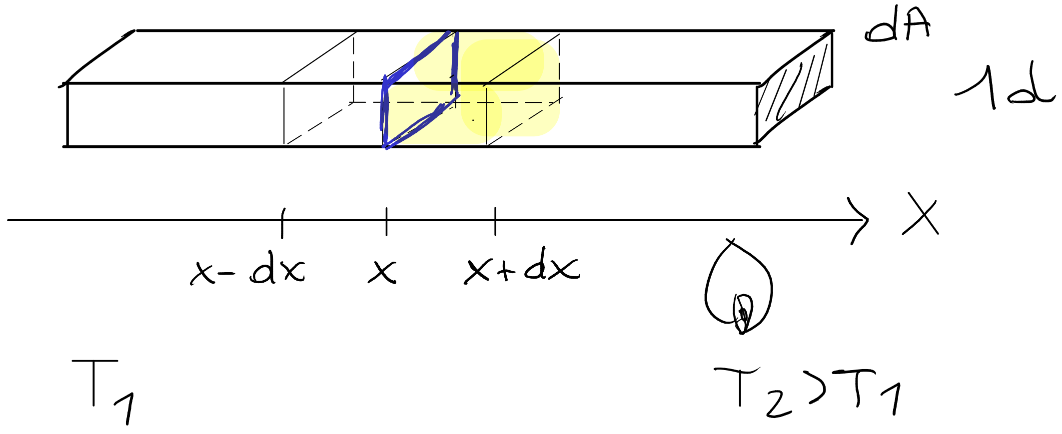
$$\tau_0 = \frac{l_0}{c} = \frac{10^{-9}\ \mathrm{m}}{3\times10^{2}\ \mathrm{m/s}}$$

$$\approx 10^{-12}\ \mathrm{s}$$

Conduzione termica:

scambi energetici sotto forma di calore in fluidi o solidi **a riposo**

Legge di Fourier: empirica



Calore scambiato dal sistema in x con quello in x − dx durante l'intervallo dt

$$\delta Q \sim dA\, dt\, \frac{dT}{dx}$$

(δQ_-)

$$dT = T(x) - T(x-dx)$$

$dT > 0 \quad \delta Q < 0$

$$\delta Q = -\lambda\, dA\, dt\, \frac{dT}{dx}$$

$\uparrow$

conducibilità termica

Joseph Fourier
1768 – 1830

$$\frac{\delta Q}{dt} = I_t$$ corrente termica — SI: $\frac{J}{s} = W$

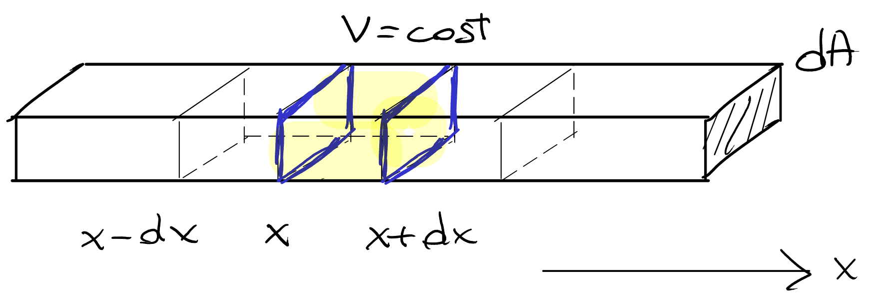
$$\frac{\delta Q}{dt\, dA} = J_t$$ densità di corrente termica — SI: $\frac{J}{s\, m^2} = \frac{W}{m^2}$

$$J_t = -\lambda \frac{dT}{dx}$$ legge di Fourier

λ SI: $\frac{W}{m \cdot K}$

Conservazione dell'energia : bilancio energetico per il sistema in x

$$\boxed{dE_c + dU = \delta W + \delta Q}$$



$$\underset{=0}{dE_c} + dU = \underset{=0}{\delta W} + \delta Q = \underset{\text{con sx}}{\delta Q_-(x)} + \underset{\text{con dx}}{\delta Q_+(x)}$$

