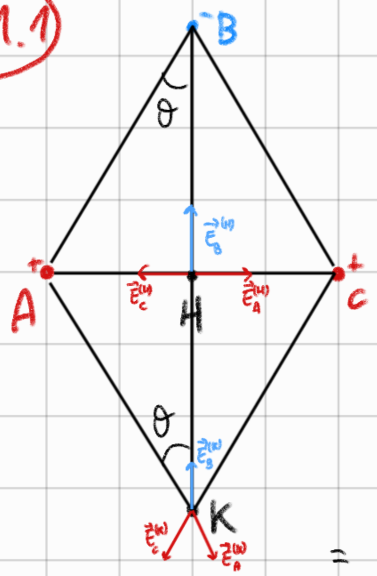
11.1



$\overline{AC} = 6\ cm \qquad \overline{AB} = \overline{BC} = 5{,}0\ cm = \overline{AK} = \overline{CK}$

$\overline{AH} = 3\ cm = \overline{HC}$

$\overline{BH} = \overline{HK} = \sqrt{5^2 - 3^2}\ cm = 4\ cm. \Rightarrow \overline{BK} = 8\ cm.$

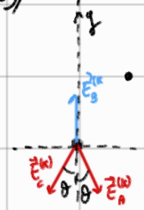
(a) • $V^{(H)} = \underbrace{V_A^{(H)}}_{\text{pot. generato da A in H}} + \underbrace{V_B^{(H)}}_{\text{pot. generato da B in H}} + \underbrace{V_C^{(H)}}_{\text{pot. generato da C in H}} = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{\overline{AH}} + \frac{1}{4\pi\varepsilon_0}\frac{q_B}{\overline{BH}} + \frac{1}{4\pi\varepsilon_0}\frac{q_C}{\overline{CH}} =$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{\overline{AH}} - \frac{2q}{\overline{BH}} + \frac{q}{\overline{CH}}\right) \underset{\overline{AH}=\overline{CH}}{=} \frac{\cancel{2}q}{\cancel{2}4\pi\varepsilon_0}\left(\frac{1}{\overline{AH}} - \frac{1}{\overline{BH}}\right) = 479\ V.$$

• $\vec{E}^{(H)} = \vec{E}_A^{(H)} + \vec{E}_B^{(H)} + \vec{E}_C^{(H)}$ con $|\vec{E}_A^{(H)}| = \frac{1}{4\pi\varepsilon_0}\frac{|q_A|}{\overline{AH}^2}$ $\quad |\vec{E}_B^{(H)}| = \frac{1}{4\pi\varepsilon_0}\frac{|q_B|}{\overline{HB}^2}$ $\quad |\vec{E}_C^{(H)}| = \frac{1}{4\pi\varepsilon_0}\frac{|q_C|}{\overline{HC}^2}$ poiché $|q_A| = |q_C|$ e $\overline{AH} = \overline{CH}$ ottengo $|\vec{E}_A^{(H)}| = |\vec{E}_C^{(H)}|$ ma giacciono su stessa direzione e versi opposti $\Rightarrow \vec{E}_A^{(H)} + \vec{E}_C^{(H)} = \vec{0}$

$\Rightarrow \vec{E}^{(H)} = \vec{E}_B^{(H)} \Rightarrow |\vec{E}^{(H)}| = |\vec{E}_B^{(H)}| = \frac{1}{4\pi\varepsilon_0}\frac{2q}{\overline{HB}^2} = 35{,}9\ \frac{kV}{m}$ e diretto verso B sulla retta BH.

(b) • $V^{(K)} = V_A^{(K)} + V_B^{(K)} + V_C^{(K)} = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{\overline{AK}} + \frac{1}{4\pi\varepsilon_0}\frac{q_B}{\overline{BK}} + \frac{1}{4\pi\varepsilon_0}\frac{q_C}{\overline{CK}} = \frac{q}{2\pi\varepsilon_0}\left(\frac{1}{\overline{AK}} - \frac{1}{\overline{BK}}\right) = 431\ V.$



• $\vec{E}^{(K)} = \vec{E}_A^{(K)} + \vec{E}_B^{(K)} + \vec{E}_C^{(K)}$

poiché $q_A = q_C$ e $\overline{AK} = \overline{KC}$, anche qui $\vec{E}_A^{(K)}, \vec{E}_C^{(K)}$ hanno lo stesso modulo. Per geometria le componenti X si annullano a vicenda. Quindi:

