Es. 1:

$$\int_0^{\frac{\pi}{2}} x \cdot \operatorname{sen}(x) \cdot \cos(x) \qquad \text{PER PARTI:} \quad \int_a^b dx\, f(x)\, g(x) = f(x)\,\overset{\circledast\circledast}{G(x)}\Big|_a^b - \int_a^b dx\, f'(x)\,\overset{\circledast}{G}(x)$$

DEF. $g(x) = \operatorname{sen}(x)\cos(x) = \frac{1}{2}\operatorname{sen}(2x)$

DEF: $f(x) = x$

DEVO INTEGRARE QUI $\circledast\circledast$

$$\Rightarrow \int_0^{\frac{\pi}{2}} dx\ x\left(\frac{1}{2}\operatorname{sen}(2x)\right) = \frac{1}{2}\int_0^{\frac{\pi}{2}} dx\ x \cdot (\operatorname{sen}(2x)) = \frac{1}{2}\cdot\frac{1}{2}\int_0^{\frac{\pi}{2}} dx\ x\ (\overset{h'(x)}{2} \cdot \operatorname{sen}(\overset{h(x)}{2x}))$$

$$= \frac{1}{4}\left[x \cdot (-\cos(2x))\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} dx\ (1)\cdot(-\cos(2x))\right] = \frac{1}{4}\left[-x \cdot \cos(2x)\Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} dx\ \cos(2x)\right]$$

HO DERIVATO! $\circledast$

$$= \frac{1}{4}\left[-x \cdot \cos(2x)\Big|_0^{\frac{\pi}{2}} + \frac{1}{2}\int_0^{\frac{\pi}{2}} dx\ 2 \cdot \cos(2x)\right] = \frac{1}{4}\left[-x \cdot \cos(2x)\Big|_0^{\frac{\pi}{2}} + \frac{1}{2}\operatorname{sen}(2x)\Big|_0^{\frac{\pi}{2}}\right]$$

$$= \frac{1}{4}\left[\left(-\frac{\pi}{2}\cdot\cos(\pi)\right) - 0 + \frac{1}{2}\cdot\cancel{(\operatorname{sen}(\pi) - \operatorname{sen}(0))}\right] = \boxed{+\frac{\pi}{8}}\ ✓$$

ES. 2:

$$\int_1^e dx \, \frac{1}{x \cdot \ln(x)} = \int_1^e dx \, \frac{1}{x} \cdot \frac{1}{\ln(x)}$$

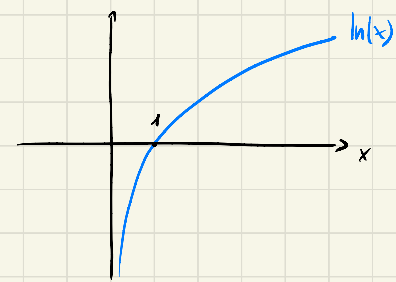
ABBIAMO UN PROBLEMA NELL'INTERVALLO DI INTEGRAZIONE PERCHÉ $\lim_{x \to 1^+} \frac{1}{x \cdot \ln(x)} = +\infty$

$\Rightarrow$ DOBBIAMO STUDIARE L'INTEGRALE CON UNO DEI 2 ESTREMI VARIABILE:

$$\lim_{a \to 1^+} \int_a^e dx \, \frac{1}{x \cdot \ln(x)} = \lim_{a \to 1^+} \int_a^e dx \, \frac{1}{x} \cdot \frac{1}{\ln(x)} = \lim_{a \to 1^+} \int_a^e dx \, \frac{\overbrace{\tfrac{1}{x}}^{f'(x)}}{\underbrace{\ln(x)}_{f(x)}} = \lim_{a \to 1^+} \ln(\ln(x)) \Big|_a^e$$

$$= \lim_{a \to 1^+} \left(\ln(1) - \ln(\ln(a)) \right) = - \lim_{a \to 1^+} \ln(\underbrace{\ln(a)}_{0^+}) = [+\infty]$$

L'INTEGRALE NON CONVERGE!



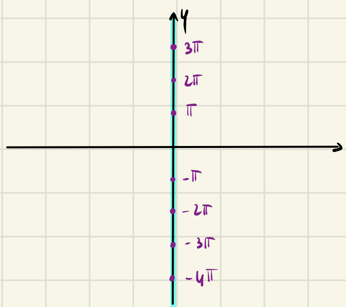
ES. 3: i) STUDIARE I PTI CRITICI DI $f(x,y) = x \cdot sen(y)$ (DOMINIO: $\mathbb{R}^2$)

I PTI CRITICI SODDISFANO $\overline{\nabla} f = 0$

$$\overline{\nabla} f(x,y) = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right] \qquad \frac{\partial f}{\partial x} = sen(y) \qquad \frac{\partial f}{\partial y} = x \cdot \cos(y) \qquad \Rightarrow \boxed{\overline{\nabla} f(x,y) = \left[sen(y),\ x \cdot \cos(y) \right]}$$

IN CHE PTI SI ANNULLA IL GRADIENTE?

$$\begin{cases} sen(y) = 0 & \Rightarrow \\ x \cdot \cos(y) = 0 & \Rightarrow \end{cases} \left[\begin{array}{l} \left[y = k \cdot \pi \quad \text{CON} \quad k = 0, \pm 1, \pm 2, \dots \right] \\ x = 0 \end{array} \right] \text{①} \checkmark$$

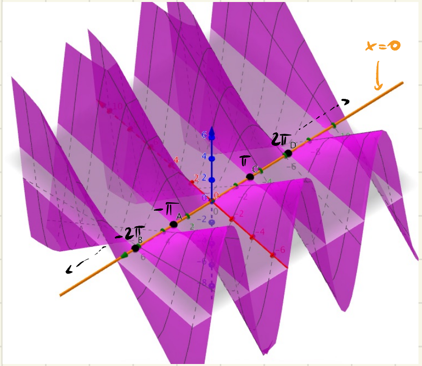


¡CI SONO INFINITI PTI CRITICI!