$$dU = \delta Q_-(x) - \delta Q_-(x+dx) = J_t(x)\, dA\, dt - J_t(x+dx)\, dA\, dt$$

$$dU = -\, dA\, dt\, [J_t(x+dx) - J_t(x)]$$

$$dU = C_v\, dT + \underset{=0}{\frac{\partial U}{\partial V}\Big|_T dV} = C_v\, dT \qquad C_v = m c_v = \rho V c_v = \rho c_v\, dA\, dx$$

$$\rho c_v \cancel{dA}\, dx\, dT = -\cancel{dA}\, dt\, [J_t(x+dx) - J_t(x)]$$

$$\frac{dT}{dt} = -\frac{1}{\rho c_v} \frac{J_t(x+dx) - J_t(x)}{dx} = -\frac{1}{\rho c_v} \frac{dJ_t}{dx}$$

Inserisco la legge di Fourier : $J_t = -\lambda \dfrac{dT}{dx}$

$$\frac{dT}{dt} = -\frac{1}{\rho c_v}\frac{d}{dx}\left(-\lambda\frac{dT}{dx}\right) = \frac{\lambda}{\rho c_v}\frac{d^2T}{dx^2} \qquad T = T(x, t)$$

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_v}\frac{\partial^2 T}{\partial x^2}$$ equazione del calore $\rightarrow$ $$\frac{\partial T}{\partial t} = D_t \frac{\partial^2 T}{\partial x^2}$$

D_t = diffusività termica

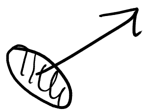
- $m\dfrac{d^2x}{dt^2} = \sum F_x$ $\qquad t \rightarrow t' = -t \Rightarrow \dfrac{d}{dt'} = -\dfrac{d}{dt}$
- $\dfrac{\partial T}{\partial t} = D_t \dfrac{\partial^2 T}{\partial x^2}$ $\rightarrow$ eq. diff. alle derivate parziali $\qquad -\dfrac{\partial T}{\partial t'} = D_t\dfrac{\partial^2 T}{\partial x^2}$

$\rightarrow$ processo irreversibile!

Random walk

$$\frac{\partial p}{\partial t} = D\frac{\partial^2 p}{\partial x^2}$$ D = coefficiente di diffusione

In 3d: $\vec{J}_t = J_{tx}\,\vec{e}_x + J_{ty}\,\vec{e}_y + J_{tz}\,\vec{e}_z$



$\vec{J}_t$

$$J_{tx} = -\lambda\frac{\partial T}{\partial x}$$

$$f(x,y,z) \quad \to \quad \vec{\nabla} f = \frac{\partial f}{\partial x}\vec{e}_x + \frac{\partial f}{\partial y}\vec{e}_y + \frac{\partial f}{\partial z}\vec{e}_z$$

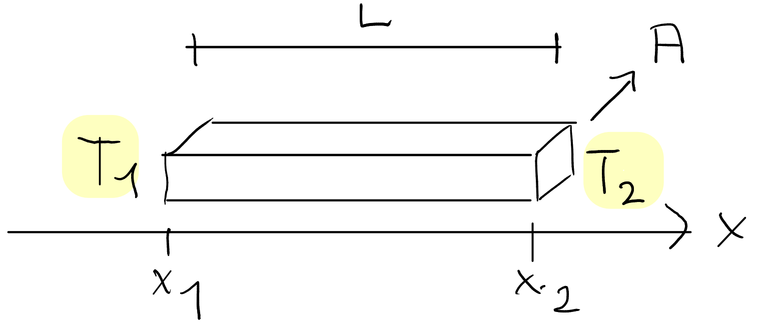
Legge di Fourier : $\vec{J}_t = -\lambda \vec{\nabla} T$

