19/04/2023

RIPASSO DERIVATE:

- $$\frac{d(e^{f(x)})}{dx} = \frac{de^{f}}{df} \cdot \frac{df}{dx} = e^{f(x)} \cdot f'(x)$$ Ex. $$e^{-x^2} \rightsquigarrow e^{-x^2} \cdot (-2x) = -2x \cdot e^{-x^2}$$

- $$\frac{d(\ln(f(x)))}{dx} = \frac{d\ln}{df} \cdot \frac{df}{dx} = \frac{1}{f(x)} \cdot f'(x)$$ Ex. $$\ln(\cos(x)) \rightsquigarrow \frac{1}{\cos(x)} \cdot (-\mathrm{sen}(x)) = -\tan(x)$$

- $$\frac{d}{dx}(f(x))^n = \frac{d(f(x))^n}{df} \cdot \frac{df}{dx} = n \cdot (f(x))^{n-1} \cdot f'(x)$$ Ex. $$\mathrm{sen}^2(x) \rightsquigarrow 2 \cdot \mathrm{sen}(x) \cdot (\cos x) = \mathrm{sen}(2x)$$

- $$\frac{d}{dx}(\cos(f(x))) = \frac{d\cos(f(x))}{df} \cdot \frac{df}{dx} = -\mathrm{sen}(f(x)) \cdot f'(x)$$ Ex. $$\cos(ax) \rightsquigarrow -\mathrm{sen}(ax) \cdot (a) = -a \cdot \mathrm{sen}(x)$$

- $$\frac{d}{dx}\left(\sqrt{f(x)}\right) = \frac{d(\sqrt{f(x)})}{df} \cdot \frac{df}{dx} = \frac{1}{2\sqrt{f(x)}} \cdot f'(x)$$ Ex. $$\sqrt{\ln(x)} = \frac{1}{2\sqrt{\ln(x)}} \cdot \left(\frac{1}{x}\right) = \frac{1}{2x\sqrt{\ln(x}}$$

REGOLA GENERALE:

$$g(f(x)) \rightsquigarrow \frac{dg}{dx} = \frac{dg}{df} \cdot \frac{df}{dx}$$

$$h(g(f(x))) \rightsquigarrow \frac{dh}{dx} = \frac{dh}{dg} \cdot \frac{dg}{df} \cdot \frac{df}{dx}$$

"**CHAIN RULE**"

ES. 1 : $\displaystyle\int_0^1 dx\, x^3 e^{1-x^4}$ ABBIAMO $e^{f(x)}$ CON $f(x) = 1-x^4$.

↳ CALCOLIAMO $\dfrac{d}{dx}\left(e^{1-x^4}\right) = e^{1-x^4}\cdot(0-4x^3) = -4x^3 e^{1-x^4}$

MOLTIPLICO E DIVIDO PER (-4) ⇝ $\dfrac{1}{(-4)}\displaystyle\int_0^1 dx\,(-4x^3)\, e^{1-x^4} = -\frac{1}{4}\int_0^1 dx\, \underbrace{(-4x^3)\, e^{1-x^4}}_{\frac{d}{dx}\left(e^{1-x^4}\right)}$

DAL TH. FONDAMENTALE DEL CALCOLO INTEGRALE : $\displaystyle\int_a^b dx\, f'(x) = f(b) - f(a)$

$$-\frac{1}{4}\int_0^1 dx\, \frac{d}{dx}\left(e^{1-x^4}\right) = -\frac{e^{1-x^4}}{4}\Bigg|_0^1 = -\frac{1}{4}\left(e^{1-1} - e^{1}\right) = \boxed{\frac{e}{4} - \frac{1}{4}}$$ ✓

