ES. 1:

$$\lim_{(x,y)\to(0,0)} \frac{x\cdot \operatorname{sen}(xy)}{x^2+y^2} \qquad \operatorname{sen}(xy) \sim x\cdot y \quad \text{PER TAYLOR}$$

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{x^2\cdot y}{x^2+y^2}$$

SONO: $x^2+y^2 \geq 0 \Rightarrow \begin{matrix} x^2+y^2 \geq x^2 \\ x^2+y^2 \geq y^2 \end{matrix} \Rightarrow \dfrac{1}{x^2+y^2} \leq \dfrac{1}{x^2}$

$$\Rightarrow \frac{x^2 y}{x^2+y^2} \leq \frac{x^2 y}{x^2} \Rightarrow \lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2+y^2} \leq \lim_{(x,y)\to(0,0)} \frac{\cancel{x^2} y}{\cancel{x^2}}$$

CHIAMIAMO $C_1(x,y) = y$ IL PRIMO CARABINIERE

SE $C_2(x,y) = -y$? $\rightsquigarrow$ BENE, DEVO AVERE:

$$\begin{cases} \lim\limits_{(x,y)\to(x_0,y_0)} C_1(x,y) = L \in \mathbb{R} \\ \lim\limits_{(x,y)\to(x_0,y_0)} C_2(x,y) = L \in \mathbb{R} \end{cases}$$

HANNO LO STESSO LIMITE I MIEI CARABINIERI?

$$\left.\begin{matrix} \lim\limits_{(x,y)\to(0,0)} C_1(x,y) = \lim\limits_{(x,y)\to(0,0)} y = 0 \\ \lim\limits_{(x,y)\to(0,0)} C_2(x,y) = \lim\limits_{(x,y)\to(0,0)} (-y) = 0 \end{matrix}\right\} \checkmark$$

$$\Rightarrow \lim_{(x,y)} c_2(x,y) \leq \lim_{(x,y)} \frac{x^2y}{x^2+y^2} \leq \lim_{(x,y)\to(0,0)} c_1(x,y)$$

$$\lim_{(x,y)} c_2(x,y) \downarrow 0 \qquad \lim_{(x,y)\to(0,0)} c_1(x,y) \downarrow 0$$

$$\left[\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2} = 0\right]$$

ES 2: $f(x,y) = \dfrac{1}{x^2+2y^2}$ STUDIARE PTI DI MIN E MAX

i) $D = \mathbb{R}^2 \setminus \{(0,0)\}$

ii) TROVARE PTI STAZIONARI $\Rightarrow \overline{\nabla} f(x,y) = 0$

$$\overline{\nabla} f(x,y) = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right] = ?$$

$$\frac{\partial f}{\partial x} = \frac{\partial_x(1)(x^2+2y^2) - (1)\partial_x(x^2+2y^2)}{(x^2+2y^2)^2} = \frac{0\cdot(x^2+2y^2) - 2x}{(x^2+2y^2)^2} = -\frac{2x}{(x^2+2y^2)^2}$$

$$\frac{\partial f}{\partial y} = -\frac{4y}{(x^2+2y^2)^2}$$

$$\Rightarrow \quad \vec{\nabla} f(x,y) = 0 \quad \Rightarrow \quad \left[-\frac{2x}{(x^2+2y^2)^2} \, , \, -\frac{4y}{(x^2+2y^2)^2} \right]$$

$$\Rightarrow \begin{cases} -2x = 0 \\ -4y = 0 \end{cases} \quad \Rightarrow \quad \left[(x,y) = (0,0) \right] \rightsquigarrow \text{MA } (0,0) \text{ È ESCLUSO DAL DOMINIO}$$

$\Rightarrow$ NON HA PTI STAZIONARI.

ES. 3:

$$\int dx \; x \cdot \text{sen}^2(x) \qquad \text{sen}^2(x) = \frac{1-\cos(2x)}{2}$$

$$\int dx \; x \left(\frac{1}{2} - \frac{\cos(2x)}{2} \right) = \frac{1}{2}\frac{x^2}{2} - \frac{1}{2}\int dx \; x \cdot \cos(2x) = \frac{x^2}{4} - \frac{1}{4}\int dx \; x \; 2\cos(2x)$$

$$= \frac{x^2}{4} - \frac{1}{4}\left[x \cdot \text{sen}(2x) - \int dx \; \text{sen}(2x) \right]$$

$$= \frac{x^2}{4} - \frac{1}{4}\left[x \cdot \text{sen}(2x) + \frac{1}{2}\cos(2x) \right]$$

ES. 4: LUNGHEZZA CURVA DI $f(x) = 2\sqrt{7x^3}$ TRA $x = 0$ E $x = 1$

HO 2 FORMULE: ⊛ $L_\gamma = \int_a^b dx \sqrt{1 + (f(x))^2}$ O $L_\gamma = \int_\gamma dt \sqrt{\dot{x}^2(t) + \dot{y}^2(t)}$

USIAMO LA ⊛:

$$f(x) = 2\sqrt{7} \cdot x^{\frac{3}{2}} \Rightarrow f'(x) = \cancel{2}\sqrt{7} \cdot \frac{3}{\cancel{2}} \sqrt{x} = 3\sqrt{7}\sqrt{x}$$

$$\Rightarrow \int_0^1 dx \sqrt{1 + (3\sqrt{7}\sqrt{x})^2} = \int_0^1 dx \sqrt{1 + 63x} = \int_0^1 dx \, (\overbrace{1 + 63x}^{g(x)})^{\frac{1}{2}} \qquad g'(x) = 0 + 63 = 63$$

$$= \frac{1}{63} \int_0^1 dx \, \underbrace{(63)}_{g'} \, \underbrace{(1 + 63x)}_{g}^{\frac{1}{2}} = \frac{1}{63} \cdot \frac{(1 + 63x)^{\frac{3}{2}}}{\frac{3}{2}} \Bigg|_0^1 = \frac{2}{189} \left[(1 + 63)^{\frac{3}{2}} - (1)^{\frac{3}{2}} \right] = \boxed{\frac{146}{27}}$$