Equazione del calore : $$\frac{\partial T}{\partial t} = D_T \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = D_T \underbrace{\vec{\nabla} \cdot}_{\text{divergenza}} \vec{\nabla} T = D_T \overbrace{\nabla^2}^{\text{laplaciano}} T$$

Conduzione termica in regime stazionario

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_v}\frac{\partial^2 T}{\partial x^2} \qquad \text{regime stazionario: } \frac{\partial}{\partial t} = 0 \quad \Rightarrow \quad \frac{d^2 T}{dx^2} = 0 \qquad \left(\frac{d^2 x}{dt^2} = 0\right)$$

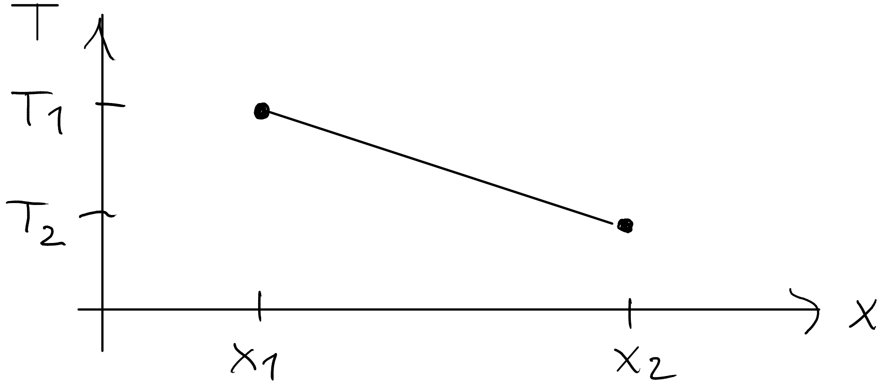
omogeneo



$$\frac{dT}{dx} = c$$

$$\int_{T_1}^{T_2} dT = c\int_{x_1}^{x_2} dx \rightarrow T_2 - T_1 = c(x_2 - x_1)$$

$$\rightarrow T_2 = c(x_2 - x_1) + T_1$$



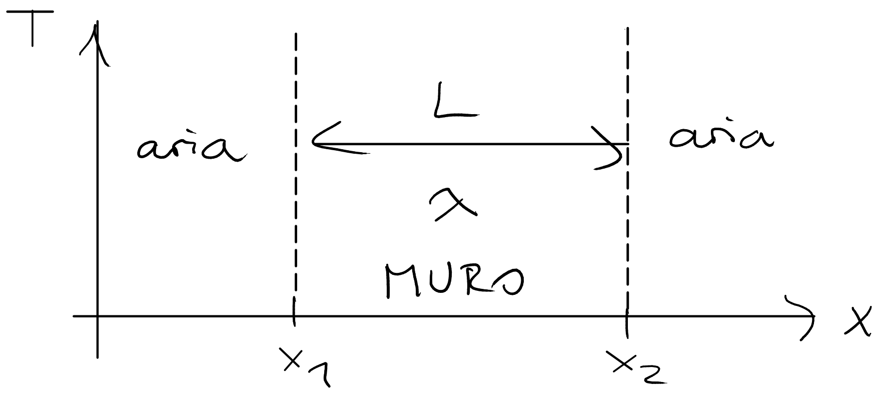
costanti

Legge di Fourier: $J_t = -\lambda \frac{dT}{dx} \Rightarrow c = -\frac{J_t}{\lambda}$