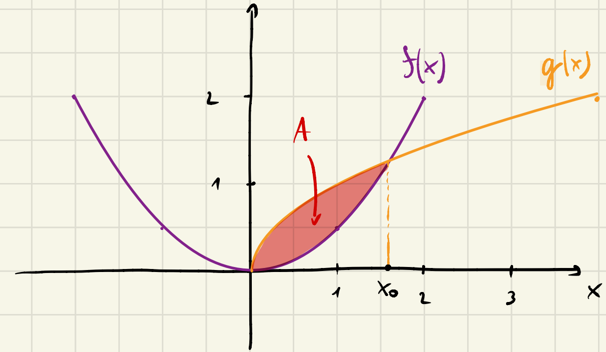
ALTERNATIVA : $\displaystyle\int_0^1 dx\, x^3 e^{1-x^4}$ SOSTITUZIONE : $z = 1-x^4 \Rightarrow dz = -4x^3 dx$ ⇝ $dx = \dfrac{dz}{-4x^3}$

$1-(1)^4$ → 0

$1-(0)^4$ → 1

$$\Rightarrow \int_1^0 \frac{dz}{-4\cancel{x^3}}\cdot \cancel{x^3}\, e^{+z} = -\frac{1}{4}\int_1^0 dz\, e^{+z} = \frac{1}{4}\int_0^1 dz\, e^{z} = \frac{1}{4}\left(e^1 - e^0\right) = \frac{e}{4} - \frac{1}{4}$$

ES. 2 : CALCOLARE AREA DELIMITATA DA : $f(x) = \frac{x^2}{2}$ $g(x) = \sqrt{x}$



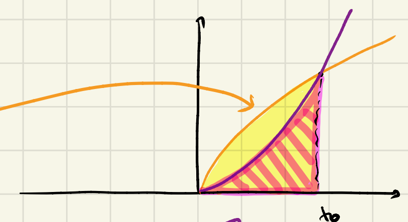
DOBBIAMO TROVARE x_0 ...

$f(x_0) = g(x_0) \rightsquigarrow \frac{x_0^2}{4} = \sqrt{x_0}\ ,\ \frac{x_0^{\cancel{4}\,3}}{4} = \cancel{x_0}\ ,\ x_0^3 = 4$

$\left[\, x_0 = \sqrt[3]{4} \,\right]$

L'INTEGRALE È L'AREA SOTTESA DA UNA FUNZIONE => SE IO FACCIO L'AREA SOTTESA DA $g(x)$ MENO AREA SOTTESA DA $f(x)$ TROVO A.

A_g A_f



$$A_g = \int_0^{x_0} dx\, g(x) \qquad A_f = \int_0^{x_0} dx\, f(x)$$

$$\Rightarrow \quad A = A_g - A_f = \int_0^{x_0} dx\, g(x) - \int_0^{x_0} dx\, f(x) = \int_0^{x_0} dx\, \sqrt{x} - \int_0^{x_0} dx\, \frac{x^2}{2}$$

$$A_g = \int_0^{x_0} dx\, \sqrt{x} = \int_0^{x_0} dx\, (x)^{\frac{1}{2}}$$

<u>RICORDA</u>: $\int dx\, x^{\alpha} = \frac{x^{\alpha+1}}{\alpha+1}$

$$= \left. \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right|_0^{x_0} = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \left. \frac{2}{3} x^{\frac{3}{2}} \right|_0^{x_0} = \frac{2}{3} x_0^{\frac{3}{2}} = \frac{2}{3}\left(\sqrt[3]{4}\right)^{\frac{3}{2}} = \frac{2}{3}(4)^{\frac{1}{3}\cdot\frac{3}{2}} = \frac{2}{3}\cdot 2 = \frac{4}{3}$$

$$A_f = +\frac{1}{2}\int_0^{x_0} dx\, x^2 = \left. \frac{1}{2}\cdot\frac{x^3}{3} \right|_0^{x_0} = \frac{1}{6}\cdot 4 = \frac{4}{6} = \frac{2}{3}$$

$$\left[A = \frac{4}{3} - \frac{2}{3} = \frac{2}{3} \right] \checkmark$$

<u>ES. 3</u>: CALCOLARE SE ESISTE $\lim_{(x,y)\to(1,0)} \frac{(x-1)^2 \ln(y+1)}{y^2+(x-1)^2}$

i) VEDIAMO CHE FA $\frac{0}{0}$ ...

