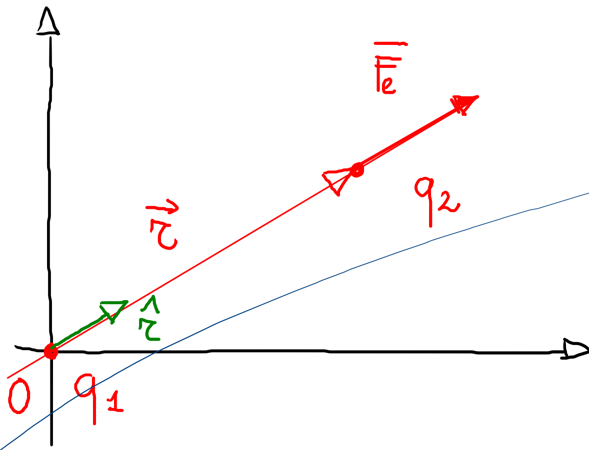
FORZA ELETTROSTATICA

$e = +1{,}602 \cdot 10^{-19}$ C carica di un protone

C ← Coulomb unità SI



$$\vec{F_e} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot \frac{q_1 \cdot q_2}{r^2}\,\hat{r}$$

$r = |\vec{r}|$

$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r}$

$\varepsilon_0 = 8{,}854 \cdot 10^{-12}\ \frac{C^2}{m^2 \cdot N}$

numero puro

$\varepsilon_r = 1$ nel vuoto

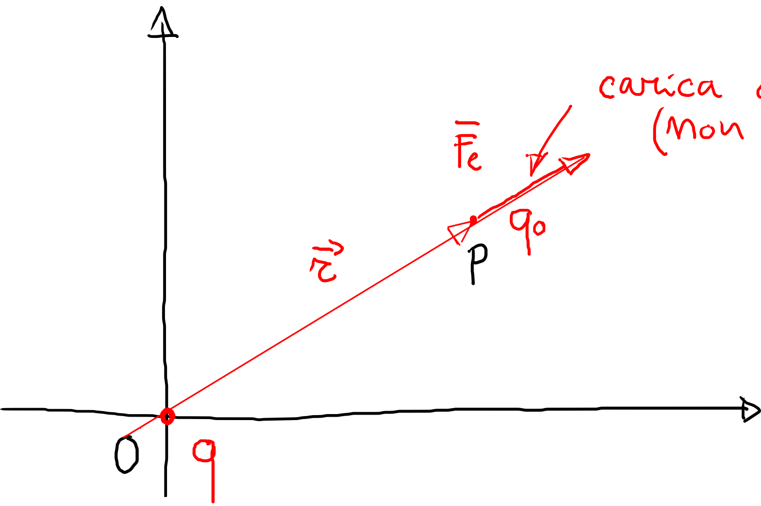
$\varepsilon_r > 1$ nei materiali dielettrici

ε_0 costante dielettrica del vuoto

ε_r " " relativa

$$\frac{1}{4\pi\varepsilon_0} = k \cong 9 \cdot 10^9\ \frac{Nm^2}{C^2}$$

CAMPO ELETTRICO



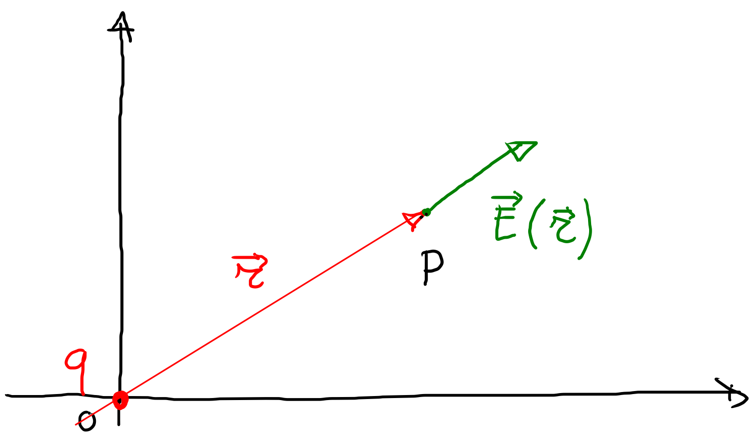
carica di prova, piccola, positiva
(non altera l'ambiente)

$$\vec{F}_e = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\,\frac{q\,q_0}{r^2}\,\hat{r}$$

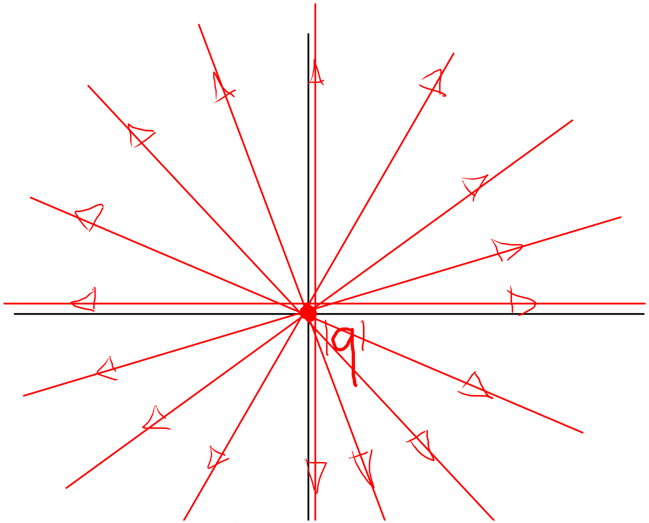
$$\vec{F}_e = q_0 \cdot \underbrace{\left(\frac{1}{4\pi\varepsilon_0\varepsilon_r}\,\frac{q}{r^2}\,\hat{r}\right)}_{\vec{E}(\vec{r})}$$

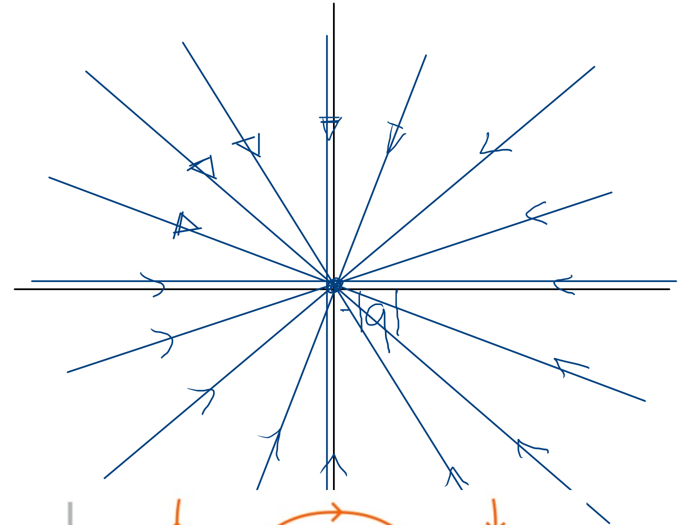
$$\vec{F}_e = q_0 \cdot \vec{E}(\vec{r})$$