Corrente termica: $I_t = A J_t \qquad J_t = \frac{I_t}{A}$

$$T_2 = -\frac{J_t}{\lambda}L + T_1 = -\frac{I_t}{\lambda A}L + T_1$$

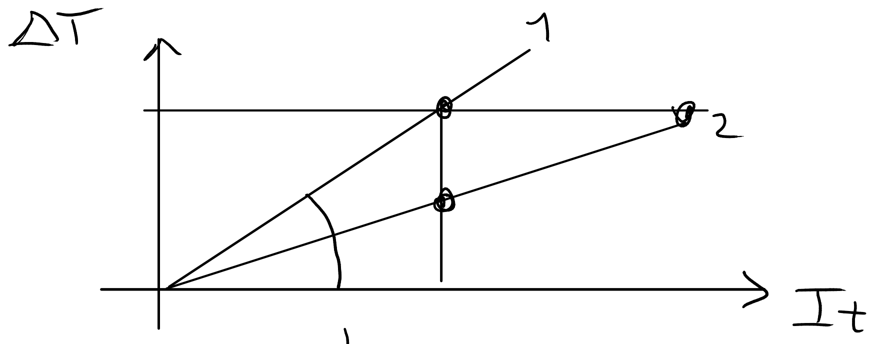
$$\frac{L}{\lambda A} I_t = T_1 - T_2 = \Delta T$$



$$I_t \sim \Delta T$$

$$\Delta T \sim I_t$$

$$\Delta T = \frac{L}{\lambda A} I_t$$



resistenza termica

Resistenza termica : SI : $\frac{K}{W}$

$$\Delta T = R I$$

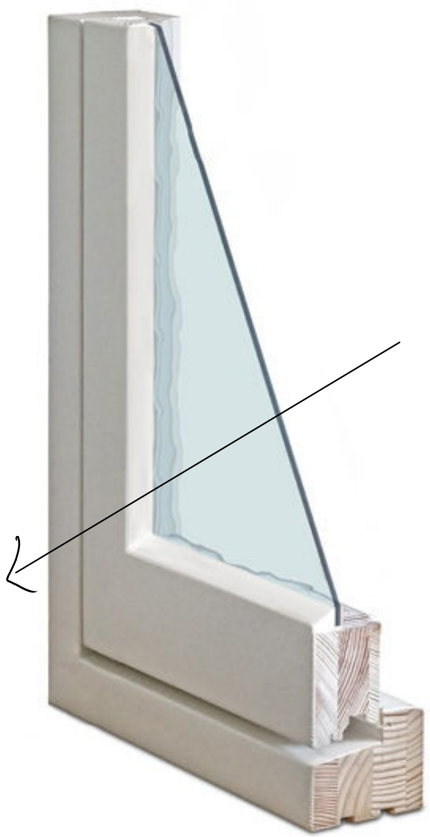
$$R_t = \frac{L}{\lambda A} \Rightarrow \Delta T = R_t I_t = R I_t$$

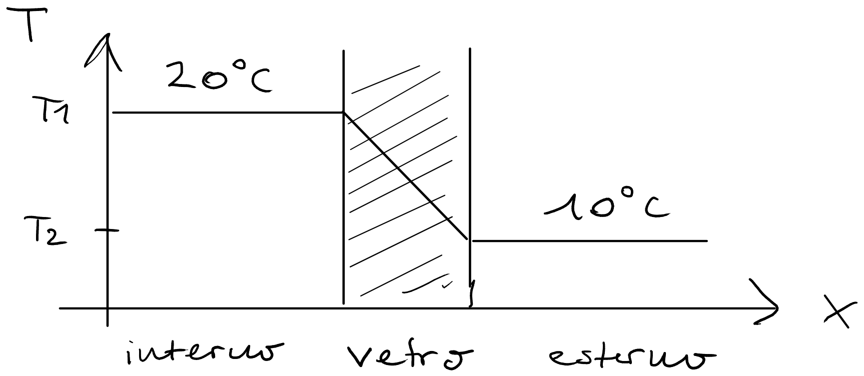
$$(\Delta V = R I)$$

Es.: vetro singolo

$A = 5\ m^2 \qquad L = 5\ mm \qquad \lambda = 1\ \frac{W}{mK}$

$$R = \frac{L}{\lambda A} = \frac{\cancel{5}\times 10^{-3}\ m}{1\ \frac{W}{mK}\times \cancel{5}\ m^2} = 10^{-3}\ \frac{K}{W}$$





$$I_t = \frac{\Delta T}{R} = \frac{10\ K}{10^{-3}\ \frac{K}{W}} = 10^4\ W = 10\ kW$$

$$Q = I_t \cdot \Delta t = 10\ kW \times 1\ h = 10\ \underbrace{kWh}_{\text{ENERGIA}}$$