ii) VISTO CHE NON È ATTORNO A $(0,0)$ USIAMO LE RETTE: TROVIAMO IL FASCIO DI RETTE CHE PASSA PER $(1,0)$

$\hookrightarrow (y-0) = m(x-1) \Rightarrow y = m(x-1)$

$$\Rightarrow \lim_{x\to 1} \frac{(x-1)^2 \ln(mx-m+1)}{m^2(x-1)^2+(x-1)^2} = ?$$

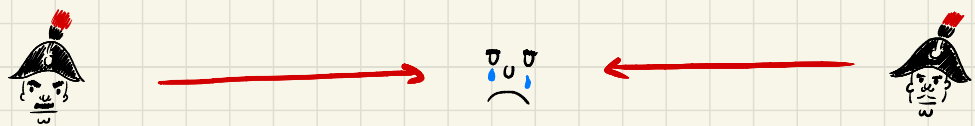
$$\lim_{x\to 1} \frac{\cancel{(x-1)^2}\ln(mx-m+1)}{(1+m^2)\cancel{(x-1)^2}} = \frac{1}{(1+m^2)}\lim_{x\to 1}\ln(mx-m+1) = \frac{1}{1+m^2}\cdot\ln(1) = 0$$

iii) PROVIAMO CON LE STIME!

↳ $(x-1)^2+y^2$ È UNA QUANTITÀ SEMPRE POSITIVA $\Rightarrow$ $(x-1)^2 \le (x-1)^2+y^2$ SEMPRE!

$$\Rightarrow \frac{(x-1)^2}{(x-1)^2+y^2} \le \frac{(x-1)^2}{(x-1)^2} \quad \text{E} \quad \frac{(x-1)^2}{(x-1)^2+y^2} \ge -\frac{(x-1)^2}{(x-1)^2}$$

<u>CARABINIERI</u>:



$$\Rightarrow \underbrace{\lim_{(x,y)\to(1,0)} -\frac{\cancel{(x-1)^2}\ln(1+y)}{\cancel{(x-1)^2}}}_{\downarrow\ 0} \le \lim_{(x,y)\to(1,0)} \frac{(x-1)^2\ln(1+y)}{(x-1)^2+y^2} \le \underbrace{\lim_{(x,y)\to(1,0)} \frac{\cancel{(x-1)^2}\ln(1+y)}{\cancel{(x-1)^2}}}_{\downarrow\ 0}$$

$$\Rightarrow \left[\lim_{(x,y)\to(1,0)} \frac{(x-1)^2\ln(1+y)}{(x-1)^2+y^2} \to 0\right]$$

PIANO TANGENTE IN $(2,0)$?

RICORDA: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)\cdot g(x) - f(x)g'(x)}{g^2(x)}$

$$f(2,0) = \frac{(2-1)^2 \ln(1)}{(2-1)^2 + 0^2} = 0$$

EQN. PIANO TANGENTE: $z - f(2,0) = \nabla f\big|_{2,0} \cdot \begin{bmatrix} x-2 \\ y-0 \end{bmatrix}$ $\qquad \nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right] = ?$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left[\ln(1+y)\cdot\frac{(x-1)^2}{(x-1)^2+y^2}\right] = \ln(1+y)\cdot\left\{\frac{2(x-1)\cdot[(x-1)^2+y^2] - (x-1)^2(2(x-1)+0)}{((x-1)^2+y^2)^2}\right\}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[(x-1)^2\cdot\frac{\ln(1+y)}{(x-1)^2+y^2}\right] = (x-1)^2\cdot\left\{\frac{\frac{1}{1+y}\cdot[(x-1)^2+y^2] - \ln(1+y)\cdot[2y]}{((x-1)^2+y^2)}\right\}$$

$$\frac{\partial f}{\partial x}\bigg|_{2,0} = 0 \qquad \frac{\partial f}{\partial y}\bigg|_{2,0} = 1\cdot\left\{\frac{1-0}{1}\right\} = 1$$

$$\Rightarrow \quad z - 0 = [0, 1]\cdot\begin{bmatrix} x-2 \\ y \end{bmatrix} \quad \Rightarrow \quad \boxed{z = y}$$