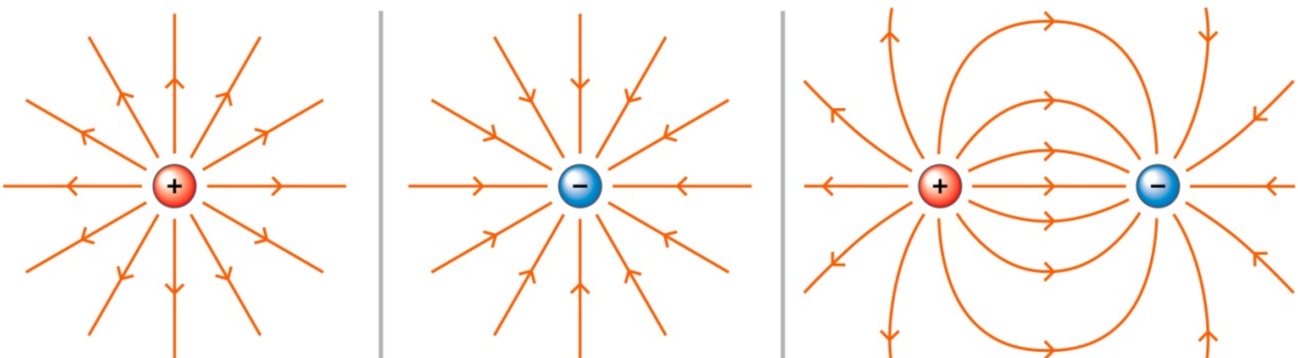
campo elettrico
generato da q
in tutto lo spazio



$$\boxed{\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\,\frac{q}{r^2}\,\hat{r}}$$





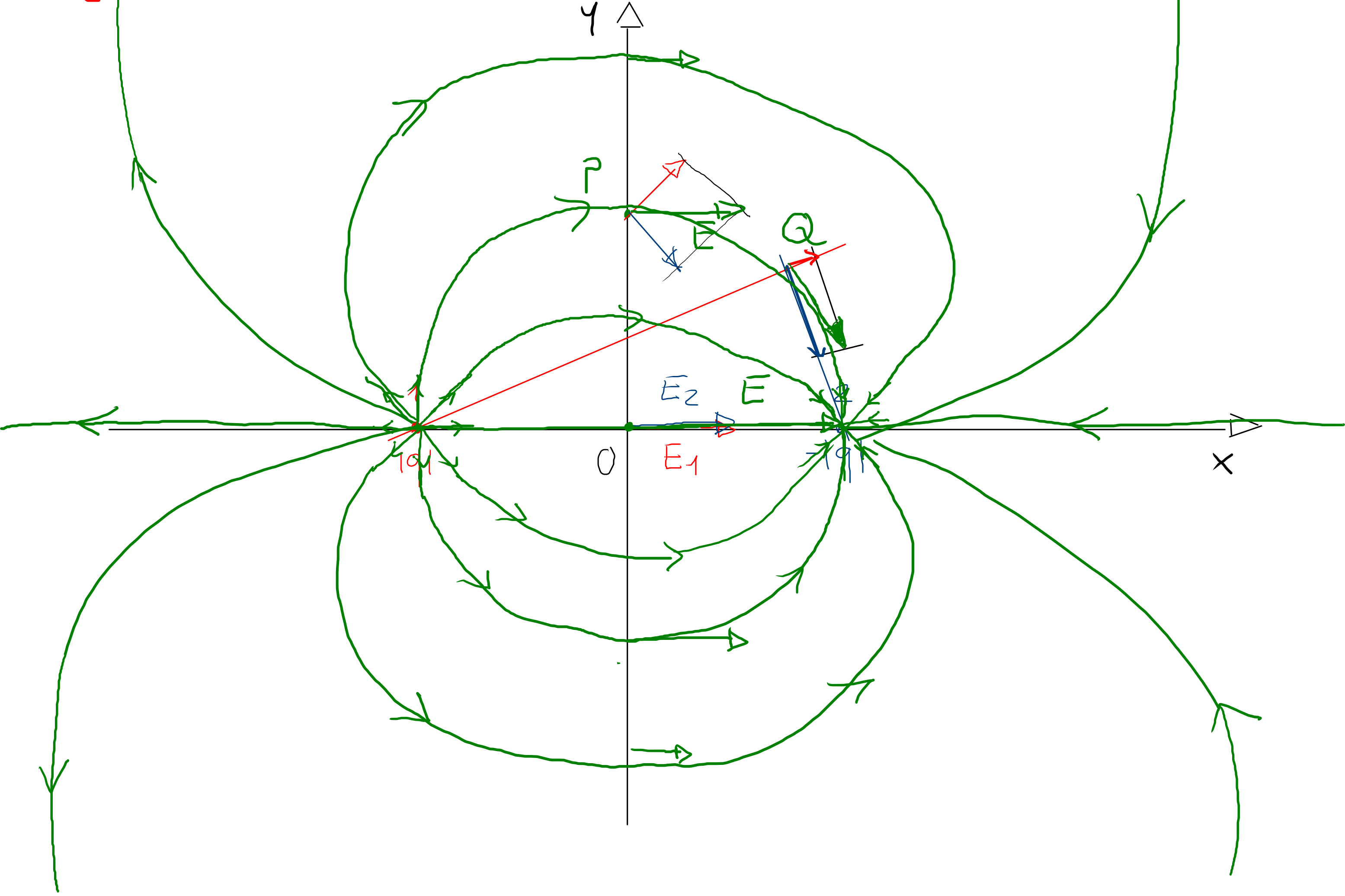


a Linee di forza del campo elettrico creato da una carica positiva.

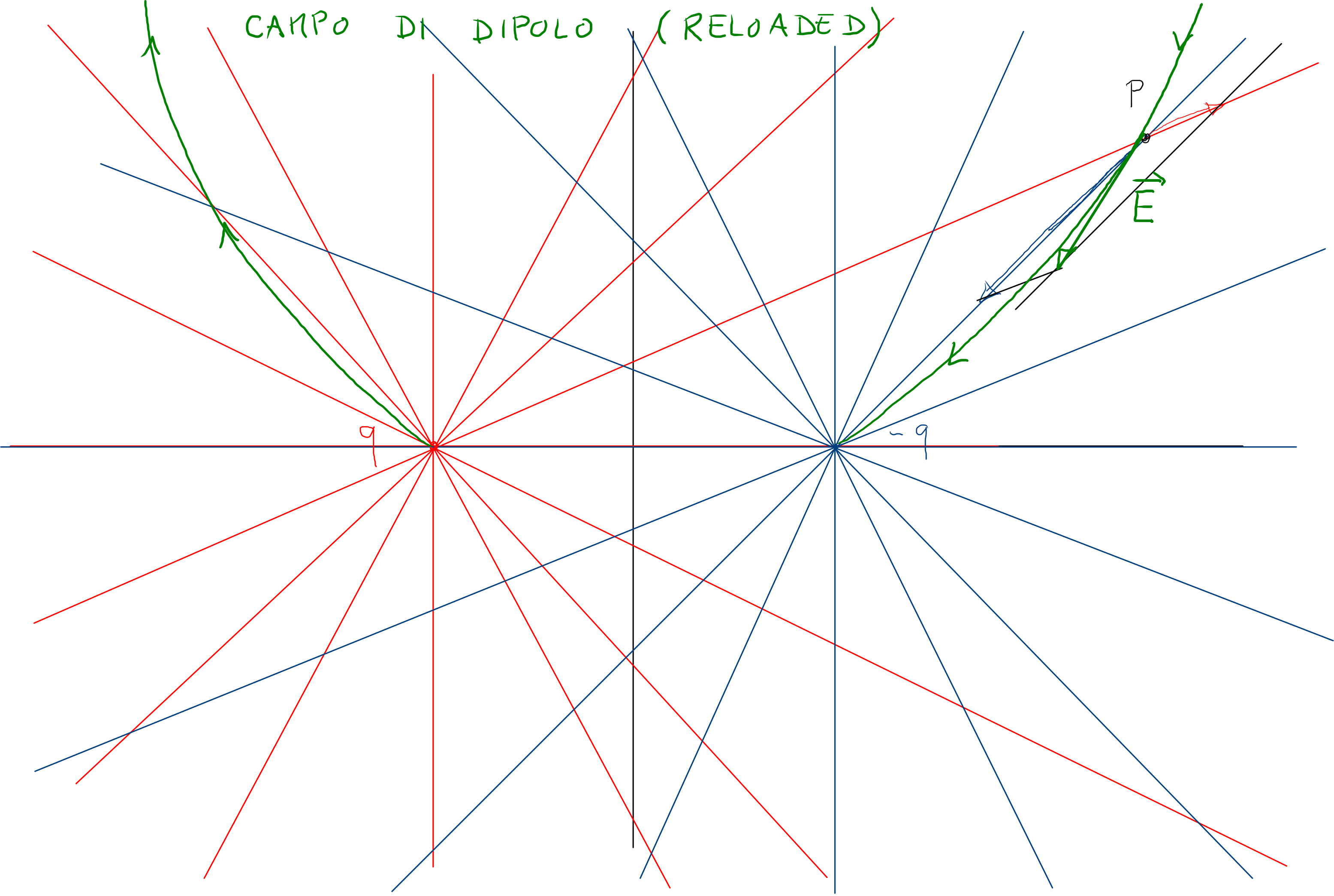
b Linee di forza del campo creato da una carica negativa.

c Linee di forza del campo creato da due cariche uguali e opposte (dipolo elettrico).