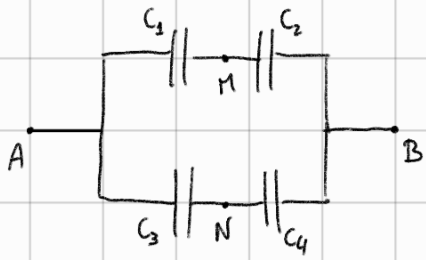
• $(\vec{E}^{(K)})_x = (\vec{E}_A^{(K)})_x + \underbrace{(\vec{E}_B^{(K)})_x}_{=0 \text{ per costruzione}} + (\vec{E}_C^{(K)})_x = 0$ $\qquad$ • $(\vec{E}^{(K)})_y = (\vec{E}_A^{(K)})_y + (\vec{E}_B^{(K)})_y + (\vec{E}_C^{(K)})_y$

con $(\vec{E}_B^{(K)})_y = |\vec{E}_B^{(K)}| = \frac{1}{4\pi\varepsilon_0}\frac{2q}{\overline{BK}^2}$ $\qquad (\vec{E}_A^{(K)})_y = (\vec{E}_C^{(K)})_y = -|\vec{E}_A^{(K)}|\underbrace{\cos\theta}_{\frac{\overline{HK}}{\overline{AK}}} = -\frac{q}{4\pi\varepsilon_0}\frac{1}{\overline{AK}^2}\cdot\frac{\overline{HK}}{\overline{AK}} = -\frac{q\,\overline{HK}}{4\pi\varepsilon_0\overline{AK}^3}$

$\Rightarrow (\vec{E}^{(K)})_y = \frac{2q}{4\pi\varepsilon_0}\left(\frac{1}{\overline{BK}^2} - \frac{\overline{HK}}{\overline{AK}^3}\right) = -9{,}42\ \frac{kV}{m}$

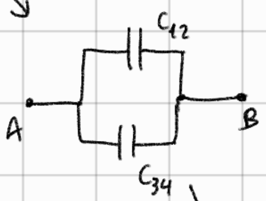
$\Rightarrow \vec{E}^{(K)}$ ha modulo $9{,}42\ \frac{kV}{m}$ ed è diretto lungo la retta HK verso il basso.

11.2



(a) C_1, C_2 in serie $\Rightarrow \frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{12} = \frac{C_1 C_2}{C_1 + C_2}$

$\Rightarrow$

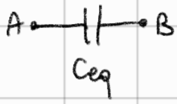


C_3, C_4 in serie $\Rightarrow \frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4} \Rightarrow C_{34} = \frac{C_3 C_4}{C_3 + C_4}$

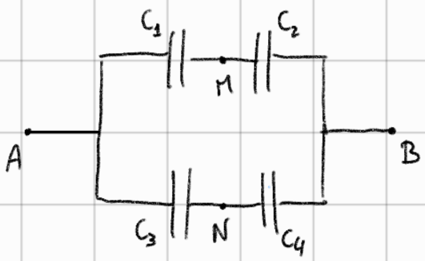
Ho ridotto il circuito a:

$\Rightarrow C_{12}, C_{34}$ in parallelo $\Rightarrow C_{1234} = C_{12} + C_{34} = \boxed{\frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4} = C_{eq}}$

$= 2{,}67 \text{ nF}.$



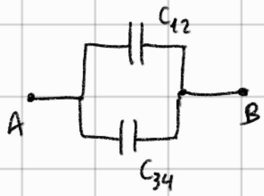
(b)



$\Delta V_1 = \frac{Q_1}{C_1} = V_M - V_A \qquad \Delta V_2 = \frac{Q_2}{C_2} = V_B - V_M$

$\Delta V_3 = \frac{Q_3}{C_4} = V_N - V_A \qquad \Delta V_4 = \frac{Q_4}{C_4} = V_B - V_N$

Valuto $V_M - V_N = (V_M - V_A) + (V_A - V_N) = \frac{Q_1}{C_1} - \frac{Q_3}{C_3}$



- C_1, C_2 SERIE $\Rightarrow Q_1 = Q_2 = Q_{12} = \Delta V_{AB} C_{12}$

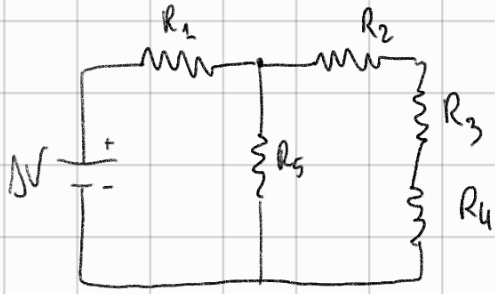
analogamente $Q_3 = Q_4 = Q_{34} = \Delta V_{AB} C_{34}$

$$\Rightarrow V_M - V_N = \frac{1}{C_1} \Delta V_{AB} C_{12} - \frac{1}{C_3} \Delta V_{AB} C_{34} = \Delta V_{AB} \left(\frac{C_{12}}{C_1} - \frac{C_{34}}{C_3} \right) = \Delta V_{AB} \underbrace{\left(\frac{1}{\cancel{C_1}} \frac{\cancel{C_1} C_2}{C_1 + C_2} - \frac{1}{\cancel{C_3}} \frac{\cancel{C_3} C_4}{C_3 + C_4} \right)}_{=0 \text{ inserendo i dati}} = 0$$

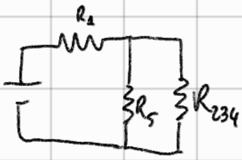
$\Rightarrow V_M = V_N.$

#11.3

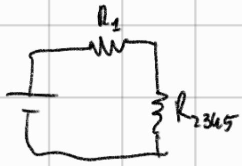
(a)