Elettricità: 0.05 € / kWh

Gas: 0.5 € / Smc → 0.05 € / kWh

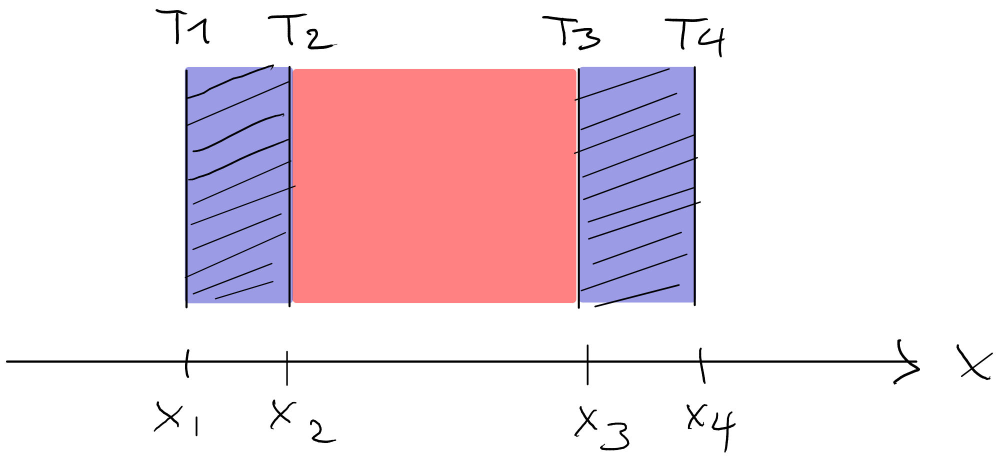
} 5c / kWh

1 giorno: 240 kWh → costo: $5\times 10^{-2}\ \frac{€}{kWh} \times 240\ kWh = 12\ €$

1 mese: → costo: 360 €

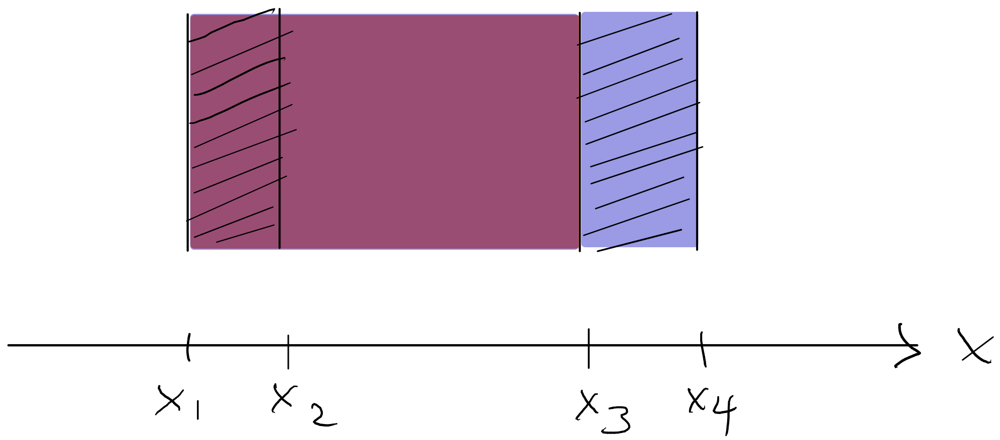
Resistenze termiche in serie

Regime stazionario : I_t è la stessa in ogni punto (costante)



$$\begin{cases} T_1 - T_2 = R_{12}\, I_t \\ T_2 - T_3 = R_{23}\, I_t \end{cases}$$

$$\left.\begin{aligned} T_1 - T_3 &= (R_{12} + R_{23})\, I_t \\ \Delta T &= R_{13}\, I_t \end{aligned}\right\} \quad R_{13} = R_{12} + R_{23}$$