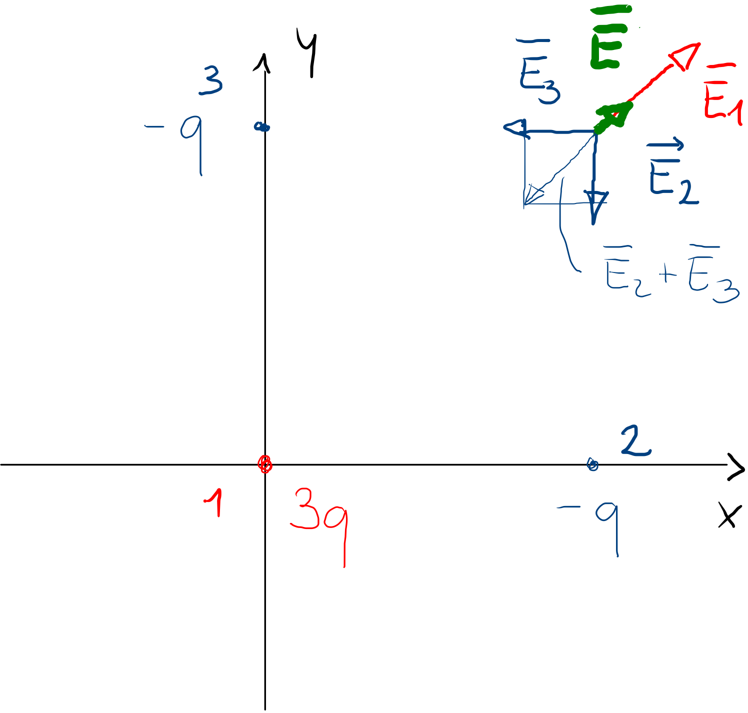
CAMPO DI DIPOLO



CAMPO DI DIPOLO (RELOADED)

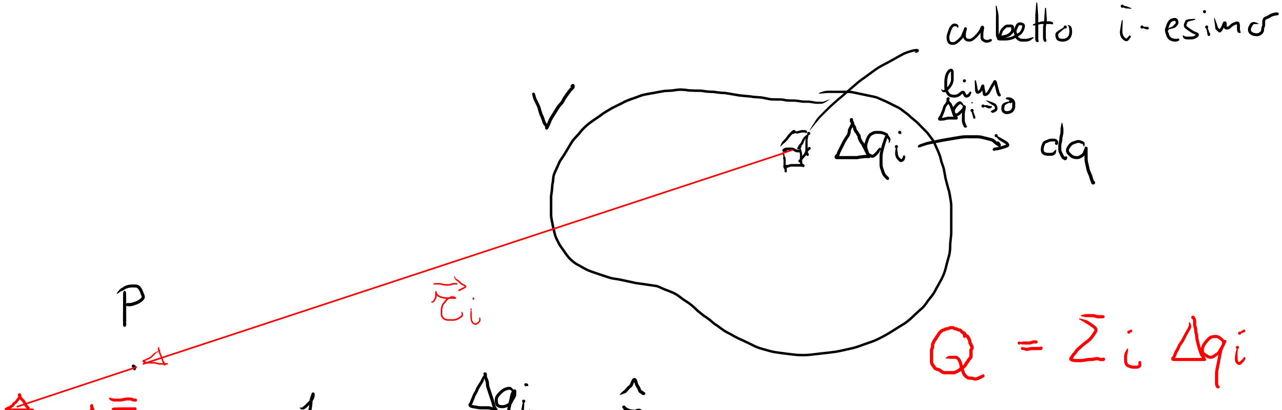


ESEMPIO : DISTRIBUZIONE DI CARICHE PUNTIFORMI



$$\overline{E} = \vec{E}_1 + \overline{E}_2 + \overline{E}_3$$

DISTRIBUZIONE GENERICA DI CARICHE



$$Q = \sum_i \Delta q_i$$

$$\Delta \vec{E}_i = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot \frac{\Delta q_i}{r_i^2}\, \hat{r}_i$$

$$\vec{E} = \sum_i \Delta \vec{E}_i = \sum_i \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{\Delta q_i}{r_i^2}\, \hat{r}_i = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \sum_i \frac{\Delta q_i}{r_i^2}\, \hat{r}_i$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \int_V \frac{dq}{r^2}\, \hat{r}$$

- CASI PARTICOLARI -

$$\vec{E} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \int_V \frac{dq}{r^2}\hat{r}$$

1) Q è distribuita uniformemente in V : $\rho_e = \frac{Q}{V}$

$$dq = \rho_e \, dV$$

2) Q è distribuita uniformemente su superficie S

$$\sigma = \frac{Q}{S} \qquad dq = \sigma \cdot dS$$

3) Q è distribuita uniformemente su lunghezza ℓ

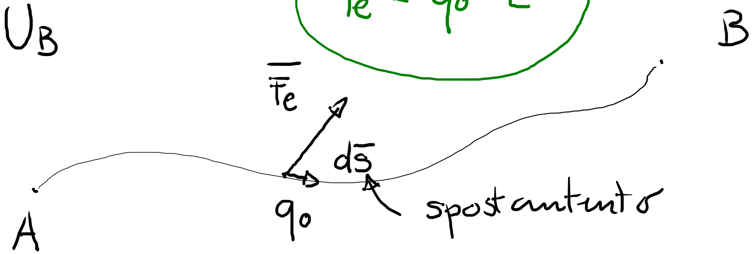
$$\lambda = \frac{Q}{\ell} \qquad dq = \lambda \cdot d\ell$$

ENERGIA POTENZIALE ELETTRICA

$$\boxed{\vec{F}_e = q_0 \cdot \vec{E}}$$

nota $[\vec{E}] = \dfrac{N}{C} = \dfrac{N \cdot m}{C \cdot m} = \dfrac{J}{C \cdot m} = \dfrac{V}{m}$

$$* \quad \mathcal{L} = -\Delta U = U_A - U_B$$



$$** \quad \mathcal{L} = \int_A^B \vec{F}_e \cdot d\vec{s} = \int_A^B q_0 \vec{E} \cdot d\vec{s} = q_0 \int_A^B \vec{E} \cdot d\vec{s}$$