- R_2, R_3, R_4 serie $\Rightarrow R_{234} = R_2 + R_3 + R_4 \Rightarrow$



- R_5, R_{234} in parallelo $\Rightarrow \dfrac{1}{R_{2345}} = \dfrac{1}{R_5} + \dfrac{1}{R_{234}} \Rightarrow R_{2345} = \dfrac{R_{234} R_5}{R_{234} + R_5} \Rightarrow$



- R_1, R_{2345} in serie $\Rightarrow R_{eq} = R_1 + R_{2345} = R_1 + \dfrac{R_{234} R_5}{R_{2345} + R_5} = \boxed{R_1 + \dfrac{(R_2+R_3+R_4)R_5}{R_2+R_3+R_4+R_5}} = 100\ \Omega$

(b) CORRENTE EROGATA DAL GENERATORE: $i = \dfrac{\Delta V}{R_{eq}} = 0,80\ A \Rightarrow$

=

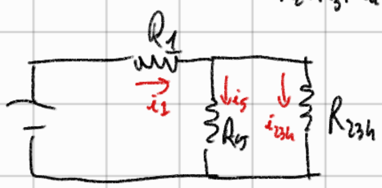
- R_{eq} è la serie di R_1 e R_{2345} $\Rightarrow \boxed{i_1 = 0,80\ A}$ corrente in R_1 $\Rightarrow \Delta V_1 = R_1 i_1$ (legge di ohm) $\Rightarrow \boxed{\Delta V_1 = 64\ V}$

diff potenziale ai capi di R_1

NB: resistenze in serie ⇒ stessa corrente.

da cui anche $i_{2345} = 0,80\ A$ e $\Delta V_{2345} = i_{2345} \cdot R_{2345} = i \cdot \dfrac{(R_2+R_3+R_4)R_5}{R_2+R_3+R_4+R_5} = 16\ V$

- R_{2345} è il parallelo di R_{234} e R_5 :



da cui $\Delta V_5 = \Delta V_{234} = \Delta V_{2345} = 16\ V$ → NB: resistenze in parallelo ⇒ stessa ΔV

$\Rightarrow i_5 = \dfrac{\Delta V_5}{R_5} \Rightarrow \boxed{i_5 = 0,4\ A}$ e $\boxed{\Delta V_5 = 16\ V}$

di contro $i_{234} = \dfrac{\Delta V_{234}}{R_{234}} = \dfrac{\Delta V_{234}}{R_2+R_3+R_4} = 0,4\ A$

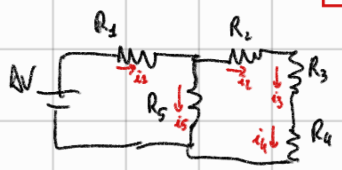
- R_{234} è serie di R_2, R_3, R_4 $\Rightarrow \boxed{i_2 = i_3 = i_4 = 0,4\ A} = i_{234} \Rightarrow$

$\Delta V_2 = i_2 \cdot R_2 \Rightarrow \boxed{\Delta V_2 = 4\ V}$

$\Delta V_3 = i_3 \cdot R_3 \Rightarrow \boxed{\Delta V_3 = 4\ V}$

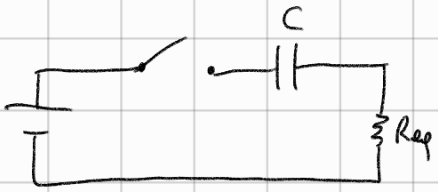
$\Delta V_4 = i_4 \cdot R_4 \Rightarrow \boxed{\Delta V_4 = 8\ V}$

$\Rightarrow$



#11.4

circuito equivalente a



con $R_{eq} = \frac{R_1 R_2}{R_1+R_2}$ $\left(R_1, R_2 \text{ parallelo: } \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}\right)$ $\Rightarrow R_{eq} = \frac{R^{\cancel{2}}}{2\cancel{R}} = \frac{1}{2}R = 3{,}00\ M\Omega$

costante di tempo: $\tau = R_{eq} \cdot C = 3 \cdot \cancel{10^6}\,\Omega \cdot 0{,}350 \cdot \cancel{10^{-6}}\,F = 1{,}05\,s$

(a) chiudo interruttore → si accumula carica nel tempo: $q(t) = (1-e^{-t/\tau})Q$ con $Q = C\Delta V$

$\Rightarrow$ • $q(t_1) = (1-e^{-t_1/\tau})\,C\Delta V = 10\,\mu C$

• $q(t_2) = (1-e^{-t_2/\tau})\,C\Delta V = 26\,\mu C$

(b) $i(t) = e^{-t/\tau} I$ con $I = \frac{\Delta V}{R_{eq}}$.

$i(t)$ attraversa R_{eq}, se voglio corrente in ciascun resistore devo dividere per 2 $(R_1 = R_2)$

$\Rightarrow i(t_1) = \frac{1}{2}\frac{\Delta V}{R} e^{-t_1/\tau} = 9{,}0\,\mu A$

$i(t_2) = \frac{1}{2}\frac{\Delta V}{R} e^{-t_2/\tau} = 1{,}25\,\mu A.$