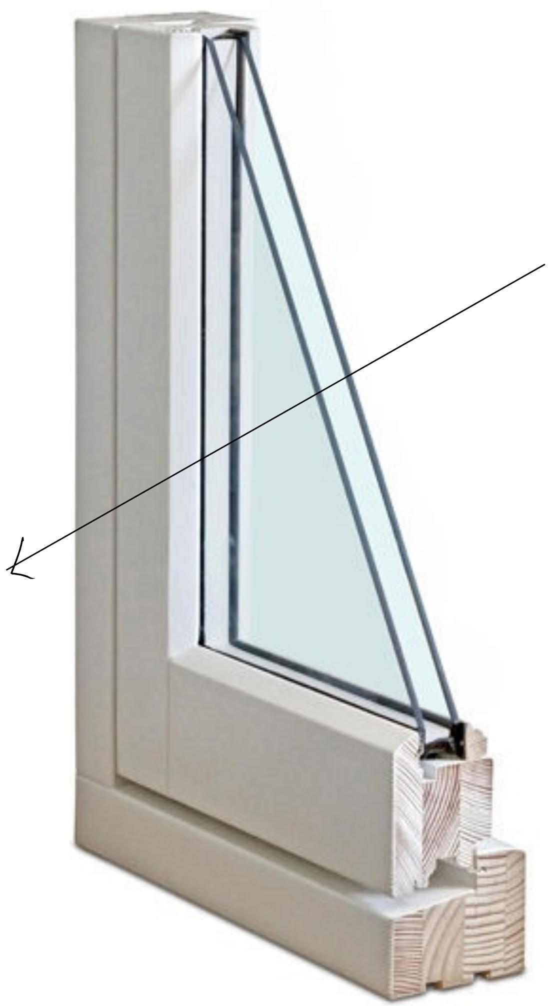
$$\begin{cases} T_1 - T_3 = R_{13}\, I_t \\ T_3 - T_4 = R_{34}\, I_t \end{cases}$$

$$T_1 - T_4 = (R_{13} + R_{34})\, I_t \rightarrow R_{14} = R_{12} + R_{23} + R_{34}$$

$$R_{tot} = \sum_{i=1}^{N} R_i$$ le resistenze termiche in SERIE si sommano

Es: doppio vetro

$A = 5\ m^2$ $L = 5\ mm$ $\lambda_{aria} = 26 \times 10^{-3}\ \frac{W}{mK}$ $x_3 - x_2 = 10\ mm$



$R_{12} = R_{34} = 10^{-3}\ \frac{K}{W}$ singolo pannello

$$R_{23} = \frac{x_3 - x_2}{\lambda_{aria} A} = \frac{10^{-2}\ m}{26 \times 10^{-3}\ \frac{W}{mK} \times 5\ m^2} = 8 \times 10^{-2}\ \frac{K}{W}$$

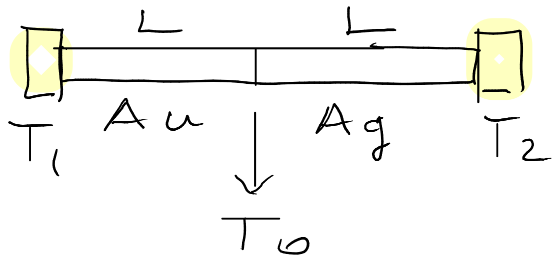
$$= 80\ R_{12}$$

$$R_{tot} = 2 \times 10^{-3}\ \frac{K}{W} + 8 \times 10^{-2}\ \frac{K}{W} = 8{,}2 \times 10^{-2}\ \frac{K}{W}$$

$$\Delta T = R_{tot}\ I_t$$

$$I_t = \frac{\Delta T}{R_{tot}} = \frac{10\ K}{8{,}2 \times 10^{-2}\ K/W} = 120\ W \approx \frac{I_t^{(1)}}{80}$$

