ES 1:

$$\int_0^{\frac{\pi}{2}} dx\ \text{sen}(2x)\cos(x)$$

$$\cos\left(\frac{\alpha}{2}\right) = +\sqrt{\frac{1+\cos(\alpha)}{2}} \Rightarrow \cos(x) = \frac{1}{\sqrt{2}}\sqrt{1+\cos(2x)}$$

$$\int_0^{\frac{\pi}{2}} dx\ \text{sen}(2x)\ \frac{1}{\sqrt{2}}\sqrt{1+\cos(2x)} = \frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}} dx\ \text{sen}(2x)\sqrt{1+\cos(2x)}$$

$$f(x) = 1+\cos(2x)$$

$$f'(x) = 0 - 2\,\text{sen}(2x) \overset{!}{=} -2\cdot\text{sen}(2x)$$

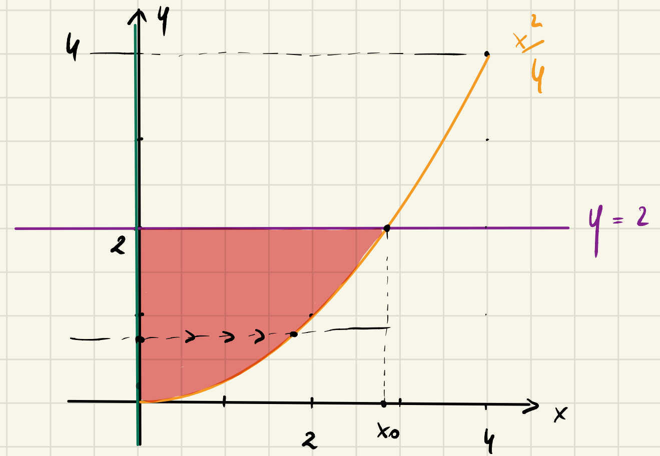
$$\Rightarrow \frac{1}{(-2)\sqrt{2}}\int_0^{\frac{\pi}{2}} dx\ \underbrace{(-2\cdot\text{sen}(2x))}_{f'(x)}\cdot\underbrace{(1+\cos(2x))}_{f(x)}^{\frac{1}{2}}$$

Uso $\int dx\ f'(x)(f(x))^{\alpha} = \frac{(f(x))^{\alpha+1}}{\alpha+1}$ se $\alpha \neq 1$.

$$\Rightarrow -\frac{1}{\cancel{2}\sqrt{2}}\ \frac{(1+\cos(2x))^{\frac{3}{2}}}{\frac{3}{\cancel{2}}}\Bigg|_0^{\frac{\pi}{2}} = -\frac{1}{3\sqrt{2}}(1+\cos(2x))^{\frac{3}{2}}\Bigg|_0^{\frac{\pi}{2}} =$$

$$= -\frac{1}{3\sqrt{2}}\left[\underbrace{\left(1+\cos\left(\cancel{2}\cdot\frac{\pi}{\cancel{2}}\right)\right)^{\frac{3}{2}}}_{=0} - (1+\cos(0))^{\frac{3}{2}}\right] = \frac{1}{3\sqrt{2}}(2)^{\frac{3}{2}} = \boxed{\frac{2}{3}}$$ ✓

ES. 2: CALCOLARE $\iint_D dx\,dy\; 5x^3\cos(y^3)$ D È LA REGIONE DELIMITATA DALL'ASSE y, DALLA RETTA $y=2$ E DALLA PARABOLA $y=x^2/4$.



$x_0 = ?$ $\quad 2 = \dfrac{x_0^2}{4}$, $\; x_0^2 = 8$

$\Rightarrow x_0 = \sqrt{8} = 2\sqrt{2}$

HO 2 ALTERNATIVE:

$$\int_0^2 dy \left(\int_0^{2\sqrt{y}} dx\; 5x^3\cos(y^3)\right) \quad \text{o} \quad \int_0^{2\sqrt{2}} dx \left(\int_{x^2/4}^{2} dy\; 5x^3\cos(y^3)\right)$$

$\downarrow$

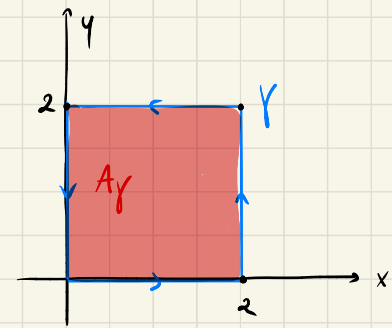
$$\int_0^2 dy\,\cos(y^3)\left(\underbrace{\int_0^{2\sqrt{y}} dx\; 5x^3}_{1}\right)$$

1 $\quad 5\int_0^{2\sqrt{y}} dx\; x^3 = \dfrac{5x^4}{4}\Big|_0^{2\sqrt{y}} = \dfrac{5}{\cancel{4}}\cdot \overset{4}{\cancel{16}}\, y^2 = 20y^2$

$$\Rightarrow 20\int_0^2 dy\; y^2\cos(y^3) \qquad f(y) = y^3 \qquad f'(y) = 3y^2$$

$$\Rightarrow \frac{20}{3}\int_0^2 dy\, \underbrace{(3y^2)}_{f'(y)} \cos(\underbrace{y^3}_{f(y)}) = \frac{20}{3}\, \text{sen}(y^3)\Big|_0^2 = \left[\frac{20}{3}\, \text{sen}(8)\right]$$

ES. 3: CALCOLARE $\oint_\gamma \left((x^3 - xy^3)\, dx + (y^2 - 2xy)\, dy\right) = ?$ CON γ IL PERIMETRO DEL QUADRATO CON LATI $[0,2] \times [0,2]$



GAUSS - GREEN:

$$\left[\oint_\gamma \left(A(x,y)\, dx + B(x,y)\, dy\right) = \int_{A_\gamma} dx\,dy \left[\frac{\partial B}{\partial x} - \frac{\partial A}{\partial y}\right]\right]$$

INTEGRALE SUL CAMMINO

INTEGRALE SULL'AREA

NEL NOSTRO CASO:

$$A(x,y) = x^3 - xy^3$$

$$B(x,y) = y^2 - 2xy$$

$$\frac{\partial B}{\partial x} = 0 - 2y \qquad \frac{\partial A}{\partial y} = 0 - x \cdot 3y^2 = -3xy^2$$

$$\Rightarrow \oint_\gamma \left((x^3 - xy^3)\,dx + (y^2 - 2xy)\,dy \right) = \int_{A_\gamma} dx\,dy \left(-2y - (-3xy^2) \right) = \int_{A_\gamma} dx\,dy \left(-2y + 3xy^2 \right)$$

$$= \int_0^2 dx \left(\int_0^2 dy \left(-2y + 3xy^2 \right) \right) = \int_0^2 dx \int_0^2 dy\,(-2y) + \int_0^2 dx \int_0^2 dy\; 3xy^2$$

$$= \int_0^2 dx \left(-\cancel{2}\frac{y^2}{\cancel{2}} \right)\Big|_0^2 + \int_0^2 dx \left(\cancel{3}x\frac{y^3}{\cancel{3}} \right)\Big|_0^2 = \int_0^2 dx\,(-4 - (-0)) + \int_0^2 dx\; x\,(8 - 0)$$

$$= -4\int_0^2 dx + 8\int_0^2 dx\; x = -4x\Big|_0^2 + \overset{4}{\cancel{8}}\frac{x^2}{\cancel{2}}\Big|_0^2 = -4\cdot(2-0) + 4\cdot(4-0) = -8 + 16 = \boxed{8} \checkmark$$