DI CHE NATURA SONO? MASSIMI, MINIMI O SELLE?

↳ STUDIAMO L'HESSIANO:

$$Hf(x,y) = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$$



$$\frac{\partial^2 f}{\partial x^2} = 0 \qquad \frac{\partial^2 f}{\partial y \partial x} = \cos(y) \qquad \frac{\partial^2 f}{\partial x \partial y} = \cos(y) \qquad \frac{\partial^2 f}{\partial y^2} = -x \cdot \operatorname{sen}(y)$$

$$\Rightarrow \quad Hf(x,y) = \begin{bmatrix} 0 & \cos(y) \\ \cos(y) & -x \cdot \operatorname{sen}(y) \end{bmatrix}$$

ADESSO DOBBIAMO VALUTARLO NEI PTI CRITICI!

k = PARI $\Rightarrow 1$

k = DISPARI $\Rightarrow -1$

$$Hf(0, k\pi) = \begin{bmatrix} 0 & \cos(k\pi) \\ \cos(k\pi) & 0 \end{bmatrix} = \begin{bmatrix} 0 & (-1)^k \\ (-1)^k & 0 \end{bmatrix}$$

NON È DIAGONALE!

$\rightsquigarrow$ DIAGONALIZZIAMO:

$$\det \begin{bmatrix} -\lambda & (-1)^k \\ (-1)^k & -\lambda \end{bmatrix} = \lambda^2 - (-1)^k \cdot (-1)^k = 0$$

$$\overset{!}{=} \lambda^2 - (-1)^{2k} = \lambda^2 - 1 = 0$$

SE k = PARI $\Rightarrow (-1)^{2k} = 1$

SE k = DISPARI $\Rightarrow (-1)^{2k} = 1$

$$\Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1 \Rightarrow Hf(0, k\pi) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\boxed{\text{GLI AUTOVALORI HANNO SEGNO OPPOSTO, ALLORA SONO TUTTI PTI. SELLA!}}$$

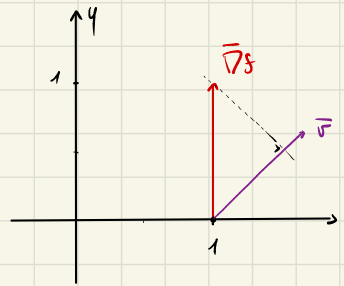
✓

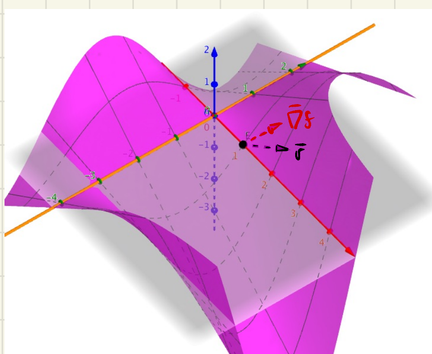
ii) DERIVATA DIREZIONALE LUNGO $\bar{v} = \left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$ IN $(x_0, y_0) = (1, 0)$

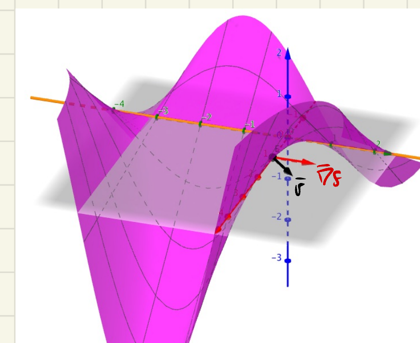
$\bar{v}$ È NORMALIZZATO ? $|\bar{v}| = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{2}{4} + \frac{2}{4}} = 1$ ✓

$$\frac{\partial f}{\partial \bar{v}} = \bar{\nabla} f \Big|_{(x_0,y_0)} \cdot \bar{v} = \left[\operatorname{sen}(y),\ x \cdot \cos(y) \right]\Big|_{(1,0)} \cdot \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} = \left[0,\ 1 \right] \cdot \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$= 0 \cdot \frac{\sqrt{2}}{2} + 1 \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}}{2}}$$ ✓







iii) EQN PIANO TANGENTE IN $\begin{array}{ccc} x_0 & y_0 & z_0 \\ (1, & 0, & 0) \end{array}$

$$z - f(x_0, y_0) = \bar{\nabla} f \Big|_{(x_0,y_0)} \cdot \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix} \Rightarrow z - 0 = [0, 1] \cdot \begin{bmatrix} x - 1 \\ y - 0 \end{bmatrix} \Rightarrow z = 0 \cdot (x-1) + 1 \cdot y$$

$$\Rightarrow \boxed{z = y} \checkmark$$