prodotto scalare

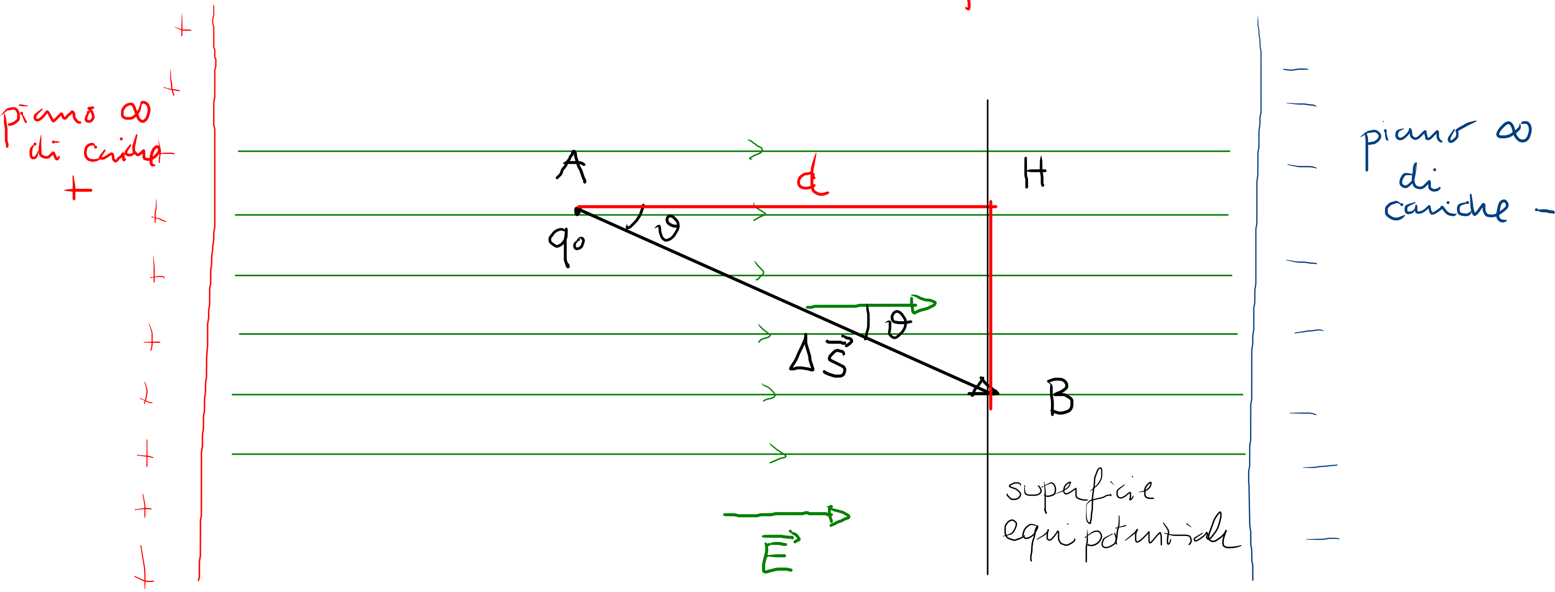
$$* = ** \Rightarrow \Delta U = U_B - U_A = -q_0 \int_A^B \vec{E} \cdot d\vec{s}$$

POTENZIALE ELETTRICO V

$$V = \frac{U}{q_0} \qquad \Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

$$[V] = \frac{J}{C} = \text{Volt}$$

ESEMPIO ΔV in un campo uniforme



$$\Delta V = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s} = -\vec{E}\cdot\Delta\vec{S} = -\underbrace{|\vec{E}|}_{E}\cdot\underbrace{|\Delta\vec{S}|\cdot\cos\theta}_{d} = -E\cdot d$$

$$\Delta U = q_0\cdot\Delta V = -q_0 E d$$

$$\mathcal{L} = -\Delta U = q_0 E d$$

VALORE DEL POTENZIALE IN P

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \vec{E} \cdot d\vec{s} = V_B - V_A$$

$B \rightarrow P$

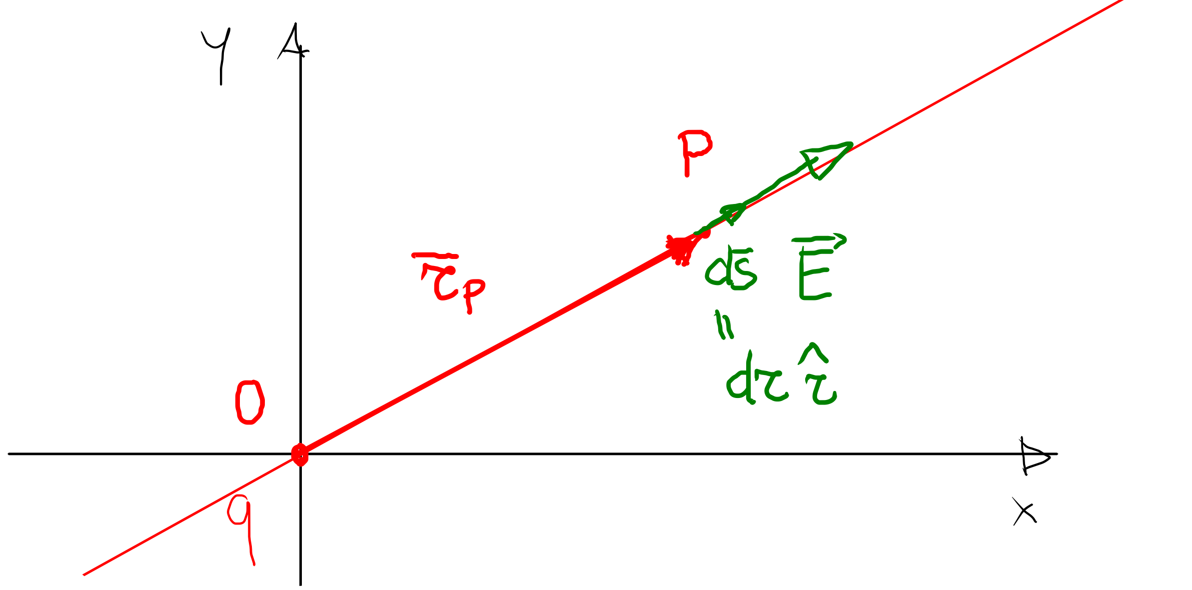
$A \rightarrow \infty \quad \Rightarrow \quad V_A = 0$

$$-\int_\infty^P \vec{E} \cdot d\vec{s} = V_P \qquad \left[\ V_P = \int_P^\infty \vec{E} \cdot d\vec{s}\ \right]$$

$$\boxed{V_P = -\int_\infty^P \vec{E} \cdot d\vec{s}}$$

V_P lavoro per unità di carica fatto CONTRO le F elettrostatiche per portare una carica di prova q_0 dall'∞ a P

POTENZIALE DI q IN O



$$\boxed{\vec E(\vec r) = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\,\frac{q}{r^2}\,\hat r}$$

$$V(\vec r) = -\int_\infty^P \vec E\cdot d\vec s$$ versione "fisica" ☺

$$= \int_P^\infty \vec E\cdot d\vec s$$ versione matematica ⇒ utile x fare i conti

$$V(\vec r_P) = \int_P^\infty \vec E\cdot d\vec s = \int_P^\infty \frac{1}{4\pi\varepsilon_0\varepsilon_r}\,\frac{q}{r^2}\,\hat r\cdot dr\,\hat r = \frac{q}{4\pi\varepsilon_0\varepsilon_r}\int_P^\infty \frac{1}{r^2}\,dr\,\underbrace{(\hat r\cdot\hat r)}_{1}$$

$$= \frac{q}{4\pi\varepsilon_0\varepsilon_r}\left[-\frac{1}{r}\right]_P^\infty = \frac{q}{4\pi\varepsilon_0\varepsilon_r}\left[-\cancel{\frac{1}{\infty}}+\frac{1}{r_P}\right]$$

$$\Rightarrow \boxed{V(\vec r) = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\,\frac{q}{r}}$$

$$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = -x^{-1} = -\frac{1}{x}$$

$$\int x^n\,dx = \frac{1}{n+1}\,x^{n+1}$$

$$\frac{d\,x^n}{dx} = n\,x^{n-1}$$

$$\vec a\cdot\vec a = |\vec a|^2$$

POTENZIALE E CAMPO ELETTRICO (approfondimento)