Es.: barre di Au e Ag in serie



$T_1 = 80°$

$T_2 = 30°$

λ_{Au}, λ_{Ag}, A

regime stazionario

$T_0 = ?$

$$\begin{cases} T_1 - T_0 = R_{Au} I_t \\ T_0 - T_2 = R_{Ag} I_t \end{cases}$$

$$\frac{T_1 - T_0}{R_{Au}} = \frac{T_0 - T_2}{R_{Ag}}$$

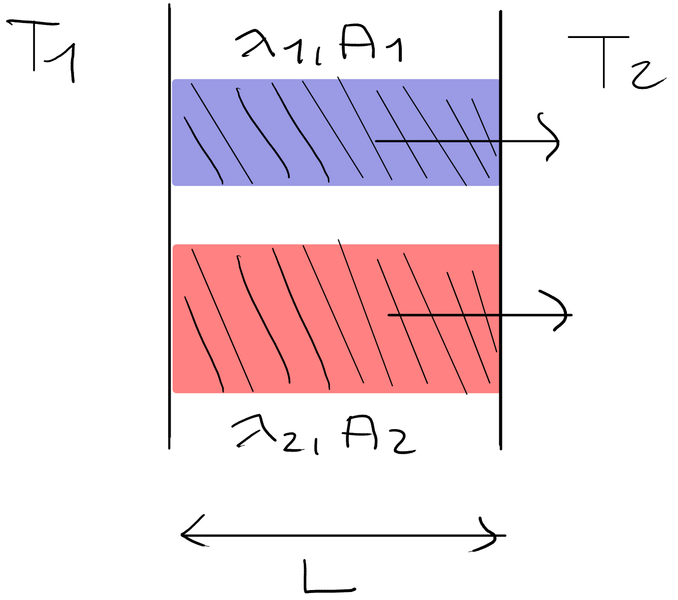
$$R_{Ag}(T_1 - T_0) = R_{Au}(T_0 - T_2)$$

$$(R_{Au} + R_{Ag})T_0 = R_{Ag}T_1 + R_{Au}T_2$$

$$T_0 = \frac{R_{Ag}T_1 + R_{Au}T_2}{R_{Au} + R_{Ag}} \rightarrow \text{media ponderata}$$

$$\frac{\sum w_i x_i}{\sum w_i}$$

Resistenze termiche in parallelo



Regime stazionario. Corrente totale: $\boxed{I_{tot} = I_1 + I_2}$

$$\Delta T = R_1 I_1 \Rightarrow I_1 = \frac{\Delta T}{R_1}$$

$$\Delta T = R_2 I_2 \Rightarrow I_2 = \frac{\Delta T}{R_2}$$

$$I_{tot} = \frac{\Delta T}{R_1} + \frac{\Delta T}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\Delta T$$

$$I_{tot} = \frac{1}{R_{tot}} \Delta T$$

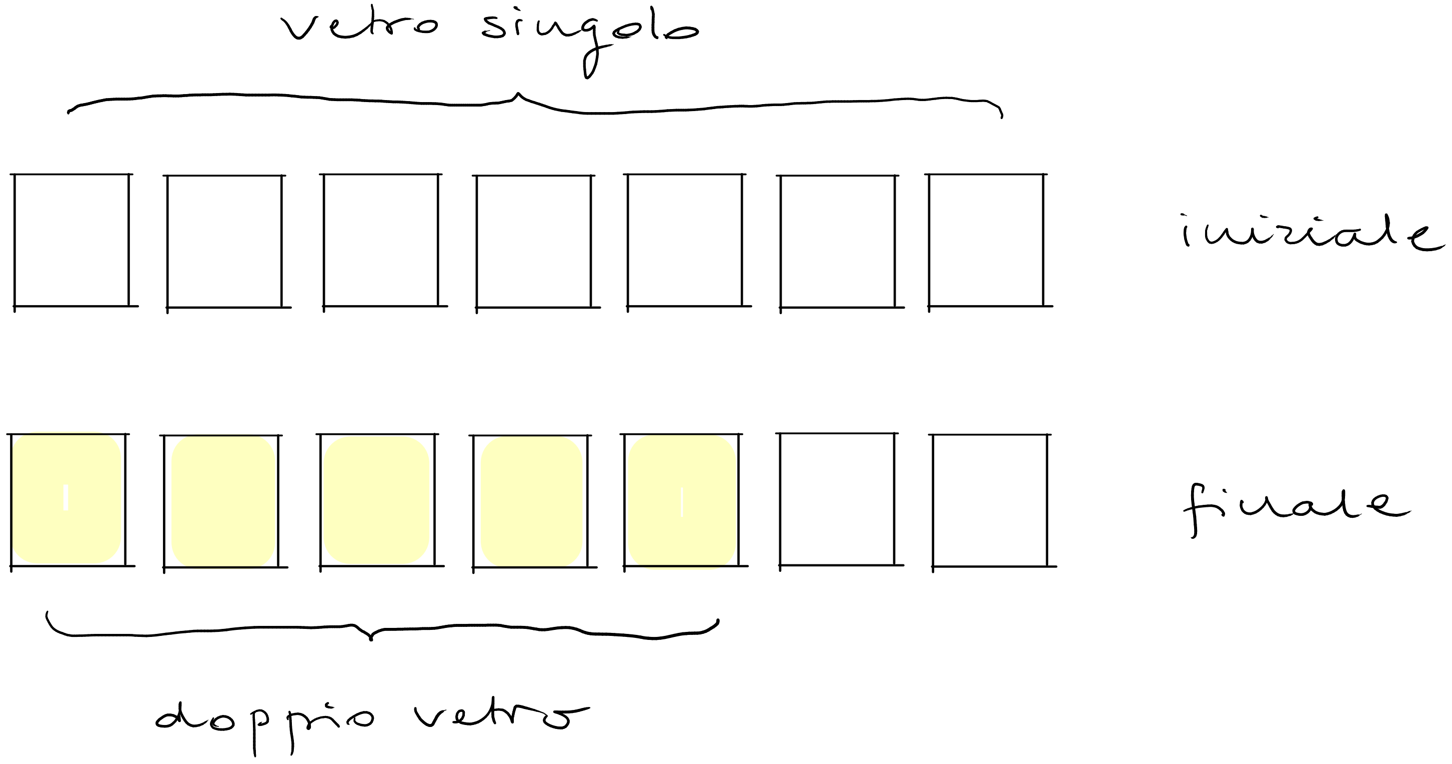
$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \frac{1}{R_{tot}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

delle resistenze termiche in parallelo si sommano i reciproci

ES.: $R_1 = R_2 = R$

serie: $R_{tot} = 2R$ parallelo: $\frac{1}{R_{tot}} = 2\frac{1}{R} \Rightarrow R_{tot} = \frac{R}{2}$

Es.: risparmio energetico casa coslovich



$$R_{2v} = 100\, R_{1v}$$

$$\varepsilon = \frac{I_{tf}}{I_{ti}} = ?$$

se $R_{1v} \approx 10^{-3}\ \frac{K}{W}$

$\Delta T = 10\ K$

trova $\Delta I_t = I_{tf} - I_{ti} = ?$

Assumendo un costo di 0,05 €/kWh, trova il risparmio giornaliero.