$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$

$$dV = -\vec{E} \cdot d\vec{s}$$ per uno spostamento infinitesimo $d\vec{s}$

In 1D: $d\vec{s} \rightarrow dx \cdot \hat{i}$

$\vec{E} \rightarrow E_x \cdot \hat{i}$

$$dV = -E_x \cdot dx$$

$$E_x = -\frac{dV}{dx}$$

$$\vec{E} = -\frac{dV}{dx}\,\hat{i}$$

In 3D: $$\vec{E} = -\left(\frac{\partial V}{\partial x} \cdot \hat{i} + \frac{\partial V}{\partial y} \cdot \hat{j} + \frac{\partial V}{\partial z}\,\hat{k}\right)$$

$$\boxed{\vec{E} = -\vec{\nabla} V}$$

CONDUTTORI (metalli)

Le cariche sono libere di muoversi

$\Rightarrow$ non devo fare lavoro contro le forze elettriche

$\Rightarrow$ <u>tutto il conduttore si trova allo stesso potenziale.</u>

$Q = 0$ (conduttore neutro) $\Rightarrow V = 0$

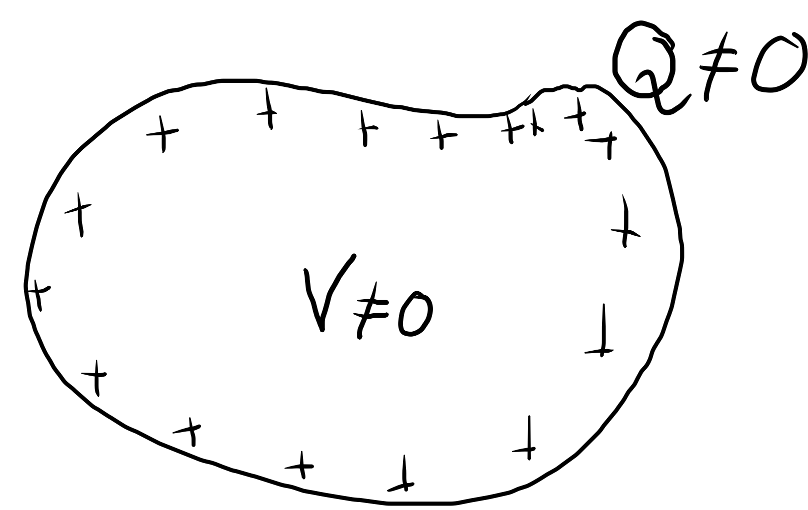
$Q \neq 0 \qquad\qquad \Rightarrow V \neq 0$

$Q = C \cdot V$

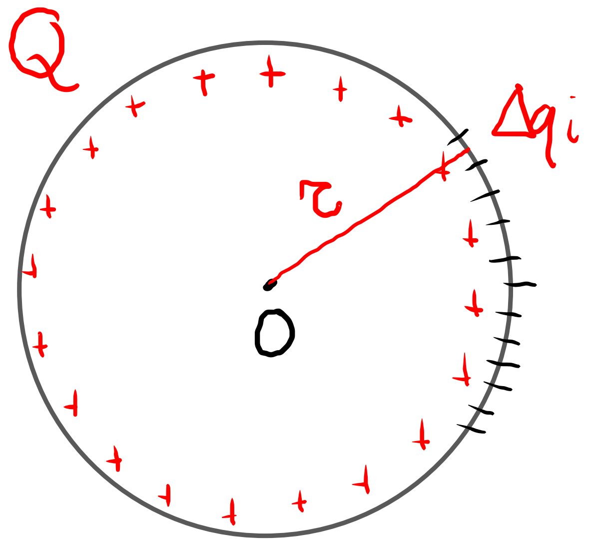
Capacità del conduttore

$[C] = \frac{[Q]}{[V]} = \frac{C}{V} = \text{Farad} = F$

Coulomb!



CAPACITÀ DI UNA SFERA METALLICA



$$Q = CV$$

$$C = \frac{Q}{V}$$

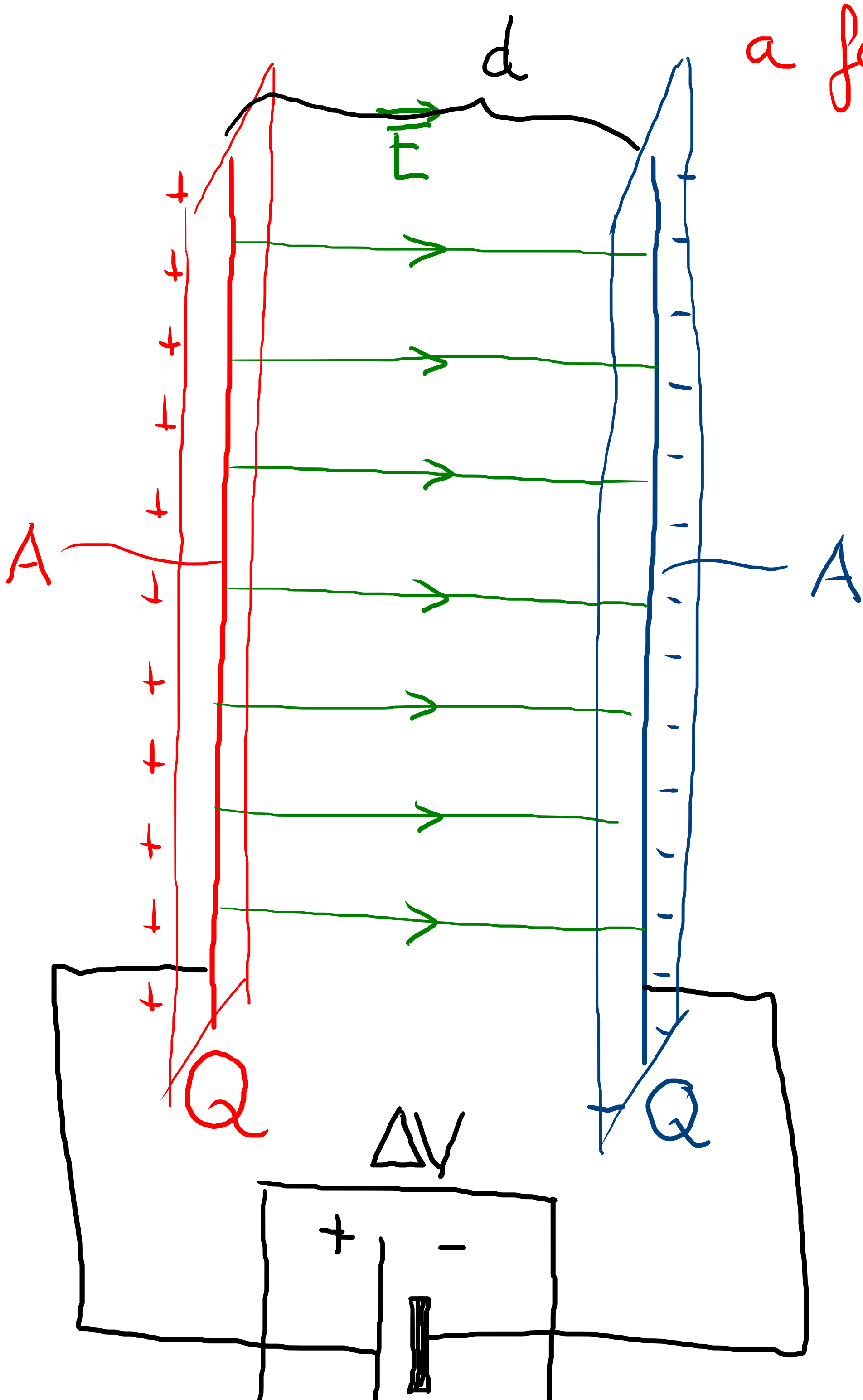
V è lo stesso in tutti i punti $\Rightarrow$ lo calcolo in O
$(\bar{E} = -\nabla V)$

$$V(O) = \sum_i \Delta V_i \quad \text{con} \quad \Delta V_i = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\,\frac{\Delta q_i}{r}$$

$$= \sum_i \frac{1}{4\pi\varepsilon_0\varepsilon_r}\,\frac{\Delta q_i}{r} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\cdot\frac{1}{r}\underbrace{\sum_i \Delta q_i}_{=Q}$$

$$V = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\,\frac{Q}{r} \Rightarrow C = \frac{Q}{V} = 4\pi\varepsilon_0\varepsilon_r \cdot r$$

CAPACITÀ DI UN CONDENSATORE
a facce piane e parallele



generatore di tensione ideale

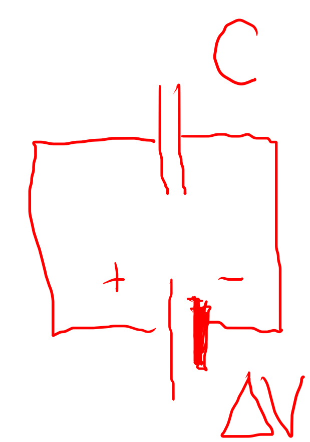
$$E = \frac{1}{\varepsilon_0 \varepsilon_r} \frac{Q}{A}$$

area delle armature

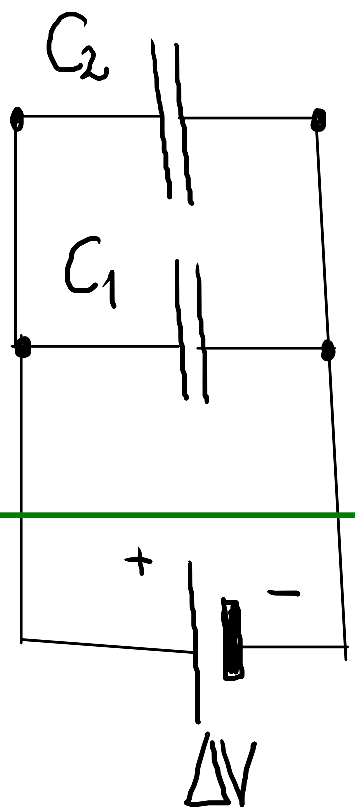
$$\Delta V = E \cdot d$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{E\,d} = \varepsilon_0 \varepsilon_r \frac{A}{\cancel{Q}} \cdot \frac{\cancel{Q}}{d}$$

$$\boxed{C = \varepsilon_0 \varepsilon_r \frac{A}{d}}$$

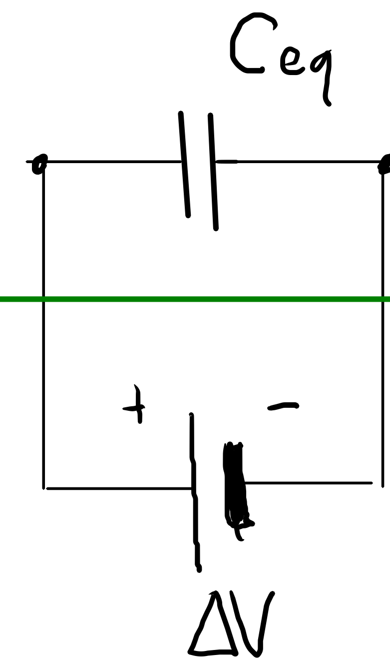


CONDENSATORI IN PARALLELO



$$\left.\begin{aligned} Q_1 &= C_1\,\Delta V_1 \\ Q_2 &= C_2\,\Delta V_2 \end{aligned}\right\} *$$

$$Q_{eq} = C_{eq} \cdot \Delta V$$



1) $\Delta V_1 = \Delta V_2 = \Delta V$

---

2) $\underline{Q_1 + Q_2 = Q = Q_{eq}}$

C_{eq} è equivalente al parallelo C_1 e C_2

Da 2) $\quad Q_1 + Q_2 = Q_{eq}$

$* \quad C_1 \Delta V_1 + C_2 \Delta V_2 = C_{eq} \Delta V$

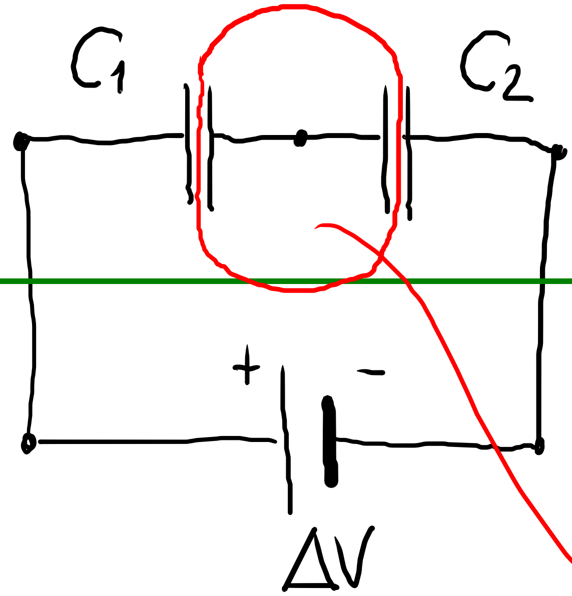
con 1) $\quad C_1 \cancel{\Delta V} + C_2 \cancel{\Delta V} = C_{eq} \cancel{\Delta V}$

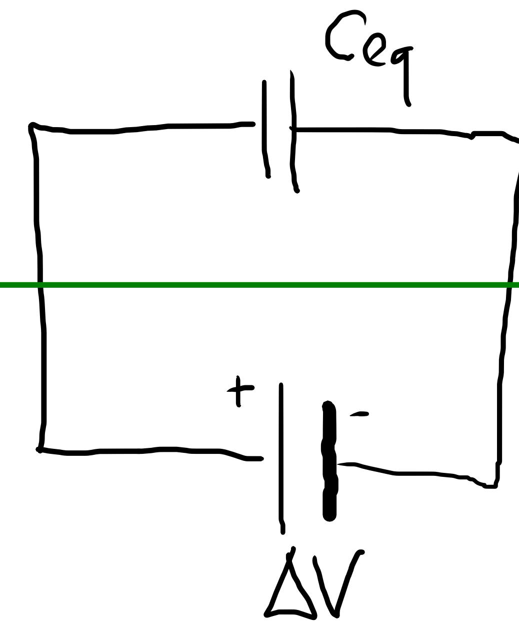
$$\boxed{C_{eq} = C_1 + C_2}$$

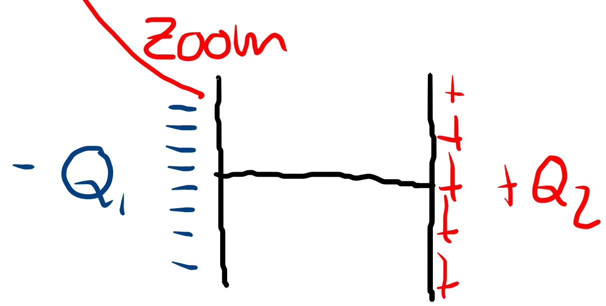
per n cond. in //

$$C_{eq} = C_1 + C_2 + \cdots + C_n$$

CONDENSATORI IN SERIE







$$* \begin{cases} Q_1 = C_1 \Delta V_1 \\ Q_2 = C_2 \Delta V_2 \end{cases}$$

é neutra

$$Q_{eq} = C_{eq} \Delta V$$

1) $\Delta V_1 + \Delta V_2 = \Delta V$

2) $Q_1 = Q_2 = Q_{eq} = Q$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

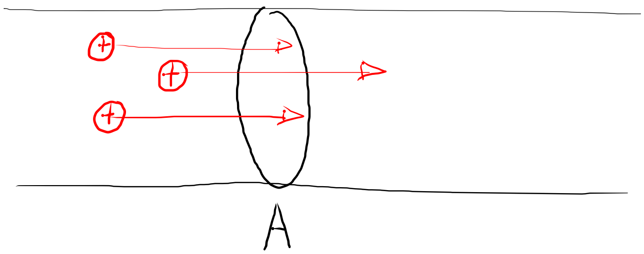
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots\dots + \frac{1}{C_n}$$

1) $\Delta V_1 + \Delta V_2 = \Delta V$

$$* \quad \frac{Q_1}{C_1} + \frac{Q_2}{C_2} = \frac{Q_{eq}}{C_{eq}}$$

con 2) $\dfrac{\cancel{Q}}{C_1} + \dfrac{\cancel{Q}}{C_2} = \dfrac{\cancel{Q}}{C_{eq}}$

CORRENTE ELETTRICA CONTINUA



ΔQ attraversa A
in Δt

$$I_m = \frac{\Delta Q}{\Delta t}$$ intensità media di corrente elettrica

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t}$$ intensità di corrente elettrica (istantanea)

$$[I] = \frac{C}{s} = A$$ Ampere (FONDAMENTALE IN SI)

$$1\,C = 1A \cdot 1s$$

LEGGI DI OHM

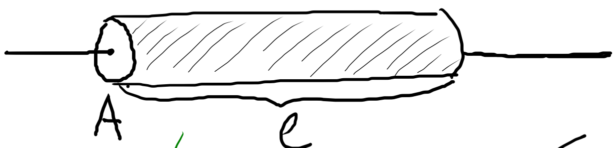
1) ΔV che si misura ai capi di una resistenza R

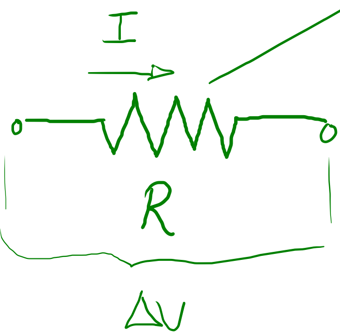
I corrente che attraversa la resistenza R

$$\Delta V = R I$$

$$[R] = \frac{[\Delta V]}{[I]} = \frac{V}{A} = \Omega \quad \text{(Ohm)}$$

2)



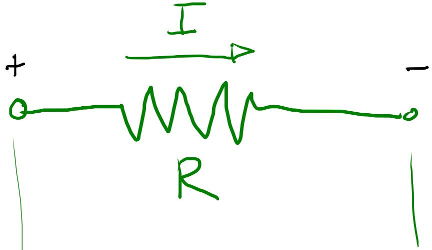


resistività del materiale

$$R = \rho \frac{\ell}{A}$$

$$[\rho] = \frac{[R][A]}{[\ell]} = \Omega \cdot m$$

POTENZA TRASFERITA AL RESISTORE



$$\Delta V = \frac{\Delta U}{q}$$

variazione di en. potenziale elettrica

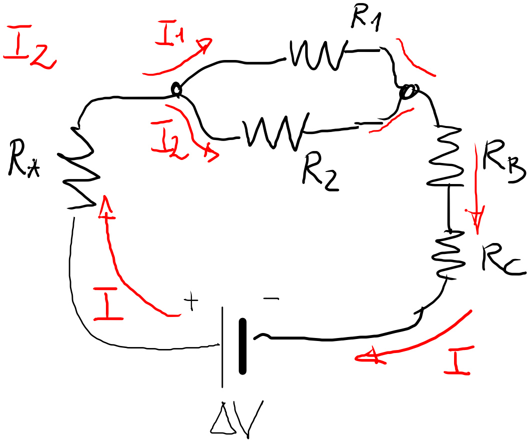
$$P = \frac{\Delta U}{\Delta t} = \frac{q\,\Delta V}{\Delta t} = \frac{q}{\Delta t} \cdot \Delta V = I \cdot \Delta V$$

$$P = I \cdot \Delta V = R I^2 = \frac{\Delta V^2}{R}$$

Circuiti in corrente elettrica continua

$I = I_1 + I_2$

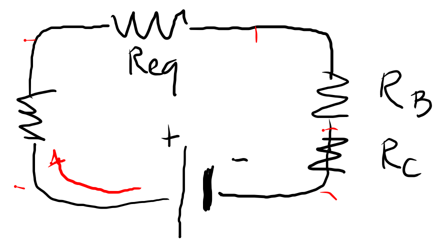
(1)



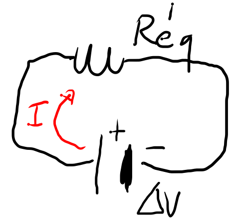
per semplificare il circuito $\Rightarrow$
resistenze in serie ed in parallelo

(2) R_1 e R_2 in parallelo $\Rightarrow$ R_{eq}, tale che $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

(2)



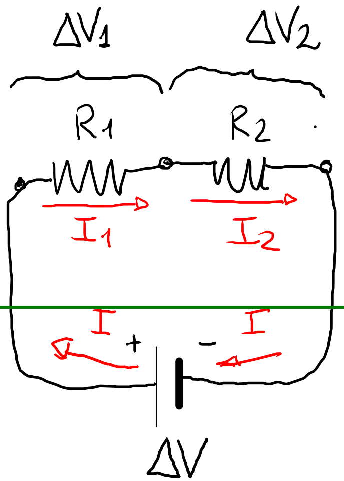
(3)



$I = \frac{\Delta V}{R'_{eq}}$

(3) R_A, R_{eq}, R_B, R_C sono tutte in serie $\Rightarrow$ $R'_{eq} = R_A + R_{eq} + R_B + R_C$

RESISTENZE IN SERIE (attraversate dalla stessa corrente)



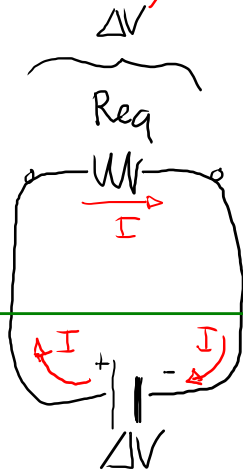
$$I_1 = I_2 = I$$

Per la legge di Ohm

$$\Delta V_1 = R_1 I_1 = R_1 I$$

$$\Delta V_2 = R_2 I_2 = R_2 I$$

$$\Delta V = R_{eq}\, I$$

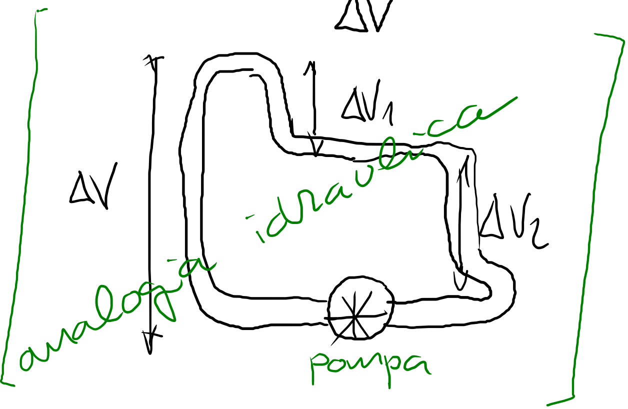


$$\Delta V = \Delta V_1 + \Delta V_2$$

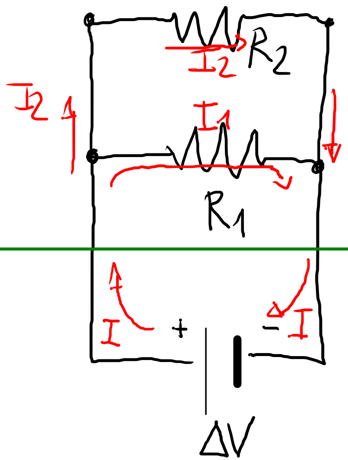
$$R_{eq} \cdot \cancel{I} = R_1 \cancel{I} + R_2 \cancel{I}$$

$$R_{eq} = R_1 + R_2$$

$$R_{eq} = R_1 + R_2 + \ldots\ldots + R_n$$

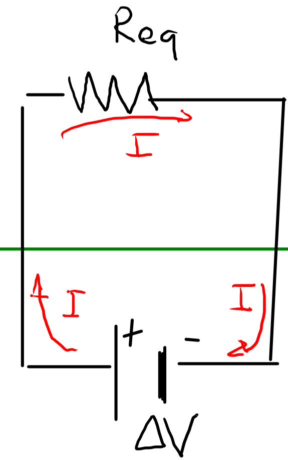


RESISTENZE IN PARALLELO



- $I = I_1 + I_2$

$\Delta V = \Delta V_1 = \Delta V_2$



$\Delta V_1 = R_1 I_1$

$* \quad I_1 = \frac{\Delta V_1}{R_1} = \frac{\Delta V}{R_1}$

$\Delta V_2 = R_2 I_2$

$** \quad I_2 = \frac{\Delta V_2}{R_2} = \frac{\Delta V}{R_2}$

$\cdot\cdot \quad I = \frac{\Delta V}{R_{eq}}$

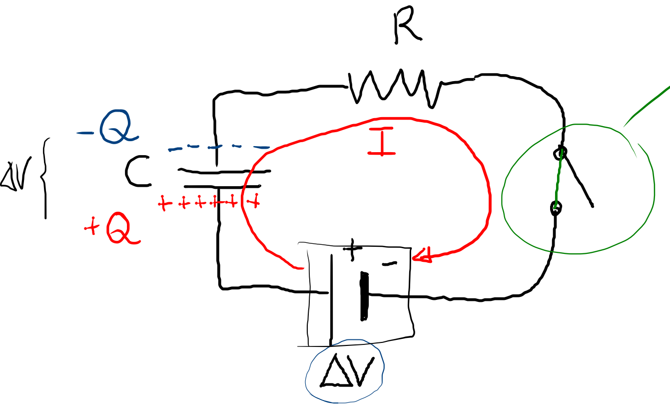
$I = I_1 + I_2$

$\frac{\cancel{\Delta V}}{R_{eq}} = \frac{\cancel{\Delta V}}{R_1} + \frac{\cancel{\Delta V}}{R_2}$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$

CIRCUITO RC (carica)



interruttore aperto
(non circola corrente)

$t=0$ si chiude $\Rightarrow$ circola corrente

Alla fine sul C troveremo Q:

$$Q = C\Delta V$$

e la corrente smetterà di circolare

In un certo istante t

$$q(t) = Q\left(1 - e^{-\frac{t}{RC}}\right)$$

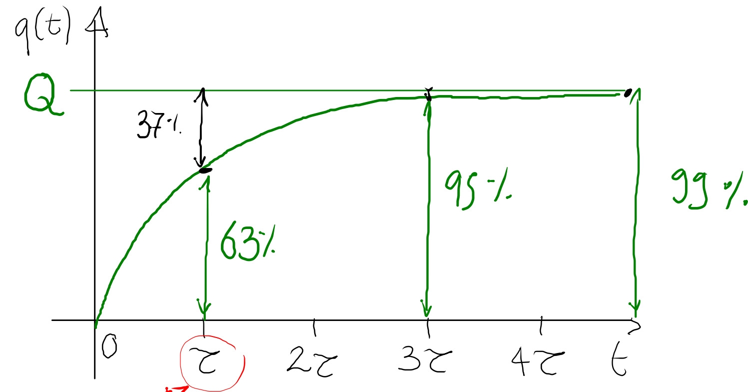
è un tempo!

τ

$e^{-1} = 0,37$

$e^{-3} = 0,05$

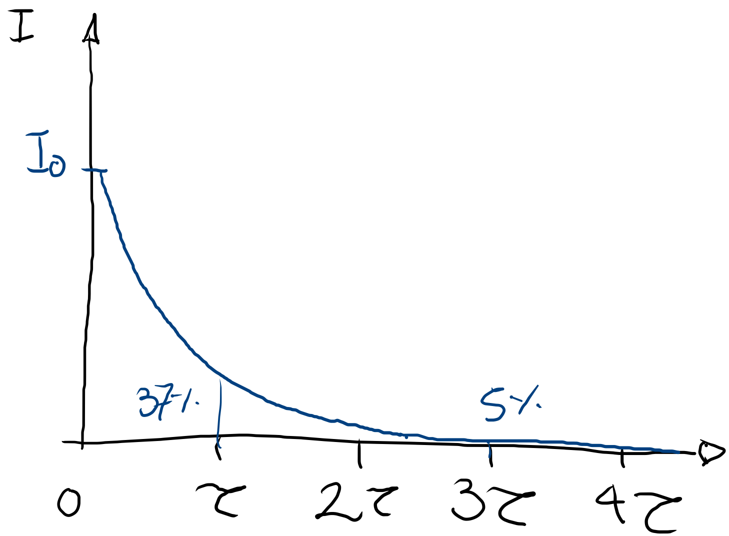
$e^{-5} = 0,007$



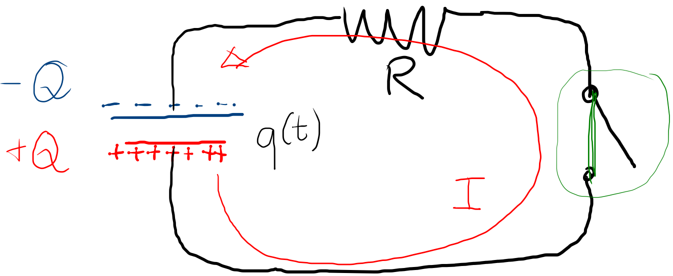
$$I = \frac{dq}{dt} = \frac{d}{dt}\,q(t) = \frac{d}{dt}\left[Q\left(1 - e^{-\frac{t}{RC}}\right)\right]$$

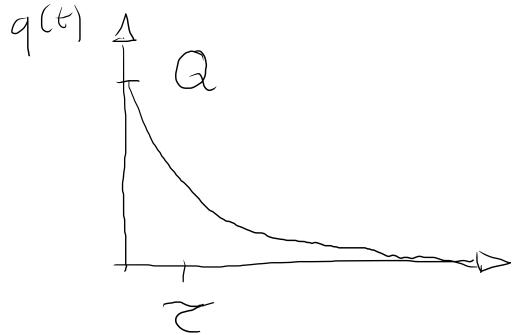
$$= Q\left[\left(-e^{-\frac{t}{RC}}\right)\cdot\left(-\frac{1}{RC}\right)\right]$$

$$I(t) = \underbrace{\frac{Q}{RC}}_{I_0}\, e^{-\frac{t}{RC}}$$



CIRCUITO RC (scarica)





$$q(t) = Q\, e^{-\frac{t}{RC}}$$

$$I = \frac{d}{dt} q(t) = \frac{d}{dt}\left(Q\, e^{-\frac{t}{RC}}\right) = Q\, e^{-\frac{t}{RC}} \cdot \left(-\frac{1}{RC}\right)$$

$$= -\frac{Q}{RC}\, e^{-\frac{t}{RC}} = -I_0\, e^{-\frac{t}{RC}}$$

la corrente ha verso opposto alla fase di carica